# *R*-parity violating flavor symmetries, recent neutrino data, and absolute neutrino mass scale

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We study the role of a very general type of flavor symmetry in controlling the strength of $R$-parity violation in supersymmetric models. We assume that only leptons are charged under a global symmetry whose breaking induces lepton number (and, hence, $R$-parity) violation. The charge assignments of leptons under this symmetry are such that the total number of independent lepton number violating couplings is reduced from 39 to 6. The most severe constraints on these flavor-correlated couplings arise from neutrino masses and mixing as well as from the nonobservation of $K_L \to \mu e$. We find that such a scenario predicts an almost vanishing smallest neutrino mass eigenvalue, allowing the upcoming generation of neutrinoless double beta decay experiments to shed light on the hierarchy.

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## I. INTRODUCTION

$R$-parity in supersymmetry is a discrete symmetry which is defined as $R_p = (-1)^{3B+L+2S}$, where $B$, $L$, and $S$ are the baryon number, lepton number, and spin of the particle, respectively (see Refs. [1–4]). All standard model particles have $R_p = 1$, while all superparticles have $R_p = -1$. The assumption of $R$-parity conservation in supersymmetric models is quite *ad hoc*, as this is not supported by any deep underlying principle. Historically, it was imposed to keep the proton stable. However, proton decay requires a simultaneous presence of $B$ and $L$ violation. Therefore, dropping all $R$-parity violating (RPV) couplings in one go is certainly overkill [1(c),5]. Still, in conventional supersymmetric theories $R_p$ conservation is imposed primarily for the sake of convenience, as otherwise the number of independent parameters in the minimal supersymmetric standard model, which is already very large and difficult to handle, is augmented by a set of new RPV parameters. Moreover, conserved $R_p$ implies that the lightest supersymmetric particle is stable, which leads to a plausible dark matter candidate, and also attributes supersymmetry with a characteristic missing energy signature in colliders. On the other hand, if lepton number is violated, one distinct advantage is that neutrino masses can be generated via a perfectly renormalizable interaction [6–8] without the need of introducing any right-handed neutrinos.

The most general superpotential with explicit RPV couplings is given by [3]

$$W_{\rm RPV} = \mu_i L_i H_u + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^C + \lambda'_{ijk} L_i Q_j D_k^C + \frac{1}{2}\lambda''_{ijk} U_i^C D_j^C D_k^C, \quad (1)$$

where $i$, $j$, and $k$ are the three quark and lepton generation indices. Here, $Q_i$ and $L_i$ are SU(2)-doublet quark and lepton superfields, respectively; $D_i^C$ and $E_i^C$ are SU(2)-singlet superfields for down-type quarks and charged leptons, respectively; and, $H_u$ is the Higgs superfield that generates the mass of the up-type quarks. This introduces 48 new couplings: three $\mu_i$-type, nine $\lambda_{ijk}$-type (note the antisymmetry in the first two indices), 27 $\lambda'_{ijk}$-type, and nine $\lambda''_{ijk}$-type couplings (note the antisymmetry in the last two indices). Only the $\lambda''$ couplings are $B$ violating; the rest are all $L$ violating. Besides, new RPV soft terms appear which introduce more unknown parameters. We do not explicitly write down these soft terms but will mention the relevant ones in appropriate places. Dealing with so many new parameters substantially reduces the predictivity of the model. At this point there are two ways to proceed. One may either take one or two RPV couplings as nonvanishing at a time and study their implications or apply some suitable flavor symmetry to relate one coupling to another [9,10]. We shall take the latter approach in this paper (see also [11] which does not differentiate between lepton doublet and down-type Higgs superfields for a different approach following a similar spirit). We show that with a simple flavor hypothesis we can bring down the number of totally independent RPV (more specifically, $L$ violating) couplings to only six. These couplings induce neutrino Majorana masses, and if the neutrino mixing matrix is tribimaximal (TBM) then the number of independent $L$ violating couplings can be further reduced to four, a scenario which prefers an inverse neutrino mass hierarchy.

