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Dark-matter admixed neutron stars

S.-C. Leung, M.-C. Chu, and L.-M. Lin

Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Hong Kong, China
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We study the hydrostatic equilibrium configuration of an admixture of degenerate dark matter and normal nuclear matter by using a general relativistic two-fluid formalism. We consider non-self-annihilating dark matter particles of mass $\sim$1 GeV. The mass-radius relations and moments of inertia of these dark-matter admixed neutron stars are investigated and the stability of these stars is demonstrated by performing a radial perturbation analysis. We find a new class of compact stars which consists of a small normal matter core with radius of a few kilometers embedded in a ten-kilometer-sized dark matter halo. These stellar objects may be observed as extraordinarily small neutron stars that are incompatible with realistic nuclear matter models.

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I. INTRODUCTION

By now, the existence of dark matter (DM) has been well established, with a large amount of evidence such as galactic rotation curves, cosmological structure, and gravitational lensing. However, the properties of DM, including their mass and interactions, are still unknown. It is thus of great interest to constrain the properties of DM through direct or indirect methods.

Direct methods search for the signals of DM-nucleus scatterings in Earth-based detectors. The latest experimental results are not conclusive. The data from the DAMA [1] and CoGeNT [2] experiments are consistent with detecting light DM particles with mass $\sim$10 GeV, which are incompatible with the null results from CDMS [3] and XENON [4]. Nevertheless, it has recently been suggested that isospin-violating DM may be the key to reconciling the experimental results [5,6]. On the other hand, indirect methods are based on the effects of DM on the properties of stellar objects such as the Sun. For example, the effects of low-mass ($\sim$ 5 GeV) asymmetric DM particles on the solar composition, oscillations, and neutrino fluxes have been considered recently [7–9].

One indirect method that is gaining attention in recent years is to study the effects of DM on compact stars. The effects due to different DM models have been considered. For example, self-annihilating DM inside compact stars can heat the stars, and hence affect the cooling properties of compact stars [10,11]. On the other hand, non-self-annihilating DM, such as asymmetric DM [12] and mirror matter [13], would simply accumulate inside the stars and affect the stellar structure. Constraints have been set by connecting the observed properties of compact stars with DM parameters.

It should be noted that neutron stars with a DM core are inherently two-fluid systems where the normal matter (NM) and DM couple essentially only through gravity. The technique used in recent studies of the structure of these dark-matter admixed neutron stars (DANS) is based on an *ad hoc* separation of the Tolman-Oppenheimer-Volkoff (TOV) equation into two different sets for the normal and dark components inside the star [13,14]. This approach is motivated by the similarity of the structure equations between the relativistic and Newtonian ones, but it is not derived from first principle. In fact, a general relativistic two-fluid formalism is available [15] and has been employed in the study of superfluid neutron stars (e.g., [16,17]), where the two fluids are normal and superfluid nuclear matter. This approach is not only more appealing from a theoretical point of view, but it also provides the ability to extend easily the study of dynamical properties of these stars in a self-consistent general relativistic framework. Here, we study the structure and stability of DANS in general. Besides the scenario where a DM core exists inside a neutron star, we also study the scenario where NM is in the core of a DM dominated compact star. It should be pointed out that the main focus of this paper is to study the equilibrium properties and observational signatures of these theoretical objects (see also[18] for a study of compact stars made of fermionic DM). The formation process of these objects requires further investigation.

II. FORMULATION

To study a two-fluid compact star, we adopt the formulation given in [16], which was initially constructed to study general relativistic superfluid neutron stars. Here we shall briefly summarize the formalism and refer the reader to [16] for more details. The central quantity of the two-fluid formalism is the master function $\Lambda(n^2, p^2, x^2)$, which is formed by three scalars, $n^2 = -n_\alpha n^\alpha$, $p^2 = -p_\alpha p^\alpha$, and $x^2 = -n_\alpha p^\alpha$. The four vectors n^α and p^α are the conserved NM and DM number density currents, respectively. The master function is a two-fluid analog of the equation of state (EOS) and $-\Lambda$ is taken to be the thermodynamic energy density.

