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Effect of nuclear interactions of neutral kaons on *CP* asymmetry measurements

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We examine the effect of the difference in nuclear interactions of K^0 and $\bar{K}^0$ mesons on the measurement of CP asymmetry for experiments at e^+e^- colliders—charm and B-meson factories. We find that this effect on CP asymmetry can be as large as 0.3%, and therefore sufficiently significant in interpreting measurements of CP asymmetry when neutral kaons are present in the final state.

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Modern high-statistics B factories discovered the joint violation of charge-conjugation and parity (CP) in B-meson decay modes. In some B^0 decays [1], large CP violation induced by $B^0 - \bar{B}^0$ mixing is observed to be consistent with the predictions of the standard model (SM) and the Kobayashi-Maskawa ansatz [2]. Smaller, direct CP violations, attributed to the interference of different amplitudes, but without mixing have also been reported [3,4]. SM predictions for the direct CP violation in many charmed-meson decays are typically of $\mathcal{O}(10^{-3})$ [5]. However, the present accuracy of measurements of CP asymmetry in D meson decays is close to their SM expectations. For example, in the decay $D^+ \to K_S^0\pi^+$ [6], the statistical sensitivity on the measured CP asymmetry (of $\approx 0.2\%$) [7] is slightly smaller than the effect expected in the SM of $(0.332 \pm 0.006)\%$ from $K^0 - \bar{K}^0$ mixing [8]. Experiments at future high-luminosity B factories and at the LHC are likely to reach the sensitivity needed to observe CP violation in some D decay modes.

The measured asymmetries of B or D mesons for decays which have K_S^0 in their final states can be mimicked (or diluted) by differences between K^0 and $\bar{K}^0$ interactions with detector material. The probability of an inelastic interaction of a neutral kaon in the detector depends on the strangeness of the kaon at any point along its path, which is due to oscillations in kaon strangeness and different nuclear cross sections for K^0 and $\bar{K}^0$. Hence, the total efficiency to observe a final state K_S^0 differs from that expected for either K^0 or $\bar{K}^0$. This effect is related to the coherent regeneration of neutral kaons [9]. This kind of contribution may be non-negligible for precise measurements of direct CP violation in B and D decays, and also important in the determination of ϕ_3 in the $B^+ \to D^0K^+ \to (K_S^0\pi^+\pi^-)_DK^+$ transition [10] and in a precise measurement of $D^0 - \bar{D}^0$ mixing in the $K_S^0\pi^+\pi^-$ final state, as the Dalitz distribution would be distorted by the K^0 interaction.

In this paper, we evaluate the effect of the difference in nuclear interactions of neutral kaons on measurements of CP asymmetry performed at charm and B factories, or will be carried out at the near future high-luminosity B factories. Our study represents an extension and more detailed description of the method used to estimate the effect of $K^0/\bar{K}^0$ interactions in material in Ref. [7]. We also note that the detector-simulation program GEANT4 [11], commonly used in high energy physics experiments, does not take into account the effect considered in this paper, as the K^0 and $\bar{K}^0$ are projected onto the K_S^0 or K_L^0 components at their production point rather than at their points of $\pi\pi$ decay. The time-dependent $K^0 - \bar{K}^0$ oscillations are thereby ignored in GEANT4. A similar effect in $D^0 - \bar{D}^0$ oscillations was found to be small in the mass and lifetime differences between D^0 and $\bar{D}^0$ [12]. The aim of this paper is to approximately estimate the magnitude of the effect due to the difference in K^0 and $\bar{K}^0$ nuclear interactions under conditions of current and future experiments, and bring this issue to the simulation developers for possible inclusion in programs such as GEANT4. The method and result can serve as an estimate of systematic uncertainty for measurements neglecting the effect, or as a starting point for more refined calculations to be used in the future experiments in order to correct for the effect.

