M theory on deformed superspace

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In this paper we will analyze a noncommutative deformation of the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory in $N = 1$ superspace formalism. We will then analyze the Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetries for this deformed ABJM theory, and its linear as well as nonlinear gauges. We will show that the sum of the gauge fixing term and the ghost term for this deformed ABJM theory can be expressed as a combination of the total BRST and the total anti-BRST variation, in Landau and nonlinear gauges. We will show that in Landau and Curci-Ferrari gauges deformed ABJM theory is invariant under an additional set of symmetry transformations. We will also discuss the effect that the addition of a bare mass term has on this theory.

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I. INTRODUCTION

The construction of an action for M theory at low energies with manifest $N = 8$ superconformal symmetry has led to the discovery of the Bagger and Lambert action with a Lie 3-algebra [1–5]. However, only one example of such a 3-algebra is known and so far the rank of the gauge group has not been increased. But a $U(N)_k \times U(N)_{-k}$ superconformal Chern-Simons-matter theory with level k and $-k$ with arbitrary rank and $N = 6$ supersymmetry has been constructed [6]. This theory called the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory is thought to describe the world volume of N $M2$ membranes placed at the singularity of R^8/Z_k as it may be possible to enhance the supersymmetry to $N = 8$ supersymmetry [7]. Furthermore, if this is done then a $SO(8)$ R symmetry at Chern-Simons levels $k = 1, 2$ will also exist for this model.

Chern-Simons theory in $N = 1$ superspace formalism has also been used in analyzing the low energy approximation of the action for M theory with $N = 8$ supersymmetry [8]. This was done by constructing a manifestly $SO(7)$ invariant superpotential which for specially chosen couplings reproduced the Bagger and Lambert action [2,3]. Hence for these values of the coupling constants full $SO(8)$ symmetry was restored. Chern-Simons theory in $N = 1$ superspace formalism has also been used for studying the ABJM theory [9]. By using the Higgs mechanism, higher-order terms that occur in the low energy approximation of the action for M theory have been analyzed in this $N = 1$ superspace formalism.

The presence of an NS antisymmetric tensor background is a source of spacetime noncommutativity in string theory [10,11]. Now as string theory introduces noncommutativity in spacetime, so field theories with spacetime noncommutativity have been thoroughly studied [12–17]. The extension of spacetime noncommutativity to superspace noncommutativity is related to the presence of other background fields. The RR field strength backgrounds give rise to θ-θ type deformations [18,19] and a gravitino background gives rise to x-θ deformation [20]. As superspace noncommutativity also arises in string theory, field theories with superspace noncommutativity have also been thoroughly studied [19–25]. However, the presence of θ-θ deformation breaks half of the supersymmetry. As we want to retain all the supersymmetry in our theory, we do not include the θ-θ deformation of the ABJM theory in this paper. It may be noted that even though this is the first work on noncommutative deformation of the ABJM theory, we analyze both the x-x and x-θ deformations at the same time. This is because we use a similar formalism to analyze both these deformations.

Because of the duality between M theory and IIA string theory, we expect that a deformation of the superalgebra on the string theory side will also correspond to some deformation of the superalgebra on the M-theory side. It is interesting to note that a three-form field strength occurs naturally in M theory. Besides that, $M2$-branes in M theory can end on $M5$-branes. In this sense $M5$-branes in M theory act as analogous objects to a D-brane in string theory. So we expect that coupling the ABJM theory to a background three-form field could lead to a noncommutative deformation of its superalgebra, just like a background two-form field strength leads to a noncommutative deformation of the superalgebra of D-branes. This can be useful in describing the physics of $M2$-branes ending on $M5$-branes. It may be noted that action for a single $M5$-brane can be derived by demanding the κ symmetry of the open membrane ending on it [26] Thus the analysis of ABJM theory coupled to a background three-form field strength might give some useful insights into understanding the dynamics of multiple $M5$-branes. This will be interesting because even though the action for a single $M5$-brane is known, the action for multiple $M5$-branes is not known [27–31].

We will thus analyze the noncommutative deformations of the ABJM theory that are expected to occur due to the

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coupling of the ABJM theory with the background three-form field strength that occurs naturally in the M theory. As the ABJM theory is composed of two Chern-Simons theories suitably coupled to matter fields, a Seiberg-Witten map will hold for the noncommutative ABJM theory because it is known to hold for noncommutative Chern-Simons theories [32,33]. We thus analyze this noncommutative ABJM theory by relating the noncommutative fields in it to ordinary commutative fields. The product of these noncommutative fields will then induce a star product for the ordinary commutative fields.

The Becchi-Rouet-Stora-Tyutin (BRST) and the anti-BRST symmetries for gauge theories have been thoroughly studied [34]. In fact it is known that for the Yang-Mills theories in Landau and nonlinear gauges the algebra generated by the BRST and the anti-BRST transformations along with FP conjugation is a subalgebra of a larger algebra called Nakanishi-Ojima algebra [35–38]. The effect of the addition of a bare mass term on the BRST and the anti-BRST symmetries has also been analyzed in the nonlinear gauges [39]. The BRST symmetry for the Chern-Simons theory has also been thoroughly investigated [40,41]. The BRST symmetry of $N = 1$ Abelian Chern-Simons theory [42] and $N = 1$ non-Abelian Chern-Simons theory [43] has been analyzed in the superspace formalism. The BRST symmetry of noncommutative pure Chern-Simons theory has also been analyzed [44,45]. We will analyze the BRST and the anti-BRST symmetries of this deformed ABJM theory. The main focus of this paper will be the generalization of some known results about the BRST and the anti-BRST symmetries in Yang-Mills theories to this deformed ABJM theory. In particular, we will show that, in certain gauges, the sum of this deformed ABJM theory along with a gauge fixing term and a ghost term is invariant under a set of symmetry transformations which obey $SL(2, R)$ algebra. This is similar to the invariance of Yang-Mills theories under the Nakanishi-Ojima algebra [35]. Furthermore, it is known that the evolution of the S matrix in the Yang-Mills theories in the massive Curci-Ferrari gauge is not unitary because the bare mass term breaks the nilpotency of the BRST and the anti-BRST transformations [39]. We will show that a similar result holds for this deformed ABJM theory in the massive Curci-Ferrari gauge. Thus we will show that for ABJM theory the unitarity of the S matrix is violated in the massive Curci-Ferrari gauge due to the breaking of the nilpotency of the BRST and the anti-BRST transformations.

