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Phenomenology of gravitational aether as a solution to the old cosmological constant problem

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One of the deepest and most long-standing mysteries in physics has been the huge discrepancy between the observed vacuum density and our expectations from theories of high energy physics, which has been dubbed the *old* cosmological constant problem. One proposal to address this puzzle at the semiclassical level is to decouple quantum vacuum from spacetime geometry via a modification of gravity that includes an incompressible fluid, known as *gravitational aether*. In this paper, we discuss classical predictions of this theory along with its compatibility with cosmological and experimental tests of gravity. We argue that deviations from general relativity (GR) in this theory are sourced by pressure or vorticity. In particular, the theory predicts that the gravitational constant for radiation is 33% larger than that of nonrelativistic matter, which is preferred by (most) cosmic microwave background (CMB), Ly-α forest, and ^{7}Li primordial abundance observations, while being consistent with other cosmological tests at $\sim 2\sigma$ level. It is further shown that all parametrized post-newtonian parameters have the standard GR values aside from the anomalous coupling to pressure ζ_4, which has not been directly measured. A more subtle prediction of this model (assuming irrotational aether) is that the (intrinsic) gravitomagnetic effect is 33% larger than GR prediction. This is consistent with current limits from LAGEOS and Gravity Probe B at $\sim 2\sigma$ level.

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I. INTRODUCTION

The discovery of recent acceleration of cosmic expansion was one of the most surprising findings in modern cosmology [1,2]. The standard cosmological model (also known as the concordance model) drives this expansion with a cosmological constant (CC). While the CC is consistent with (nearly) all current cosmological observations, it requires an extreme fine-tuning of more than 60 orders of magnitude, known as the cosmological constant problem [3]. More precisely, a covariant regularization of the vacuum state energy of a quantum field theory (QFT), if it exists, acts just like the CC in linear order, but has a value many orders of magnitude larger than what is inferred from observations.

If the QFT prediction of the cosmological constant is considered reasonable (and in lieu of an extreme fine-tuning), there is no choice but to abandon the idea that vacuum energy should gravitate. This, however, requires modifying Einstein's theory of gravity, in which all sources of energy gravitate. Attempts in this direction have been proposed in the context of massive gravity [4], or brane-world models of extra dimensions such as cascading gravity [5,6], or supersymmetric large extra dimensions (e.g., [7]). However, efforts to find explicit cosmological solutions that degravitate vacuum energy have proved difficult (e.g., [8,9]).

In [10], one of us proposed a novel approach to modified gravity in which the QFT vacuum quantum fluctuations (of linear order in the metric) are decoupled from gravity through the introduction of an incompressible perfect fluid called the gravitational aether. In this model, the right-hand side of the Einstein field equation is modified as

$$(8\pi G')^{-1}G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}T^{\alpha}_{\alpha}g_{\mu\nu} + T'_{\mu\nu} \tag{1}$$

$$T'_{\mu\nu} = p'(u'_{\mu}u'_{\nu} + g_{\mu\nu}), \tag{2}$$

where G' is the (only) constant of the theory and $T'_{\mu\nu}$ is the aether fluid which has pressure p' and four-velocity u'. Our metric signature is $(-,+,+,+)$. Aether is constrained by requiring the conservation of the energy-momentum tensor $T_{\mu\nu}$, and the Bianchi identity:

$$\nabla^{\mu}T'_{\mu\nu} = \frac{1}{4}\nabla_{\nu}T, \tag{3}$$

which can be written in a similar form to the relativistic hydrodynamic equations:

$$p'\nabla\cdot\mathbf{u}' = -\frac{1}{4}\dot{T}, \tag{4}$$

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$$p'\dot{\mathbf{u}}' = -\nabla_\perp\left(p' - \frac{T}{4}\right), \tag{5}$$

where $\dot{} \equiv \mathbf{u}' \cdot \nabla$, and $\nabla_\perp$ is the gradient normal to $\mathbf{u}'$ four-vector. Notice that Eqs. (4) and (5) can be combined to find a parabolic equation for the evolution of $\mathbf{u}'$, which generically has a well-defined initial value problem, at least for a finite time [11].