Let us consider production of some meson $\mathcal{P}$ and its antimeson $\bar{\mathcal{P}}$ in e^+e^- collisions, each followed by its decay into states containing a neutral kaon, and observed through the $K_S^0 \to \pi^+\pi^-$ or $\pi^0\pi^0$ mode:

$$\mathcal{P} \to K_S^0 + X, \qquad \bar{\mathcal{P}} \to K_S^0 + \bar{X}.$$

$\mathcal{P}$ can be a charmed or B meson. For certain charmed meson decays, there is a small contribution from doubly Cabibbo-suppressed decays that we ignore, in our main calculation, but assign a systematic uncertainty for this assumption. The CP asymmetry in the $\mathcal{P}$ decays is defined as

$$A_{CP}^{\mathcal{P}\to K_S^0+X} = \frac{\int d\Gamma^{\mathcal{P}\to K_S^0+X} - \int d\Gamma^{\bar{\mathcal{P}}\to K_S^0+\bar{X}}}{\int d\Gamma^{\mathcal{P}\to K_S^0+X} + \int d\Gamma^{\bar{\mathcal{P}}\to K_S^0+\bar{X}}}, \tag{1}$$

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where Γ denotes the partial decay width. We assume that the production point is surrounded by a cylindrical structure of material, typically used in a collider detector environment, such as a beam pipe and several thin layers of vertex detectors.

To obtain the time development of neutral kaons in matter, we use the calculation carried out in Refs. [13,14]. The time evolution of amplitudes in the K_L^0 and K_S^0 basis, as given in Ref. [14], becomes

$$\alpha_{\rm L}(t) = e^{-i\Sigma\cdot t}\Big[\alpha_{\rm L}^0\cos\Big(\frac{\Delta\lambda}{2}\sqrt{1+4r^2}t\Big) - i\frac{\alpha_{\rm L}^0+2r\alpha_{\rm S}^0}{\sqrt{1+4r^2}}\sin\Big(\frac{\Delta\lambda}{2}\sqrt{1+4r^2}t\Big)\Big],$$
$$\alpha_{\rm S}(t) = e^{-i\Sigma\cdot t}\Big[\alpha_{\rm S}^0\cos\Big(\frac{\Delta\lambda}{2}\sqrt{1+4r^2}t\Big) + i\frac{\alpha_{\rm S}^0-2r\alpha_{\rm L}^0}{\sqrt{1+4r^2}}\sin\Big(\frac{\Delta\lambda}{2}\sqrt{1+4r^2}t\Big)\Big],$$

where

$$\Sigma \equiv \frac{1}{2}(\lambda_L+\lambda_S+\chi+\bar{\chi}),$$
$$\Delta\lambda = \lambda_L-\lambda_S = \Delta m - \frac{i}{2}\Delta\Gamma = (m_L-m_S) - \frac{i}{2}(\Gamma_L-\Gamma_S),$$
$$\Delta\chi = \chi-\bar{\chi} = -\frac{2\pi\mathcal{N}}{m}\Delta f = -\frac{2\pi\mathcal{N}}{m}(f-\bar{f}). \quad (2)$$

The quantities $\alpha_{\rm L}(t)$ and $\alpha_{\rm S}(t)$ are the amplitudes for finding states as a K_L^0 and K_S^0 at some time t, respectively, and $\alpha_{\rm L}^0$ and $\alpha_{\rm S}^0$ are those states at $t=0$, where t refers to their proper times. The masses m_L and m_S, and decay widths Γ_L and Γ_S refer to K_L^0 and K_S^0, respectively. The quantity m in $\Delta\chi$ denotes the mass of the K^0 and $\bar{K}^0$. The volume density of material is $\mathcal{N}\equiv\frac{\rho N_A}{M}$, where ρ is the mass density. N_A is Avogadro's number, and M is the mean molar mass. The quantities f and $\bar{f}$ are the forward scattering amplitudes of K^0 and $\bar{K}^0$, respectively. The parameter r is called the regeneration parameter, defined as $r=\frac{1}{2}\frac{\Delta\chi}{\Delta\lambda}$, and its magnitude is generally small, typically in the order of 10^{-2}. Expanding $\alpha_{\rm L}(t)$ and $\alpha_{\rm S}(t)$ up to the first order in r, we obtain

$$\alpha_{\rm L}(t) = \xi_L(t)\alpha_{\rm L}^0 + \zeta(t)\alpha_{\rm S}^0 r,$$
$$\alpha_{\rm S}(t) = \xi_S(t)\alpha_{\rm S}^0 + \zeta(t)\alpha_{\rm L}^0 r, \quad (3)$$

where $\xi_{L,S}(t) = \frac{1}{2}\,e^{-\frac{i}{2}(\chi+\bar{\chi})t}e^{-i\lambda_{L,S}t}$ and $\zeta(t) = \frac{1}{2}\,e^{-\frac{i}{2}(\chi+\bar{\chi})t}\times(e^{-i\lambda_L t}-e^{-i\lambda_S t})$. From these relations, the amplitudes following the passage of several layers of detector material can be obtained iteratively as follows:

