PHYSICAL REVIEW D **84**, 106004 (2011)

Structure function of holographic quark-gluon plasma: Sakai-Sugimoto model versus its noncritical version

Yan Yan Bu* and Jin Min Yang†

Institute of Theoretical Physics, Academia Sinica, Beijing 100190, People's Republic of China

(Received 20 August 2011; published 4 November 2011)

Motivated by recent studies of deep inelastic scattering off the $\mathcal{N} = 4$ super-Yang-Mills (SYM) plasma, holographically dual to an $\mathrm{AdS}_5 \times S^5$ black hole, we use the spacelike flavor current to probe the internal structure of one holographic quark-gluon plasma, which is described by the Sakai-Sugimoto model at high temperature phase (i.e., the chiral-symmetric phase). The plasma structure function is extracted from the retarded flavor current-current correlator. Our main aim in this paper is to explore the effect of nonconformality on these physical quantities. As usual, our study is under the supergravity approximation and the limit of large color number. Although the Sakai-Sugimoto model is nonconformal, which makes the calculations more involved than the well-studied $\mathcal{N} = 4$ SYM case, the result seems to indicate that the nonconformality has little essential effect on the physical picture of the internal structure of holographic plasma, which is consistent with the intuition from the asymptotic freedom of QCD at high energy. While the physical picture underlying our investigation is same as the deep inelastic scattering off the $\mathcal{N} = 4$ SYM plasma with(out) flavor, the plasma structure functions are quantitatively different, especially their scaling dependence on the temperature, which can be recognized as model dependent. As a comparison, we also do the same analysis for the noncritical version of the Sakai-Sugimoto model which is conformal in the sense that it has a constant dilaton vacuum. The result for this noncritical model is quite similar to the conformal $\mathcal{N} = 4$ SYM plasma. We therefore attribute the above difference to the effect of nonconformality of the Sakai-Sugimoto model.

DOI: 10.1103/PhysRevD.84.106004 PACS numbers: 11.25.Tq, 12.38.Lg, 12.38.Mh

I. INTRODUCTION

In heavy-ion collision, which is now experimentally studied at the Relativistic Heavy Ion Collider (RHIC) and the LHC, the so-called quark-gluon plasma seems to be strongly interacting and behaves like a perfect liquid [1], which is greatly different from the previously recognized weakly-coupled quark-gluon plasma. This brings to an urgent stage nonperturbative investigations of hadronic matter at high temperature and high density produced in heavy-ion collision. Although the lattice method can calculate some properties of a strongly coupled system, it can just extract some static quantities such as the hadron mass spectrum and thermodynamical behavior. What is worse is that when adding finite density or chemical potential into the thermal QCD, the lattice calculation usually confronts the notorious sign problem. Therefore, improvement in the theoretical understanding of strongly coupled quark-gluon plasma (sQGP) should not only go beyond the traditional perturbative QCD (pQCD) approach but also reveal some properties out of equilibrium, such as transport properties, dispersion relation, high-energy scattering, and so on.

Gauge/gravity duality [2] states that strong-coupling gauge theory can be mapped to weak-coupling gravity with a negative cosmological constant in the limit of large 't Hooft coupling and large N_c, where N_c is the number of color of gauge theory. Although the gravity dual of the realistic QCD has not yet been established, one expects this duality to be of great importance in understanding some nonperturbative properties of QCD or at least some universal features of a strongly coupled system. In fact, over the last decade, using the gauge/gravity duality technique to study properties of sQGP has gained great success [3]. However, most of the studies have focused on the static or hydrodynamic properties at large scale or long time compared to the inversion of the temperature of the system. Therefore, it is of great interest to study the hard probe of the plasma and reveal its internal structure, which should be in contrast with the parton picture of a single hadron in pQCD. Studies on this topic[1] were originally proposed in [6] for $\mathcal{R}$-current scattering off a dilaton hadron in a hard-wall model and later generalized to the plasma case without flavor in Ref. [7] and with flavor in Ref. [8].

The main lesson[2] from these investigations is that at high energy the $\mathcal{R}$ current or flavor current probes the partonic behavior of the plasma, giving nonvanishing plasma structure function, while at low energy the current is not

*yybu@itp.ac.cn
†jmyang@itp.ac.cn

[1]For studies on deep inelastic scattering under the technique of gauge/gravity duality, see, e.g., Refs. [4,5].

[2]For details on discussions of the structure function and the partonic picture of holographic quark-gluon plasma, see Refs. [7–9].

absorbed by the plasma, indicating a vanishing contribution to the plasma structure function.[3] Besides the plasma structure function, another important quantity is the so-called plasma saturation momentum, which is the critical energy defining the transition from weakly quasielastic scattering to high-energy deep inelastic scattering. In other words, the partonic picture for holographic plasma exists only when all partons have transverse momenta below the saturation momentum.

Since previous studies focused mainly on the D3-brane geometry, which is dual to the conformal $\mathcal{N} = 4$ SYM, we here[4] use one nonconformal gravity dual model of QCD to probe the effect of the nonconformality on the plasma structure. To be specific, the model under consideration here is the transversely intersecting D4/D8/$\overline{\text{D8}}$ brane system, which is now usually referred to as the Sakai-Sugimoto model [10]. It is one of the most successful holographic QCD models from the top-down approach in realizing some phenomena of low-energy QCD such as confinement/deconfinement phase transition,[5] non-Abelian chiral symmetry breaking, and vector meson dominance. The temperature under the gauge/gravity duality approach can be realized by extending the color brane geometry to a black hole [12]. The high-temperature phase means that the chiral symmetry is restoring, which is denoted by the parallel profile of flavor D8/$\overline{\text{D8}}$ branes. Note that the temperature will be much smaller than the four-momentum of the flavor current, since in this paper we are focusing on high-energy scattering to probe the internal structure of holographic plasma. The setup in this paper is similar to that of Ref. [8], but because the induced metric on the flavor worldvolume reduces to an $\text{AdS}_5 \times S^4$ black hole, it is essentially the same as that of the $\mathcal{R}$-current deep inelastic scattering (DIS) off the $\mathcal{N} = 4$ SYM plasma. Thus, the procedure here is similar to that of Ref. [7] and much simpler than that in Ref. [8]. To extract the structure function, we need to study the flavor-current propagation in holographic plasma. According to gauge/gravity duality, this can be achieved by studying the $U(1)$ flavor gauge field in curved spacetime. The corresponding action and equation of motion determining this dynamic are encoded in a Maxwell theory in curved five-dimensional spacetime. On the other hand, the parton structure function has a standard definition in quantum field theory, which is encoded in the retarded current-current correlator. One main task of gauge/gravity duality is to calculate the retarded Green function for finite-temperature field theory from dual gravity following the recipe in Ref. [13]. In this sense, the information of the structure function is totally encoded in the solution of the above-mentioned Maxwell equation in a five-dimensional curved spacetime of asymptotic AdS type. One additional key point under this prescription is that the solution should obey the incoming wave boundary condition at the horizon, reflecting the full-absorption characteristic of a black hole. Once the equation of motion for the flavor $U(1)$ gauge field is turned into a Schrödinger type of equation, it will be found that their behavior is very similar to that of the $\mathcal{N} = 4$ SYM case. This is natural since the setup of holographic models is very general. This may also be recognized as one universal characteristic of gauge/gravity duality.

Another motivation for our study in this paper is to probe some universal features of the structure of holographic plasma, described by the intersecting D-brane system in the conformal or nonconformal case. We have learned that both the procedure and the physical picture for these calculations are universal, which is in part due to the unified approach of gauge/gravity duality in dealing with strongly coupled problems. The specific form of the structure function is model dependent, which may allow us to resort to the experiments to judge which model is much closer to physical reality. For the two structure functions, we find, that in the nonconformal background, they also satisfy the Callan-Gross relation in the limit of a large Bjorken variable (defined later), which is same as in the $\mathcal{N} = 4$ SYM case. Therefore, it is reasonable to conjecture that this relation should be universal for holographic plasma described by the intersecting D-brane system, having nothing to do with the conformality of the holographic background.