## II. A GENERIC FLAVOR MODEL

We assume that the Yukawa structure leading to the masses and mixing of quarks and charged leptons is fixed by some unspecified global symmetry. This symmetry also ensures baryon number conservation. There is a second global symmetry ($X$), an Abelian horizontal symmetry, which is at the center of our attention. *Only* leptons are charged under $X$, such that for each generation $i$,

 

$$Q_X(L_i) = -Q_X(E_i^C). \tag{2}$$

We assume that the $Q_X$ charges of different generations are all positive. The horizontal symmetry is explicitly broken by a small parameter $\varepsilon < 1$, whose charge under $X$ is $Q_X(\varepsilon) = -1$. If the total charge of a given superpotential term is $n$, then the term is suppressed by $\varepsilon^n$. As an example, if $Q = Z_N$, then the suppression would be $\varepsilon^{n(\mathrm{mod}N)}$ [9].

Now we look at the consequences of Eq. (2) for the 48 RPV couplings of Eq. (1). Since $B$ number is conserved, all the $\lambda''$ couplings vanish right away. Since only leptons are charged under $X$, it follows that $Q_X(L_iQ_jD_k^C) = Q_X(L_iH_u) = Q_X(L_i)$, and hence $\lambda'_i \equiv \lambda'_{ijk} \simeq \tilde{\mu}_i \equiv \mu_i/\mu$, where the supersymmetry preserving $\mu$ parameter is assumed to be of the same order as the supersymmetry breaking soft masses ($\tilde{m}$). Turning our attention to the $L_iL_jE_k^C$ operator, we notice that when $j = k$, the same argument as above leads to $\lambda_{ijj} \simeq \lambda'_{ijk} \simeq \tilde{\mu}_i$. Thus 39 *a priori* independent $L$ violating couplings basically boil down to only six,

$$\tilde{\mu}_i(\simeq \lambda'_{ijk} \simeq \lambda_{ijj}), \qquad \lambda_{123}, \lambda_{132}, \lambda_{231}. \tag{3}$$

Thanks to the flavor symmetry, the $L$ violating bilinear soft parameters $B_i$ would be aligned to the corresponding superpotential parameters $\mu_i$ as well, i.e. $\tilde{B}_i \equiv B_i/\tilde{m}^2 \simeq \tilde{\mu}_i$. It should be noted that when we say that two couplings are related, we mean that they have a common suppression factor $\epsilon^{Q_X}$. Indeed, there are order-one uncertainties in the actual coefficients of the operators, for which reason we have used a near-equality sign in Eq. (3). Now we come to the relative sizes of the $L$ violating couplings. The suppression would depend on the sum of $Q_X$ charges of the associated lepton fields as a power of $\varepsilon$. More specifically,

$$\begin{aligned} \tilde{\mu}_i &\simeq \tilde{B}_i \simeq \lambda'_i \sim \varepsilon^{Q_X(L_i)}, \\ \lambda_{ijk} &\sim \varepsilon^{\{Q_X(L_i)+Q_X(L_j)+Q_X(E_k^C)\}}. \end{aligned} \tag{4}$$

Eventually, we shall provide a specific demonstration with $Q = Z_{N_1} \times Z_{N_2}$ [9], which means there are all together six charges for the three lepton generations.

Many RPV couplings which are not so strongly constrained may now be related by Eq. (4) to the ones which are severely constrained by experiments. The existing bounds on the individual and product couplings can be found in the reviews [3].

## III. NEUTRINO MASSES AND MIXING

One of the high points of $R$-parity violation is that it generates neutrino masses and mixing through a perfectly renormalizable interaction without the need of introducing any right-handed neutrino. This has already been studied at various levels of detail [6–8]. In this work we will follow the notation of [8]. The neutrino masses, in the basis in which all the sneutrino vacuum expectation values vanish, can be written as

$$\begin{aligned} m_{ij} \approx{}& \frac{\cos^2\beta}{\tilde{m}}\mu_i\mu_j + \frac{g^2}{64\pi^2\cos^2\beta}\frac{B_iB_j}{\tilde{m}^3} + \frac{g^2}{64\pi^2\cos\beta} \\ &\times \frac{\mu_iB_j + \mu_jB_i}{\tilde{m}^2} + \sum_k \frac{3}{16\pi^2} g m_{d_k} \frac{\mu_i\lambda'_{jkk} + \mu_j\lambda'_{ikk}}{\tilde{m}} \\ &+ \sum_k \frac{1}{16\pi^2} g m_{e_k} \frac{\mu_i\lambda_{jkk} + \mu_j\lambda_{ikk}}{\tilde{m}} \\ &+ \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk}\lambda'_{jkl} \frac{m_{d_l}m_{d_k}}{\tilde{m}_q^2} \mu\tan\beta \\ &+ \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk}\lambda_{jkl} \frac{m_{e_l}m_{e_k}}{\tilde{m}^2} \mu\tan\beta, \end{aligned} \tag{5}$$

