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Higgs decays to dark matter: Beyond the minimal model

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We examine the interplay between Higgs mediation of dark-matter annihilation and scattering on one hand and the invisible Higgs decay width on the other, in a generic class of models utilizing the Higgs portal. We find that, while the invisible width of the Higgs to dark matter is now constrained for a minimal singlet scalar dark matter particle by experiments such as XENON100, this conclusion is not robust within more generic examples of Higgs mediation. We present a survey of simple dark matter scenarios with $m_{\rm DM} < m_h/2$ and Higgs portal mediation, where direct-detection signatures are suppressed, while the Higgs width is still dominated by decays to dark matter.

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I. INTRODUCTION

The past year has seen impressive progress toward an understanding of electroweak symmetry breaking at the LHC and the Tevatron. The allowed mass range for its simplest manifestation—the standard model (SM) Higgs boson—is now limited to 114–145 GeV [1], which is of course indirectly favored by the global precision electroweak fit. This low mass region is notoriously difficult to probe at the LHC and may also be vulnerable to non-SM decay modes that can hide the Higgs by suppressing conventional decays even in the simplest extensions of the standard model, see, *e.g.*, [2–4]. Thus, while the absence of SM-type Higgs decay signatures over the full mass range could be interpreted as evidence in favor of a nonperturbative mechanism for electroweak symmetry breaking, a plausible alternative hypothesis would be that a perturbative Higgs scalar is present but with non-SM decay channels that make it hard to identify [2–4]. Among these scenarios, Higgs decays to dark matter (DM) through the so-called Higgs portal comprise a distinct and natural possibility. While first identified many years ago [5], it is only recently that both Higgs searches [1] and dark matter direct-detection efforts [6] have reached the level of sensitivity required to make an experimental test of this possibility a reality.

As the LHC approaches the sensitivity level required to directly observe a SM Higgs in the low mass range, underground direct-detection experiments probing nuclear recoils of weakly-interacting massive particle (WIMP) dark matter are also reaching the important threshold of Higgs-mediated scattering, $\sigma_{\rm SI} \sim 10^{-45} - 10^{-43}$ cm^2 [6]. This coincidence naturally focuses attention on the possible interplay between these two probes of the Higgs sector and its interactions. In particular, the question arises as to whether a putative invisible Higgs width may be constrained by the presence or absence of any direct-detection signal. As recently emphasized in the literature for the most economical DM model—a singlet scalar with Higgs-mediated interactions [5,7,8]—combining the collider limits on a SM-like Higgs with the direct-detection constraints indeed leads to significant restrictions on any invisible Higgs branching in the low mass $m_h \sim 120$ GeV region [9–12]. In the present paper, we will examine the generality of this conclusion in a simple but more generic class of Higgs-mediated dark matter interactions, finding that it is far from robust. Indeed, we observe that many scenarios for Higgs mediation in the dark sector, beyond the minimal singlet scalar, allow for a significant invisible Higgs branching while being poorly constrained by the results of direct-detection experiments.

In order to focus on invisible Higgs decays, we will consider dark matter (and scalar mediators) whose mass is below $m_h/2$, i.e. below 50–60 GeV for the light Higgs region. For such relatively light states, effective field theory dictates that the largest couplings will be through renormalizable operators, and in this case, the Higgs portal:

$$\mathcal{L}_{\rm portal} = H^\dagger H(A_i S_i + \lambda_{ij} S_i S_j), \tag{1}$$

where H is the SM Higgs doublet, and we allow for at least two real SM-singlet scalar fields S_i. These scalars, and other components of the hidden sector, could also transform under larger representations of a hidden sector (gauge) group or indeed be composite operators, but for simplicity we focus on perturbative singlets as representative of the basic physics involved. The lightest scalar S may be a dark matter candidate itself or may mediate the interactions of another stable dark matter species in the hidden sector. For the latter case, we enumerate the renormalizable possibilities below for a hidden sector fermion N,

$$\mathcal{L}_{\rm hid} = (m_N + \alpha_i S_i)\bar{N}N + \beta_i S\bar{N}i\gamma_5 N - V(S_i). \tag{2}$$

The potential term may include multiple scalars, and it is assumed that it satisfies the usual requirements of vacuum stability. The minimal model contains just one scalar field, and the Higgs portal $\lambda_{\rm min} H^\dagger H S^2$ regulates the abundance of S as WIMP DM. It also provides a rigid link between the invisible Higgs branching ratio and the DM scattering cross section [5,8].