II. DEFORMATION OF ABJM THEORY

In this section we will deform the superspace of ABJM theory without breaking any supersymmetry. To do so we define θ^a as a two-component Grassmann parameter and let $y^\mu = x^\mu + \theta^a(\gamma^\mu)_a^b\theta_b$. Then we promote them to operators $\hat\theta^a$ and $\hat y^\mu$ such that they satisfy the following deformed superspace algebra [20],

$$[\hat y^\mu, \hat y^\nu] = B^{\mu\nu}, \qquad [\hat y^\mu, \hat\theta^a] = A^{\mu a}. \tag{1}$$

This is the most general deformation that we can have without breaking any supersymmetry [19]. We use Weyl ordering and express the Fourier transformation of superfields on this deformed superspace as

$$\hat X(\hat y, \hat\theta) = \int d^4k \int d^2\pi e^{-ik\hat y - \pi\hat\theta} X(k, \pi). \tag{2}$$

Now we have a one to one map between a function of $\hat\theta$, $\hat y$ and a function of ordinary superspace coordinates θ, y via

$$X(y, \theta) = \int d^4k \int d^2\pi e^{-iky - \pi\theta} X(k, \pi). \tag{3}$$

Now as we have a one to one map between superfields on this deformed superspace and superfields on the undeformed superspace, we can define the product of two superfields on this deformed superspace. To do that we can express the product of two superfields $\hat X(\hat y, \hat\theta)\hat Z(\hat y, \hat\theta)$ on this deformed superspace as

$$\begin{aligned}\hat X(\hat y, \hat\theta)\hat Z(\hat y, \hat\theta) = \int d^4k_1 d^4k_2 \int d^2\pi_1 d^2\pi_2 \exp \\ - i((k_1 + k_2)\hat y + (\pi_1 + \pi_2)\hat\theta) \\ \times \exp(i\Delta)X(k_1, \pi_1)Z(k_2, \pi_2),\end{aligned} \tag{4}$$

where

$$\exp(i\Delta) = \exp - \frac{i}{2}(B^{\mu\nu}k^2_\mu k^1_\nu + A^{\mu a}(\pi^2_a k^1_\mu - k^2_\mu \pi^1_a)). \tag{5}$$

So we can now define the star product between ordinary functions as follows:

$$\begin{aligned}X(y, \theta) \star Z(y, \theta) = \exp - \frac{i}{2}(B^{\mu\nu}\partial^2_\mu\partial^1_\nu + A^{\mu a}(\partial^2_a\partial^1_\mu - \partial^2_\mu\partial^1_a)) \\ \times X(y_1, \theta_1)Z(y_2, \theta_2)|_{y_1=y_2=y, \theta_1=\theta_2=\theta}.\end{aligned} \tag{6}$$

The star product reduces to the usual Moyal star product for the bosonic noncommutativity in the limit $A^{a\mu} = 0$. Furthermore, when $B^{\mu\nu} = A^{a\mu} = 0$, then the star product reduces to the ordinary product. It is also useful to define the following bracket

$$2[X, Z]_\star = X \star Z \pm Z \star X, \tag{7}$$

where the relative sign is negative unless both the fields are fermionic.

Now we construct the classical Lagrangian density with the gauge group $U(N)_k \times U(N)_{-k}$ [9], on this deformed superspace,

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}, \tag{8}$$

where $\mathcal{L}_{CS}$ and $\tilde{\mathcal{L}}_{CS}$ are deformed Chern-Simons theories with gauge groups $U(N)_k$ and $U(N)_{-k}$, respectively. They can thus be expressed as

$$\mathcal{L}_{CS} = \frac{k}{2\pi}\int d^2\theta\, \mathrm{Tr}\Big[\Gamma^a \star \omega_a + \frac{i}{3}[\Gamma^a, \Gamma^b]_\star \star D_b\Gamma_a + \frac{1}{3}[\Gamma^a, \Gamma^b]_\star \star [\Gamma_a, \Gamma_b]_\star\Big]_|,$$
$$\tilde{\mathcal{L}}_{CS} = \frac{k}{2\pi}\int d^2\theta\, \mathrm{Tr}\Big[\tilde{\Gamma}^a \star \tilde{\omega}_a + \frac{i}{3}[\tilde{\Gamma}^a, \tilde{\Gamma}^b]_\star \star D_b\tilde{\Gamma}_a + \frac{1}{3}[\tilde{\Gamma}^a, \tilde{\Gamma}^b]_\star \star [\tilde{\Gamma}_a, \tilde{\Gamma}_b]_\star\Big]_|, \tag{9}$$

where k is an integer [46] and

$$\omega_a = \tfrac{1}{2}D^b D_a \Gamma_b - i[\Gamma^b, D_b\Gamma_a]_\star - \tfrac{2}{3}[\Gamma^b, [\Gamma_b, \Gamma_a]_\star]_\star,$$
$$\tilde{\omega}_a = \tfrac{1}{2}D^b D_a \tilde{\Gamma}_b - i[\tilde{\Gamma}^b, D_b\tilde{\Gamma}_a]_\star - \tfrac{2}{3}[\tilde{\Gamma}^b, [\tilde{\Gamma}_b, \tilde{\Gamma}_a]_\star]_\star. \tag{10}$$