Since the flavor gauge field we are focusing on in this paper can also be considered as the gravity dual of a vector meson, our study can then be regarded as the completion of previous studies of mesonic quasinormal modes in Ref. [14]. Here, we extend these studies to high-momentum and high-frequency limits, in contrast to previous hydrodynamic behavior or just the high-frequency limit. However, here we will not go into the detailed numeric extraction of mesonic quasinormal frequency in high-frequency and high-momentum limits, and leave this task for future investigation.

We will also do the same analysis for the noncritical version of the Sakai-Sugimoto model for comparison. This noncritical model was proposed in Ref. [15] to overcome some drawbacks, which are general in critical string theory,

[3]When taking into account the nonperturbative tunneling effect of current when encountering a narrow and high potential barrier, an exponentially suppressed structure function can be obtained.

[4]As in the literature, for simplification, we do not consider charge density, which can be modeled by the time component of the flavor gauge field in present construction, and just leave this problem for further investigation.

[5]Recently, one paper [11] appeared which proves that the previous interpretation of transition between the anti–de Sitter (AdS) soliton and the black D4-brane as the strong-coupling continuation of the confinement/deconfinement transition in four dimensional Yang-Mills theory is not valid. The authors there proposed an alternative gravity dual of the confinement/deconfinement transition. For details on this topic, see the original work [11].

of the critical Sakai-Sugimoto model. This model can be thought of as a conformal one, so we expect the related results in this model should be more similar to the $\mathcal{N}=4$ SYM case. This expectation has been confirmed by the fact that the structure function for the noncritical model has the same scaling dependence on temperature as that of the $\mathcal{N}=4$ SYM case. This is another difference between the Sakai-Sugimoto model and its noncritical version.

Another striking point is that the stringy imprints have appeared in the final results of the structure functions for both models considered in this paper. Seemingly, this would lean toward the successful aspects of the Sakai-Sugimoto model. However, if we recall that the holographic plasma is quite different from the realistic sQGP, we should just take these unsatisfactory features as nonuniversal ones and focus on some universal features that emerge from the different holographic models.

The rest of this paper is organized as follows. In Sec. II, we first give a brief overview of the Sakai-Sugimoto model as well as its noncritical version, and then we give the basic equations for later calculations. In Sec. III, we present detailed extraction of the structure function for the Sakai-Sugimoto model and list the main results for its noncritical version. Then we have a brief discussion about the results. Section IV is devoted to a short summary and some open questions.

II. OVERVIEW OF MODELS AND BASIC EQUATIONS

In this section we first recapitulate the Sakai-Sugimoto model and its noncritical version [10,15,16]. Then we turn to the definitions for the plasma structure function in terms of physical quantity in standard field theory. We will also state basic equations determining the flavor-current propagation in these plasmas from the viewpoint of dual gravity; the equations are essential for later extraction of the structure function. We will follow the notation conventions in Refs. [7,8].

A. Sakai-Sugimoto model versus its noncritical version

The bulk background geometry of the Sakai-Sugimoto model is given by a 10-dimensional supergravity description of N_c coincident D4-branes in type IIA superstring theory compactified on a circle. According to Ref. [12], there are two different metrics for this supergravity, representing two different phases of holographic QCD. The transition between these two phases is interpreted as the deconfinement phase transition. Here, we just focus on the high-temperature deconfining phase, described by the following backgrounds[6]:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}(-f(u)dt^2 + d\vec{x}^2 + dx_4^2) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \tag{1}$$

$$e^{\phi} = g_s\left(\frac{u}{R}\right)^{3/4}, \quad f(u) = 1 - \left(\frac{u_T}{u}\right)^3, \quad t \sim t + \beta = t + \frac{4\pi R^{3/2}}{3u_T^{1/2}}. \tag{2}$$

The temperature of the holographic plasma dual to the above background is

$$T = \frac{1}{\beta} = \frac{3u_T^{1/2}}{4\pi R^{3/2}}. \tag{3}$$

The curvature radius R of the background is related to the string coupling g_s and string length l_s via

$$R^3 = \pi g_s N_c l_s^3 \equiv \pi\lambda l_s^3. \tag{4}$$

Here in the second equality we have defined the 't Hooft coupling constant λ from the dual gravity side as $\lambda \equiv g_s N_c$.

The above gravity background is dual to the gluon sector, and the quark sector can be introduced in quenched approximation by adding N_f pairs of D8- and $\overline{\text{D8}}$-flavor branes to the above geometry and making them transverse to the circle along x_4. In the quenched limit, $N_f \ll N_c$, the backreaction of the flavor branes on the background geometry can be neglected. Dynamics of the flavor sector is encoded in the Dirac-Born-Infeld (DBI) plus the Chern-Simons actions for the flavor branes in the above background. However, the Chern-Simons term will be exactly zero in this paper, as there is no background for the $U(1)$ gauge field on the flavor branes.

Chiral phase transition for the flavor sector in this deconfined phase can happen, and it has a beautiful geometric explanation: the parallel profile of the flavor D8- and $\overline{\text{D8}}$-branes stands for the chiral-restoring phase while the connected U-shaped profile stands for the chirally broken phase. In general, high temperature corresponds to the chiral-restoring phase while low temperature corresponds to the chirally broken phase. (The details on this topic can be found in Ref. [17].) In the high-temperature phase, the induced metric on the flavor branes has the following standard AdS form:

$$ds_8^2 = \left(\frac{u}{R}\right)^{3/2}(-f(u)dt^2 + d\vec{x}^2) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right). \tag{5}$$

The other model we are concerned about here is the noncritical version of the Sakai-Sugimoto model. It is

[6]Because of the periodic identification of the time coordinate, it should be Euclidean time. However, we here use the Minkowskian signature for later convenience when we extract the Minkowski-space retarded Green function.

based on the supergravity description of N_c coincident D4-branes in six dimensions with one dimension compactified on a circle, as in the Sakai-Sugimoto model. The corresponding background geometry takes the following form in the high-temperature phase:

$$ds^2 = \left(\frac{u}{R}\right)^2(-f(u)dt^2 + d\vec{x}^2 + dx_4^2) + \left(\frac{R}{u}\right)^2 \frac{du^2}{f(u)}, \quad (6)$$

$$e^{\phi} = \frac{2\sqrt{2}}{\sqrt{3}N_c}, \qquad R^2 = \tfrac{15}{2}, \qquad f(u) = 1 - \left(\frac{u_T}{u}\right)^5,$$
$$t \sim t + \beta = t + \frac{4\pi R^2}{5u_T}. \quad (7)$$

The flavor quark sector can be introduced by adding N_f pairs of D4- and $\overline{\text{D4}}$-branes into the above background geometry. One important feature of this model is that it does not have the undesired internal space, which may introduce the unwanted KK modes. Another striking characteristic of this model is that it has a constant dilaton vacuum as for the D3-brane geometry in 10 dimensions, which should signal that this model is a conformal one. As in the Sakai-Sugimoto model, the chiral-restoring phase means that the induced metric on the flavor branes takes the form

$$ds_4^2 = \left(\frac{u}{R}\right)^2(-f(u)dt^2 + d\vec{x}^2) + \left(\frac{R}{u}\right)^2 \frac{du^2}{f(u)}. \quad (8)$$

For convenience in later calculations, we now rescale the radial coordinate u to make it dimensionless by a transformation $u_T/u = r$. Then the induced geometry on the flavor branes takes the following simplified versions:

$$ds_8^2 = \left(\frac{u_T}{R}\right)^{3/2} r^{-(3/2)}(-f(r)dt^2 + d\vec{x}^2) + R^{3/2}u_T^{1/2}r^{-(5/2)}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right), \quad (9)$$

$$e^{\phi} = g_s\left(\frac{u_T}{R}\right)^{3/4} r^{-(3/4)}, \qquad f(r) = 1 - r^3; \quad (10)$$

$$ds_4^2 = \left(\frac{u_T}{R}\right)^2 \frac{1}{r^2}(-f(r)dt^2 + d\vec{x}^2) + \left(\frac{R}{r}\right)^2 \frac{dr^2}{f(r)}, \quad (11)$$

$$e^{\phi} = \frac{2\sqrt{2}}{\sqrt{3}N_c}, \qquad f(r) = 1 - r^5. \quad (12)$$

Note that under the new coordinates, the interval for radial coordinate r is located in a finite regime: $r \in [0, 1]$, and the horizon is now at $r = 1$, while the boundary is at $r = 0$, which will make later analysis convenient.

