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# Duality-invariant Einstein-Planck relation and the speed of light at very short wavelengths

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We propose a generalized Einstein-Planck relation for photons which is invariant under the change $\lambda/al_P$ to $al_P/\lambda$, $\lambda$ being the photon wavelength, $l_P$ Planck's length, and $a$ a numerical constant. This yields a wavelength-dependent speed of light $v(\lambda) = c/(1 + a^2(l_P/\lambda)^2)$, with $c$ the usual speed of light *in vacuo*, indicating that the speed of light should decrease for sufficiently short wavelengths. We discuss the conceptual differences with the previous proposals related to a possible decrease of the speed of light for very short wavelengths based on quantum fluctuations of the space-time, as well as its consequences on Heisenberg's uncertainty principle up to second order in $l_P$.

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The formulation of quantum mechanics at small spatial scales comparable to the Planck scale is a topic of much interest in quantum gravity [1,2]. In this letter we point out an heuristic generalization of the Einstein-Planck relation based on the idea of duality invariance with respect to the change from $\lambda/al_P$ to $al_P/\lambda$, $\lambda$ being the photon wavelength, $l_P$ the Planck length, and $a$ a numerical constant. This idea is analogous to the duality invariance relations proposed in superstring theories concerning the compact dimensions [3,4] but it refers to the actual spatial dimensions. Though up to now there are not compelling reasons to accept such an equation, we think that its physical consequences and its symmetry make it worthy of attention, because of their relation with the speed of light at short wavelengths, and with Heisenberg's uncertainty principle; we will also comment on the differences with other proposals.

The main motivation of our analysis is the current interest in the speed of light *in vacuo* at very short wavelengths. Indeed, it is suspected that in these conditions the speed of light should be smaller than the usual speed of light $c$ for common wavelengths [5–13]. Experiments on extremely energetic photons coming from astrophysical sources are being explored through the observations of gamma ray bursts and active galactic nuclei with gamma-ray telescopes, such as MAGIC or Fermi satellite [5–7]. Such a hypothetical slowing of light is attributed, in a common interpretation, to the quantum fluctuations of space at very small lengths [8–13]. The short-wavelength photons would be able to probe such small distances, and, in so doing, they would not follow a straight path, but a slightly distorted one, implying a longer path length than the straight one and, therefore, their delay with respect to longer wavelength photons emitted in a same burst would be interpreted as an apparent slower speed of propagation. Measurements and interpretations of this idea are worth of consideration, because they provide a possible way to measure some physical effects at quantum-gravity scale.

In the mentioned interpretation, the actual speed of light is always $c$, and the apparently lower speed of ultraenergetic photons would be due, in fact, to the lengthening of their path. In principle, it is not necessary that the small obstacles are random quantum fluctuations; they could also be possible small-scale defects of space, as for instance, some small cosmic string loops or walls [14,15]. Yet another possibility would be that the observed bursts emitted first the longer wavelength photons and afterwards the short-wavelength ones, by some unknown mechanism, but both sets of photons travelling with the same speed and along the same path; in this case, the observed delay would not be due to path quantum fluctuations, but to a source origin.

The possibility we consider here is yet another one, different from the previous ones, namely, that the speed of light is intrinsically lower at very short wavelengths. In this line of thought—which does not dismiss the other possibilities mentioned above, each of which could have its own independent contribution—we consider the possibility of a duality-invariant relation between small and big spatial scales, manifested on a generalized form of the Einstein-Planck relation and on the dispersion relation of electromagnetic waves.