For a static and spherically symmetric spacetime $ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, the structure equations for a two-fluid compact star are given by [16]

$$A_0^0 p' + B_0^0 n' + \frac{1}{2}(Bn + Ap)\nu' = 0,$$
$$C_0^0 p' + A_0^0 n' + \frac{1}{2}(An + Cp)\nu' = 0,$$
$$\lambda' = \frac{1 - e^{\lambda}}{r} - 8\pi r e^{\lambda}\Lambda,$$
$$\nu' = -\frac{1 - e^{\lambda}}{r} + 8\pi r e^{\lambda}\Psi, \tag{1}$$

where the primes indicate derivative with respect to r, and the coefficients A, B, C, A_0^0, B_0^0, and C_0^0 are functions of the master function. Their expressions are given by Eqs. (3) and (25) in [16]. The generalized pressure Ψ is computed by Eq. (18) in [16].

The EOS information $P = P(\rho)$ (with P and ρ being the pressure and energy density, respectively) needed in the standard relativistic-star calculation based on the TOV equation is now replaced by the master function $\Lambda(n^2, p^2, x^2)$. We assume no interaction between NM and DM except for gravitation. The master function does not depend on the scalar $x^2 = -n_\alpha p^\alpha$ and is separable in the sense that $\Lambda(n^2, p^2) = \Lambda_{\rm NM}(n^2) + \Lambda_{\rm DM}(p^2)$, $\Lambda_{\rm NM}(n^2)$ and $\Lambda_{\rm DM}(p^2)$ being the negative of energy densities of NM and DM at a given number density, respectively. We use the APR EOS [19] for NM and assume that the DM component of the star is formed by non-self-annihilating DM governed by an ideal Fermi gas. As discussed earlier, DM candidates in the mass range of a few GeV are of great interest recently. We shall thus consider fermionic DM particles of mass $m_X \sim 1$ GeV in this work.

III. RESULTS

In Fig. 1, we show the mass-radius relations of DANS for different amount of DM specified by the parameter $\epsilon = M_{\rm DM}/(M_{\rm NM} + M_{\rm DM})$. Here, $M_{\rm NM}$ and $M_{\rm DM}$ are calculated by the product of the particle mass and total number of particles for NM and DM in the star. They may be referred to as the baryonic masses for NM and DM (though it should be noted that DM is nonbaryonic). In the figure, M is the gravitational mass and R is the radius of the star. The DM particle mass $m_X = 1$ GeV is fixed. The case $\epsilon = 0$ corresponds to ordinary neutron star models constructed using the APR EOS without DM. We see that the existence of a DM core would lower the maximum stable mass allowed by this EOS. The DANS would also have smaller radii. For the maximum stable mass configuration, M and R are decreased by about 35% and 9%, respectively, as ϵ changes from 0 to 0.2. For the case $\epsilon = 0.8$, the stars are DM dominated compact stars, with NM concentrated in the core. It is seen clearly that the mass-radius relation of

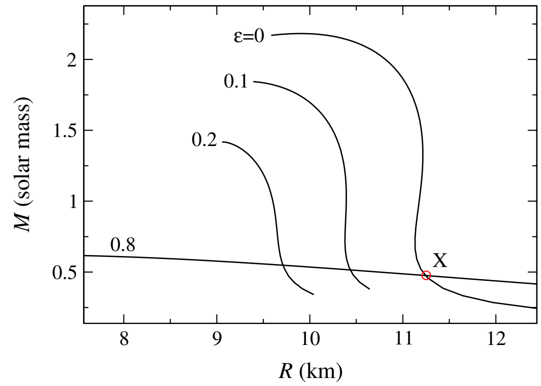

FIG. 1 (color online). Mass-radius relations for different amount of DM specified by the parameter ϵ (see text). The DM particle mass is $m_X = 1$ GeV. The density profiles for the ordinary neutron star ($\epsilon = 0$) and DM dominated star ($\epsilon = 0.8$) at the point X are shown in Fig. 2.

these stars is different from that of NM dominated stars qualitatively.