Various combinations of the portal and hidden sector couplings (A, λ, α/β) will determine the relic density and scattering cross section of dark matter. Our primary strategy will be to constrain these parameters by requiring that the relic dark-matter abundance is regulated by the annihilation at freeze-out either of DM itself or of its mediators, and then to explore the interplay between the existing constraints and future sensitivity in direct-detection and the invisible decay width $\Gamma(h \to 2\mathrm{DM})$. To assess the importance of this decay channel, we will characterize the invisible Higgs branching with the following figure of merit [8], which approximates the dilution of all visible Higgs decay modes in the low m_h regime:

$$R_{\mathrm{vis}} = \frac{\Gamma(h \to b\bar{b})}{\Gamma(h \to 2\mathrm{DM}) + \Gamma(h \to b\bar{b})} \sim \frac{Y_b^2}{Y_b^2 + \frac{2}{3}\lambda_{\mathrm{min}}^2 v^2/m_h^2}. \tag{3}$$

In this expression, v is the electroweak vacuum expectation value, the Yukawa coupling of the b-quark is normalized at the m_h scale, and the phase space factors are neglected. As the DM mass is taken below $m_{\mathrm{DM}} \sim 40$ GeV, the invisible Higgs decay channel becomes dominant. Within background-dominated LHC Higgs searches, a detection of the Higgs boson via its conventional decay modes would then require increasing the size of the Higgs data set by at least a factor of $1/R_{\mathrm{vis}}^2$.

In the next section, we outline a series of specific model scenarios falling within the general Higgs-mediated setting of Eqs. (1) and (2), focusing on the link between the invisible Higgs width and the direct-detection sensitivity.

II. SCENARIOS AND SIGNATURES

The Higgs portal allows for a number of Higgs-mediated dark matter scenarios, where the set of induced $h-\mathrm{DM}$ couplings determines both the direct scattering cross section and the invisible Higgs width. Below, we detail a series of modules (or simplified models [13]) which encode the basic physics. Many of these modules can be embedded as part of more comprehensive UV theories.

A. WIMPs and the pseudoscalar Higgs portal

The WIMP scenario of fermionic DM mediated by the Higgs portal has been discussed before (see, *e.g.*, Refs. [14–16]), focusing on its CP-conserving version (although, see Ref. [17]). Here, we consider a CP-odd combination of the trilinear Higgs portal with a pseudoscalar coupling,

$$\mathcal{L} = (H^\dagger H)(AS + \lambda S^2) + \beta S\bar{N}i\gamma_5 N, \tag{4}$$

which, on integrating out the heavier scalar S and taking the unimportant coupling λ to be small, leads to

$$\mathcal{L}_{\mathrm{eff}} = \lambda_h h\bar{N}i\gamma_5 N. \tag{5}$$

The effective Higgs coupling λ_h results from $S-h$ mixing induced by the $ASH^\dagger H$ term in the Lagrangian and is taken to satisfy the freeze-out condition,

$$\langle\sigma v\rangle_{\bar{N}N\to\mathrm{SM}} \simeq \frac{3\lambda_h^2}{4\pi}\left(\frac{m_b}{m_h}\right)^2 \frac{m_N^2}{m_h^4} \sim 1\ \mathrm{pb}. \tag{6}$$

This requires

$$\lambda_h^2 \sim 10 \times \left(\frac{20\ \mathrm{GeV}}{m_N}\right)^2, \tag{7}$$

where we have taken $m_N \sim 20$ GeV, which is close to the lower bound given by the perturbative threshold, $\lambda_h^2 \sim 4\pi$. With $m_h \sim \mathcal{O}(120)$ GeV, this scenario has a limited range for the DM mass, where λ_h is always significantly larger than the b-quark Yukawa coupling, leading to $R_{\mathrm{vis}} \ll 1$ and the possibility of an $O(10^3)$ suppression of all visible Higgs decay modes.