Before concluding this subsection, we give a short comment about one general feature of the above models. Whether or not there is background on the flavor branes, fluctuations of the flavor gauge field and scalar mode in the chiral-restoring phase will always decouple, which together with exact AdS forms of the induced metrics will greatly simplify later calculations. This is also one reason why we choose the transversely intersecting D-brane systems for study. As mentioned in Sec. I, the plasma structure function is completely encoded in the dynamics of the flavor $U(1)$ gauge field propagating through the above-mentioned geometry, which is described by the DBI actions on the flavor branes.

B. DIS: Field theoretical definitions

Deep inelastic scattering in QCD is a powerful tool for exploring the hadron structure. Here we mainly focus on the electromagnetic mediation between the charged lepton and the hadron. The basic objective of DIS is to compute the retarded current-current correlator defined by

$$\Pi_{\mu\nu}(k) \equiv i\int d^4x e^{-ik\cdot x}\theta(x_0)\langle[J_\mu(x), J_\nu(0)]\rangle, \quad (13)$$

where k is the four-momentum of the electromagnetic current, $\langle\ldots\rangle$ means quantum vacuum expectation, and $J_\mu(x)$ is the mediated electromagnetic current. When considering the lepton scattering off the plasma, the hadron should be replaced by the plasma system, and the vacuum polarization tensor Eq. (13) is modified to

$$\Pi_{\mu\nu}(k, T) \equiv i\int d^4x e^{-ik\cdot x}\theta(x_0)\langle[J_\mu(x), J_\nu(0)]\rangle_T, \quad (14)$$

where $\langle\ldots\rangle_T$ means the thermal expectation value in the plasma system.

Although the gravity dual of the $SU(N)\times U(1)_{\text{e.m.}}$ gauge theory has not been established, we can use a nondynamical electromagnetic field to model the photon just as in condensed matter physics given that the electromagnetic coupling constant is very small. In the present context, the electromagnetic current is replaced by the flavor $U(1)$ current and also denoted as $J_\mu(x)$. Now we list the general structure of the thermal polarization tensor defined in Eq. (14). According to the gauge symmetry and rotation symmetry of thermal field theory, it can be decomposed into two scalar functions as

$$\Pi_{\mu\nu}(k, T) = \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{Q^2}\right)\Pi_1(x, Q^2) + \left(n_\mu - k_\mu\frac{n\cdot k}{Q^2}\right)\left(n_\nu - k_\nu\frac{n\cdot k}{Q^2}\right)\Pi_2(x, Q^2), \quad (15)$$

where $\eta_{\mu\nu} = (-1, 1, 1, 1)$, $Q^2 \equiv k^\mu k_\mu$ is the current virtuality, and n_μ is the four-velocity of the plasma, which will be chosen as $n_\mu = (-1, 0, 0, 0)$ to signal that the plasma is at rest. We have also defined the Bjorken variable as $x = -Q^2/[2(n\cdot k)T]$. Then the DIS structure function of the plasma can be extracted from the polarization tensor as

$$F_1 \equiv \frac{1}{2\pi}\,\mathrm{Im}\Pi_1, \qquad F_2 \equiv \frac{-(n\cdot k)}{2\pi T}\,\mathrm{Im}\Pi_2. \tag{16}$$

For the sake of later convenience, we now proceed to express the above two functions in terms of the longitudinal and transverse polarization tensors Π_{LL} and Π_{yy} that will be introduced in Sec. III:

$$F_1 = \frac{1}{2\pi}\,\mathrm{Im}\Pi_{yy} = \frac{1}{2\pi}\,\mathrm{Im}\Pi_{zz}, \tag{17}$$

$$F_2 = \frac{\omega^2}{q^2}\left(\frac{Q^2 x}{\pi}\,\mathrm{Im}\Pi_{LL} + 2xF_1\right). \tag{18}$$

In obtaining these two expressions, we have used the flavor current momentum defined in Eq. (25) and assumed that the plasma is at rest. From Eq. (18), we can see that in the interesting kinematic regime, $\omega^2/q^2 = 1 - Q^2/q^2 \simeq 1$, F_2 can be simplified further to

$$F_2 \simeq \frac{Q^2 x}{\pi}\,\mathrm{Im}\Pi_{LL} + 2xF_1. \tag{19}$$

If the first term in Eq. (19) could be negligible, we then straightforwardly come to the familiar Callan-Gross relation $F_2 \simeq 2xF_1$.

In particle physics, the structure function has been studied by the operator product expansion technique for specific hard processes. The parton model is suitable for the weak-coupling regime of high-energy QCD, while in the present case, for holographic quark-gluon plasma, which has been thought of as strongly coupled, we resort to its gravity dual for calculations of above quantities. There is a standard prescription [13] for calculating the retarded Green function such as Eq. (14) under the approach of gauge/gravity duality. The current J_μ couples to its source $A_\mu(x, r=0)$ as

$$S_{\mathrm{int}} = \int d^4x J_\mu A^\mu(x, r=0), \tag{20}$$

where $A_\mu(x, r)$ is the flavor $U(1)$ gauge field introduced in the next subsection. The main idea under this prescription is to invert the operator Green function on the field theory side to a dual field Green function on the gravity side, whose calculations just need classical gravity action. More details on this prescription can be found in Ref. [13]. Now we explicitly write the expression for the polarization tensor defined in Eq. (14) in terms of the variables on the dual gravity side:

$$\Pi_{\mu\nu}(k, T) \equiv \frac{\delta^2 S}{\delta A_\mu(-k)\delta A_\nu(k)}\Bigg|_{r=0}, \tag{21}$$

where S is the on-shell action defined in Eqs. (26) and (34). Later, it will be found that, due to the coupling of A_x and A_t, we have to express the on-shell action in terms of longitudinal mode A_L and transverse modes A_y, A_z as defined in the next subsection.

C. Basic equations: Flavor current propagation in plasma

As in many works on applications of gauge/gravity duality to strong-coupling problems, we choose the radial gauge for gauge potential, i.e., $A_r = 0$. This is enough for the noncritical case, while for the Sakai-Sugimoto model we also have internal symmetry on Ω_4 space. For brevity, we also set the gauge potential along it to zero and assume that the gauge potential does not depend on the internal coordinate. As to the action, we just retain it to quadratic order in the gauge field fluctuation, which is enough for the propagation of the flavor current. In the following, we will write just the main equations for the gauge fields. (For details on their extraction, see Refs. [14,18].)