In Fig. 1, the circle marked by X is the intersection point between the curves $\epsilon = 0$ and 0.8. While the ordinary neutron star ($\epsilon = 0$) and dark-matter dominated star ($\epsilon = 0.8$) at X have the same M and R, their internal structures in fact differ significantly. Figure 2 shows the density profiles of these two stellar configurations, the upper (lower) panel corresponding to the ordinary neutron star (DANS) model. For the case $\epsilon = 0.8$ (lower panel), it is seen that a small NM core is embedded in a ten-kilometer sized DM halo. Since M and R of the two stars are the same, it would seem impossible to distinguish them based on their gravitational effects on other nearby stellar objects. However, the visible radius of the DM dominated star (defined by the radius of the NM core) is $0.56R$ and the total mass enclosed in the NM core is $0.72M$. The two stars can be distinguished by measuring the gravitational redshift of spectral lines, since

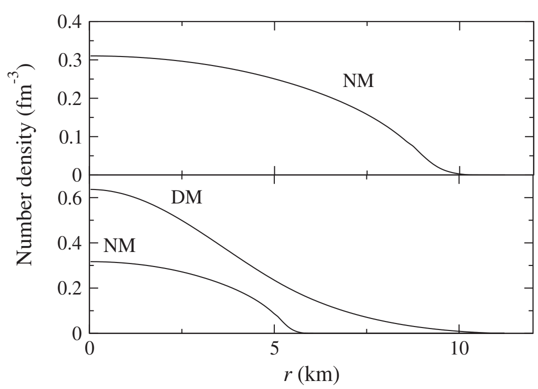

FIG. 2. Upper panel: Density profile for the ordinary neutron star ($\epsilon = 0$) at the point X in Fig. 1. Lower panel: NM and DM density profiles for the DM dominated star ($\epsilon = 0.8$) at the same point.

that produced near the surface of the NM core will be about 30% larger than that of ordinary neutron stars.

In order to check the stability of these stars, we have solved the set of equations for radial perturbations of a two-fluid compact stars developed in [16]. Similar to the one-fluid case, the problem is to solve for the eigenvalues ω^2, where ω is the oscillation frequency of the star. We will present the details of our calculations and analysis elsewhere. Here we show our main results in Fig. 3, where M is plotted against the central energy density in the upper panel for the DM dominated sequence ($\epsilon = 0.8$) in Fig. 1. The lower panel plots the squared frequency of the fundamental mode ω_0^2 for the same sequence. Similar to the one-fluid study for the stability of ordinary neutron stars, the point $\omega_0^2 = 0$ marks the onset of instability. Figure 3 shows that ω_0^2 passes through zero at the central density corresponding to the maximum mass configuration. Beyond this critical central density, the stars are unstable against radial perturbations. For lower central densities, such as the DM dominated star at the point X in Fig. 1, the stellar configurations are all stable.

Besides the gravitational mass and radius, it is also interesting to consider the moment of inertia I of DANS since it is measurable and plays an important role in the physics of neutron stars. Bejger and Haensel [20] discovered an (approximately) EOS-independent formula relating I, M and R. For ordinary neutron stars, they found that

$$\tilde{I} = \begin{cases} z/(0.1 + 2z) & \text{if } z \le 0.1, \\ 2(1 + 5z)/9 & \text{if } z > 0.1, \end{cases} \tag{2}$$

where the scaled moment of inertia $\tilde{I} = I/MR^2$ and $z = (M/M_\odot)(\text{km}/R)$. This universal formula was obtained by fitting a large set of realistic EOS models for nuclear matter, including the APR EOS used in this work. The moment of inertia of a rotating star in general relativity is commonly defined by $I = J/\Omega$, where J and Ω are the angular momentum and angular velocity, respectively. In the slow rotation limit, J scales with Ω linearly and I is determined only by the nonrotating background quantities [21].