Turning to direct detection (see Fig. 1(a)), we observe that the pseudoscalar density $\bar{N}i\gamma_5 N$ vanishes in the nonrelativistic limit, which suppresses the elastic DM-nucleon scattering cross section by an additional factor of $(v/c)^2 \sim 10^{-6}$,

$$\sigma_p^{\mathrm{eq}} \simeq \frac{1}{2\pi}(v/c)^2 \times \frac{g_{hpp}^2\lambda_h^2 m_p^2}{m_h^4} \times \left(\frac{Am_p}{Am_p + m_N}\right)^2 \lesssim 10^{-48}\ \mathrm{cm}^2 \times \lambda_h^2. \tag{8}$$

Note that (distinct from σ_{SI}) this is an *equivalent* DM-nucleon–scattering cross section derived from the DM-nucleus cross section with A nucleons. One observes that not only is this cross section well below the current level of direct-detection sensitivity, but it may be below the potentially irreducible background due to the solar neutrino recoil signal [18].

At this point, we should try to address the question of how natural it is to have a dominant CP-odd coupling for DM, given the fact that CP violation is small (or flavor-suppressed) in the observable sector of the SM. Unfortunately, this question does not have a clear-cut answer in the model (5) due to the super-renormalizable nature of the portal $ASH^\dagger H$. Indeed, if S is a pseudoscalar in the dark sector, the $S\bar{N}i\gamma_5 N$ coupling conserves all discrete symmetries. With A the CP-violating coupling, even a value $A \sim O(M_W)$ may be "small" in the sense that $A \ll \Lambda_{\mathrm{UV}}$, given that the UV cutoff of this theory is unknown. Changing the CP charge assignment by taking S to be a scalar, we see that the source of CP violation, now shifted to β, is also well-sequestered from any visible sector observables due to the need for Higgs mediation. Thus, a fermionic WIMP interacting via this pseudoscalar Higgs portal is a viable possibility, and naturally leads to a large invisible Higgs width, $R_{\mathrm{vis}} \ll 1$, while having suppressed signatures for direct detection. We note in passing