Here the superderivative D_a is given by

$$D_a = \partial_a + (\gamma^\mu \partial_\mu)^b_a \theta_b, \tag{11}$$

and "|" means that the quantity is evaluated at $\theta_a = 0$. In component form the Γ_a and $\tilde{\Gamma}_a$ are given by

$$\Gamma_a = \chi_a + B\theta_a + \tfrac{1}{2}(\gamma^\mu)_a A_\mu + i\theta^2[\lambda_a - \tfrac{1}{2}(\gamma^\mu \partial_\mu \chi)_a],$$
$$\tilde{\Gamma}_a = \tilde{\chi}_a + \tilde{B}\theta_a + \tfrac{1}{2}(\gamma^\mu)_a \tilde{A}_\mu + i\theta^2[\tilde{\lambda}_a - \tfrac{1}{2}(\gamma^\mu \partial_\mu \tilde{\chi})_a]. \tag{12}$$

The Lagrangian density for the matter fields is given by

$$\mathcal{L}_M = \frac{1}{4}\int d^2\theta\, \mathrm{Tr}\Big[[\nabla^a_{(X)} \star X^{I\dagger} \star \nabla_{a(X)} \star X_I] + [\nabla^a_{(Y)} \star Y^{I\dagger} \star \nabla_{a(Y)} \star Y_I] + \frac{16\pi}{k}\mathcal{V}_\star\Big]_|, \tag{13}$$

where

$$\begin{aligned}\nabla_{(X)a} \star X^I &= D_a X^I + i\Gamma_a \star X^I - i\tilde{\Gamma}_a \star X^I,\\ \nabla_{(X)a} \star X^{I\dagger} &= D_a X^{I\dagger} - i\Gamma_a \star X^{I\dagger} + i\tilde{\Gamma}_a \star X^{I\dagger},\\ \nabla_{(Y)a} \star Y^I &= D_a Y^I - i\Gamma_a \star Y^I + i\tilde{\Gamma}_a \star Y^I,\\ \nabla_{(Y)a} \star Y^{I\dagger} &= D_a Y^{I\dagger} + i\Gamma_a \star Y^{I\dagger} - i\tilde{\Gamma}_a \star Y^{I\dagger},\end{aligned} \tag{14}$$

and $\mathcal{V}_\star$ is the potential term given by

$$\mathcal{V}_\star = \epsilon^{IJ}\epsilon_{KL}[X_I \star Y^K \star X_J \star Y^L] + \epsilon_{IJ}\epsilon^{KL}[X^{\dagger I} \star Y^\dagger_K \star X^{\dagger J} \star Y^\dagger_L]. \tag{15}$$

This model reduces to the regular ABJM theory when $B^{\mu\nu} = A^{\mu a} = 0$.

III. LINEAR GAUGE

All the degrees of freedom in the Lagrangian density for this deformed ABJM theory are not physical because it is invariant under the following gauge transformations,

$$\begin{aligned}\delta\Gamma_a &= \nabla_a \star \Lambda,\\ \delta\tilde{\Gamma}_a &= \tilde{\nabla}_a \star \tilde{\Lambda},\\ \delta X^I &= i(\Lambda - \tilde{\Lambda}) \star X^I,\\ \delta X^{I\dagger} &= -i(\Lambda - \tilde{\Lambda}) \star X^{I\dagger} \delta Y^I = -i(\Lambda - \tilde{\Lambda}) \star Y^I,\\ \delta Y^{I\dagger} &= i(\Lambda - \tilde{\Lambda}) \star Y^{I\dagger},\end{aligned} \tag{16}$$

where

$$\nabla_a = D_a - i\Gamma_a, \qquad \tilde{\nabla}_a = D_a - i\tilde{\Gamma}_a. \tag{17}$$

So we have to fix a gauge before doing any calculations. This can be done by choosing the following gauge fixing conditions,

$$D^a \star \Gamma_a = 0, \qquad D^a \star \tilde{\Gamma}_a = 0. \tag{18}$$

These gauge fixing conditions can be incorporated at the quantum level by adding the following gauge fixing term to the original Lagrangian density,

$$\mathcal{L}_{gf} = \int d^2\theta\, \mathrm{Tr}\Big[b \star (D^a\Gamma_a) + \frac{\alpha}{2} b \star b - i\tilde{b} \star (D^a\tilde{\Gamma}_a) + \frac{\alpha}{2}\tilde{b} \star \tilde{b}\Big]_|. \tag{19}$$

The ghost terms corresponding to this gauge fixing term can be written as

$$\mathcal{L}_{gh} = \int d^2\theta\, \mathrm{Tr}[\bar{c} \star D^a\nabla_a \star c - \tilde{\bar{c}} \star D^a\tilde{\nabla}_a \star \tilde{c}]_|. \tag{20}$$

The total Lagrangian density obtained by the addition of the original classical Lagrangian density, the gauge fixing term, and the ghost term is invariant under the following BRST transformations,

$$\begin{aligned}&s\Gamma_a = \nabla_a \star c, \qquad s\tilde{\Gamma}_a = \tilde{\nabla}_a \star \tilde{c},\\ &sc = -[c, c]_\star, \qquad s\tilde{\bar{c}} = -\tilde{b} - 2[\tilde{\bar{c}}, \tilde{c}]_\star,\\ &s\bar{c} = b, \qquad s\tilde{c} = -[\tilde{c}, \tilde{c}]_\star,\\ &sb = 0, \qquad s\tilde{b} = -[\tilde{b}, \tilde{\bar{c}}]_\star,\\ &sX^I = i(c - \tilde{c}) \star X^I, \qquad sX^{I\dagger} = -i(c - \tilde{c}) \star X^{I\dagger},\\ &sY^I = -i(c - \tilde{c}) \star Y^I, \qquad sY^{I\dagger} = i(c - \tilde{c}) \star Y^{I\dagger}.\end{aligned} \tag{21}$$