In the two-fluid case, individual NM and DM angular momenta ($J_{\rm NM}$ and $J_{\rm DM}$) can be defined [17]. In the slow rotation limit, the moments of inertia can also be defined by $I_{\rm NM} = J_{\rm NM}/\Omega_{\rm NM}$ and $I_{\rm DM} = J_{\rm DM}/\Omega_{\rm DM}$, where $\Omega_{\rm NM}$ and $\Omega_{\rm DM}$ are, respectively, the angular velocity of NM and DM. $I_{\rm NM}$ and $I_{\rm DM}$ depend on the nonrotating background quantities of NM and DM separately. However, their definitions are meaningful only when the two fluids are noninteracting (as we assume in this work). In the general situation, the angular momentum of each fluid would contain a contribution which depends on the coefficient A in Eq. (1) and the relative velocity $\Omega_{\rm NM} - \Omega_{\rm DM}$ [17]. The coefficient A vanishes only when the master function Λ is independent of the scalar product $x^2 = -n_\alpha p^\alpha$.

With the moments of inertia of NM and DM defined individually as above, we can further define the total moment of inertia of DANS by $I = I_{\rm NM} + I_{\rm DM}$. In Fig. 4, we plot the scaled moment of inertia $\tilde{I}$ against compactness M/R for the DANS sequences shown in Fig. 1. As in Fig. 1, the solid lines are sequences for different amount of DM specified by ϵ. The case $\epsilon = 0$ corresponds to ordinary neutron stars. The dashed line corresponds to Eq. (2). The three vertical lines (with arrows) at $M/R = 0.05$, 0.1 and 0.15 represent the range of values of $\tilde{I}$ obtained by the large set of EOS models which were used to obtain Eq. (2). They can be regarded as the error bars of Eq. (2) at those values of M/R. The circles in the figure correspond to the ordinary neutron star ($\epsilon = 0$) and DM dominated star ($\epsilon = 0.8$) at the point X in Fig. 1. While the scaled moment of inertia of ordinary neutron stars can be modeled approximately by Eq. (2), Fig. 4 shows that $\tilde{I}$ of DANS depends sensitively on the amount of DM. In particular, for the DM dominated sequence $\epsilon = 0.8$, the value of $\tilde{I}$ is significantly smaller than that allowed for ordinary neutron stars with the same compactness. This

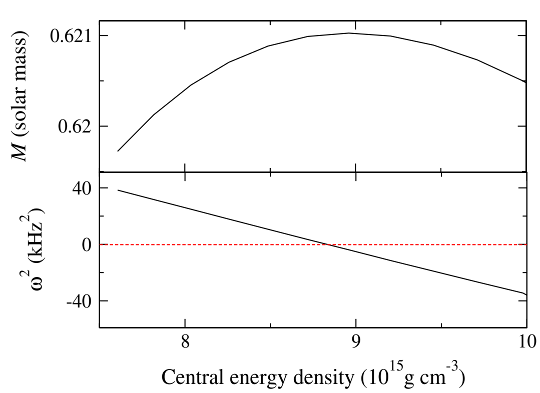

FIG. 3 (color online). Gravitational mass M (upper panel) and squared frequency of the fundamental mode ω_0^2 (lower panel) are plotted against the central energy density for the DM dominated sequence ($\epsilon = 0.8$) in Fig. 1.

