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$B \to X_d\gamma$ and constraints on new physics

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We combine recent progress in measuring the branching ratio of the decay $B \to X_d\gamma$ with the discovery that hadronic uncertainties in the CP-averaged branching ratio drop out to a large extent. Implications of these improvements on the size of possible new physics effects are investigated. We find the updated SM prediction for the CP-averaged branching ratio to be $\langle \mathrm{Br}[B \to X_d\gamma]^{\mathrm{SM}}_{E_\gamma > 1.6\ \mathrm{GeV}}\rangle = 1.54^{+0.26}_{-0.31} \times 10^{-5}$, which should be compared with the experimental value of $\langle \mathrm{Br}[B \to X_d\gamma]^{\mathrm{exp}}_{E_\gamma > 1.6\ \mathrm{GeV}}\rangle = (1.41 \pm 0.57) \times 10^{-5}$. After performing a model independent analysis, we consider different new physics models: the Minimal Supersymmetric Standard Model with generic sources of flavor violation, the two Higgs doublet model of type III, and a model with right-handed charged currents. It is found that the constraints on the supersymmetry parameters δ^d_{13} have improved and that the absolute value of the right-handed quark mixing matrix element $|V^R_{td}|$ must be smaller than 1.5×10^{-4}.

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I. INTRODUCTION

In the past, the main focus has been on the inclusive decay $B \to X_s\gamma$ while its analog with a down quark in the final state, $B \to X_d\gamma$, received much less attention. The reason for this was that both the experimental measurement $\mathrm{Br}[B \to X_s\gamma]^{\mathrm{exp}}_{E_\gamma > 1.6\ \mathrm{GeV}} = (3.60 \pm 0.23) \times 10^{-4}$ [1] and the standard model prediction (it is now known to next-to-next-to-leading order (NNLO) precision) $\mathrm{Br}[B \to X_s\gamma]^{\mathrm{SM}}_{E_\gamma > 1.6\ \mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$ [2,3] of this decay were significantly better compared to $B \to X_d\gamma$. However, this situation has changed recently:

(i) The new CP-averaged branching ratio $\langle \mathrm{Br}[B \to X_d\gamma]^{\mathrm{exp}}_{E_\gamma > 1.6\ \mathrm{GeV}}\rangle = (1.41 \pm 0.57) \times 10^{-5}$ of the *BABAR* Collaboration [4,5] (CP averaging is denoted by $\langle \ldots \rangle$ throughout this article) is more precise than the previous one and the photon cut is lower which reduces the error of the extrapolation to 1.6 GeV.[1] Furthermore, there are good experimental prospects for this decay: the analysis of existing BELLE data and the future super-B factories [6,7] will allow for a more precise determination of this branching ratio.

(ii) The theory prediction for the standard model (SM) contribution has been calculated in Ref. [8] and the next-to-leading order (NLO) QCD corrections can be found in Ref. [9]. As in the case of $B \to X_s\gamma$, also $B \to X_d\gamma$ suffers from hadronic uncertainties, but for the latter the nonperturbative contributions from up-quark loops are not Cabibbo-Kobayashi-Maskawa (CKM) suppressed which magnifies the error of the theory prediction. However, it has been only recently realized that most of these uncertainties drop out in the CP-averaged branching ratio [10,11]. Thus, the SM prediction for $B \to X_d\gamma$ can in principle be calculated with the same accuracy as $B \to X_s\gamma$.

(iii) In addition, the error in the determination of the CKM element V_{td} has constantly decreased in the last years [12,13]. This further reduces the uncertainty of the SM contribution to $B \to X_d\gamma$ which depends quadratically on V_{td}. The uncertainty coming from the determination of V_{td} now only induces an error in the SM branching ratio of approximately 10% if one varies the value of V_{td} within its 95% C.L. region.

These significant improvements and promising prospects on the theoretical as well as on the experimental side motivate us to perform an updated analysis of $B \to X_d\gamma$ and the constraints placed from this decay.