This total Lagrangian density is also invariant under the following anti-BRST transformations,

$$\begin{aligned}&\bar{s}\Gamma_a = \nabla_a \star \bar{c}, \qquad \bar{s}\tilde{\Gamma}_a = \tilde{\nabla}_a \star \tilde{\bar{c}},\\ &\bar{s}c = -b - 2[\bar{c}, c]_\star, \qquad [\bar{s}, \tilde{c}]_\star = \tilde{b},\\ &\bar{s}\,\bar{c} = -[\bar{c}, \bar{c}]_\star, \qquad \bar{s}\,\tilde{\bar{c}} = -[\tilde{\bar{c}}, \tilde{\bar{c}}]_\star,\\ &\bar{s}b = -[b, c]_\star, \qquad \bar{s}\,\tilde{b} = 0,\\ &\bar{s}X^I = i(\bar{c} - \tilde{\bar{c}}) \star X^I, \qquad \bar{s}X^{I\dagger} = -i(\bar{c} - \tilde{\bar{c}}) \star X^{I\dagger},\\ &\bar{s}Y^I = -i(\bar{c} - \tilde{\bar{c}}) \star Y^I, \qquad \bar{s}Y^{I\dagger} = i(\bar{c} - \tilde{\bar{c}}) \star Y^{I\dagger}.\end{aligned} \tag{22}$$

Both these sets of transformations are nilpotent:

$$[s, s]_\star = [\bar{s}, \bar{s}]_\star = 0. \tag{23}$$

In fact they also satisfy $[s, \bar{s}]_\star = 0$. Here star product means that any product of fields in the transformation be treated as a star product. We can now express the sum of the gauge fixing term and the ghost term as

$$\begin{aligned}\mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} &= -\int d^2\theta \bar{s}\, \text{Tr}\Big[c \star \Big(D^a\Gamma_a - \frac{i\alpha}{2} b\Big) \\ &\quad - \tilde{c} \star \Big(D^a\tilde{\Gamma}_a - \frac{i\alpha}{2}\tilde{b}\Big)\Big]_| \\ &= \int d^2\theta s\, \text{Tr}\Big[\bar{c} \star \Big(D^a\Gamma_a - \frac{\alpha}{2} b\Big) \\ &\quad - \tilde{\bar{c}} \star \Big(D^a\tilde{\Gamma}_a - \frac{\alpha}{2}\tilde{b}\Big)\Big]_|.\end{aligned} \tag{24}$$

Thus the sum of the gauge fixing term and ghost term can be expressed as a total BRST or a total anti-BRST variation. In the Landau gauge, $\alpha = 0$, and so we have

$$\begin{aligned}\mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{gf}} &= \int d^2\theta s\, \text{Tr}[\bar{c} \star (D^a\Gamma_a) - \tilde{\bar{c}} \star (D^a\tilde{\Gamma}_a)]_| \\ &= \int d^2\theta \bar{s}\, \text{Tr}[c \star (D^a\Gamma_a) - \tilde{c} \star (D^a\tilde{\Gamma}_a)]_|.\end{aligned} \tag{25}$$

In fact in the Landau gauge we can express the sum of the gauge fixing term and the ghost term as a combination of the total BRST and the total anti-BRST variation. Thus in the Landau gauge the sum of the gauge fixing term and the ghost term is given by

$$\begin{aligned}\mathcal{L}_{\text{gh}} + \mathcal{L}_{\text{gf}} &= -\frac{1}{2}\int d^2\theta s\bar{s}\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a]_| \\ &= \frac{1}{2}\int d^2\theta \bar{s}s\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a]_|.\end{aligned} \tag{26}$$

IV. NONLINEAR GAUGES

For Yang-Mills theories in the Curci-Ferrari gauge the sum of the gauge fixing term and the ghost term can also be expressed as a combination of the total BRST and the total anti-BRST variation, for any value of α [35]. In this section we will show that the sum of the gauge fixing term and the ghost term for this deformed ABJM theory in the Curci-Ferrari gauge can also be expressed as a combination of a total BRST and a total anti-BRST variation, for any value of α. The BRST transformations for the deformed ABJM theory in the Curci-Ferrari gauge are given by

$$\begin{aligned}s\Gamma_a &= \nabla_a \star c, & sb &= -[b, c]_\star - [\bar{c}, [c, c]_\star]_\star, \\ sc &= -[c, c]_\star, & s\bar{c} &= b - [\bar{c}, c]_\star, \\ s\tilde{\Gamma}_a &= \tilde{\nabla}_a \star \tilde{c}, & s\tilde{b} &= -[\tilde{b}, \tilde{c}]_\star - [\tilde{\bar{c}}, [\tilde{c}, \tilde{c}]_\star]_\star, \\ s\tilde{c} &= -[\tilde{c}, \tilde{c}]_\star, & s\tilde{\bar{c}} &= \tilde{b} - [\tilde{\bar{c}}, \tilde{c}]_\star, \\ sX^I &= i(c - \tilde{c}) \star X^I, & sX^{I\dagger} &= -i(c - \tilde{c}) \star X^{I\dagger}, \\ sY^I &= -i(\bar{c} - \tilde{\bar{c}}) \star Y^I, & sY^{I\dagger} &= i(\bar{c} - \tilde{\bar{c}}) \star Y^{I\dagger}.\end{aligned} \tag{27}$$

The anti-BRST transformations for this theory in the Curci-Ferrari gauge are given by

$$\begin{aligned}\bar{s}\Gamma_a &= \nabla_a \star \bar{c}, & \bar{s}b &= -[b, \bar{c}]_\star + [c, [\bar{c}, \bar{c}]_\star]_\star, \\ \bar{s}\,\bar{c} &= -[\bar{c}, \bar{c}]_\star, & \bar{s}c &= -b - [\bar{c}, c]_\star, \\ \bar{s}\tilde{\Gamma}_a &= \tilde{\nabla}_a \star \tilde{\bar{c}}, & \bar{s}\,\tilde{b} &= -[\tilde{b}, \tilde{\bar{c}}]_\star + [\tilde{c}, [\tilde{\bar{c}}, \tilde{\bar{c}}]_\star]_\star, \\ \bar{s}\,\tilde{\bar{c}} &= -[\tilde{\bar{c}}, \tilde{\bar{c}}]_\star, & \bar{s}\,\tilde{c} &= -\tilde{b} - [\tilde{\bar{c}}, \tilde{c}]_\star, \\ \bar{s}X^I &= i(\bar{c} - \tilde{\bar{c}}) \star X^I, & \bar{s}X^{I\dagger} &= -i(\bar{c} - \tilde{\bar{c}}) \star X^{I\dagger}, \\ \bar{s}Y^I &= -i(\bar{c} - \tilde{\bar{c}}) \star Y^I, & \bar{s}Y^{I\dagger} &= i(\bar{c} - \tilde{\bar{c}}) \star Y^{I\dagger}.\end{aligned} \tag{28}$$