$$\alpha_{\rm L}(t_j) = \xi_L(t_j-t_{j-1})\alpha_{\rm L}(t_{j-1}) + \zeta(t_j-t_{j-1})\alpha_{\rm S}(t_{j-1})r_j$$
$$\alpha_{\rm S}(t_j) = \xi_S(t_j-t_{j-1})\alpha_{\rm S}(t_{j-1}) + \zeta(t_j-t_{j-1})\alpha_{\rm L}(t_{j-1})r_j \quad (4)$$

where index j refers to the layer of material last penetrated. This follows because neutral kaons pass through several layers of vacuum and material before they decay. The number of terms in Eq. (4) increases rapidly as the number of detector layers increases, and it is squared when $|\alpha_{\rm L,S}(t)|^2$ are computed to obtain the probability. We evaluate all terms using the symbolic calculation program MATHEMATICA [15]. Our dilution effect ($A_{\mathcal{D}}$) can be extracted from the total asymmetry (A_T), which is incorporated in K^0 regeneration. Without CP violation in $\mathcal{P}\to K_S^0+X$ decay itself, the A_T in the decay can be expressed as

$$A_T^{\mathcal{P}\to K_S^0+X} \equiv \frac{\int R(t)d\Gamma^{\mathcal{P}\to K_S^0+X} - \int \bar{R}(t)d\Gamma^{\bar{\mathcal{P}}\to K_S^0+\bar{X}}}{\int R(t)d\Gamma^{\mathcal{P}\to K_S^0+X} + \int \bar{R}(t)d\Gamma^{\bar{\mathcal{P}}\to K_S^0+\bar{X}}} \cong A_{CP}^{K^0} + A_{\mathcal{D}} + A_{\rm int}, \quad (5)$$

where $R(t)$ and $\bar{R}(t)$, the two-pion decay rates for initial K^0 and $\bar{K}^0$, respectively, can be expressed as $R_S|\alpha_{\rm S}(t)+\eta\alpha_{\rm L}(t)|^2$. R_S is the time-independent decay rate of the K_S^0 eigenstate, and the ratio of amplitudes $\eta=\mathcal{M}(K_L^0\to\pi^+\pi^-)/\mathcal{M}(K_S^0\to\pi^+\pi^-)$. The first term in Eq. (5), $A_{CP}^{K^0}$, is the asymmetry due to $K^0-\bar{K}^0$ mixing which is not of primary interest in this paper, and thus can be subtracted. The third term, $A_{\rm int}$ is the asymmetry from interference between the CP violation in K^0 mixing and the material related amplitudes, and is expected to be of $\mathcal{O}(|r\eta|)\approx 10^{-5}$. We estimate the third term numerically as $\approx 10^{-6}$, and therefore ignore it. Hence, A_T reduces to $A_{\mathcal{D}}$ if the CP violation effect due to $K^0-\bar{K}^0$ mixing in A_T is removed, thereby setting the parameter $\eta=0$. Approximating $\Re\mathfrak{e}(\Delta f)/\Im\mathfrak{m}(\Delta f)=1$, and $\Delta m\approx\frac{1}{2}\Delta\Gamma$, $A_{\mathcal{D}}$ can be expressed as

$$A_{\mathcal{D}} \propto -\Im\mathfrak{m}(\Delta f) \propto \sigma(\bar{K}^0N) - \sigma(K^0N), \quad (6)$$

where N refers to the atomic nucleon of the detector material.

To compute results for Eq. (6) taking into account effects of nuclear screening [16], we adopt an empirical scaling law based on measurements in C, Al, Cu, Sn, and Pb for neutral kaon momenta ($p^{\bar{K}^0}$) between 20 and 140 GeV/c [17]:

$$\Delta\sigma(\bar{K}^0N) \equiv \sigma(\bar{K}^0N) - \sigma(K^0N) = \frac{23.2A^{0.758\pm0.003}}{[p^{\bar{K}^0}({\rm GeV}/c)]^{0.614}}\ {\rm mb}, \quad (7)$$

where A is the atomic number and 0.758 accounts for nuclear screening. The scaling of $A^{0.758}$ in Eq. (7) also describes Pb, Cu, and C data quite well down to 5 GeV/c