II. EFFECTIVE HAMILTONIAN AND SM PREDICTION

In the SM the effective Hamiltonian governing $\bar{B} \to X_d\gamma$ is given by

$$\mathcal{H}_{\mathrm{eff}} = \frac{-4G_F}{\sqrt{2}} V^\star_{td} V_{tb} \left[\sum_{i=1}^{8} C_i O_i + \epsilon_d \sum_{i=1}^{2} C_i (O_i - O_i^u) \right], \tag{1}$$

where O_1^u, O_2^u, O_1, ..., O_6 are four-quark operators, $\epsilon_d = \frac{V^*_{ud}V_{ub}}{V^*_{td}V_{tb}}$ and the (chromo)magnetic operator (O_8) O_7 is given by

[1] Note that in the International Conference on High Energy Physics (ICHEP) 2010 update of HFAG the value for $B \to X_d\gamma$ is not extrapolated from the photon cut of 2.26 GeV used in the *BABAR* measurement to a cut of 1.6 GeV even though this was done in previous updates. It can thus be misleading to compare the value of HFAG with the one of Particle Data Group (quoted in the HFAG analysis) since the latter one has been extrapolated down to 1.6 GeV. We obtained the value quoted above by using the extrapolation of HFAG. In order to be conservative we doubled the error given in *www.slac.stanford.edu/xorg/hfag/rare/ichep10/radll/btosg.pdf* .

$$O_7 = \frac{e}{16\pi^2} m_b(\mu)(\bar{d}_L \sigma_{\mu\nu} b_R) F^{\mu\nu},$$
$$O_8 = \frac{g_s}{16\pi^2} m_b(\mu)(\bar{d}_L T^a \sigma_{\mu\nu} b_R) G^{a\mu\nu}. \tag{2}$$

In the presence of new physics (NP), additional operators may appear. We assume that the only sizeable NP contributions enter through $O_{7,8}^{(\prime)}$ which is the case for the models under consideration in this article. The operators $O'_{7,8}$ are obtained by exchanging L with R and vice versa in the unprimed operators. The NLO decay width can thus be written as

$$\mathrm{Br}[\bar{B} \to X_d \gamma] = \mathcal{N} \left| \frac{V_{td}^{\star} V_{tb}}{V_{cb}} \right|^2 (P + N + P'). \tag{3}$$

Here P contains the perturbative SM contributions and the NP contributions to $O_{7,8}$ while P' contains only the NP contributions to $O'_{7,8}$. N denotes the nonperturbative corrections and $\mathcal{N} \approx 2.5 \times 10^{-3}$ is a numerical prefactor (see Ref. [14] for details).

Before turning our attention to NP, we update the SM prediction for the CP averaged branching ratio $\langle \mathrm{Br}[B \to X_d \gamma]_{\mathrm{SM}} \rangle$ of Refs. [9,14] by using the improved determination of the CKM-element V_{td} and the reduced nonperturbative uncertainties which are estimated to be at most 5% (as for $B \to X_s \gamma$) [10,11]. The remaining leading uncertainty stems from renormalization scheme dependence of the ratio m_c/m_b (approximately 15%) which is supplemented by a 3.5% scale ambiguity and a 6% parametric uncertainty [14]. In addition there is still a 10% change in the branching ratio if one varies V_{td} within its 95% C.L. region. Adding all these uncertainties in quadrature, we get

$$\langle \mathrm{Br}[B \to X_d \gamma]^{\mathrm{SM}}_{E_\gamma > 1.6\ \mathrm{GeV}} \rangle = 1.54^{+0.26}_{-0.31} \times 10^{-5}. \tag{4}$$

Comparing this with the experimental value of $\langle \mathrm{Br}[B \to X_d \gamma]^{\mathrm{exp}}_{E_\gamma > 1.6\ \mathrm{GeV}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$, we see that the SM prediction well within the experimental 1σ range.