This modification of Einstein Eqs. (1) and (2), if self-consistent and in agreement with other experimental bounds on gravity, could potentially constitute a solution to the old cosmological constant problem. We will show in this paper that none of these experimental bounds, as yet, rule out this theory (at $\sim 2\sigma$ level) and that it is surprisingly similar to general relativity [12].

Nevertheless, the new cosmological constant problem, i.e., the present-day acceleration of cosmic expansion is not addressed by the original gravitational aether proposal. In [13,14], it is argued that quantum gravity effects in the presence of astrophysical black holes can naturally explain this phenomenon. In this proposal, the formation of black holes leads to a negative aether pressure, that is set by the horizon temperature of the black holes. However, in the present work we only focus on phenomenological implications of the *classical* gravitational aether scenario, and defer study of black hole-dark energy connection, which could be potentially very important on cosmological scales at late times. Instead, we use a standard cosmological constant to model the late-time acceleration of cosmic expansion. Throughout the paper we set the speed of light $c = 1$.

II. COSMOLOGICAL CONSTRAINTS ON GRAVITATIONAL AETHER

If the energy-momentum tensor of matter, $T_{\mu\nu}$, can be approximated by a perfect fluid with constant equation of state $p = w\rho$ and four-velocity u_μ, direct substitution into Eq. (1) shows that if: $u'_\mu = u_\mu$, $p' = \frac{(1+w)(3w-1)}{4}\rho$, then the solutions to the gravitational aether theory are identical to those of general relativity (GR) with a renormalized gravitational constant:

$$G_N \to G_{\rm eff} = (1+w)G_N, \tag{6}$$

where $G_N = 3G'/4$. In other words, the gravitational coupling is not a constant anymore, and can change significantly for fluids with relativistic pressure. Not surprisingly, for vacuum equation of state $w = -1$, $G_{\rm eff} = 0$, which implies that vacuum does not gravitate.

In particular, in the case of homogeneous Friedmann-Lemaître-Robertson-Walker cosmology where the perfect fluid approximation is valid, this theory predicts that the effective G that relates geometry to the matter density ρ in Friedmann equation is different in the matter and radiation eras

$$\frac{G_N}{G_R} \equiv \frac{G_{\rm eff}(w=0)}{G_{\rm eff}(w=1/3)} = \frac{3}{4}. \tag{7}$$

This is the first cosmological prediction of this theory: radiation energy gravitates more strongly than nonrelativistic matter. The expansion history in the radiation era depends on the product $G\rho_{\rm rad}$, and is constrained through different observational probes. The constraints are often described as the bound on the effective number of neutrinos $N_{\nu,{\rm eff}}$, which quantifies the total radiation density $\rho_{\rm rad}$. However, assuming only photons (that are constrained by CMB observation) and three neutrino species, with no more light particles left over from the very early universe, we can translate the constraints to those on $G_{\rm eff}$ by requiring $G_{\rm eff}\rho_{\rm rad}(N_\nu = 3) = G_N\rho_{\rm rad}(N_\nu = 3 + \Delta N_\nu)$. In particular, based on standard thermal decoupling of neutrinos, Eq. (7) can be translated to $\Delta N_\nu = 2.5$, at least for a homogeneous universe [15]. Using this correspondence, we can now discuss cosmological constraints on the gravitational aether scenario.

A. Big bang nucleosynthesis (BBN)

It has been known that the increase of the gravitational constant at around $T = \mathcal{O}(1)$ MeV epoch induces earlier freeze-out of the neutron to proton ratio because of a speedup effect of the increased cosmic expansion. This raises the abundance of ^{4}He sensitively and deuterium (D) mildly, and can lower the abundance of ^{7}Be through ^{7}Be$(n, p)^7$Li$(p, \alpha)^4$He (note that the second p is thermal proton). For a relatively large baryon to photon ratio $\eta \gtrsim 3 \times 10^{-10}$, the dominant mode to produce ^{7}Li is the electron capture of ^{7}Be at a later epoch through $^7{\rm Be} + e^- \to {}^7{\rm Li} + \nu_e$. Therefore, the decrease of ^{7}Be makes the fitting better because so far any observational ^{7}Li abundances have been so low that they could not have agreed with theoretical prediction in standard BBN (SBBN) at better than 3σ [16].