where $m_{d_i}$ ($m_{e_i}$) denote the masses of the down-type quarks (charged leptons). A comment on the approximations made above is in order. We have denoted the squark masses by $\tilde{m}_q$ and assumed them to be somewhat heavier than a common mass scale $\tilde{m}$ assumed for the sleptons and weak gauginos/Higgsinos. This approximation may be crude but is good enough for our order-of-magnitude estimate of the RPV couplings. In Eq. (5), the first three terms account for the tree-level and one-loop contributions from bilinear couplings only, the following two sums represent the one-loop contributions from both bilinear and trilinear couplings, while the last two sums stand for one-loop contributions from trilinear couplings only. The possibility of large left-right squark/slepton mixing which may be induced by large $\tan\beta(\equiv v_u/v_d)$ has been taken into account in the purely trilinear loop dynamics. The tree-level $\mu_i\mu_j$ contribution generates a rank-one mass matrix and therefore yields only one mass eigenvalue. Since, in our case, $B_i$, $\lambda'_i$, $\lambda_{ijj}$ are all proportional to $\mu_i$, even after including their contributions the rank-one nature of the mass matrix does not change. What breaks the alignment and yields more nonvanishing eigenvalues is the contribution from the purely trilinear loops involving $\lambda_{ijk}(i \neq j \neq k)$, since these couplings are not aligned with $\mu_i$.

This leaves us with the remaining three couplings, namely, $\lambda_{123}$, $\lambda_{132}$, and $\lambda_{231}$, no two indices of which are the same, for generating the second *mandatory* and the third *optional* nonvanishing neutrino masses and the three mixing angles (two large and one small). Note that the existing bounds on $\lambda_{ijk}$ with $i \neq j \neq k$ are relatively less stringent—see Table I.

Different low-energy processes, especially some lepton flavor violating decays, yield important constraints on trilinear product couplings [12–14]. Because of the smallness of *most* of the couplings as shown in the first row of Table I, these constraints are in almost all cases easily satisfied. The bounds emerging from the nonobservation of $K_L^0 \to \mu\bar{e}/e\bar{\mu}$ [13,14], namely,

$$\lambda_{ijk}\lambda'_{lmn} < 6.7 \times 10^{-9} m^2_{\tilde{\nu}_{L3}}/(100\ \mathrm{GeV})^2, \tag{6}$$

with the combinations $(ijk)(lmn)$: (312)(312), (312) × (321), (321)(312), (321)(321), play a crucial role in

TABLE I. List of the six independent couplings and the standard couplings they are related to by the flavor symmetry $X$. The three $\tilde{\mu}_i$ couplings are of the same order of magnitude as 36 out of 39 *a priori* independent RPV couplings. A mass of 100 GeV is assumed for the superparticles exchanged in the processes involved. These superparticles are indicated within first bracket right after the bounds (the weak gaugino mass $M_\chi$ and the three scalar leptons $\tilde{\ell}_R$). The entries in the square brackets specify the different observables from which the bounds originate. Here, $R_\tau = \Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$.

| Our couplings | Related to | Existing limits (sources) | Reference |
|---|---|---|---|
| $\tilde{\mu}_i$ | $\tilde{\mu}_i$, $\lambda'_{ijk}$, $\lambda_{ijj}$ | $1.5 \times 10^{-6}(M_{\tilde{\chi}})[m_{\nu_i}]$ | [7] |
| $\lambda_{123}$ | $\lambda_{123}$ | $0.03(\tilde{\tau}_R)[V_{ud}]$ | [4] |
| $\lambda_{132}$ | $\lambda_{132}$ | $0.03(\tilde{\mu}_R)[R_\tau]$ | [4] |
| $\lambda_{231}$ | $\lambda_{231}$ | $0.05(\tilde{e}_R)[R_\tau]$ | [4] |

neutrino mass/mixing model building in our scenario, as we shall see later. Because of the specific interconnections among RPV couplings owing to the flavor symmetry, Eq. (6) leads to the following limits:

$$\lambda_{132}\lambda'_3, \qquad \lambda_{231}\lambda'_3 < 6.7 \times 10^{-9} m^2_{\tilde{\nu}_{L3}}/(100\ \text{GeV})^2. \quad (7)$$