For the Sakai-Sugimoto model, the DBI action for the fluctuation of the flavor $U(1)$ gauge field takes the following form after integrating out the Ω_4 space:

$$S_8 = -\frac{(2\pi l_s^2)^2}{4} T_8 N_f V_{S^4} \int d^4x dr \sqrt{-g_{\mathrm{eff}}}\, g^{MN} g^{PQ} F_{MP} F_{NQ}, \tag{22}$$

$$\sqrt{-g_{\mathrm{eff}}} \equiv e^{-\phi}\sqrt{-\det g_5}\, g_{S^4}^2 = g_s^{-1} R^{3/2} u_T^{7/2} r^{-(9/2)}, \tag{23}$$

where $F_{MP} = \partial_M A_P - \partial_P A_M$ is the field strength of the gauge field A_M, g^{MN} is the inversion of the induced metric in Eq. (9), the index M, N and so on just need to run the former five coordinates in Eq. (9), T_8 is the D8-brane tension, and V_{S^4} is the volume of unit sphere Ω_4. The equation of motion (EOM) for the gauge field A_M followed from this action is of Maxwell type:

$$\partial_M(\sqrt{-g_{\mathrm{eff}}}\, g^{MN} g^{PQ} F_{NQ}) = 0. \tag{24}$$

Now we turn to the momentum space by doing the following partial Fourier transformation of the gauge field:

$$A_\mu(x, r) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} A_\mu(k_\mu, r), \qquad k_\mu = (-\omega, q, 0, 0). \tag{25}$$

Without loss of generality, we have chosen the spatial momentum along just one spatial direction as done in the literature. Then, in the partial momentum space, the on-shell action turns into the following form:

$$S_8 = -\frac{(2\pi l_s^2)^2}{2} T_8 N_f V_{S^4} \int \frac{d^4k}{(2\pi)^4} \times \sqrt{-g_{\mathrm{eff}}}\, g^{rr}\{g^{tt} A_t(-k, r)\partial_r A_t(k, r) + g^{ii} A_i(-k, r)\partial_r A_i(k, r)\}|_{r=0}^{r=1} \quad (i = x, y, z), \tag{26}$$

while Eq. (24) can be explicitly cast into the following three forms:

$$\omega A_t' + q f(r) A_x' = 0, \tag{27}$$

$$A_y'' + \frac{\partial_r(\sqrt{-g_{\rm eff}} g^{rr} g^{yy})}{\sqrt{-g_{\rm eff}} g^{rr} g^{yy}} A_y' - \frac{g^{yy}}{g^{rr}}\left(q^2 - \frac{\omega^2}{f(r)}\right) A_y = 0, \tag{28}$$

$$\begin{aligned} A_t''' &+ \left[\frac{2\partial_r(\sqrt{-g_{\rm eff}} g^{rr} g^{tt})}{\sqrt{-g_{\rm eff}} g^{rr} g^{tt}} - \frac{\partial_r(\sqrt{-g_{\rm eff}} g^{tt} g^{xx})}{\sqrt{-g_{\rm eff}} g^{tt} g^{xx}}\right] A_t'' \\ &+ \left\{\frac{\partial_r^2(\sqrt{-g_{\rm eff}} g^{rr} g^{tt})}{\sqrt{-g_{\rm eff}} g^{rr} g^{tt}} - \frac{g^{xx}}{g^{rr}}\left(q^2 - \frac{\omega^2}{f(r)}\right) \right. \\ &\left. - \frac{\partial_r(\sqrt{-g_{\rm eff}} g^{tt} g^{xx})}{\sqrt{-g_{\rm eff}} g^{tt} g^{xx}} \frac{\partial_r(\sqrt{-g_{\rm eff}} g^{rr} g^{tt})}{\sqrt{-g_{\rm eff}} g^{rr} g^{tt}}\right\} A_t' = 0. \end{aligned} \tag{29}$$

The component A_z satisfies the same equation as the A_y component in Eq. (28), and we refer to them as transverse modes. Another important relation between A_x and A_t is

$$\partial_r(\sqrt{-g_{\rm eff}} g^{rr} g^{tt} A_t') - \sqrt{-g_{\rm eff}} g^{tt} g^{xx} q(\omega A_x + q A_t) = 0. \tag{30}$$

In the above equations, prime denotes the derivative with respect to the radial coordinate r. Once the induced metric is inserted into Eqs. (28)–(30), they can be simplified to the following forms:

$$a'' + \left[\frac{1}{2r} + \frac{f'(r)}{f(r)}\right] a' + \left[\frac{\tilde\omega^2 - \tilde q^2 f(r)}{r f^2(r)} - \frac{f'(r)}{2r f(r)}\right] a = 0, \tag{31}$$

$$A_y'' + \left[-\frac{1}{2r} + \frac{f'(r)}{f(r)}\right] A_y' - \frac{\tilde q^2 f(r) - \tilde\omega^2}{r f^2(r)} A_y = 0, \tag{32}$$

$$A_{\rm L} = \frac{u_T}{R^3} q^{-1} r^{3/2} f(r) (r^{-1/2} a)', \tag{33}$$

where $(\tilde\omega, \tilde q) \equiv \frac{3}{4\pi T}(\omega, q)$ are dimensionless variables, a denotes A_t', and A_L is the longitudinal mode defined as $A_L \equiv q A_t + \omega A_x$. Note that we have written Eqs. (27)–(30) in a general form so that they should also be suitable for the noncritical model being studied later.

Similarly, we could write similar equations for the noncritical version of the Sakai-Sugimoto model, but for brevity we just list the main results like Eqs. (26) and (31)–(33):

$$\begin{aligned} S_4 = &-\frac{(2\pi l_s^2)^2}{2} \frac{T_4 N_f}{e^\phi} \frac{u_T^2}{R^3} \int \frac{d^4k}{(2\pi)^4} \frac{1}{r} \\ &\times [f(r) A_i(-k, r) \partial_r A_i(k, r) \\ &- A_t(-k, r) \partial_r A_t(k, r)]|_{r=0}^{r=1} \qquad (i = x, y, z), \end{aligned} \tag{34}$$

$$a'' + \left[\frac{f'(r)}{f(r)} - \frac{1}{r}\right] a' + \left[\frac{1}{r^2} - \frac{f'(r)}{r f(r)} + \frac{\tilde\omega^2 - \tilde q^2 f(r)}{f^2(r)}\right] a = 0, \tag{35}$$

$$A_y'' + \left[\frac{f'(r)}{f(r)} - \frac{1}{r}\right] A_y' + \frac{\tilde\omega^2 - \tilde q^2 f(r)}{f^2(r)} A_y = 0, \tag{36}$$

$$A_L \equiv \frac{u_T^2}{R^4} \frac{1}{q} r f(r) \left(\frac{a}{r}\right)'. \tag{37}$$

Here the only difference from the Sakai-Sugimoto model in notation lies in the specific definitions of the dimensionless variables $\tilde\omega$ and $\tilde q$:

$$\tilde\omega \equiv \frac{\omega}{0.8\pi T}, \qquad \tilde q \equiv \frac{q}{0.8\pi T}. \tag{38}$$

We now express the flavor brane actions in terms of $A_{\rm L}$,A_y, A_z and a, as mentioned in last subsection. Because similar calculations have been done in the literature many times, we here merely list the final results. For the Sakai-Sugimoto model, we have

$$\begin{aligned} S_8 = &-\frac{(2\pi l_s^2)^2}{2} T_8 N_f V_{S^4} \int \frac{d^4k}{(2\pi)^4} \\ &\times \sqrt{-g_{\rm eff}} g^{rr} \{g^{tt} q^{-1} A_L(-k, r) a(k, r) \\ &+ g^{ii} A_i(-k, r) \partial_r A_i(k, r)\}|_{r=0}^{r=1} \qquad (i = y, z), \end{aligned} \tag{39}$$

and for the noncritical model

$$\begin{aligned} S_4 = &-\frac{(2\pi l_s^2)^2}{2} \frac{T_4 N_f}{e^\phi} \frac{u_T^2}{R^3} \\ &\times \int \frac{d^4k}{(2\pi)^4} r^{-1} \{-q^{-1} A_L(-k, r) a(k, r) \\ &+ f(r) A_i(-k, r) \partial_r A_i(k, r)\}|_{r=0}^{r=1} \qquad (i = y, z). \end{aligned} \tag{40}$$

Some remarks are due about these equations which will determine later calculations. Explicitly, these equations are more involved than the related ones in Ref. [7], which have already been confirmed in Ref. [8] for D3/D7-brane setup. We can attribute these complexities to the introduction of the flavor branes and the nonconformality of the model. However, once these EOMs are turned into standard Schrödinger types, and taking high-momentum and high-frequency limits, we will find that these complications will automatically disappear and the equations are qualitatively in common with those of Ref. [7].