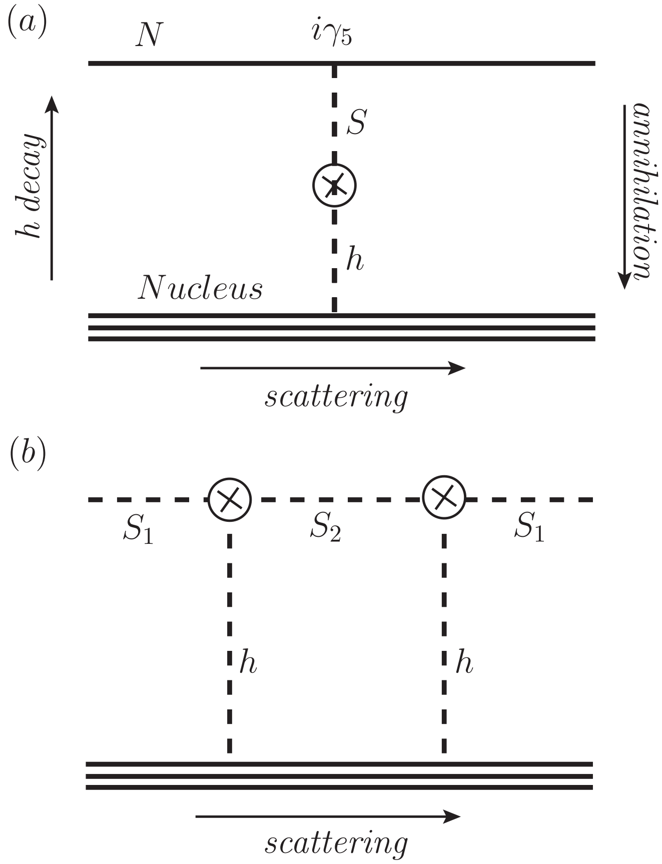

FIG. 1. Schematic illustration of the scattering, annihilation, and Higgs decay processes for: (a) fermionic WIMP interactions mediated via the pseudoscalar portal; and (b) split scalar WIMPs, showing loop-level elastic scattering which can dominate over tree-level exchange for $\lambda_{12} \gg \lambda_{11}$.

that were such a scenario realized, there would be an observable indirect signature through DM annihilation within overdense regions in the galactic halo [19] (see, *e.g.*, Ref. [20] for analyses in the minimal model).

B. Split WIMPs

Multicomponent WIMPs with a small splitting between the states have been examined in some detail in recent years, primarily in connection with inelastic DM models [21]. Here, we consider two real scalar WIMPs coupled through the Higgs portal,

$$\mathcal{L} = \sum_{i,j=1,2} S_i\left(\Lambda_{ij}(H^\dagger H) - \frac{1}{2}M_{ij}^2\right)S_j, \tag{9}$$

where Λ and M^2 are the 2×2 real symmetric matrices. DM stability requires a suitable dark symmetry, which forbids the A term couplings. A $\mathbb{Z}_2$ acting as $S_i \to -S_i$ is one minimal option, among many alternatives. After electroweak symmetry breaking and without the loss of generality, we can choose the mass terms to be diagonal and write the effective Higgs-DM Lagrangian as

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}(m_1^2 S_1^2 + m_2^2 S_2^2) + h(\lambda_{11}S_1^2 + 2\lambda_{12}S_1S_2 + \lambda_{22}S_2^2), \tag{10}$$

where we also assume $m_2 > m_1$. In the present paper, we will not attempt to analyze the full parameter space for m_1, m_2 and λ_{ij}. Instead, we will concentrate on the case where the mass splitting is relatively small,

$$\Delta m = m_2 - m_1 \lesssim 0.1 m_1, \tag{11}$$

and the couplings exhibit the hierarchical pattern,

$$\lambda_{22} \gg \lambda_{12}, \lambda_{11}. \tag{12}$$

Choosing this pattern of couplings, which we will justify below, ensures that the model has the following properties:

(i) S_1 is a stable DM candidate, while S_2 is unstable, $S_2 \to S_1 + \mathrm{SM}$.

(ii) The cosmological abundance of S_1 is controlled via coannihilation: $S_1 + \mathrm{SM} \to S_2 + \mathrm{SM}$ followed by $S_2 + S_2 \to \mathrm{SM}$.

(iii) The elastic scattering cross section of S_1 on nucleons is suppressed relative to the minimal model by $(\lambda_{11}/\lambda_{22})^2$ (or loop-suppressed if λ_{11} is sufficiently small [22], as exhibited in Fig. 1(b)). If the mass splitting Δm is in the keV range or below, an inelastic component to scattering is also present but suppressed by $(\lambda_{12}/\lambda_{22})^2$

(iv) For DM masses below roughly 40 GeV, the Higgs decay is totally dominated by $h \to 2S_2$. Depending on the size of λ_{12}, the subsequent decay $S_2 \to S_1 + \mathrm{SM}$ may or may not happen within the detector volume, resulting in either a "buried" or an invisible Higgs decay signature.