Our duality-invariant proposal for the Einstein-Planck equation consists in writing the energy of photons in terms of the wavelength as

$$E = \frac{hc}{al_P}\left(\frac{\lambda}{al_P} + \frac{al_P}{\lambda}\right)^{-1}. \qquad (1)$$

For $\lambda$ much longer than $al_P$, it reduces to the Einstein-Planck equation written in terms of $\lambda$, namely $E = hc/\lambda$; whereas it is invariant with respect to the change $\lambda/al_P$ to $al_P/\lambda$ inside the parentheses. For $\lambda \ll al_P$ it yields $E = (c^4/a^2G)\lambda$, which is analogous to the energy of cosmic string loops of length $\lambda$. In [16] we have explored a thermodynamic duality between a gas of photons and

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a gas of cosmic string loops related, respectively, to these two extreme behaviours. Intuitively, the crossover from usual photon relation to a gravitation dominated behavior may be imagined through the fact that when the wave length of a photon becomes smaller than the Schwarzschild radius corresponding to its energy $E$, namely $R = 2GE/c^4$, gravitational effects should dominate. Thus for $\lambda$ shorter than $l_c^2 \approx 2Gh/c^3$, one could expect that the photon could modify the geometry of space by producing either a black hole or a cosmic string loop of radius $l = 2GE/c^4$.

Though we have modified the Einstein-Planck relation in terms of the wavelength, when expressed in terms of frequency we keep its usual form, which we consider fundamental, namely

$$E = hf. \tag{2}$$

Comparison of (1) and (2) yields for the dispersion relation of photons the duality-invariant form

$$f = \frac{c}{al_P}\frac{1}{\frac{\lambda}{al_P} + \frac{al_P}{\lambda}}, \tag{3}$$

yielding for the phase speed of light $V(\lambda)$ the relation

$$V(\lambda) = f\lambda = \frac{c}{1 + (al_p/\lambda)^2}. \tag{4}$$

Thus, according to this relation, the speed of light *in vacuo* would depend on the wavelength, and it would be smaller than $c$ for wavelengths of the order of $al_P$ or less. For the sake of comparison with other approaches and with observations, it is convenient to express the velocity in terms of the photon energy; up to second order in $E/E_P$, $E$ being the photon energy and $E_P$ the Planck energy, expression (4) yields

$$V(E) = c\left[1 - a^2\left(\frac{E}{E_P}\right)^2\right]. \tag{5}$$

Now, we compare with other formalisms describing a wavelength dependence of the speed of light. The photon dispersion relation is written as [5]

$$c^2p^2 = E^2\left[1 + \eta\frac{E}{E_P} + \xi\left(\frac{E}{E_P}\right)^2 + \ldots\right], \tag{6}$$

with $\eta$ and $\xi$ being constant numerical parameters. One often takes the first-order or the second-order approximations, depending on the model of space-time quantum fluctuations being used. Since recent observational analyses [5–7] seem to exclude first-order effects, or at least to put severe constraints on their value, we take only the second-order expression for the energy-dependent speed of light following from (6), namely

$$V = c\left[1 - \xi\left(\frac{E}{E_P}\right)^2\right]. \tag{7}$$

Then, the parameter $\xi$ in (7) can be identified (up to the second order) with our parameter $a^2$ in (5). Current observational results based on the observations of the energy peaks of astronomical flares from gamma ray bursts and from active galactic nuclei [5–7] indicate that $\xi = a^2 < 3.6 \cdot 10^{16}$ [5]. This indicates that the modifications to the speed of light could begin at length scales some 6 or 7 orders of magnitude higher than the Planck length scale ($l_P = 1,62 \cdot 10^{-35}$ m) or at energy scales some 6 or 7 orders of magnitude lower than the Planck energy ($E_P = 1.22 \cdot 10^{19}$ GeV).

The two proposals for the reduction of the speed of light, namely, a foamy space-time microstructure related to quantum fluctuations, and a dual-invariant generalization of Einstein-Planck relation, do not exclude each other, but they are conceptually different. For instance, in (6) the photon momentum is considered to be given by the usual form $p = h/\lambda$, whereas in the duality-invariant proposal (1) it is assumed to be

$$p = \frac{h}{al_P}\frac{1}{\frac{\lambda}{al_P} + \frac{al_P}{\lambda}}, \tag{8}$$

in such a way that the relation $cp = E$ for photons is assumed to retain its validity at any wavelength, in contrast to (6).