We now proceed by following Ref. [7] to turn these EOMs into standard Schrödinger types and discuss some general features of effective potentials, respectively. We will find that the general discussions for potentials will reveal some physical pictures for the DIS processes being considered in this paper.

The equations obeyed by new fields have the following common form irrespective of the models:

$$-\psi_L'' + V_L(r)\psi_L = 0 \qquad \text{(for time component } a\text{)}, \tag{41}$$

$$-\psi_T'' + V_T(r)\psi_T = 0 \quad \text{(for transverse component } A_y \text{ or } A_z\text{)}. \tag{42}$$

For the Sakai-Sugimoto model, the explicit field transformations and effective potentials are as follows:

$$a = \sqrt{\frac{1}{(1-r^3)r^{1/2}}}\psi_L, \tag{43}$$

$$V_L(r) = \frac{1}{r(1-r^3)^2}\left[\frac{1}{16r}(-3 - 78r^3 + 45r^6) + K^2 - \tilde{q}^2 r^3\right]; \tag{44}$$

$$A_y = \sqrt{\frac{r^{1/2}}{1-r^3}}\psi_T, \tag{45}$$

$$V_T(r) = \frac{1}{r(1-r^3)^2}\left[\frac{1}{16r}(5 - 46r^3 + 5r^6) + K^2 - \tilde{q}^2 r^3\right]. \tag{46}$$

While for the noncritical model, we have following respective ones:

$$a = \sqrt{\frac{r}{(1-r^5)}}\psi_L, \tag{47}$$

$$V_L(r) = \frac{1}{(1-r^5)^2}\left[\frac{1}{4r^2}(-1 - 48r^5 + 24r^{10}) + K^2 - \tilde{q}^2 r^5\right]; \tag{48}$$

$$A_y = \sqrt{\frac{r}{(1-r^5)}}\psi_T, \tag{49}$$

$$V_T(r) = \frac{1}{(1-r^5)^2}\left[\frac{1}{4r^2}(3 + 8r^{10} - 36r^5) + K^2 - \tilde{q}^2 r^5\right]. \tag{50}$$

In these equations, we have defined the dimensionless current virtuality as $K^2 \equiv \tilde{q}^2 - \tilde{\omega}^2$. We will take this virtuality to be spacelike, which amounts to saying that the process considered here is like the lepton deep inelastic scattering off the proton. But as noted in Ref. [7], the final results will also be suitable for timelike virtuality if we take the high-momentum limit (especially if $\tilde{q}^2 \gg K^2$). Since we are interested in the internal structure of holographic quark-gluon plasma, we should use a high-energy probe to explore this just like the DIS processes in pQCD. One basic difference between the plasma and a single hadron is that the former has an intrinsic scale (temperature). In short, we are focusing on the following kinematic parameter space:

$$\tilde{\omega} \gg 1, \qquad \tilde{q} \gg 1, \qquad K^2 \gg 1. \tag{51}$$

Under the above kinematics, these effective potentials can be further approximated as follows.[7] For the Sakai-Sugimoto model, we have

$$V_L(r) = \frac{1}{r(1-r^3)^2}\left[-\frac{3}{16r} + K^2 - \tilde{q}^2 r^3\right], \tag{52}$$

$$V_T(r) = \frac{1}{r(1-r^3)^2}\left[\frac{5}{16r} + K^2 - \tilde{q}^2 r^3\right]; \tag{53}$$

while for the noncritical model, we have

$$V_L(r) = \frac{1}{(1-r^5)^2}\left[-\frac{1}{4r^2} + K^2 - \tilde{q}^2 r^5\right], \tag{54}$$

$$V_T(r) = \frac{1}{(1-r^5)^2}\left[\frac{3}{4r^2} + K^2 - \tilde{q}^2 r^5\right]. \tag{55}$$

Equations (41), (42), and (52)–(55) are the main ingredients for later extraction of the structure functions of holographic plasma.

We have seen that, in the interesting kinematic regime symbolized by Eq. (51), for both the Sakai-Sugimoto model and its noncritical version, the effective potentials[8] for longitudinal as well as transverse modes are qualitatively similar to those of the $\mathcal{N} = 4$ SYM case with(out) flavors [7,8]. More explicitly, the maximum of the longitudinal potential can be positive (corresponding to potential barrier), negative (with no barrier at all), or zero according to the value of $\tilde{q}/K^4$ (for the Sakai-Sugimoto model) or $\tilde{q}^2/K^7$ (for the noncritical model). While effective, the potentials for the transverse modes are more involved because they start from a positive infinity and then fall to negative infinity very rapidly, which may complicate later analysis using WKB approximation to construct the wave function ψ_T. Recalling the fact that the dilaton vacuum for the Sakai-Sugimoto model is not a constant while for the noncritical version it is constant, these facts together seem to indicate that the nonconformality of the Sakai-Sugimoto model is not essential to the physical picture of the high-energy scattering process. This is a by-product of the general behavior analysis for the

[7]For the following four potentials, we have ignored polynomials of r in their final expressions. Strictly speaking, this is not right; but since we are concerned, in particular, with their behavior near the boundary $r = 0$, which directly determines the polarization tensor, and their behavior at the horizon $r = 1$ is just for the incoming wave boundary condition, we believe this choice is reasonable.

[8]We should have plotted these effective potentials for illustration as in Refs. [7,8], but we skip it here for brevity since the plots are basically the same.

effective potentials; later we will confirm this observation by direct extraction of the structure functions for both holographic models.

Before concluding this section, we briefly summarize the physical picture in Ref. [7] governing high-energy DIS from the viewpoint of nonrelativistic quantum mechanics. For the longitudinal mode, when the potential barrier builds in (corresponding to the small spatial-momentum case), the wave function cannot be imposed on the incoming wave boundary condition at the horizon due to the high and narrow barrier, which indicates zero structure function. When taking into account the nonperturbative tunneling effect, a small exponentially suppressed structure function can be obtained. Therefore, in the following sections, we focus merely on the high spatial-momentum limit. In this regime, the wave function will be complex and an incoming wave boundary condition can be imposed at the horizon. Moreover, in this high-energy regime, a partonic picture for the plasma exists.

III. STRUCTURE FUNCTION OF HOLOGRAPHIC QUARK-GLUON PLASMA

Now we have all the elements to calculate the polarization tensor defined in Eq. (14). As mentioned above, we should focus on the high-momentum kinematics besides the one defined in Eq. (51). This means that the K^2 terms in effective potentials can be ignored as well, which makes semianalytical solutions for these Schrödinger equations possible. In this section, we follow the standard WKB approach in nonrelativistic quantum mechanics to construct these solutions. We present the calculations in detail for the Sakai-Sugimoto model and then list the final results for its noncritical version.

We first discuss the longitudinal mode. Under the high-momentum approximation discussed in the previous paragraph, the Schrödinger equation for longitudinal mode then takes the following simple form near $r \simeq 0$:

$$\psi_L''(r) + \left(\frac{3}{16r^2} + \tilde{q}^2 r^2\right)\psi_L(r) = 0. \tag{56}$$

Its general solution is a linear combination of the Bessel and Neumann functions:

$$\psi_L(r \simeq 0) = c_1 \tilde{q}^{1/4} r^{1/2} J_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) + c_2 \tilde{q}^{1/4} r^{1/2} N_{1/8}\left(\frac{\tilde{q}r^2}{2}\right), \tag{57}$$

and the constants c_1, c_2 will be determined by imposing an incoming wave boundary condition at the horizon, which requires matches of solutions at different regimes.