Both these sets of transformations are also nilpotent:

$$[s, s]_\star = [\bar{s}, \bar{s}]_\star = 0. \tag{29}$$

In fact they also satisfy $[s, \bar{s}]_\star = 0$. We can now write the sum of the gauge fixing term and the ghost term for this deformed ABJM theory as a combination of a total BRST and a total anti-BRST variation, as

$$\begin{aligned}\mathcal{L}'_g &= \frac{1}{2}\int d^2\theta s\bar{s}\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a - \alpha\bar{c} \star c \\ &\quad + \alpha\tilde{\bar{c}} \star \tilde{c}]_| \\ &= -\frac{1}{2}\int d^2\theta \bar{s}s\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a \\ &\quad - \alpha\bar{c} \star c_a + \alpha\tilde{\bar{c}} \star \tilde{c}]_|.\end{aligned} \tag{30}$$

In Yang-Mills theory the effect of the addition of a bare mass to the sum of the gauge fixing term and the ghost term has been analyzed [39]. We can also generalize the Curci-Ferrari gauge in the deformed ABJM theory to the massive Curci-Ferrari gauge by the addition of a similar bare mass term. Thus we can also write the massive Curci-Ferrari type of Lagrangian density for the deformed ABJM theory as

$$\begin{aligned}\mathcal{L}'_g &= -\frac{1}{2}\int d^2\theta[\bar{s}s + im^2]\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a \\ &\quad - \alpha\bar{c} \star c + \alpha\tilde{\bar{c}} \star \tilde{c}]_| \\ &= \frac{1}{2}\int d^2\theta[s\bar{s} - im^2]\, \text{Tr}[\Gamma^a \star \Gamma_a - \tilde{\Gamma}^a \star \tilde{\Gamma}_a \\ &\quad - \alpha\bar{c} \star c + \alpha\tilde{\bar{c}} \star \tilde{c}]_|.\end{aligned} \tag{31}$$

The BRST transformations for the deformed ABJM theory in this massive Curci-Ferrari gauge are given by

$$
\begin{aligned}
&s\Gamma_a = \nabla_a \star c, \qquad sb = im^2 c - [b, c]_\star - [\bar{c}, [c, c]_\star]_\star,\\
&sc = -[c, c]_\star, \qquad s\bar{c} = b - [\bar{c}, c]_\star,\\
&s\tilde{\Gamma}_a = \tilde{\nabla}_a \star \tilde{c},\\
&s\tilde{b} = im^2\tilde{c} - [\tilde{b}, \tilde{c}]_\star - [\tilde{\bar{c}}, [\tilde{c}, \tilde{c}]_\star]_\star s\tilde{c} = -[\tilde{c}, \tilde{c}]_\star,\\
&s\tilde{\bar{c}} = \tilde{b} - [\tilde{\bar{c}}, \tilde{c}]_\star, \qquad sX^I = i(c - \tilde{c}) \star X^I,\\
&sX^{I\dagger} = -i(c - \tilde{c}) \star X^{I\dagger}, \qquad sY^I = -i(\bar{c} - \tilde{\bar{c}}) \star Y^I,\\
&sY^{I\dagger} = i(\bar{c} - \tilde{\bar{c}}) \star Y^{I\dagger}.
\end{aligned}
\tag{32}
$$

Similarly the anti-BRST transformations for the deformed ABJM theory in this massive Curci-Ferrari gauge are given by

$$
\begin{aligned}
&\bar{s}\Gamma_a = \nabla_a \star \bar{c}, \qquad \bar{s}b = im^2\bar{c} - [b, \bar{c}]_\star + [c, [\bar{c}, \bar{c}]_\star]_\star,\\
&\bar{s}\,\bar{c} = -[\bar{c}, \bar{c}]_\star, \qquad \bar{s}c = -b - [\bar{c}, c]_\star,\\
&\bar{s}\tilde{\Gamma}_a = \tilde{\nabla}_a \star \tilde{\bar{c}}, \qquad \bar{s}\,\tilde{b} = im^2\tilde{\bar{c}} - [\tilde{b}, \tilde{\bar{c}}]_\star + [\tilde{c}, [\tilde{\bar{c}}, \tilde{\bar{c}}]_\star]_\star,\\
&\bar{s}\,\tilde{\bar{c}} = -[\tilde{\bar{c}}, \tilde{\bar{c}}]_\star, \qquad \bar{s}\,\tilde{c} = -\tilde{b} - [\tilde{\bar{c}}, \tilde{c}]_\star,\\
&\bar{s}X^I = i(\bar{c} - \tilde{\bar{c}}) \star X^I, \qquad \bar{s}X^{I\dagger} = -i(\bar{c} - \tilde{\bar{c}}) \star X^{I\dagger},\\
&\bar{s}Y^I = -i(\bar{c} - \tilde{\bar{c}}) \star Y^I, \qquad \bar{s}Y^{I\dagger} = i(\bar{c} - \tilde{\bar{c}}) \star Y^{I\dagger}.
\end{aligned}
\tag{33}
$$

The addition of a bare mass term breaks the nilpotency of the BRST and the anti-BRST transformations. The BRST and the anti-BRST transformations now satisfy

$$[s, s]_\star = [\bar{s}, \bar{s}]_\star \sim 2im^2. \tag{34}$$

However, in the zero mass limit, the nilpotency of the BRST and the anti-BRST transformations is restored.