III. CONSTRAINTS ON NEW PHYSICS

Since the SM contribution to $B \to X_d \gamma$ is not only loop but also chirality suppressed, this decay is, just as $B \to X_s \gamma$, very sensitive to new sources of flavor and chirality violation which occur in most NP models. In order to include NP into the calculation of the branching ratio we rely on the NLO formula of Ref. [14].

The constraints in this section are obtained by demanding that branching ratio, including NP contributions, should lie within the 2σ range of the experimental values if not indicated otherwise. In order to give a conservative estimate we add the theory error and the experimental one linearly. Further we define

$$C_{7,8}^{\mathrm{NP}} = C_{7,8} - C_{7,8}^{\mathrm{SM}}, \qquad C_{7,8}^{\prime\mathrm{NP}} = C'_{7,8}. \tag{5}$$

A. Model independent analysis

First, we can constrain the Wilson coefficients $C_7^{\mathrm{NP}(\prime)}$ and $C_8^{\mathrm{NP}(\prime)}$ at the scale M_W. In the left plot of Fig. 1 we show the 1σ and 2σ allowed region in the $Re[C_7^{\mathrm{NP}}/C_7^{\mathrm{SM}}] - Im[C_7^{\mathrm{NP}}/C_7^{\mathrm{SM}}]$ plane for $C_7^{\mathrm{NP}}/C_8^{\mathrm{NP}} = C_7^{\mathrm{SM}}/C_8^{\mathrm{SM}}$. Clearly the size of constructive contributions is very limited, but large destructive contributions are still possible. The primed operators always give a constructive contribution to the branching ratio and thus their possible size is rather limited (see right plot of Fig. 1). Note that $|C_{7,8}^{\mathrm{NP}}|$ can easily several times larger than $|C_{7,8}^{\mathrm{SM}}|$ in the case of destructing

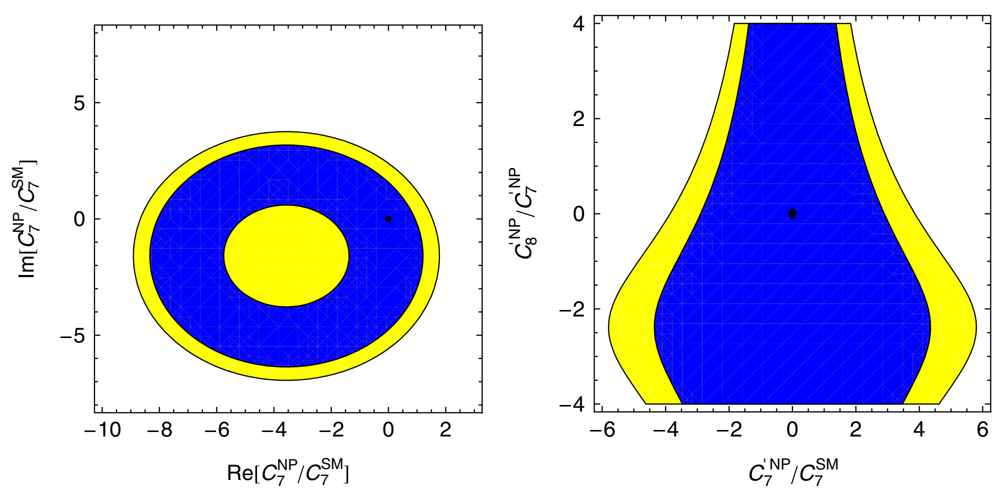

FIG. 1 (color online). The blue (yellow) region agrees with the measured branching ratio at the 1σ (2σ) level. Left plot: Allowed region in the $\mathrm{Re}[C_7^{\mathrm{NP}}/C_7^{\mathrm{SM}}] - \mathrm{Im}[C_7^{\mathrm{NP}}/C_7^{\mathrm{SM}}]$ plane for $C_7^{\mathrm{NP}}/C_8^{\mathrm{NP}} = C_7^{\mathrm{SM}}/C_8^{\mathrm{SM}}$ and $C_{7,8}^{\prime\mathrm{NP}} = 0$. Right plot: Allowed region in the $C_7^{\prime\mathrm{NP}}/C_7^{\mathrm{SM}} - C_8^{\prime\mathrm{NP}}/C_7^{\prime\mathrm{NP}}$ plane for $C_8^{\prime\mathrm{NP}}/C_7^{\prime\mathrm{NP}} \in \mathbb{R}$. Note that the constraints in the right plot are independent of the phase of $C_7^{\prime\mathrm{NP}}$. In both plots the SM point is marked by a black dot.