Near $r \simeq 1$, the Schrödinger equation can be approximated as

$$\psi_L''(r) + \frac{\tilde{q}^2}{9(1-r)^2}\psi_L(r) = 0. \tag{58}$$

Its general solution is

$$\psi_L(r \simeq 1) = c_3(1-r)^{(1/2)-(i(\tilde{q}/3))} + c_4(1-r)^{(1/2)+(i(\tilde{q}/3))}. \tag{59}$$

Recalling Eqs. (43) and (45) and imposing an incoming wave boundary condition at the horizon make us to conclude that $c_4 = 0$, thus leaving the general solution near the horizon to be

$$\psi_L(r \simeq 1) = c_3(1-r)^{(1/2)-i(\tilde{q}/3)}. \tag{60}$$

Besides, we have to study in detail the solution suitable for an intermediate regime far from the singularities at $r = 0$ and $r = 1$. In this regime, the Schrödinger equation is approximated as

$$\psi_L''(r) + \frac{\tilde{q}^2 r^2}{(1-r^3)^2}\psi_L(r) = 0. \tag{61}$$

For convenience, we now define the so-called canonical momentum $p(r)$ and action $s(r)$:

$$p(r) = \frac{\tilde{q}r}{1-r^3}, \qquad s(r) = \int_0^r p(r)dr = \int_0^r dr\frac{\tilde{q}r}{1-r^3}.$$

Then the two linear independent solutions in the intermediate regime under WKB approximation are

$$\psi_L^{(1)}(r) = \frac{1}{\sqrt{p(r)}}\cos[s(r) + \phi_1], \tag{62}$$

$$\psi_L^{(2)}(r) = \frac{1}{\sqrt{p(r)}}\sin[s(r) + \phi_2]. \tag{63}$$

The asymptotic behaviors for $p(r)$ and $s(r)$ at singularities are necessary for the solution matching underlying different regimes:

$$p(r \simeq 0) \simeq \tilde{q}r \quad \text{and} \quad p(r \simeq 1) \simeq \frac{\tilde{q}}{3(1-r)}, \tag{64}$$

$$s(r \simeq 0) \simeq \frac{\tilde{q}r^2}{2} \quad \text{and}$$
$$s(r \simeq 1) \simeq -\frac{\tilde{q}}{3}\log(1-r) + \text{constant}. \tag{65}$$

The next step is to match these solutions to determine c_1, c_2, and c_3. In doing this, we need the asymptotic expansion for the Bessel or Neumann function for a very large variable[9]:

$$J_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) \simeq \sqrt{\frac{4}{\pi\tilde{q}r^2}}\cos\left(\frac{\tilde{q}r^2}{2} - \frac{\pi}{16} - \frac{\pi}{4}\right), \tag{66}$$

[9] Although r is small here, $\tilde{q}$ is large and we therefore regard $\tilde{q}r^2/2$ to be large enough.

$$N_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) \simeq \sqrt{\frac{4}{\pi\tilde{q}r^2}}\sin\left(\frac{\tilde{q}r^2}{2} - \frac{\pi}{16} - \frac{\pi}{4}\right). \tag{67}$$

It is then easily found that, if we choose $\phi_1 = \phi_2 = -5\pi/16$ and $c_2 = ic_1$, the matching of solutions in different regimes is accomplished. Note that the condition $c_2 = ic_1$ is the main result from the solution matching, which is also a direct reflection of the incoming wave boundary condition imposed at the horizon.

Now we easily arrive at the boundary behavior for the wave function $\psi_L(r)$ as

$$\psi_L(r \simeq 0) = c_1\tilde{q}^{1/4}r^{1/2}J_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) + ic_1\tilde{q}^{1/4}r^{1/2}N_{1/8}\left(\frac{\tilde{q}r^2}{2}\right) \equiv c_1\tilde{q}^{1/4}r^{1/2}H^{(1)}_{1/8}\left(\frac{\tilde{q}r^2}{2}\right). \tag{68}$$

In the second line of Eq. (68), we have written the solution as the first-kind Hankel function with order $1/8$.

The effective potential for transverse mode is more involved, but the analysis under WKB approximation is similar; therefore we here list just the final results. Matching of solutions in three different regimes (near horizon, near boundary, and intermediate regime far from these singularities) results in the following boundary behavior for transverse modes A_y and A_z:

$$\psi_T(r \simeq 0) = c_1^i\tilde{q}^{1/4}\sqrt{\frac{r^{3/2}}{1-r^3}}H^{(1)}_{3/8}(\tilde{q}r^2/2). \tag{69}$$

The only undetermined constants c_1 and c_1^i can be expressed in terms of boundary values of the gauge field $A_L(r=0) \equiv A_L(0)$ or $A_{y,z}(r=0) \equiv A_i(0)$, respectively. This can be achieved by using Eqs. (33), (43), and (45), and the final results essential for later extraction of structure functions are listed below. For longitudinal mode, we have

$$A_L(r \simeq 0) = c_1\frac{u_T}{R^3}q^{-1}\tilde{q}^{1/4}r^{3/2}(1-r^3) \times \left\{\sqrt{\frac{1}{(1-r^3)r^{1/2}}}H^{(1)}_{1/8}\left(\frac{\tilde{q}r^2}{2}\right)\right\}', \tag{70}$$

$$a(r \simeq 0) = c_1\tilde{q}^{1/4}\sqrt{\frac{r^{1/2}}{(1-r^3)}}H^{(1)}_{1/8}\left(\frac{\tilde{q}r^2}{2}\right), \tag{71}$$

$$c_1 = \frac{-i\pi 2^{3/4}q}{\Gamma(1/8)\tilde{q}^{1/8}}\frac{R^3}{u_T}A_L(0); \tag{72}$$

and for transverse modes A_i ($i = y, z$), we have

$$A_i(r \simeq 0) = c_1^i\tilde{q}^{1/4}\sqrt{\frac{r^{3/2}}{1-r^3}}H^{(1)}_{3/8}\left(\frac{\tilde{q}r^2}{2}\right), \tag{73}$$

$$c_1^i = \frac{i\pi\tilde{q}^{1/8}}{2^{3/4}\Gamma(3/8)}A_i(0). \tag{74}$$

With these solutions, we can derive the expressions for the thermal polarization tensor defined in Eq. (14):

$$\mathrm{Im}\,\Pi_{LL}(k,T) = \frac{\sqrt{2}l_s}{12\Gamma^2(1/8)g_s}\lambda N_f N_c T\tilde{q}^{1/4}, \tag{75}$$

$$\mathrm{Im}\,\Pi_{yy}(k,T) = \mathrm{Im}\Pi_{zz}(k,T) = \frac{\sqrt{2}\pi l_s}{54\Gamma^2(3/8)g_s}\lambda N_f N_c T^3\tilde{q}^{3/4}, \tag{76}$$

and other components are exactly vanishing. Referring to the above results for the polarization tensor, we have listed just the imaginary parts that are directly related to the structure function. In fact, the real part of the polarization tensor are divergent and therefore need to be regularized. (Here, however, we have skipped these details.) Moreover, we here for simplification assume the regularization will not introduce new terms into the imaginary parts. Therefore, the structure functions F_1 and F_2 are easily derived via Eqs. (17)–(19):

$$F_1(k,T) = \frac{\sqrt{2}l_s}{108\Gamma^2(3/8)g_s}\lambda N_f N_c T^3\tilde{q}^{3/4} \simeq \frac{l_s}{g_s}\lambda N_f N_c T^3\tilde{q}^{3/4}, \tag{77}$$

$$F_2(k,T) = \frac{\omega^2}{q^2}\left[\frac{Q^2x}{\pi}\frac{\sqrt{2}l_s}{12\Gamma^2(1/8)g_s}\lambda N_f N_c T\tilde{q}^{1/4} + 2xF_1\right] \simeq 2xF_1. \tag{78}$$

In the first line of Eq. (78), one can easily show that the first term can be ignored when comparing to the second one in the interesting kinematic regime $\tilde{q}/K^4 \gg 1$. Following Ref. [7], we now express the two structure functions in terms of the Bjorken variable x defined in Sec. II B and the flavor current virtuality Q^2 as

$$F_1(x,Q^2) = \frac{\sqrt{2}l_s}{108\Gamma^2(3/8)g_s}\lambda N_f N_c T^3\left(\frac{3Q^2}{8\pi x T^2}\right)^{3/4} \sim \lambda N_f N_c T^3\left(\frac{3Q^2}{8\pi x T^2}\right)^{3/4}, \tag{79}$$

$$F_2(x,Q^2) \simeq 2xF_1(x,Q^2) \sim 2\lambda N_f N_c T^3 x\left(\frac{3Q^2}{8\pi x T^2}\right)^{3/4}. \tag{80}$$

In obtaining the above two equations, we have used one approximate relation $\tilde{\omega} \simeq \tilde{q}$ to express the spatial momentum as $q \simeq Q^2/(2xT)$.