In this study, we adopt the following observational light element abundances as primordial values: the mass fraction of ^{4}He, $Y_p = 0.2561 \pm 0.0108$ (stat) [17], the deuterium to hydrogen ratio, ${\rm D/H} = (2.80 \pm 0.20) \times 10^{-5}$ [18], and the ^{7}Li to hydrogen ratio ${\rm Log}_{10}(^7{\rm Li/H}) = -9.63 \pm 0.06$ [19,20]. Theoretical errors come from experimental uncertainties in cross sections [16,22,23] and neutron lifetime [24,25].

Comparing theoretical prediction with the observational light element abundances provides a constraint on G_N/G_R. Figure 1 shows the results of a comprehensive analysis for ^{4}He, D, and ^{7}Li. We also plotted a band for baryon to photon ratio, η which was reported from CMB observations by WMAP 7-year, $\eta = (6.225 \pm 0.170) \times 10^{-10}$ in the case of $G_N/G_R = 1$ [26]. Then we can see that every light element agrees with the gravitational aether theory within 2σ. It is notable that ^{7}Li in this theory fits the data better than that in SBBN. Performing χ^2 fitting for three

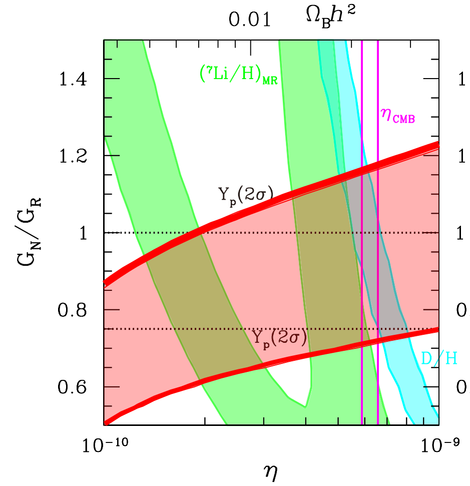

FIG. 1 (color online). Allowed regions with 2σ lines for D/H, Y_p and ^{7}Li/H are shown. The upper and lower horizontal dashed lines indicate GR and gravitational aether predictions, respectively. The thickness of Y_p means the uncertainty in measurements of neutron lifetime [24,25]. We can translate the vertical axis into ΔN_ν by using a relation $G_N/G_R \simeq 1/(1 + 0.135\Delta N_\nu)$.

elements with three degrees of freedom, however, the model is allowed only at 99.7% (3σ) in total.

However, notice that the main discrepancy is with deuterium abundance observed in quasar absorption lines, which suffer from an unexplained scatter. Moreover, deuterium could be depleted by absorption onto dust grains that would make its primordial value closer to our prediction (see [27] for a discussion).