V. NAKANISHI-OJIMA ALGEBRA

In Yang-Mills theory it is known that whenever the sum of the gauge fixing term and the ghost term can be written as a combination of the total BRST and the total anti-BRST variation, the total Lagrangian density is invariant under a set of symmetry transformations which obey $SL(2, R)$ algebra [35]. Now for the deformed ABJM theory in the Landau and nonlinear gauges, the sum of the gauge fixing term and ghost term is expressed as a combination of the total BRST and the total anti-BRST variation, so we expect the total Lagrangian density for this deformed ABJM theory will also be invariant under a set of symmetry transformations which obey $SL(2, R)$ algebra. In fact in these gauges the deformed ABJM theory is also invariant under the following transformations,

$$
\begin{aligned}
&\delta_1 b = [c, c]_\star, \qquad \delta_1 \tilde{b} = [\tilde{c}, \tilde{c}]_\star,\\
&\delta_1 c = 0, \qquad \delta_1 \tilde{c} = 0,\\
&\delta_1 \bar{c} = c, \qquad \delta_1 \tilde{\bar{c}} = \tilde{c},\\
&\delta_1 \Gamma_a = 0, \qquad \delta_1 \tilde{\Gamma}_a = 0,\\
&\delta_1 X^I = 0, \qquad \delta_1 X^{I\dagger} = 0,\\
&\delta_2 b = [\bar{c}, \bar{c}]_\star, \qquad \delta_2 \tilde{b} = [\tilde{\bar{c}}, \tilde{\bar{c}}]_\star,\\
&\delta_2 c = \bar{c}, \qquad \delta_2 \tilde{c} = \tilde{\bar{c}},\\
&\delta_2 \bar{c} = 0, \qquad \delta_2 \tilde{\bar{c}} = 0,\\
&\delta_2 X^I = 0, \qquad \delta_2 X^{I\dagger} = 0.
\end{aligned}
\tag{35}
$$

In the Landau and Curci-Ferrari gauges these transformations, the BRST transformation, and the anti-BRST transformation along with the FP conjugation form the Nakanishi-Ojima $SL(2, R)$ algebra,

$$
\begin{aligned}
&[s, s]_\star = 0, \qquad [\bar{s}, \bar{s}]_\star = 0,\\
&[s, \bar{s}]_\star = 0, \qquad [\delta_1, \delta_2]_\star = -2\delta_{FP}\\
&[\delta_1, \delta_{FP}]_\star = -4\delta_1, \qquad [\delta_2, \delta_{FP}]_\star = 4\delta_2,\\
&[s, \delta_{FP}]_\star = -2s, \qquad [\bar{s}, \delta_{FP}]_\star = 2\bar{s},\\
&[s, \delta_1]_\star = 0, \qquad [\bar{s}, \delta_1]_\star = -2s,\\
&[s, \delta_2]_\star = 2\bar{s}, \qquad [\bar{s}, \delta_2]_\star = 0.
\end{aligned}
\tag{36}
$$

This algebra gets modified due to the presence of the bare mass term in the massive Curci-Ferrari gauge. This is because the nilpotency of both the BRST and the anti-BRST transformations is broken by the addition of a bare mass term. However, even though the nilpotency of the BRST and the anti-BRST transformations is broken, the FP conjugation is not broken in the massive Curci-Ferrari gauge. Thus we are able construct an algebra for the set of symmetric transformations in the massive Curci-Ferrari gauge. This algebra for the set of symmetric transformations in the massive Curci-Ferrari gauge is given by

$$
\begin{aligned}
&[s, s]_\star = -2im^2\delta_1, \qquad [\bar{s}, \bar{s}]_\star = 2im^2\delta_2,\\
&[s, \bar{s}]_\star = 2im^2\delta_{FP}, \qquad [\delta_1, \delta_2]_\star = -2\delta_{FP}\\
&[\delta_1, \delta_{FP}]_\star = -4\delta_1, \qquad [\delta_2, \delta_{FP}]_\star = 4\delta_2,\\
&[s, \delta_{FP}]_\star = -2s, \qquad [\bar{s}, \delta_{FP}]_\star = 2\bar{s}, \qquad [s, \delta_1]_\star = 0,\\
&[\bar{s}, \delta_1]_\star = -2s, \qquad [s, \delta_2]_\star = 2\bar{s}, \qquad [\bar{s}, \delta_2]_\star = 0.
\end{aligned}
\tag{37}
$$

VI. CONSERVED CHARGES

In conventional commutative field theories, for every symmetry under which the Lagrangian density is invariant there is a conserved charge obtained from a divergenceless current associated with that symmetry of the theory. In noncommutative field theories even though the variation of the action vanishes for all local parameters of transformation, the divergence of the current need not vanish. However, for conventional noncommutative field theories the divergence of the current is equal to the Moyal bracket of some functions [47]. This Moyal bracket vanishes for the spacelike noncommutativity when we integrate on the continuity equation over all spatial coordinates in order to obtain the time variation of the charge [48]. Consequently, the charge associated to a symmetry transformation commutes with the Hamiltonian of the theory in this case. A similar result will hold for the star bracket if we are again restricted to spacelike noncommutativity. Here again the charge associated with a symmetry transformation will commute with the Hamiltonian of the theory. So from now on we shall be restricted to discussions of spacelike noncommutativity. So for two local functions X and Z associated with a symmetry, the divergence of the current will be given as

$$[X, Y]_\star = 2\mathcal{D}^\mu \star J_\mu. \tag{38}$$

where $\mathcal{D}^\mu$ is the ordinary covariant derivative. As we have restricted the discussion to spacelike noncommutativity, we get

$$\int d^3y[X, Z]_\star = 0. \tag{39}$$

Now the conserved charge is given by

$$Q = \int d^3y J^0 = 0. \tag{40}$$

Now we define the current associated with the noncommutative BRST symmetry $J^\mu_{(B)}$ and noncommutative anti-BRST symmetry $\bar{J}^\mu_{(B)}$ as follows:

$$\begin{aligned}
2J^\mu_{(B)} = \int \mathrm{d}^2\theta \mathrm{Tr}\bigg[&\frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \Gamma_b} \star s\Gamma_b + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu c} \star sc \\
&+ \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \bar{c}} \star s\bar{c} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu b} \star sb + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{\Gamma}_a} \star s\tilde{\Gamma}_a \\
&+ \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{c}} \star s\tilde{c} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{\bar{c}}} \star s\tilde{\bar{c}} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{b}} \star s\tilde{b}\bigg], \\
2\bar{J}^\mu_{(B)} = \int \mathrm{d}^2\theta \mathrm{Tr}\bigg[&\frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \Gamma_b} \star \bar{s}\Gamma_b + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu c} \star \bar{s}c \\
&+ \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \bar{c}} \star \bar{s}\,\bar{c} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu b} \star \bar{s}b + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{\Gamma}_a} \star \bar{s}\tilde{\Gamma}_a \\
&+ \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{c}} \star \bar{s}\,\tilde{c} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{\bar{c}}} \star \bar{s}\,\tilde{\bar{c}} + \frac{\partial L_{\mathrm{eff}}}{\partial \mathcal{D}_\mu \tilde{b}} \star \bar{s}\,\tilde{b}\bigg],
\end{aligned} \tag{41}$$

where

$$\int d^2\theta[L_{\mathrm{eff}}]_| = \mathcal{L}_c + \mathcal{L}_{\mathrm{gh}} + \mathcal{L}_{\mathrm{gf}}.$$

Hence, the BRST charge Q_B and anti-BRST charge $\bar{Q}_B$ associated with the currents $J^\mu_{(B)}$ and $\bar{J}^\mu_{(B)}$ are conserved,

$$Q_B = \int d^3y J^0_{(B)} = 0, \qquad \bar{Q}_B = \int d^3y \bar{J}^0_{(B)} = 0. \tag{42}$$

The BRST charge Q_B and anti-BRST charge $\bar{Q}_B$ are both nilpotent for all gauges except the massive Curci-Ferrari gauge,

$$Q_B^2 = \bar{Q}_B^2 = 0. \tag{43}$$

However, for the massive Curci-Ferrari gauge these charges are not nilpotent:

$$Q_B^2 \neq 0, \qquad \bar{Q}_B^2 \neq 0. \tag{44}$$

The nilpotency of Q_B and $\bar{Q}_B$ is very important to isolate the physical Hilbert space and prove the unitarity of the S matrix. This is what will be done in the next section.

VII. PHYSICAL SUBSPACE

The total Lagrangian which is formed by the sum of the original Lagrangian, the gauge fixing term, and the ghost term is invariant under the noncommutative BRST and the noncommutative anti-BRST transformations. As the charges Q_B and $\bar{Q}_B$ are nilpotent for all gauges except the massive Curci-Ferrari gauge, so their action on any field twice will vanish for all gauges except the massive Curci-Ferrari gauge. So for any state $|\phi\rangle$ in a gauge other than the massive Curci-Ferrari gauge, we have

$$Q_B^2|\phi\rangle = 0, \qquad \bar{Q}_B^2|\phi\rangle = 0. \tag{45}$$

We shall now restrict our discussion to gauges other than the massive Curci-Ferrari gauge. The physical states $|\phi_p\rangle$ can now be defined as states that are annihilated by Q_B:

$$Q_B|\phi_p\rangle = 0. \tag{46}$$

We can also define the physical states as states that are annihilated by $\bar{Q}_B$:

$$\bar{Q}_B|\phi_p\rangle = 0. \tag{47}$$

We will obtain the same result by using any of these as the definition for the physical sates. Now as we get the same physical result by using either the noncommutative BRST or the noncommutative anti-BRST charge, we will denote them both by Q, so Q represents both Q_B and $\bar{Q}_B$. Thus the physical states $Q|\phi_p\rangle$ are annihilated by Q,

$$Q|\phi_p\rangle = 0. \tag{48}$$

This criterion divides the Fock space into three parts, $\mathcal{H}_0$, $\mathcal{H}_1$, and $\mathcal{H}_2$. The space $\mathcal{H}_1$, comprises of those states that are not annihilated by Q. So if $|\phi_1\rangle$ is any state in $\mathcal{H}_1$, then we have

$$Q|\phi_1\rangle \neq 0. \tag{49}$$

The space $\mathcal{H}_2$ comprises those states that are obtained by the action of Q on states belonging to $\mathcal{H}_1$. So if $|\phi_2\rangle$ is any state in $\mathcal{H}_2$, then we have

$$|\phi_2\rangle = Q|\phi_1\rangle. \tag{50}$$

Thus we have

$$Q|\phi_2\rangle = Q^2|\phi_1\rangle = 0. \tag{51}$$

So all the states in $\mathcal{H}_2$ are annihilated by Q. The space $\mathcal{H}_0$ comprises of those states that are annihilated by Q and are not obtained by the action of Q on any state belonging to $\mathcal{H}_1$. So if $|\phi_0\rangle$ is any state in $\mathcal{H}_0$, then we have

$$Q|\phi_0\rangle = 0, \tag{52}$$

$$|\phi_0\rangle \neq Q|\phi_1\rangle. \tag{53}$$

Clearly the physical states $|\phi_p\rangle$ can only belong to $\mathcal{H}_0$ or $\mathcal{H}_2$. This is because any state in $\mathcal{H}_0$ or $\mathcal{H}_2$ is annihilated by Q. However, any state in $\mathcal{H}_2$ will be orthogonal to all physical states including itself:

$$\langle\phi_p|\phi_2\rangle = \langle\phi_p|(Q|\phi_1\rangle) = (\langle\phi_p|Q)|\phi_1\rangle = 0. \tag{54}$$

Thus two physical states that differ from each other by a state in $\mathcal{H}_2$ will be indistinguishable,

$$|\phi_p\rangle = |\phi_p\rangle + |\phi_2\rangle. \tag{55}$$

So all the relevant physical states actually lie in $\mathcal{H}_0$.