Before closing this subsection, we briefly carry out similar analysis for the noncritical model. Since in the previous subsection the general procedure for using the WKB method to solve the Schrödinger problem was presented in detail, we here list just the corresponding key results. The solution for the longitudinal mode near boundary $r = 0$ behaves as

$$A_L(r \simeq 0) = C_1 \frac{u_T^2}{R^4} q^{-1} \tilde{q}^{1/7} r(1 - r^5) \times \left\{ \sqrt{\frac{1}{1 - r^5}} H_0^{(1)}\left(\frac{2}{7} \tilde{q} r^{7/2}\right) \right\}', \tag{81}$$

$$a(r \simeq 0) = C_1 \tilde{q}^{1/7} \frac{r}{\sqrt{1 - r^5}} H_0^{(1)}\left(\frac{2}{7} \tilde{q} r^{7/2}\right), \tag{82}$$

$$C_1 = -\frac{i\pi}{7} \frac{q}{\tilde{q}^{1/7}} \frac{R^4}{u_T^2} A_L(0); \tag{83}$$

and for transverse mode $A_i(r)$ $(i = y, z)$,

$$A_i(r \simeq 0) = C_1^i \tilde{q}^{1/7} \frac{r}{\sqrt{1 - r^5}} H_{2/7}^{(1)}\left(\frac{2}{7} \tilde{q} r^{7/2}\right), \tag{84}$$

$$C_1^i = \frac{i\pi \tilde{q}^{1/7}}{7^{2/7} \Gamma(2/7)} A_i(0). \tag{85}$$

The polarization tensor can be derived by inserting these solutions into the on-shell action Eq. (40):

$$\mathrm{Im}\,\Pi_{LL}(k, T) = \frac{\sqrt{3} R N_f N_c}{56\sqrt{2}\pi g_s l_s}, \tag{86}$$

$$\mathrm{Im}\,\Pi_{yy}(k, T) = \frac{7\sqrt{6}\pi R N_f N_c}{25\Gamma^2(2/7) g_s l_s} T^2 \left(\frac{\tilde{q}}{7}\right)^{4/7}. \tag{87}$$

Then the structure functions are

$$F_1(k, T) = \frac{7\sqrt{6} R N_f N_c}{50\Gamma^2(2/7) g_s l_s} T^2 \left(\frac{\tilde{q}}{7}\right)^{4/7} \simeq N_f N_c T^2 \left(\frac{\tilde{q}}{7}\right)^{4/7}, \tag{88}$$

$$F_2(k, T) = \frac{\omega^2}{q^2} \left[\frac{Q^2 x}{\pi} \frac{\sqrt{3} N_f N_c}{56\sqrt{2}\pi g_s l_s} + \frac{7\sqrt{6} R N_f N_c}{25\Gamma^2(2/7) g_s l_s} x T^2 \left(\frac{\tilde{q}}{7}\right)^{4/7} \right] \simeq 2x F_1(k, T). \tag{89}$$

In the first line of Eq. (89), one can easily show that the first term can be neglected in the interesting regime $\tilde{q}^{4/7} \gg K^2$, so we have the approximate equality of the third line as in the Sakai-Sugimoto model. We now express these results in terms of the Bjorken variable x and the flavor current virtuality Q^2 as in the Sakai-Sugimoto model:

$$F_1(x, Q^2) \simeq \frac{7\sqrt{6} R}{50\Gamma^2(2/7) g_s l_s} N_f N_c T^2 \left(\frac{5Q^2}{8\pi x T^2}\right)^{4/7} \sim N_f N_c T^2 \left(\frac{5Q^2}{8\pi x T^2}\right)^{4/7}, \tag{90}$$

$$F_2(x, Q^2) \simeq 2x F_1(x, Q^2) \sim 2 N_f N_c T^2 x \left(\frac{5Q^2}{8\pi x T^2}\right)^{4/7}. \tag{91}$$

Equations (77)–(80) and (88)–(91) comprise the main results of this paper. Now we have a short remark on these results and also a brief comparison between these two models as well as the well-investigated $\mathcal{N} = 4$ SYM case.

The first point is that we have an analogy of the Callan-Gross relation $F_2 \simeq 2xF_1$ in the interesting kinematic regime considered in this paper. In pQCD, this relation holds only at relatively large values of the Bjorken variable x, where parton structures of hadrons are dominated by the valence quarks. This relation has already been obtained in DIS off $\mathcal{N} = 4$ SYM plasma with(out) flavors, and here it also holds for the Sakai-Sugimoto model as well as its noncritical version. Since the setups of holographic dual of sQGP are very general, and, what is more, the physical picture underlying the $\mathcal{R}$ or the flavor current DIS off the plasma is quite simple and general, these surprising facts seem to indicate that it may be a general relation for holographic quark-gluon plasma.

The second key point is concerned with the nonconformal characteristic of the Sakai-Sugimoto model, which is also the main motivation for the present study. Because, in our interesting kinematic regime, the two structure functions are related to each other by the Callan-Gross relation, we then mainly focus on the function F_1. It is clear that for two models F_1 presents scaling behavior on their dependence on temperature T and dimensionless spatial momentum $\tilde{q}$. Their dependence on dimensionless momentum $\tilde{q}$ is approximately the same, while on temperature is quite different: $\sim T^3$ for the Sakai-Sugimoto model and $\sim T^2$ for the noncritical model. Recalling that the latter scaling behavior $\sim T^2$ has also been valid in Ref. [7] for the $\mathcal{N} = 4$ SYM plasma, we may think of the essential effect of the nonconformality of the Sakai-Sugimoto model as the T^3-scaling behavior of the F_1 structure function. However, this guess needs further confirmation or cancellation because the nonconformality of the Sakai-Sugimoto model is not well-controlled. Moreover, because the gauge coupling constant of strong interaction has a logarithmic running with an evolving energy scale, and simple gravity realization of this kind of gauge theory has been established by combining top-down and bottom-up approaches in Ref. [19], it may be interesting to resort to this kind of model to probe the effect of gauge coupling running on the internal structure of sQGP.

The last point we would like to stress is about the prefactors for the structure functions. Similar to Ref. [8], $N_f N_c$ counts the number of freedom of the plasma, and we here probe the quark sector. The models considered in this paper have left some stringy imprints on the quantities of the field theoretical side. (Here they are the plasma structure function.) This can be easily read from Eqs. (77), (78), (88), and (89), which have explicit dependence on the string-coupling constant g_s and the string length l_s. Moreover, the behavior concerning these two stringy parameters seems to be different between the Sakai-Sugimoto model and its noncritical version. These facts seem to be very strange and even unacceptable because we here focused on the field-side quantities, and they should not show explicit dependence on the parameters on the gravity or string side. Compared to related results of the $\mathcal{N} = 4$ SYM plasma, these undesirable behaviors have not come out there, which seems to say that the D3-brane geometry is a much better gravity dual of some quantum field theory in describing field theoretical physical quantities under the gauge/gravity duality technique. However, if we recall that the models we take here are different from realistic QCD theory, then it is acceptable that the results are counterintuitive from the viewpoint of field theoretical considerations.