We can estimate the size of the off-diagonal coupling λ_{12} needed to ensure that S_1 and S_2 stay chemically coupled at freeze-out by comparing the rate of Higgs-induced $1 \leftrightarrow 2$ conversion, $S_1 + \mathrm{SM} \to S_2 + \mathrm{SM}$, with the Hubble rate at $T \sim 0.05 m_1$. Estimating the scattering of $m_S \sim 20$ GeV S-particles on charm quarks at $T \sim 1$ GeV, we arrive at the condition

$$\lambda_{12}^2 \gtrsim 10^{-6}. \tag{13}$$

We now address the naturalness of the hierarchy (11) and (12). A simple scenario which achieves it assumes that initially the matrix Λ is dominated by one entry, $\Lambda \simeq \mathrm{diag}(0, \lambda_{22})$, and the mass matrix is also nearly diagonal with a small off-diagonal entry, $m_{12}^2 \ll m_2^2 - m_1^2$. In this case, the mixing angle required to go to the mass eigenstate basis is small, $\theta \sim m_{12}^2/(m_2^2 - m_1^2)$, and the induced hS_1S_2 and hS_1^2 couplings are suppressed: $\lambda_{12} \sim \theta\lambda_{22}$; $\lambda_{11} \sim \theta^2\lambda_{22}$. The small size of m_{12}^2 relative to $m_2^2 - m_1^2$ can arise naturally if there are separate approximate discrete symmetries for S_1 and S_2 broken by this term. We conclude that this scenario is viable and does not require an excessive tuning of parameters. In view of the coannihilation requirement (13), θ can be as small as 10^{-3}, in which case the direct-detection cross sections are suppressed by a factor of $O(\theta^4) \sim 10^{-12}$.

C. Secluded WIMPs

The secluded regime [16,23] makes use of the Lagrangian

$$\mathcal{L} = H^\dagger H(AS + \lambda S^2) + \alpha S\bar{N}N \tag{14}$$

and requires that $m_N > m_S$, so that annihilation proceeds via $\bar{N}N \to SS$, with S decaying on-shell to the SM. Requiring N to be a thermal WIMP leads to the usual restriction that α^4/m_N^2 is fixed to be $O(\text{pb})$. The Higgs can decay directly to the mediators $h \to SS$, with a width controlled by the coupling λ which does not affect the abundance of N. Taking λ above Y_b ensures $R_{\text{vis}} < 1$, so that the Higgs width to $2S$ can be sizeable. As in the split WIMP scenario, this decay will be invisible if S is sufficiently long-lived to escape the detector, while it will be buried if S decays occur inside the detector, leading to multiple jets in the final state. The WIMP-nucleon scattering cross section can be made almost arbitrarily small [16], and in practice, taking $A/v \sim 10^{-3}$ renders the scattering rate below the neutrino background for direct detection. On the other hand, this model does have indirect DM-detection signatures, which can be enhanced in the case of nonrelativistic annihilation, provided that m_S is light [23–26].

D. Super-WIMPs

A particularly "signature-poor" variant of the scenarios considered here comprises a neutral particle N sufficiently weakly coupled to the observable sector that throughout the thermal history of the Universe, it remains sparsely populated compared to other species, i.e. a super-WIMP (see, e.g, Ref. [27]). The Lagrangian can be taken in a form that closely resembles the minimal scalar DM model,

$$\mathcal{L} = \lambda(H^\dagger H)S^2 + \alpha S\bar{N}N - V(S), \tag{15}$$

where the mediator S is relatively heavy, $m_S > 2m_N$. The thermal history consists of the normal annihilation of S at the freeze-out, followed by the late decay $S \to 2N$ producing the relic dark-matter population. Assuming that the coupling constant α satisfies the criterion,

$$\alpha^2 \lesssim 10^{-24}, \tag{16}$$

the direct thermal production of N states (*e.g.* via $Sh \to N\bar{N}$) is subdominant to S decays, and N is a super-WIMP. The coupling λ required to ensure the appropriate relic abundance of S (and thus N) can then be obtained from the corresponding coupling in the minimal scalar model by the following rescaling:

$$\lambda = \frac{2m_N}{m_S} \times \lambda_{\text{min}}. \tag{17}$$

The only signature of this model is the enhanced $h \to 2S$ decay that can easily dominate over the SM channels if $2m_N/m_S$ is not too small. Unlike other super-WIMP models (*e.g.* next-to-lightest–superpartner-to-gravitino decays), the decay of $S \to 2N$ occurs completely within the dark sector and does not have additional signatures related to either big bang nucleosynthesis or the cosmic microwave background [27].