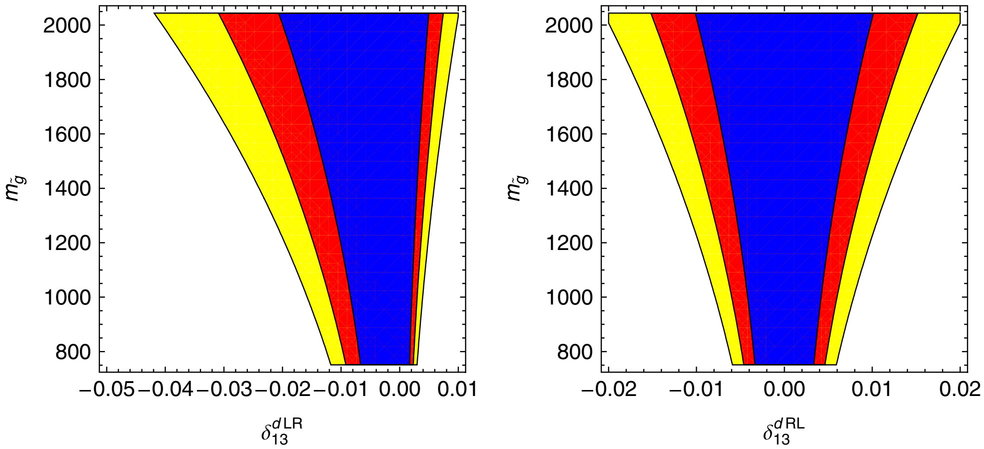

FIG. 2 (color online). Constraints on the mass insertion parameters δ_{13}^{dLR} (left plot) and δ_{13}^{dRL} (right plot) for $m_{\tilde{g}} = m_{\tilde{q}} = 1$ TeV and $\mu\tan\beta = 30$ TeV (yellow), $\mu\tan\beta = 0$ (red), $\mu\tan\beta = -30$TeV (blue). Note that we only considered the leading gluino contributions (see Ref. [28] for details). The constraints on δ_{13}^{dRL} are independent of its phase while the constraints on δ_{13}^{dLR} are given for $\mathrm{Arg}[\delta_{13}^{dRL}] = \mathrm{Arg}[V_{td}]$ and have to be scaled according to Fig. 1 otherwise. In order to take into account the chirally enhanced corrections we used the effective Feynman rules of Ref. [29].

interference. The reason for this is that we normalize $C_{7,8}^{\mathrm{NP}}$ only to $C_{7,8}^{\mathrm{SM}}$, i.e. even for $C_{7,8}^{\mathrm{NP}} = -C_{7,8}^{\mathrm{SM}}$ the branching ratio is not zero because of the contributions from the SM four-quark operators.

In models with minimal flavor violation (MFV) [15], the constraints on the Wilson coefficients $C_7^{\mathrm{NP}(\prime)}$ and $C_8^{\mathrm{NP}(\prime)}$ obtained in this section can be directly compared to the ones from $b \to s\gamma$ because the CKM elements are factored out in Eq. (1). Note that despite the recent improvements in $b \to d\gamma$ the constraints from $b \to s\gamma$ are still stronger if MFV is assumed.

B. MSSM

The generic MSSM possesses many new sources of flavor violation and constraining this flavor structure with flavour changing neutral current processes has a long and fruitful tradition [16,17]. Concerning $B \to X_d\gamma$, we are especially sensitive to the chirality flipping elements $\delta_{13}^{dLR,RL}$ [18] (but also to $\delta_{13}^{dLL,RR}$ at moderate to large values of $\tan\beta$), and we get even more stringent constraints than from $B_d - \bar{B}_d$ mixing [19]. The results are depicted in Figs. 2 and 3.