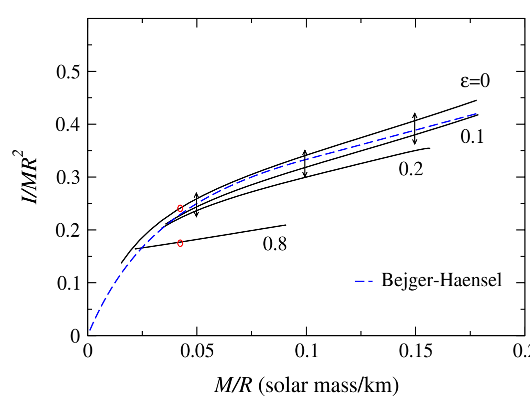

FIG. 4 (color online). Scaled moment of inertia I/MR^2 is plotted against compactness M/R for the DANS sequences shown in Fig. 1. Each sequence is labeled by the parameter ϵ. The dashed line represents Eq. (2). The circles correspond to the two stellar models ($\epsilon = 0$ and 0.8) at the point X in Fig. 1.

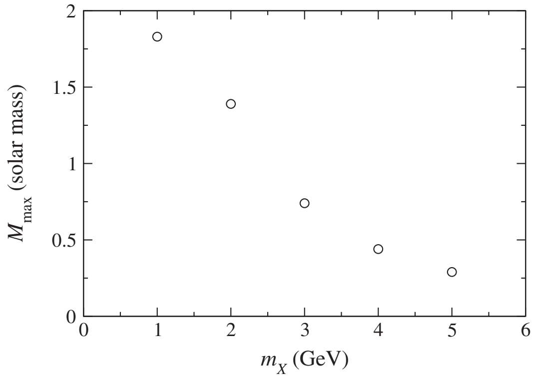

FIG. 5. Maximum stable mass $M_{\max}$ is plotted against the DM particle mass m_X for a fixed amount of DM specified by $\epsilon = 0.1$.

might lead to observational signatures of DM dominated compact stars. Further work is required to investigate the observational implications of DANS in details.

So far we have focused our study on DM particle mass $m_X = 1$ GeV. In view of the recent interest in DM candidates in the mass range of a few GeV, it is interesting to see how different DM particle masses in this range affect our results. In Fig. 5, we plot the maximum stable mass $M_{\max}$ along a sequence of compact stars with $\epsilon = 0.1$ as a function of m_X. Note that the sequence for $m_X = 1$ GeV is shown in Fig. 1. For a given proportion of DM inside the stars, Fig. 5 shows that a higher DM particle mass in general leads to a smaller maximum stable mass. This can easily be understood by noting that, since the NM and DM are assumed to be noninteracting (except through gravity), the DM core is supported only by its own degenerate pressure. It is well known that the maximum mass limit for a self-gravitating Fermi gas decreases as the particle mass increases. Hence, the onset of the collapse of a degenerate DM core is responsible for the dependence of $M_{\max}$ on m_X as seen in Fig. 5. It should also be noted that, while the pressure of NM within the DM core does not contribute to supporting the weight of the DM core, the mass of the NM fluid does enhance the collapse of the DM core.

IV. DISCUSSION AND CONCLUSIONS

In summary, we have studied the effects of a degenerate DM core formed by non-self-annihilating DM particles of mass $\sim$1 GeV upon the structure of neutron stars. The structure of these DANS depends strongly on the size of the DM core. In particular, we found a new class of compact stars which are DM dominated—a NM core embedded in a ten-kilometer sized DM halo. The stability of these stars has been checked by performing a radial perturbation analysis. These DM-dominated stars have rather different mass-radius relations and (scaled) moments of inertia compared to ordinary neutron stars. A distinctive property of these stars is their small NM core radius of about a few km, from which thermal radiation could be observed. The detection of a compact star with a thermally radiating surface of such a small size could provide a strong evidence for their existence.

Could DANS be formed in the first place? To answer this question, one needs to consider the effects of DM on the stellar formation process. In fact, the heating effects due to DM annihilation on stellar formation have been studied in recent years [22]. The result is the prediction of a new phase of stellar evolution during which a protostar is supported by DM heating. What if one replaces the annihilating DM model in [22] by non-self-annihilating DM? How would such a DM core affect the stellar evolution? Would the DM core survive the supernova explosion of massive stars and form DANS as studied in this paper? These are challenging questions that deserve further investigation.

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