If we set the numerical values of the couplings near their upper limits (see Table I), they turn out to be large enough to offset the loop suppression factors. The mass matrix entries can then be written with only six RPV couplings as

$$\begin{aligned}
m_{ee} &\approx a\mu_1\mu_1 + \frac{1}{8\pi^2}\lambda_{123}\lambda_{132}\frac{m_\mu m_\tau}{\tilde{m}^2}\mu\tan\beta,\\
m_{e\mu} &\approx a\mu_1\mu_2 + \frac{1}{8\pi^2}\lambda_{123}\lambda_{232}\frac{m_\tau m_\mu}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{213}\lambda_{131}\frac{m_\tau m_e}{\tilde{m}^2}\mu\tan\beta,\\
m_{e\tau} &\approx a\mu_1\mu_3 + \frac{1}{8\pi^2}\lambda_{132}\lambda_{323}\frac{m_\mu m_\tau}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{312}\lambda_{121}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta,\\
m_{\mu\mu} &\approx a\mu_2\mu_2 + \frac{1}{8\pi^2}\lambda_{231}\lambda_{213}\frac{m_e m_\tau}{\tilde{m}^2}\mu\tan\beta,\\
m_{\mu\tau} &\approx a\mu_2\mu_3 + \frac{1}{8\pi^2}\lambda_{231}\lambda_{313}\frac{m_\tau m_e}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{321}\lambda_{212}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta,\\
m_{\tau\tau} &\approx a\mu_3\mu_3 + \frac{1}{8\pi^2}\lambda_{312}\lambda_{321}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta,
\end{aligned} \quad (8)$$

with

$$a = \frac{\cos^2\beta}{\tilde{m}} + \sum_k \frac{3g m_{d_k}}{8\pi^2\tilde{m}^2} + \sum_k \frac{g m_{e_k}}{8\pi^2\tilde{m}^2} + \sum_{k,l}\frac{3m_{d_l}m_{d_k}}{8\pi^2\mu\tilde{m}_q^2}\tan\beta. \quad (9)$$

With this mass matrix we try to reproduce the neutrino oscillation data, namely, the two mass-squared differences ($\Delta m^2_{21}$ and $\Delta m^2_{31}$) and the three mixing angles ($\theta_{12}$, $\theta_{23}$, and $\theta_{13}$). For simplicity we assume that all the phases in the neutrino mixing matrix are zero. Since neutrino oscillation analysis probes only the mass-squared differences and not their absolute values, we need to assume the hierarchy (normal/inverted) of the masses and the size of the smallest eigenvalue to fix the other two masses. There is no lower limit on the smallest neutrino mass eigenvalue; it can still be zero.

We take the best fit values of the neutrino mass-squared differences from [15]: $\Delta m^2_{21} = 7.59 \times 10^{-5}\ \text{eV}^2$, $\Delta m^2_{31}(\text{IH}) = -2.40 \times 10^{-3}\ \text{eV}^2$, $\Delta m^2_{31}(\text{NH}) = 2.51 \times 10^{-3}\ \text{eV}^2$. Above, NH stands for normal hierarchy and IH stands for inverted hierarchy of neutrino masses. The two mixing angles $\theta_{12}$ and $\theta_{23}$ are set to their TBM values (using best fit values instead does not lead to significant changes). Very recently, two long-baseline accelerator experiments T2K and MINOS, both probing $\nu_\mu \to \nu_e$ appearance, have reported, for the first time, a nonzero measurement of $\theta_{13}$. T2K has observed six electronlike events against an estimated background of 1.5, thus discarding $\theta_{13} = 0$ at the level of $2.5\sigma$ [16]. The MINOS experiment observes 62 electronlike events against an expected 49, thus disfavoring $\theta_{13} = 0$ at $1.5\sigma$ [17]. A new global fit suggests $\sin^2\theta_{13} = 0.021(0.025) \pm 0.007$ with old (new) reactor fluxes [18]. The central value corresponds to $\theta_{13} \approx 9°$. A second global fit can be found in [19]. Both indicate $\theta_{13} > 0$ with a significance of about