C. 2 Higgs Doublet Model of type III

Despite the significant improvements in $B \to X_d\gamma$, the bounds from $B \to X_s\gamma$ are still tighter in scenarios with MFV [15]. Thus the constraints on the charged Higgs mass of a two Higgs doublet models (2HDM) of type II are still more stringent from $B \to X_s\gamma$. However in a 2HDM of type III the nonholomorphic couplings of a t and a u quark to the Higgs can be constrained. This kind of models have been considered in Refs. [20–22], where however additional assumptions on the structure of the couplings has been imposed for the phenomenological studies.

Following the notation of Ref. [23] we denote the coupling coefficients of the charged Higgs vertex (for large $\tan\beta$) as

$$\Gamma_{u_f d_i}^{LRH^\pm} = \frac{1}{v}\sum_{j=1}^{3} V_{fj}^{\mathrm{CKM}} \tan(\beta)(m_{d_i}\delta_{ji} - \tilde{\Sigma}_{jiA'\mu}^{dLR})$$
$$\Gamma_{u_f d_i}^{RLH^\pm} = \frac{1}{v}\sum_{j=1}^{3} (\tan(\beta)\tilde{\Sigma}_{fjA'\mu}^{uRL} + \cot(\beta) m_{u_f}\delta_{fj}) V_{ji}^{\mathrm{CKM}}. \tag{6}$$

In Fig. 4 we show the constraints that we get on the product $\tan(\beta)^2\tilde{\Sigma}_{31A'\mu}^{uRL}$ which therefore, up to strongly suppressed terms, depend only on the charged Higgs mass. In principle we could also consider $\Sigma_{jiA'\mu}^{dLR}$, but the bounds from $B_d \to \mu^+\mu^-$ are more stringent.

D. Right-handed charged currents

It is well known that $B \to X_s\gamma$ puts stringent constraints on models with right-handed charged currents [24,25]. We can thus also constrain the elements of the right-handed mixing matrix through $B \to X_d\gamma$.[2] We define the effective W-quark-quark vertex as

$$i\Gamma_{t,d}^{W\mu} = -i\frac{g_2}{\sqrt{2}}\gamma^\mu(V_{td}P_L + V_{td}^R P_R). \tag{7}$$

If $V_{td}^R \neq 0$ contributions to $C_{7,8}'$ are induced which necessarily enhance the branching ratio. Using the formulas of Ref. [25] and assuming the SM value for V_{td}, we get the following limit on V_{td}^R:

[2] An effective right-handed W coupling can also be induced in the MSSM [26] which then affects the determination of V_{ub} and V_{cb}. However, in this case $B \to X_d\gamma$ is not dangerous because one cannot generate a sizable V_{td}^R coupling.