[17]. The deuteron data in Ref. [18] also agree well with the prediction of Eq. (7) for $A = 2$ from 50 to 200 GeV/c. We extend the scaling down to lower momenta assuming isospin symmetry of nuclear interactions, $\sigma(\bar{K}^0\mathrm{n}) \cong \sigma(K^-\mathrm{p})$ and $\sigma(K^0\mathrm{p}) \cong \sigma(K^+\mathrm{n})$. We approximate $\sigma(K^+\mathrm{n}) \cong \sigma(K^+\mathrm{p})$ to improve the estimation of $A_{\mathcal{D}}$, and this assumption is consistent with measurements [8]. (Symbols p and n correspond to the proton and neutron, respectively.) Using experimental results for $\sigma(K^-\mathrm{p})$ and $\sigma(K^+\mathrm{p})$ from Ref. [8], we obtain $\Delta\sigma(K^-\{\mathrm{d}, \mathrm{p}\})$, where d denotes deuteron, with $\Delta\sigma(K^-d) \equiv \sigma(K^-d) - \sigma(K^+d)$ and $\Delta\sigma(K^-p) \equiv \sigma(K^-p) - \sigma(K^+p)$. Figure 1 shows $\Delta\sigma(K^-\{\mathrm{d}, \mathrm{p}\})$ (top) and the ratio of the two, $\Delta\sigma(K^-d)/\Delta\sigma(K^-p)$ (bottom), as a function of the kaon momentum. We fit the ratio of $\Delta\sigma(K^-\mathrm{d})$ to $\Delta\sigma(K^-\mathrm{p})$ using an empirical function while keeping the nuclear screening term $A^{0.758}$ fixed. The value of χ^2/d.o.f is approximately 2, indicating our modeling of the ratio of $\Delta\sigma(K^-\mathrm{d})$ to $\Delta\sigma(K^-\mathrm{p})$ is not unreasonable, so that Eq. (7), obtained in the high-momentum range, can be scaled down to 1 GeV/c and below. Using the fit, Eq. (7) is altered as follows:

$$\Delta\sigma(\bar{K}^0 N) = \frac{A^{0.758}\Delta\sigma(K^-\mathrm{p})}{1 + 1.252\mathrm{e}^{-1.841 p^{K^-}(\mathrm{GeV}/c)}}\ \mathrm{mb}, \qquad (8)$$

where p^{K^-} is the momentum of K^-. We use Eq. (8) in the numerical calculation of Eq. (6). The numerator in Eq. (8) should extrapolate the screening effect to atoms in the detector material we use in Table I, and the denominator reflects the low-momentum behavior of the difference in cross section between the proton and deuteron data. We compared our scaling method with the experimental data [19], and found a good agreement.

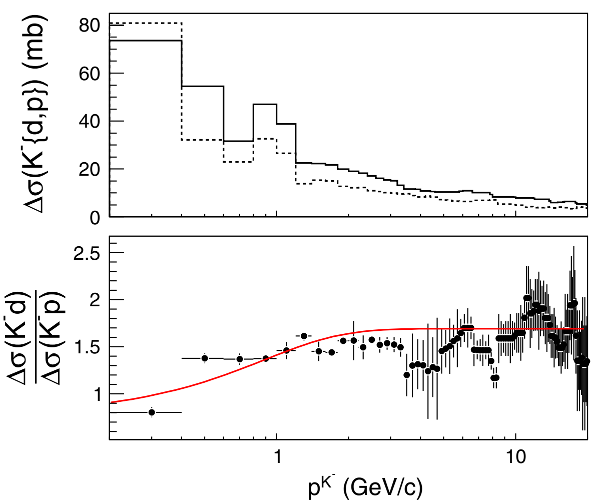

FIG. 1 (color online). The $\Delta\sigma(K^-\{d, p\}) = \sigma(K^-\{d, p\}) - \sigma(K^+\{d, p\})$ values as a function of kaon momentum, obtained from [8] for the proton (dotted lines) and the deuteron data (solid) are shown in the top plot. The ratio of two cross section differences as a function of kaon momentum is shown in the bottom plot (solid circles), together with the fit using the error function (curve).