$3\sigma$. We shall see below that in our scenario whether $\theta_{13}$ is vanishing or nonvanishing plays an important role in predicting whether neutrino mass hierarchy is *normal* [$\Delta m^2 \equiv m_3^2 - 0.5(m_2^2 + m_1^2) > 0$] or *inverted* ($\Delta m^2 < 0$). Since none of the two experiments has so far conclusively established a nonzero value of $\theta_{13}$, we take both the paradigms, namely, $\theta_{13} = 0$ and $\theta_{13} \neq 0$, and study what do they imply on the choices of RPV parameters and whether we can predict the nature of mass hierarchy.

## IV. TRIBIMAXIMAL MIXING

The TBM structure immediately implies that $|m_{e\mu}| = |m_{e\tau}|$ and $m_{\mu\mu} = m_{\tau\tau}$, regardless of whether the lightest mass eigenvalue is vanishing or not, and also irrespective of whether the neutrino mass hierarchy is normal or inverted. For our couplings this can be comfortably realized by setting $|\lambda_{123}| = |\lambda_{132}|$ and $|\mu_2| = |\mu_3|$—see Eq. (8). This means that we can parametrize the mass matrix with four independent RPV parameters instead of six, which of course improves the predictivity of the model. Dropping the terms in the loop contribution proportional to the electron mass, we obtain

$$m_{e\mu} \approx m_{e\tau} \approx a\mu_1\mu_2 - \frac{1}{8\pi^2}\lambda_{123}\mu_3\frac{m_\tau m_\mu}{\tilde{m}^2}\tan\beta. \quad (10)$$

Clearly, under this situation, the absolute values for the tree-level contributions to $m_{\mu\mu} \sim a\mu_2\mu_2$, $m_{\mu\tau} \sim a\mu_2\mu_3$, and $m_{\tau\tau} \sim a\mu_3\mu_3$ are the same. Setting all *CP* violating phases to zero, the TBM mixing matrix takes the form [20]

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

To fix the numerical values of the mass matrix from $m = U_{\rm TBM}^T \times \mathrm{diag}(m_1, m_2, m_3) \times U_{\rm TBM}$, all we need to decide is the mass hierarchy (normal or inverted) and the smallest mass eigenvalue.

*Inverted hierarchy.*—We first consider the case of inverted hierarchy with $m_3 = 0$. This choice additionally demands $m_{\mu\tau} = -m_{\mu\mu}$. One obtains

$$m = \begin{pmatrix} 4.92\times10^{-2} & 2.56\times10^{-4} & -2.56\times10^{-4} \\ 2.56\times10^{-4} & 2.47\times10^{-2} & -2.47\times10^{-2} \\ -2.56\times10^{-4} & -2.47\times10^{-2} & 2.47\times10^{-2} \end{pmatrix} \times \mathrm{eV}\ (\mathrm{IH, TBM}, m_3 = 0). \quad (12)$$

By setting $\mu_2 = -\mu_3$ and keeping $\lambda_{231} \lesssim \lambda_{123}$, we obtain a rough analytical solution using Eqs. (4) and (8),

$$|\mu_2| = |\mu_3| \approx \sqrt{a^{-1}m_{\mu\mu}},$$
$$\lambda_{123} \approx \sqrt{\frac{4\pi^2 m_{ee}\tilde{m}^2}{m_\mu m_\tau \mu\tan\beta}}, \quad (13)$$
$$\mu_1 \approx \frac{m_{e\mu} + \mu_2\lambda_{123}m_\tau m_\mu\tan\beta/(8\pi^2\tilde{m}^2)}{(a\mu_2)}.$$