Eqs. (1)–(8) could be in principle checked through particle-antiparticle production and in Compton scattering of electrons with very short-wavelength photons. According to (1), the maximum energy of photons would be $E_{\max} = hc/(2al_P)$, with $a$ less than $10^8$; therefore, this maximum should settle some limits on the mentioned processes. Here, we are proposing (8) for photons; it would be interesting to explore whether it could also be applied to massive particles, as a generalization of the de Broglie relation. If this was so, and applying the energy-momentum relation $E = (m_0^2c^4 + p^2c^2)^{1/2}$, it would also follow a maximum value for the energy of particles, for a wavelength of the order of $al_P$.

Interpretation (8) of $p$, combined with the use of naive diffraction theory in a narrow slit, leads to a modification of Heisenberg's uncertainty expression, as given by

$$\Delta p\Delta x \geq \frac{h}{1 + \frac{a^2l_P^2}{\lambda^2}} \approx h\left(1 - \frac{a^2l_P^2}{h^2}p^2\right), \tag{9}$$

with $\Delta x$ and $\Delta p$ the uncertainties in position and momentum. This suggests that for sufficiently short wavelengths the uncertainty could be reduced below the usual Heisenberg's value. Intuitively, this would follow from the idea that according to (1) the energy of very short-wavelength photons would be smaller than that considered in the usual theory, and the perturbation on the particle being observed would be less than in the usual theory.

In contrast, other modified uncertainty relations have been inspired in the possible existence of a minimal length (not necessarily related to quantum space-time fluctuations). For instance, generalized uncertainty relations based on small-angle graviton scattering at Planckian energy were proposed in Refs. [17–20], leading to the idea that string theory implies that distances smaller than the Planck length cannot be probed, and that below the Planck scale the concept of space-time may lose meaning. Other analogous proposals have arisen in the framework of quantum fluctuating space-time, or in string theories in Refs. [21–28]. The generalized form of the uncertainty relations in these contexts has usually the form

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha \frac{l_P^2}{\hbar} \Delta p, \tag{10}$$

with $\alpha$ a constant of the order 1. If $l_P = 0$ (10) reduces to Heisenberg's uncertainty principle.

In this expression, the uncertainty product $\Delta p \Delta x$ for $l_P \neq 0$ would be higher than the classical one, instead of being smaller. Result (10) is also logical in its theoretical framework, because it is expected that the uncertainty of space-time fluctuations will add to the usual quantum uncertainty in a fixed space-time.

Note that (10) implies for $\Delta x$ a minimum value $(\Delta x)_{\min} = 2\alpha^{1/2} l_P$. The intuitive argument for this minimum length under which the conventional notion of distance breaks down is that the resolution of small distances requires particles with very short wavelengths, which in the usual theory have very high energy and consequently could disturb the space-time structure being tried. Then, $(\Delta x)_{\min} > 0$ may be viewed as a quantum fuzziness of space, or, alternatively, as a consequence of the nonpointlike character of the fundamental particles in string theory.

In contrast, in our proposal (8), the situation would be much different, since the length $l$ could be arbitrarily short, since the photons or particles of wavelength shorter than $al_P$ would have lower and lower energy for shorter and shorter wavelengths. In this framework, one could have ultraviolet regularization, but not due to a minimal length, but to the change in dispersion relation implying small energies for very short wavelengths.

Note, incidentally, that what we have said would remain valid if instead of (1) one considers

$$E = \frac{hc}{al_P}\left[\left(\frac{\lambda}{al_P}\right)^n + \left(\frac{al_P}{\lambda}\right)^n\right]^{1/n} \tag{11}$$

with any arbitrary $n$ different from zero. Thus the essential point is the duality invariance rather than the specific form of (1).

Though the present proposal is certainly speculative, we think it worth of consideration until sufficient reasons to discard it arise. In the meantime, it may suggest some experiments that without it would not seem related to the wavelength-dependent speed of light. For instance, the mentioned Compton scattering experiments, on which the relation (8) between momentum and wavelength would have consequences, and the possible existence of a maximum photon energy less than Planck's energy. These two possible effects are usually not related to the wavelength dependence of the speed of light in the approach based on the foamy microstructure of space-time, but they could be naturally related to such dependence in the model we have proposed here.

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