To obtain the expected four-vectors of K_S^0 mesons in the final state, we use PYTHIA [20] and EVTGEN [21] Monte Carlo codes to simulate generation and decay of charmed and B mesons produced in e^+e^- collisions. Two kinematic cases are considered reflecting two distinct experimental environments: the first case is for a center-of-mass energy $\sqrt{s} = 10.58$ GeV and a Lorentz boost factor of $\beta\gamma = 0.425$ (B factory), and the second case is for $\sqrt{s} = 3770$ MeV with no Lorentz boost (charm factory).

The numerical values of Eq. (6) are calculated for $D^+ \to K_S^0\pi^+$, $D^0 \to K_S^0\pi^+\pi^-$, $D^0 \to K_S^0 K^+K^-$, $B^+ \to K_S^0\pi^+$, $B^0 \to K_S^0\pi^+\pi^-$, and $B^0 \to K_S^0 K^+K^-$ [6], produced in the two kinds of e^+e^- collisions described above. The choice of the decay channels is arbitrary, but intended to show a broad range of momenta that depend on decay characteristics. The first four plots in Fig. 2 show the momentum and polar angle distributions of K_S^0 mesons in the laboratory frame for served final states at $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$. The distributions in polar angle are seen to be very similar for K_S^0 from charmed and B meson decays, despite that the momentum distributions show large differences among the decay modes, which causes significant differences in the values of $A_{\mathcal{D}}$.

As for the material geometry, we choose two general detector options, summarized in Table I, that closely

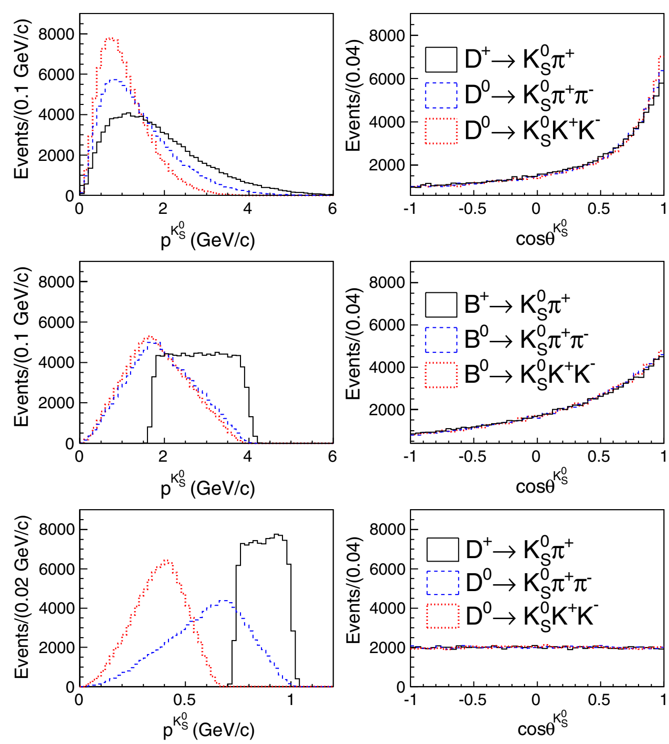

FIG. 2 (color online). K_S^0 momentum (left column) and angular distributions (right column) for different decay modes. The upper two rows are for $\sqrt{s} = 10.58$ GeV and the $\beta\gamma = 0.425$ configuration and the bottom row is for $\sqrt{s} = 3770$ MeV.

TABLE I. Two beam pipe and detector configurations selected for the study described in the text, with δ and R corresponding to the thickness and radius of the given detector component. There are two configurations of layers given for Case II.

Material	Beam pipe Be	Detector layers Si
Case I	$\delta = 1$ mm	$\delta = 300\ \mu$m
	at $R = 1.5$ cm	at $R = 2.0, 4.35, 7.0, 8.8$ cm
Case II	$\delta = 1$ mm	$\delta = 50\ \mu$m
	at $R = 1.0$ cm	at $R = 1.4, 2.2$ cm
	-	$\delta = 300\ \mu$m
	-	at $R = 3.8, 8.0, 11.5, 14.0$ cm

resemble the existing or planned B-meson and charm factories. The first option, denoted as "Case I" [22,23], reflects the current charm and B-meson-factory experiments. The second option, denoted as "Case II," reflects a proposed super B-factory experiment [24]. We apply typical geometrical acceptance criteria in calculating $A_{\mathcal{D}}$ for each case.