Putting $\tilde{m} = \mu = 100$ GeV and $\tilde{m}_q = 300$ GeV in Eq. (13) we obtain a solution (with $\mu_2 = -\mu_3$ and $\lambda_{132} = -\lambda_{123}$)

$$\tilde{\mu}_1 = 1.9\times10^{-8}, \qquad \tilde{\mu}_2 = -4.7\times10^{-6},$$
$$\lambda_{231} \sim 10^{-4}, \qquad \lambda_{123} = -3.2\times10^{-4} \quad \text{for } \tan\beta = 10. \quad (14)$$

To illustrate how this coupling pattern can arise from a flavor symmetry we are providing an exemplary flavor group for this case. However, since this choice is not necessarily unique and our conclusions do not depend on the specific flavor group, we omit this exercise for the other scenarios. In this case, the required relative suppression can be reproduced by a $Q_X = Z_4 \times Z_8$ family symmetry with a breaking parameter $\varepsilon$. The necessary charge assignments are given by

$$Q_B(L_1) = (2, 5),$$
$$Q_B(L_2) = (0, 5), \quad (15)$$
$$Q_B(L_3) = (3, 2),$$

which imply

$$Q_B(L_1L_2E_3^C) = (3, 0),$$
$$Q_B(L_1L_3E_2^C) = (1, 2), \quad (16)$$
$$Q_B(L_2L_3E_1^C) = (1, 2).$$

These assignments lead exactly to the required suppression of the couplings with $\varepsilon \approx 0.1$ as

$$\mu_2(=\mu_3) \sim \varepsilon^5, \qquad \mu_1 \sim \varepsilon^7,$$
$$\lambda_{123}(=\lambda_{132}) \sim \varepsilon^3, \qquad \lambda_{231} \sim \varepsilon^3. \quad (17)$$

In the above example, the near-equality of the magnitude of the entries in the $\mu - \tau$ block is ensured by saturating these entries with the tree-level contributions, while keeping the loop contributions suppressed. If, within the TBM framework, we now consider $m_3$ to be slightly above zero, then $m_{\mu\mu} = m_{\tau\tau} > |m_{\mu\tau}|$. To obtain $m_3 = 0.001$ eV with $\tilde{m} = \mu = 100$ GeV, we need $\tilde{\mu}_1 = 1.9\times10^{-8}$, $\tilde{\mu}_2 = -4.6\times10^{-6}$, $\tilde{\mu}_3 = 4.7\times10^{-6}$, $\lambda_{123} = -3.1\times10^{-4}$, $\lambda_{132} = -3.3\times10^{-4}$, $\lambda_{231} = 2.7\times10^{-3}$. We should note two important things: (i) These choices imply $\lambda_{231}\lambda_3' = 1.3\times10^{-8}$, which mildly overshoots the $K_L \to \mu e$ bound as shown in Eq. (7). If we increase $m_3$ further, the disagreement with the $K_L$ bounds deepens. (ii) The four-parameter scenario with $|\mu_2| = |\mu_3|$ and $|\lambda_{123}| = |\lambda_{132}|$ is not compatible with a nonvanishing absolute neutrino mass scale, i.e., we cannot fit the data assuming these "equalities" with $m_3 > 0$, because of the hierarchical nature of the charged lepton masses which appear in Eq. (8).

*Normal hierarchy.*—We now consider normal hierarchy of neutrino masses. In this case the smallest mass eigenvalue is $m_1$. Within the TBM structure, if we keep $m_1 = 0$, it follows that $m_{\mu\mu} = m_{\tau\tau} > |m_{\mu\tau}|$. The numerical values of the mass matrix entries are

$$m = \begin{pmatrix} 2.90 \times 10^{-3} & 2.90 \times 10^{-3} & -2.90 \times 10^{-3} \\ 2.90 \times 10^{-3} & 2.80 \times 10^{-2} & 2.21 \times 10^{-2} \\ -2.90 \times 10^{-3} & 2.21 \times 10^{-2} & 2.80 \times 10^{-2} \end{pmatrix} \times \text{eV (NH, TBM, } m_1 = 0). \quad (18)$$