Now if the asymptotic physical states are given by

$$|\phi_{pa,\text{out}}\rangle = |_{pa,}t\to\infty\rangle, \quad |\phi_{pb,\text{in}}\rangle = |\phi_{pb}, t\to -\infty\rangle, \tag{56}$$

then a typical $\mathcal{S}$-matrix element can be written as

$$\langle|\phi_{pa,\text{out}}|\phi_{pb,\text{in}}\rangle = \langle|\phi_{pa}|\mathcal{S}^\dagger\mathcal{S}|\phi_{pb}\rangle. \tag{57}$$

Now as the noncommutative BRST and the noncommutative anti-BRST charges are conserved charges, so they commute with the Hamiltonian and thus the time evolution of any physical state will also be annihilated by Q,

$$Q\mathcal{S}|\phi_{pb}\rangle = 0. \tag{58}$$

This implies that the states $\mathcal{S}|\phi_{pb}\rangle$ must be a linear combination of states in $\mathcal{H}_0$ and $\mathcal{H}_2$. However, as the states in $\mathcal{H}_2$ have zero inner product with one another and also with states in $\mathcal{H}_0$, so the only contributions come from states in $\mathcal{H}_0$. So we can write

$$\langle|\phi_{pa}|\mathcal{S}^\dagger\mathcal{S}|\phi_{pb}\rangle = \sum_i\langle|\phi_{pa}|\mathcal{S}^\dagger|\phi_{0,i}\rangle\langle\phi_{0,i}|\mathcal{S}|\phi_{pb}\rangle. \tag{59}$$

Since the full $\mathcal{S}$ matrix is unitary, this relation implies that the S matrix restricted to physical subspace is also unitary. It may be noted that the nilpotency of the noncommutative BRST and the noncommutative anti-BRST charges was essential for proving the unitarity of the resultant theory. Now as the noncommutative BRST and the noncommutative anti-BRST charges are not nilpotent in the massive Curci-Ferrari gauge,

$$Q_B^2|\phi\rangle \neq 0, \qquad \bar{Q}_B^2|\phi\rangle \neq 0, \tag{60}$$

so the $\mathcal{S}$ does not factorize in the massive Curci-Ferrari gauge

$$\langle|\phi_{pa}|\mathcal{S}^\dagger\mathcal{S}|\phi_{pb}\rangle \neq \sum_i\langle|\phi_{pa}|\mathcal{S}^\dagger|\phi_{0,i}\rangle\langle\phi_{0,i}|\mathcal{S}|\phi_{pb}\rangle, \tag{61}$$

and thus the resultant theory is not unitary. However, even though this noncommutative deformation is not unitary in the massive Curci-Ferrari gauge, the nilpotency is restored in the zero mass limit. Thus the unitarity is also restored in the zero mass limit.

VIII. CONCLUSION

In this paper we studied a noncommutative deformation of the ABJM theory in $N = 1$ superspace formalism. In performing our analyses the noncommutative fields were related to ordinary ones and the product of these noncommutative fields was related to a star product of ordinary fields. The main focus of the paper was to generalize some results that are known for Yang-Mills theories to this deformed ABJM theory. So we analyzed the behavior of the BRST and the anti-BRST symmetries for this deformed ABJM theory, and its linear as well as nonlinear gauges. We have expressed the sum of the gauge fixing term and the ghost term for this deformed ABJM theory as a combination of the total BRST and the total anti-BRST variation, in the Landau gauge. Furthermore, this was achieved for an arbitrary value of α by the making the BRST and the anti-BRST transformations nonlinear. The addition of a bare mass term violated the nilpotency of the BRST and the anti-BRST transformations and this in turn breaks the unitarity of the theory. We have also shown that in the Landau and Curci-Ferrari gauges the deformed ABJM theory is invariant under Nakanishi-Ojima $SL(2, R)$ algebra. We have also analyzed the effect that the addition of a bare mass term has on this algebra.

In Yang-Mills theories the presence of nonlinear terms gives rise to an effective potential whose vacuum configuration favors the formation of off-diagonal ghost condensates [49]. The ghost condensation in Yang-Mills theories also occurs in the Landau gauge [50]. The ghost condensation in Yang-Mills theories breaks the $SL(2, R)$ symmetry which exists in these gauges. It will be interesting to investigate if the ghost condensation in this deformed ABJM theory also leads to a dynamic breaking of the $SL(2, R)$ symmetry.

The infinite temporal derivatives occur in the product of fields for this noncommuative ABJM theory due to $B^{0\mu}$ and A^{0a}. This will give rise to nonlocal behavior in the deformed ABJM theory. This in general will lead to a violation of the unitarity of the deformed ABJM theory. However, if we restrict the deformation of the ABJM theory to spacelike noncommutativity, i.e., we set $B^{0\mu} = A^{0a} = 0$, then these problems will not occur. It will then be possible to construct the Norther's chargers corresponding to the BRST and the anti-BRST symmetries and use them to project out the physical states. As the nilpotency of the BRST and the anti-BRST transformations is violated in the massive Curci-Ferrari gauge, so we expect that unitarity will also be violated in that gauge, even after restricting to spacelike noncommutativity.

ABJM theory has been used to study various examples of AdS_4/CFT_3 correspondence [51–55]. In fact AdS_4/CFT_3 has also been used to analyze the fractional quantum Hall effect [56]. The fractional quantum Hall effect in ABJM theory has also been analyzed [57]. In ABJM theory $D6$-branes wrapped over $AdS_4 \times S^3/Z_2$ in type IIA superstring theory on $AdS_4 \times CP^3$ give its dual description with $N = 3$ supersymmetry. In the presence of fractional branes, the ABJM theory can model the fractional quantum Hall effect, with RR fields regarded as the external electric-magnetic field. In this model the addition of the flavor $D6$-brane describes a class of the fractional quantum Hall plateau transition. It will be interesting to analyze the fractional quantum Hall effect, with RR fields regarded as the external electric-magnetic field in the deformed superspace. We can expect that addition of the flavor $D6$-brane might describe a class of a fractional quantum Hall plateau transition in the deformed superspace ABJM theory also.

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