We calculate $A_{\mathcal{D}}$ for Case I, with $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$, for the decay modes mentioned previously, and their resultant values are summarized in Table II. We find that $A_{\mathcal{D}}$ values are $\approx 10^{-3}$ for all the above decay modes, and they are mainly affected by the beam pipe. We also plot the distributions of $A_{\mathcal{D}}$ as a function of momentum and polar angle of K_S^0 for Case I. The upper plots of Fig. 3 are the $A_{\mathcal{D}}$ distributions for $D^+ \to K_S^0\pi^+$ at $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$. The values of $A_{\mathcal{D}}$ depend strongly on K_S^0 momentum distributions as shown in Fig. 3 and are larger for smaller momenta. This can be understood from the fact that the cross section difference is larger at small momenta as shown in the upper plot of Fig. 1. We apply typical experimental selection criteria of $p^*(D^+) > 2.5$ GeV$/c$ and $p_T(\pi^+) > 0.45$ GeV$/c$ in $D^+ \to K_S^0\pi^+$ decay, where $p^*(D^+)$ and $p_T(\pi^+)$ are the momenta of D^+ in the center-of-mass frame and the transverse momenta of π^+ in the laboratory frame, respectively. We find practically no difference in $A_{\mathcal{D}}$ applying these selection criteria.

The major systematic uncertainty in this calculation is from the assumption $\Re\mathrm{e}(\Delta f)/\Im\mathrm{m}(\Delta f) = 1$. We estimate this effect using momentum-dependent values of $\Re\mathrm{e}(\Delta f)/\Im\mathrm{m}(\Delta f)$, where $\Re\mathrm{e}(\Delta f)$ is obtained from the best known values in Ref. [25] (kaon momenta available up to 2.6 GeV$/c$). The results differ from $\Re\mathrm{e}(\Delta f)/\Im\mathrm{m}(\Delta f) = 1$ by 6% when limiting the K_S^0 momentum range up to 2.6 GeV$/c$ for charmed-meson decays. For B meson decays, the effect is found to be 10%. Because of limited information on $\Re\mathrm{e}(\Delta f)$, we assign systematic uncertainties of 10% and 20% for charmed and B meson decays, respectively, for the assumption of $\Re\mathrm{e}(\Delta f)/\Im\mathrm{m}(\Delta f) = 1$. The systematic effect from the assumption that $\Delta m \approx \frac{1}{2}\Delta\Gamma$ is found to be negligible. Systematic effects due to uncertainties in modeling Eq. (8) are also found to be negligible. The systematic uncertainties from the measurements for $\sigma(K^-\mathrm{p})$ and $\sigma(K^+\mathrm{p})$ are 0.5% and 0.9%, respectively. Systematic uncertainties due to the statistical uncertainties on $\sigma(K^-\mathrm{p})$ and $\sigma(K^+\mathrm{p})$ are estimated from Monte Carlo, and found to be negligible. Other sources include uncertainties on Δm, and lifetimes of K_L^0 and K_S^0, and are also negligible. There is a contribution from doubly Cabibbo-suppressed decays of charmed mesons that is neglected in the computation of $A_{\mathcal{D}}$. According to Ref. [26], we assign a 10% systematic uncertainty to the final states with a contribution from doubly Cabibbo-suppressed decays.

In the study of the same decay channels of charmed mesons for the center-of-mass energy in the region of $\psi(3770)$, we introduce no Lorentz boost for the detector geometry described by Case I. This checks the effect of different kinematics of K_S^0 by comparing the results with those for $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$. The bottom two plots in Fig. 2 show the momentum and polar angle distributions of K_S^0 mesons for $\sqrt{s} = 3770$ MeV with no Lorentz boost, showing lower K_S^0 momentum distributions relative to those from the configuration with $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$. Larger $A_{\mathcal{D}}$ values are con-

TABLE II. Numerical estimation of $A_{\mathcal{D}}$ for three configurations. The values in parentheses are only for the beam pipe element.