B. Power spectrum of cosmological fluctuations

The gravitational aether theory can also be tested by considering the power spectrum of the CMB, just as a number of publications have recently investigated the apparent preference for extra relativistic degrees of freedom (see e.g., [28–30]). Using a modified version of Cmbeasy [31,32], we compute constraints on G_N/G_R from scalar perturbations in a scenario with three massless neutrino species (details are discussed in Appendix A). The seven-year CMB data from WMAP [26] together with small-scale observations from the Atacama Cosmology Telescope (ACT) [33] yield $G_N/G_R = 0.73^{+0.31}_{-0.21}$ at 95% confidence. Just like any additional relativistic component can be compensated by a higher fraction of dark matter in order to keep the time of matter-radiation equality constant, there is a high amount of degeneracy between G_N/G_R and $\Omega_m h^2$ and h (see Figs. 2 and 3). Recent data from the South Pole Telescope (SPT), which measured the CMB power spectrum in the multipole range $650 < \ell < 3000$ significantly tightens the constraint and yields $0.88^{+0.17}_{-0.13}$ (for the combination of ACT and SPT data we have adopted the SPT treatment of foreground nuisance parameters). A similar effect can be seen when adding baryonic acoustic oscillations (BAO) [34] and constraints on the Hubble rate. Here we adopt the value of $H_0 = 73.8 \pm 2.4\ \mathrm{km}^{-1}\,\mathrm{Mpc}^{-1}$ [35]. Then, by breaking the degeneracy between the matter content and h, the combination WMAP + ACT + BAO + Sne + Hubble results in $G_N/G_R = 0.89^{+0.13}_{-0.11}$. The supernovae data of the Union catalog [36] do not significantly contribute to this constraint. We note that both cases, i.e., adding either SPT data or adding the Hubble constraints to the basic WMAP + ACT set, move the gravitational aether value of $G_N/G_R = 0.75$ to the border or just outside of the 95% confidence interval, while the general relativity value of $G_N/G_R = 1.0$ is well compatible with all combinations of data. Consequently, the full combination of data, i.e., WMAP + ACT + SPT + Hubble + BAO + Sne, constrains G_N/G_R to $0.94^{+0.10}_{-0.09}$.

In contrast, observational constraints at lower redshifts, in particular, data of the Ly-α forest [37] prefer the aether prediction. Furthermore, additional degeneracies with, e.g., the Helium mass fraction Y_p might shift the preferred values. Combining WMAP + ACT + Sne with the Ly-α forest constraints yields, $G_N/G_R = 0.68^{+0.32}_{-0.25}$ at 95% level, with Y_p as a free parameter. However, we should note that this result is more prone to systematic uncertainties due to the quasilinear nature of the Ly-α forest. Also, including

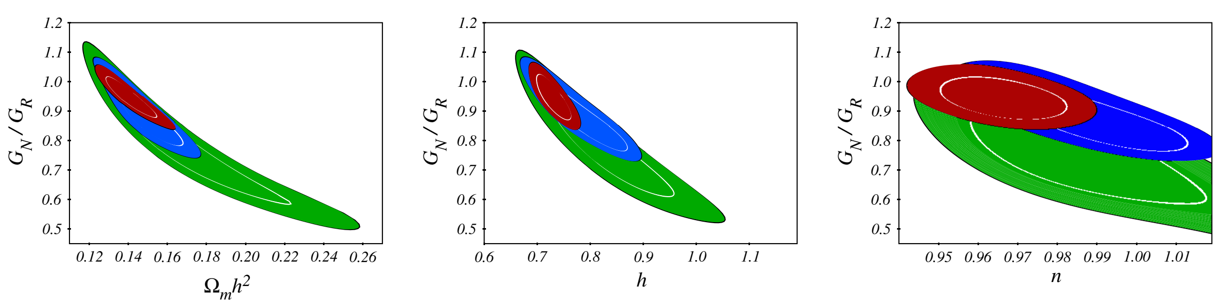

FIG. 2 (color online). Constraints at the 95% confidence level for G_N/G_R from WMAP 7-year (background, green), WMAP + ACT + SPT (middle, blue) and WMAP + ACT + SPT + Sne + BAO + Hubble data (front, red). The white lines show the 68% confidence levels. Note that the gravitational aether prediction is $G_N/G_R = 0.75$, while in general relativity $G_R = G_N$.

TABLE I. Summary of the constraints on G_N/G_R and the associated 95% confidence intervals for different combinations of observational data.

	G_N/G_R
WMAP + ACT	$0.73^{+0.31}_{-0.21}$
WMAP + ACT + SPT	$0.88^{+0.17}_{-0.13}$
WMAP + ACT + Hubble + BAO + Sne	$0.89^{+0.13}_{-0.11}$
WMAP + ACT + SPT + Hubble + BAO + Sne	$0.94^{+0.10}_{-0.09}$
WMAP + ACT + Sne + Ly $-$ α (free Y_p)	$0.68^{+0.32}_{-0.25}$

the SPT data in this combination changes this result to the higher value of $0.90^{+0.27}_{-0.23}$. A summary of the constraints with different combinations of data is provided in Table I.