The couplings needed to fit these entries are $\tilde{\mu}_1 = -5.2 \times 10^{-7}$, $\tilde{\mu}_2 = 3.9 \times 10^{-6}$, $\tilde{\mu}_3 = 5.0 \times 10^{-6}$, $\lambda_{123} = -4.4 \times 10^{-3}$, $\lambda_{132} = -1.2 \times 10^{-6}$, $\lambda_{231} = 1.0 \times 10^{-3}$. Although we are within the $K_L \to \mu e$ bound, the requirement $m_{e\mu} = -m_{e\tau}$ is realized quite differently. The relative signs of the tree-level couplings invariably imply $m_{e\mu}^{\text{tree}} \approx +m_{e\tau}^{\text{tree}}$. This difference between the experimental requirement and the tree-level contribution cannot be resolved, even keeping in mind that signs of the RPV couplings can be chosen at will and also each neutrino field can be redefined to absorb a sign. Therefore, a sign adjustment for one of the entries ($e\mu$) via a large loop contribution is needed, while the loop contribution to the other one ($e\tau$) becomes negligible. This is reflected in the large hierarchy between $\lambda_{123}$ and $\lambda_{132}$. We recall that such a sign adjustment was not required in the case of inverted hierarchy (TBM, $m_3 = 0$). If we now increase the value of $m_1$ (from zero) and try to fit normal hierarchy within the TBM framework, the $K_L \to \mu e$ bound haunts us like in the case of inverted hierarchy with $m_3 > 0$. Therefore, our most robust prediction is the tight constraint for the smallest mass eigenvalue. Thus, inverted hierarchy can be fit with four parameters, while normal hierarchy requires six parameters and a sign altering large loop correction.

## V. NONZERO $\theta_{13}$

In view of the recent T2K data which measure nonvanishing $\theta_{13}$, we study if our model is able to accommodate normal and inverted hierarchies when $\theta_{13}$ is close to its central value of 9.0°. Unlike in the case of TBM which guarantees $|m_{e\mu}| = |m_{e\tau}|$ and $m_{\mu\mu} = m_{\tau\tau}$, it is not possible to fit the data with four parameters when $\theta_{13} \neq 0$.

*Inverted hierarchy.*—First we consider the case $m_3 = 0$. The numerical entries of the mass matrix are given by

$$m = \begin{pmatrix} 4.80 \times 10^{-2} & -5.13 \times 10^{-3} & -5.63 \times 10^{-3} \\ -5.13 \times 10^{-3} & 2.53 \times 10^{-2} & -2.41 \times 10^{-2} \\ -5.63 \times 10^{-3} & -2.41 \times 10^{-2} & 2.54 \times 10^{-2} \end{pmatrix} \times \text{eV (IH, } \theta_{13} = 9.0°, m_3 = 0). \quad (19)$$

This can be fit with $\tilde{\mu}_1 = -1.1 \times 10^{-6}$, $\tilde{\mu}_2 = -4.5 \times 10^{-6}$, $\tilde{\mu}_3 = 4.8 \times 10^{-6}$, $\lambda_{123} = 9.3 \times 10^{-3}$, $\lambda_{132} = 1.1 \times 10^{-5}$, $\lambda_{231} = -1.1 \times 10^{-4}$. Two things are worth noting: (i) The magnitudes of $\lambda_{123}$ and $\lambda_{132}$ are separated by nearly three orders, while they assumed identical numerical values in the case of TBM. (ii) The tree-level contribution to $m_{e\mu}$ has the wrong sign like in the case of NH with $\theta_{13} = 0$. Again a large sign-adjusting loop contribution is needed to be in agreement with the experimental data. If we now increase the value of $m_3$, the required magnitude for $\lambda_{231}$ becomes larger, and eventually beyond $m_3 = 0.01$ eV the $K_L \to \mu e$ bound overshoots.

*Normal hierarchy.*—For $m_1 = 0$, the mass matrix entries are given by

$$m = \begin{pmatrix} 4.06 \times 10^{-3} & 8.02 \times 10^{-3} & 2.29 \times 10^{-3} \\ 8.02 \times 10^{-3} & 2.67 \times 10^{-2} & 2.16 \times 10^{-2} \\ 2.29 \times 10^{-3} & 2.16 \times 10^{-2} & 2.80 \times 10^{-2} \end{pmatrix} \times \text{eV (NH, } \theta_{13} = 9.0°, m_1 = 0). \quad (20)$$