Configurations / Decay Modes	Case I, $\sqrt{s} = 10.58$ GeV, $\beta\gamma = 0.425$ $A_{\mathcal{D}}(\times 10^{-4})$	Case I, $\sqrt{s} = 3770$ MeV $A_{\mathcal{D}}(\times 10^{-4})$	Case II, $\sqrt{s} = 10.58$ GeV, $\beta\gamma = 0.425$ $A_{\mathcal{D}}(\times 10^{-4})$
$D^+ \to K_S^0\pi^+$	10.8 (9.0)	15.9 (12.0)	8.8 (8.5)
$D^0 \to K_S^0\pi^+\pi^-$	12.9 (11.0)	17.4 (14.7)	10.5 (10.4)
$D^0 \to K_S^0K^+K^-$	15.1 (12.8)	30.6 (27.0)	12.0 (11.8)
$B^+ \to K_S^0\pi^+$	6.3 (4.5)	...	5.2 (4.3)
$B^0 \to K_S^0\pi^+\pi^-$	9.1 (7.1)	...	7.5 (6.7)
$B^0 \to K_S^0K^+K^-$	9.5 (7.4)	...	7.8 (7.0)

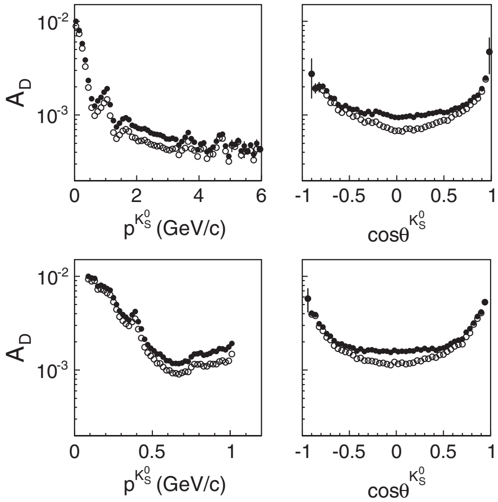

FIG. 3. Distributions of $A_{\mathcal{D}}$ as a function of K_S^0 momentum (left) and the polar angle (right) for $D^+ \to K_S^0\pi^+$ for $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$ (top) and for $D^0 \to K_S^0\pi^+\pi^-$ for $\sqrt{s} = 3770$ GeV configuration (bottom). Case I detector geometry is used in both instances.

sequently expected, which is consistent with the calculations shown in the third column of Table II. The bottom of Fig. 3 shows the distributions of $A_{\mathcal{D}}$ as a function of momentum and polar angle of K_S^0 for $D^0 \to K_S^0\pi^+\pi^-$ at $\sqrt{s} = 3770$ MeV and $\beta\gamma = 0$. We find that the $A_{\mathcal{D}}$ values are in general larger than given in the second column of Table II. Again, this reflects the K_S^0 momentum distribution shown in the bottom plot in Fig. 2, which peaks in the phase space region with the largest $\Delta\sigma(\bar{K}^0N)$. Here, the systematic uncertainty from the assumption that $\Re\mathfrak{e}(\Delta f)/\Im\mathfrak{m}(\Delta f) = 1$ is found to be 30%, and other sources are negligible.

As a final benchmark, we also evaluate $A_{\mathcal{D}}$ for the Case II configuration with $\sqrt{s} = 10.58$ GeV and $\beta\gamma = 0.425$. The results of estimating of $A_{\mathcal{D}}$ are listed in the last column of Table II. This configuration checks the effect of different geometry for detector material by comparing results from Case I with the same kinematics. We find that the contribution of the first two thin layers of Si sensors is negligible. Furthermore, the contribution of the outer Si sensors is also smaller than that of the Si sensors in Case I as their distances from the production point of neutral kaons are longer. This results in smaller dilution than for Case I. Systematic sources and effects are similar to those of due to Case I.

As shown above, the dilution effect in the calculation of $A_{\mathcal{D}}$ is most sensitive to the momentum of K_S^0, and mainly due to the beam pipe contribution. Hence, the dilution effect in very high energy experiments in the LHC environment can be smaller than the impact in experiments considered in this paper.

In summary, we estimate the dilution effect in the measurement of CP asymmetry caused by the difference in nuclear interactions of K^0 and $\bar{K}^0$ in e^+e^- collisions for several typical experimental configurations. We find that the effect can be as large as 0.3% in decays involving low-momentum neutral kaons. The estimated systematic uncertainties on the calculated $A_{\mathcal{D}}$ range in $(15 \approx 30)\%$ depending on K_S^0 momentum. We suggest that forthcoming high-sensitivity measurements of CP asymmetry involving neutral kaons in the final state should take into account the impact on the difference in K^0 and $\bar{K}^0$ strong interactions ($A_{\mathcal{D}}$).

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