E. WIMPs and the supersymmetric Higgs portal

Appropriately mixed $\tilde{B}/\tilde{H}$ neutralino dark matter candidates in the minimal supersymmetric standard model are archetypal examples of WIMPs which can undergo Higgs-mediated elastic scattering at the current direct-detection threshold (see, *e.g.* Ref. [28] for an analysis in the low-mass range). However, this example does not fit within the scenarios outlined above, primarily because it relies on $\tan\beta = \langle H_2\rangle/\langle H_1\rangle$ being large, which enhances DM annihilation mediated by the pseudoscalar Higgs A and neutralino-nucleon scattering mediated by H exchange. The invisible width of the lightest Higgs boson h decaying to neutralinos is suppressed by m_χ^2/μ^2 due to the small mixing of $\tilde{B}$ with $\tilde{H}_2$. Models with light neutralino DM also typically have a number of charged states (Higgses $H^\pm$, sfermions, etc.) near the weak scale, which allows for discovery via channels unrelated to DM. However, turning to next-to-minimal supersymmetric standard model scenarios, the chances for invisible or hidden Higgs decays are substantially higher [3].

Here, we extend the minimal-supersymmetric-standard-model particle content by singlet chiral superfields N and S, in close analogy with Eqs. (1) and (2), while requiring all superpartners of the SM fields to be heavy. An example of this extension, with a supersymmetric Higgs portal, is given by the superpotential

$$W = \lambda_S H_1 H_2 S + m_S S^2 + (M + \alpha S)N^2. \tag{18}$$

If, in addition to SM superpartners, H, A, and $H^\pm$ are heavy and $\tan\beta \gg 1$, this model reduces to the SM (with H_2 being the SM-like Higgs doublet) plus the supersymmetric multiplets of S and N. By varying the couplings, one can find regimes reproducing most of the SM Higgs portal models discussed above. The scalar potential contains the terms $V = |\mu H_2 + \lambda_S H_2 S|^2$, from which we can identify the couplings to the complex scalar S in Eq. (1) as $A = \mu\lambda_S$ and $\lambda = |\lambda_S|^2$. Choosing these couplings appropriately, we reproduce the super-WIMP and secluded WIMP models with states from the N multiplet playing the role of DM. The pseudoscalar Higgs portal can be constructed by taking α real and choosing $\arg(\mu^*\lambda_S) = \pm\pi/2$. In this case, only $\text{Im}(S)$ couples the fermionic DM candidate N to the Higgs portal via the $\bar{N}i\gamma_5 N$ bilinear, while $\text{Re}(S)$ will not couple to the Higgs portal at all. Finally, split WIMPs can be obtained by introducing multiple N_i states with small mass splittings, while allowing just one to have a large coupling to the S mediator field. In all these models, the lightest supersymmetric Higgs state h can have a significant (or dominant) invisible branching fraction directly to DM states and/or its mediators, while the direct-detection cross sections are suppressed.

III. CONCLUDING REMARKS

The simplicity of the varied Higgs portal scenarios considered above serves to underscore the point that generic models of Higgs-mediated dark matter—beyond the minimal model of scalar DM—do not imply a rigid link between the invisible Higgs decay width and the DM direct-detection signal. Thus, the absence of a signal in direct detection need not preclude a sizeable invisible Higgs width even if dark matter is predominantly Higgs-mediated. This emphasizes the important role that invisible Higgs searches could play in the eventuality that conventional Higgs signatures are found to be suppressed and/or excluded at the level of SM cross sections and decay rates.

Finally, we note that scenarios with a large invisible Higgs branching may have interesting cosmological implications, ranging from changes to the thermal history of the SM and dark sectors (see, *e.g.*, Ref. [29]) to modifications of the electroweak phase transition [30], which may impact scenarios for electroweak baryogenesis.

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