Future CMB observations by the Planck satellite, as well as ground-based observatories are expected to improve this constraint dramatically over the next five years, and thus confirm or rule out this prediction.

III. PRECISION TESTS OF GRAVITY

Gravity on millimeter to solar system scales is well described by general relativity, which has passed many precision tests on these scales with flying colors (see, e.g., [38] for a review). That is why it is hard to imagine how an order unity change in the theory such as that of Eq. (1) can be consistent with these tests, without introducing any fine-tuned parameter. In this section, we argue that nearly all these tests are with gravitational sources that have negligible *pressure* or *vorticity*, which source deviations from GR predictions in gravitational aether theory.

A. Parametrized post-Newtonian (PPN) formalism

In Sec. II, we argued that for any perfect fluid with constant equation of state, w, the solutions of gravitational aether theory are identical to those of GR with a renormalized gravitational constant $\propto (1 + w)$. However, for generic astrophysical applications, w is not constant except for pure radiation, or in the pressureless limit of $w = 0$. Focusing on the latter case, and given that pressure is 1st order in post-Newtonian expansion, we can quantify the gravitational aether theory through the PPN formalism.

The PPN formalism is defined in a weak field, slow motion limit, and describes the next-to-Newtonian-order gravitational effects in terms of a standardized set of potentials and 10 parameters. These PPN parameters will be determined by solving the field Eqs. (1) order-by-order with a perfect fluid source in a standard coordinate gauge. The conventional introductory details of the formalism will be skipped over (see [39] for a more detailed explanation of the procedure and the general PPN formalism).

To be clear, though, we will assume a nearly globally Minkowskian coordinate system and basis with respect to which, at zeroth order, the metric is the Minkowski metric ($g_{\mu\nu} = \eta_{\mu\nu}$) and the fluid four-velocity u^μ is purely timelike ($u^0 = 1$, $u^i = 0$). The stress-energy tensor is taken to have the form $T_{\mu\nu} = (\rho + \rho\Pi + p)u_\mu u_\nu + p g_{\mu\nu}$ where u_μ, ρ, Π and p are the unit four-velocity, rest-mass-energy density, internal energy density, and isotropic pressure of the fluid source, respectively. The fluid variables are assigned orders of $\rho \sim \Pi \sim \frac{p}{\rho} \sim u_i^2 \sim$ 1PN. In the weak field limit, the metric can be written as a perturbation of the Minkowski metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The components of the metric perturbations $h_{\mu\nu}$ with respect to this basis will be assumed to be of orders: $h_{00} \sim$ 1PN + 2PN, $h_{ij} \sim$ 1PN, and $h_{0i} \sim$ 1.5PN. This choice preserve the Newtonian limit while allowing one to determine just the first post-Newtonian corrections. Furthermore, the aether four-velocity u'_μ will be assumed to be of the same order as that of the matter fluid.

Solving (3) to 1PN gives $p' = -\rho/4$, which can be used in (1) to solve for g_{00} and g_{ij} to 1PN

$$h_{00} = 2U \tag{8}$$

$$h_{ij} = 2U\delta_{ij}, \tag{9}$$

where U is the Newtonian potential and the following gauge condition is imposed: $\partial_j h_{ij} = \frac{1}{2}(\partial_i h_{jj} - \partial_i h_{00})$. Comparing the continuity equations for matter and aether (i.e., the "time" component of (3) to 1.5 PN), it can be shown that