IV. SUMMARY AND OUTLOOK

In this paper, we have used a high-temperature version of the Sakai-Sugimoto model, a quite successful gravity dual model of QCD-like theory, to explore the internal structure of holographic quark-gluon plasma. The physical process we analyze here is like the well-investigated DIS in standard QCD, but with the scattered proton replaced by the plasma system and the mediated electromagnetic current simulated by the flavor current, as in Ref. [8]. We have seen that the procedure under the gauge/gravity duality technique to study DIS off holographic quark-gluon plasma is quite general and easily promoted to other gauge/gravity duality setups. The results obtained in this paper for the structure function under the Sakai-Sugimoto model is quite different from the well-studied $\mathcal{N} = 4$ SYM plasma with(out) flavors. This should be regarded as the effect of the nonconformality of the Sakai-Sugimoto model, which is the most important motivation for our present study. To confirm this, we have also chosen the noncritical version of the Sakai-Sugimoto model for a comparative study. We found that the structure functions for the latter model and the $\mathcal{N} = 4$ SYM plasma are much alike. The result of this paper seems to contradict intuition from the fact of asymptotic freedom of pQCD. But we should keep in mind that holographic quark-gluon plasma considered in this paper is a strongly coupled system, and thus we should not expect it to behave exactly as the weakly interacting regime of realistic QCD. In fact, a more realistic holographic QCD model taking care of the running of the gauge coupling constant was proposed in Ref. [19], and we expect to use this model to explore the effect of gauge coupling running on the structure of sQGP.

In realistic QCD, N_f and N_c are of $\mathcal{O}(1)$, while the applicability of the gravity dual of $SU(N_c)$ gauge theory usually requires a large N_c limit. Therefore, we cannot directly compare our results with the data from heavy-ion collision. One prescription to overcome this obstacle is to consider the flavor backreaction to the background geometry and then carry out similar calculations in the $N_f/N_c \sim 1$ limit. Although the hadronic matter produced in heavy-ion collision is of high temperature and high density, in this paper we take into account only the high-temperature element as in the literature. So, in this sense, our present models are not so realistic and need to be promoted to the high-energy and high-density quark-gluon plasma case. Fortunately, under the gauge/gravity duality setup, the matter density also has a gravity realization—$\mathcal{R}$ charge or flavor charge—which can be realized by rotating the color brane along the internal space or as the time component of the flavor gauge field, respectively. Then the analysis of DIS off quark-gluon plasma at high density and high temperature can also be carried out by including this element in the model setup. We leave these problems for future investigations.

ACKNOWLEDGMENTS

Y. Y. B. greatly thanks Zhen-Hua Zhang for useful discussions. This work was supported in part by NSFC Grant No. 10821504 and No. 10725526.

[1] E. Shuryak, Prog. Part. Nucl. Phys. **53**, 273 (2004).

[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[3] R. C. Myers and S. E. Vazquez, Classical Quantum Gravity **25**, 114008 (2008); M. P. Heller, R. A. Janik, and R. Peschanski, Acta Phys. Pol. B **39**, 3183 (2008); E. Iancu, *ibid.* **39**, 3213 (2008); S. S. Gubser and A. Karch, Annu. Rev. Nucl. Part. Sci. **59**, 145 (2009); R. A. Janik, Lect. Notes Phys. **828**, 147 (2011); J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, and U. A. Wiedemann, arXiv:1101.0618.

[4] H. Boschi-Filho and N. R. F. Braga, Phys. Lett. B **560**, 232 (2003); R. C. Brower and C.-I. Tan, Nucl. Phys. **B662**, 393 (2003); K. Kang and H. Nastase, Phys. Lett. B **624**, 125 (2005); R. C. Brower, J. Polchinski, M. J. Strassler, and C.-I. Tan, J. High Energy Phys. 12 (2007) 005; S. Lin and E.

Shuryak, Phys. Rev. D **77**, 085013 (2008); Y. Hatta, E. Iancu, and A. H. Mueller, J. High Energy Phys. 01 (2008) 026; C. A. Ballon Bayona, H. Boschi-Filho, and N. R. F. Braga, *ibid.* 03 (2008) 064; 10 (2008) 088; L. Cornalba and M. S. Costa, Phys. Rev. D **78**, 096010 (2008); B. Pire, C. Roiesnel, L. Szymanowski, and S. Wallon, Phys. Lett. B **670**, 84 (2008); C. A. Ballon Bayona, H. Boschi-Filho, and N. R. F. Braga, J. High Energy Phys. 09 (2008) 114; J.-H. Gao and B.-W. Xiao, Phys. Rev. D **80**, 015025 (2009); E. Avsar, E. Iancu, L. McLerran, and D. N. Triantafyllopoulos, J. High Energy Phys. 11 (2009) 105; J.-H. Gao and B.-W. Xiao, Phys. Rev. D **81**, 035008 (2010); B. Hassanain and M. Schvellinger, J. High Energy Phys. 04 (2010) 012.

[5] L. Cornalba, M. S. Costa, and J. Penedones, Phys. Rev. Lett. **105**, 072003 (2010); M. A. Betemps, V. P. Goncalves, and J. T. de Santana Amaral, Phys. Rev. D **81**, 094012 (2010); C. Marquet, C. Roiesnel, and S. Wallon, J. High Energy Phys. 04 (2010) 051; J.-H. Gao and Z.-G. Mou, Phys. Rev. D **81**, 096006 (2010); E. Levin and I. Potashnikova, J. High Energy Phys. 08 (2010) 112; C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, and M. A. C. Torres, J. High Energy Phys. 10, (2010) 055; C. A. B. Bayona, H. Boschi-Filho, and N. R. F. Braga, Nucl. Phys. **B851**, 66 (2011); A. Vega, I. Schmidt, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **83**, 036001 (2011); R. Nishio and T. Watari, arXiv:1105.2907; arXiv:1105.2999 [Phys. Rev. D, to be published].

[6] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88**, 031601 (2002); J. High Energy Phys. 05 (2003) 012.

[7] Y. Hatta, E. Iancu, and A. H. Mueller, J. High Energy Phys. 01 (2008) 063.

[8] C. A. Ballon Bayona, H. Boschi-Filho, and N. R. F. Braga, Phys. Rev. D **81**, 086003 (2010); E. Iancu and A. H. Mueller, J. High Energy Phys. 02 (2010) 023.

[9] Y. Hatta, E. Iancu, and A. H. Mueller, J. High Energy Phys. 05 (2008) 037.

[10] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005); **114**, 1083 (2005).

[11] G. Mandal and T. Morita, arXiv:1107.4048.

[12] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).

[13] D. T. Son and A. O. Starinets, J. High Energy Phys. 09 (2002) 042.

[14] N. Evans and E. Threlfall, Phys. Rev. D **77**, 126008 (2008).

[15] S. Kuperstein and J. Sonnenschein, J. High Energy Phys. 07 (2004) 049; 11 (2004) 026; R. Casero, A. Paredes, and J. Sonnenschein, J. High Energy Phys. 01 (2006) 127.

[16] V. Mazu and J. Sonnenschein, J. High Energy Phys. 06 (2008) 091.

[17] O. Aharony, J. Sonnenschein, and S. Yankielowicz, Ann. Phys. (N.Y.) **322**, 1420 (2007); A. Parnachev and D. A. Sahakyan, Phys. Rev. Lett. **97**, 111601 (2006).

[18] Y. Y. Bu and J. M. Yang, arXiv:1105.3646.

[19] U. Gursoy and E. Kiritsis, J. High Energy Phys. 02 (2008) 032; U. Gursoy, E. Kiritsis, and F. Nitti, J. High Energy Phys. 02 (2008) 019.