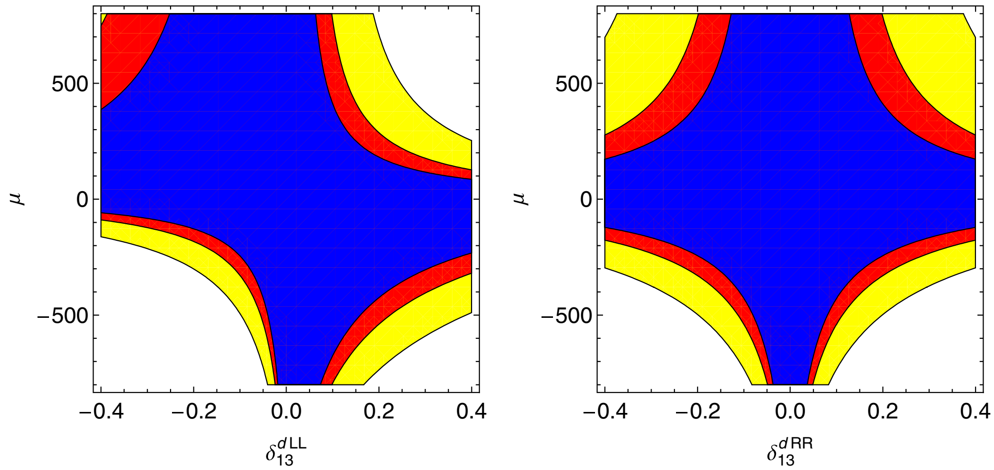

FIG. 3 (color online). Constraints on the mass insertion parameters δ_{13}^{dLR} (left plot) and δ_{13}^{dRL} (right plot) for $\tan\beta = 50$, $m_{\tilde{q}} = 1$ TeV, and $m_{\tilde{g}} = 1.5$ TeV (yellow), $m_{\tilde{g}} = 1$ TeV (red) $m_{\tilde{g}} = 0.75$ TeV (blue). The constraints on δ_{13}^{dLL} are independent of its phase while the constraints on δ_{13}^{dRR} are given for $\mathrm{Arg}[\delta_{13}^{dRR}] = \mathrm{Arg}[V_{td}]$ and have to be scaled according to Fig. 1 if the phase is different.

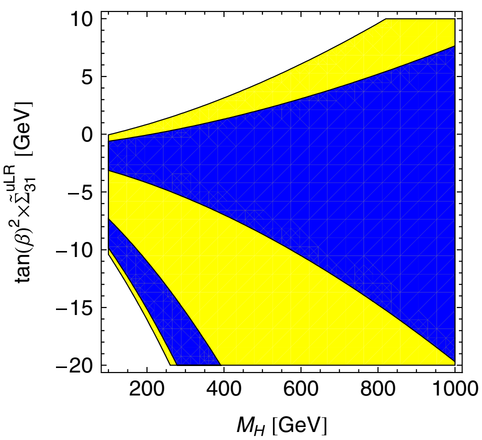

FIG. 4 (color online). Allowed regions in the $\tan(\beta)^2 \tilde{\Sigma}_{31A'\mu}^{uRL} - M_{H^+}$ plane in the 2HDM III.

$$|V_{td}^R| \leq 1.5 \times 10^{-4}. \tag{8}$$

Note that this constraint is approximately 3.5 times stronger than what is found for the best-fit solution of the right-handed CKM matrix in Ref. [27].

IV. CONCLUSIONS AND OUTLOOK

In this letter we studied the constraints on NP from the inclusive radiative decay $B \to X_d \gamma$. Including the improved determination of V_{td} and the reduced hadronic uncertainties [10,11] in the *CP*-averaged branching ratio, the new NLO SM prediction is given by $\langle \mathrm{Br}[B \to X_d \gamma]^{\mathrm{SM}}_{E_\gamma > 1.6\ \mathrm{GeV}} \rangle = 1.54^{+0.26}_{-0.31} \times 10^{-5}$. If we extrapolate the experimental value from the *BABAR* Collaboration [4] to a photon energy cut of 1.6 GeV, we get $\langle \mathrm{Br}[B \to X_d \gamma]^{\mathrm{exp}}_{E_\gamma > 1.6\ \mathrm{GeV}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$.

We found constraints on the parameters $\delta_{13}^{dLR,RL}$ of the MSSM squark mass matrices which are more stringent than the ones obtained from $B - \bar{B}$ mixing. Also, an effective right-handed W coupling to the top and down quarks is severely constrained: $|V_{td}^R| \leq 1.5 \times 10^{-4}$. This, for example, strongly disfavors the proposed best-fit solution to the right-handed CKM matrix of Ref. [27].

The significance of $B \to X_d \gamma$ can even be improved by a NNLO computation of the SM prediction which is in progress. In addition, an analysis of the existing BELLE data would be welcome in order to reduce the error of the measurement. $B \to X_d \gamma$ is also very interesting for future super-B factories which will be able to measure this decay very precisely.

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