This can be reproduced with $\tilde{\mu}_1 = 4.1 \times 10^{-7}$, $\tilde{\mu}_2 = 3.8 \times 10^{-6}$, $\tilde{\mu}_3 = 5.0 \times 10^{-6}$, $\lambda_{123} = -5.3 \times 10^{-3}$, $\lambda_{132} = -1.5 \times 10^{-6}$, $\lambda_{231} = 8.3 \times 10^{-4}$. Note that $\lambda_{231}$ is small enough to satisfy the $K_L \to \mu e$ bound. Contrary to the case of inverted hierarchy, now no large sign-flipping correction for $m_{e\mu}$ is needed. However, the difference between the values of $m_{e\mu}$ and $m_{e\tau}$ still leads to a hierarchy in the $\lambda$ couplings. Just like in the previous cases, the $K_L \to \mu e$ bound begins to be relevant as soon as $m_1$ increases to around 0.005 eV (which requires $\lambda_{231} = 1.5 \times 10^{-3}$). The main conclusion for nonzero $\theta_{13}$ is again that the smallest mass eigenvalue is required to be almost vanishing in both hierarchies. But contrary to the TBM case, now IH requires a sign adjustment, while NH does not.

## VI. COLLIDER SIGNATURES

The LHC signatures of the $\lambda_{ijk}$ couplings have recently been explored in [21]. In our scenario, only three couplings $\lambda_{ijk} (i \neq j \neq k)$ are relatively large ($10^{-3}$–$10^{-4}$) and the rest are of order $10^{-6}$. The large couplings are small enough to make sure that the RPV vertex is numerically relevant only at the end of a supersymmetry cascade when the lightest neutralino decays via a $\lambda_{ijk}$ interaction, $\tilde{\chi}_1^0 \to l^\pm l^\mp \nu$. The $\lambda_{ijk}$ couplings thus give rise to $l_i l_k$ or $l_j l_k$ final states plus missing energy. Depending on the numerical values of the corresponding $\lambda_{ijk}$ couplings, the branching ratios into the $l_i l_k$ or $l_j l_k$ channel will scale as $|\lambda_{ijk}|^2$. Thus both invariant mass distributions and number counting of the final state leptons should be a part of the search method. However, other decay channels like $\tilde{\chi}_1^0 \to W^\pm l^\mp$ and $\tilde{\chi}_1^0 \to Z\nu$ are available due to the presence of the bilinear couplings. Their role has been investigated in detail in [22].

Therefore, a detailed study of neutralino decays is important to test this and other RPV models and differentiate between them. The nonobservation of an excess in four lepton events at CMS and ATLAS so far indicates a somewhat heavier squark mass scale than the one we choose. However, scaling the slepton masses accordingly, this will not lead to any significant changes related to our work.

## VII. CONCLUSIONS

In this paper we have studied a generic and simple flavor model which reduces the number of independent couplings from 39 to 6, i.e., $\mu_i (i = 1, 2, 3)$, $\lambda_{123}$, $\lambda_{132}$, and $\lambda_{231}$. This results in an extremely predictive framework, which can reproduce the correct neutrino masses and mixings while satisfying all other low energy bounds.

In its simplest realization the scenario leads to a four-parameter model with exact tribimaximal mixing and prefers inverse hierarchy, for which a specific flavor model, viz $Z_4 \times Z_8$, has been proposed. A nonvanishing mixing angle $\theta_{13}$ can be accommodated in a six-parameter realization.

A general prediction of all possible realizations is an almost vanishing absolute mass scale for neutrinos, i.e., an essentially massless lightest neutrino. This feature is tightly related to the nonobservation of $K_L \rightarrow \mu e$ which affects many important coupling products in this framework. As a consequence, any positive signal in one of the upcoming neutrinoless double beta decay experiments would imply an inverted neutrino mass hierarchy, since for the combination of normal hierarchy and an almost vanishing absolute mass scale, the resulting $|m_{ee}|$ is beyond their sensitivity. In other words, if conclusive evidence of nonzero $\theta_{13}$ is established, then our scenario would be able to accommodate a positive signal of neutrinoless double beta decay only at the expense of a large sign-flipping correction to one of the off-diagonal elements of the mass matrix. Moreover, the flavor structure proposed here can lead to specific decays of a neutralino lightest supersymmetric particle at the LHC.

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