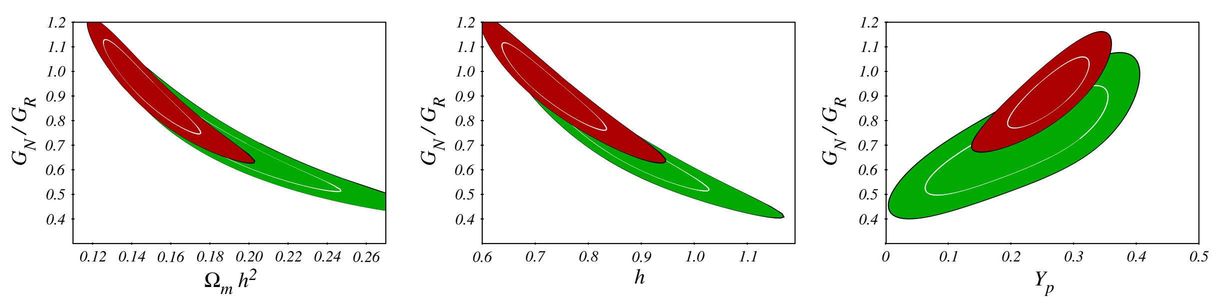

FIG. 3 (color online). Constraints at the 95% confidence level for G_N/G_R from WMAP + ACT + Sne + Ly $-$ α (background, green) and WMAP + ACT + SPT + Sne + Ly $-$ α (front, red). The white lines show the 68% confidence levels. Note that the gravitational aether prediction is $G_N/G_R = 0.75$, while in general relativity $G_R = G_N$.

$$u'^i - u^i = t^i, \tag{10}$$

where t^i satisfies $\nabla^i(t_i\rho) = 0$. This implies that the rotational component of aether is not fixed by matter within the PN expansion formalism. Here we will make the assumption that $t^i = 0$ so that aether is completely dragged by matter. We will discuss this choice further in Sec. III B.

Previously we mentioned that in this case, an exact solution for u'_μ and p' exists when matter has a constant equation of state. (It is worth noting that in the $t^i = 0$ case, higher PN equations appear to imply a nonstandard condition on the pressure $\nabla_a(u^a p) = 0$, which is satisfied for a constant equation of state.) Using this solution and an additional gauge condition $\partial_i h_{0i} = 3\partial_0 U$, the field equations can be solved for g_{0i} and g_{00} to 1.5 PN and 2 PN, respectively

$$h_{0i} = -\frac{7}{2}V_i - \frac{1}{2}W_i, \tag{11}$$

$$h_{00} = 2U - 2U^2 + 4\phi_1 + 4\phi_2 + 2\phi_3 + 6\left(1 + \frac{1}{3}\right)\phi_4, \tag{12}$$

where Appendix B includes the definition for all potentials. Collecting all the results (8), (9), (11), and (12), indicates that all metric components are as in standard GR, except for the term in g_{00} with the pressure-dependent potential ζ_4. Consulting the parametrization rubric indicates that all PPN parameters have the standard values except ζ_4, which equals

$$\zeta_4 = \frac{1}{3}, \tag{13}$$

which was already pointed out in [10]. Notice that ζ_4, i.e., the anomalous coupling of gravity to pressure is the only PPN parameter that is not measured experimentally, as one needs to probe the relationship between gravity and pressure of an object with near relativistic pressures. A notable exception is observation of neutron stars (or their mergers, via gravitational wave observations), which can potentially measure ζ_4, assuming that the uncertainties in nuclear equation of state are under control [40].

B. Gravitomagnetic effect

In the previous section, we showed that rotation of aether is not fixed by matter in the nonrelativistic regime. We further assumed that aether rotates with matter. Here we will argue that observational bounds on the gravitomagnetic effect provide a mild prefernce for this assumption.

Spacetime around a rotating object with a weak gravitational field, like Earth, can be described in terms of a set of potentials. With appropriate definitions, these potentials satisfy equations analogous to Maxwell's equations [41]. The gravitomagnetic effect describes the dragging of spacetime around a rotating object and can be quantified by a gravitomagnetic field $\vec{B}$ defined as

$$\vec{B} = -4\frac{3\vec{r}(\vec{r}\cdot\vec{S}) - \vec{S}r^2}{2r^5}, \tag{14}$$

$$S^i = 2G'\int \epsilon^i_{jk}x'^j T^{0k}_{\rm eff}\mathrm{d}^3x', \tag{15}$$

where $\vec{r}$ is the position vector measured from the center of the object, ϵ^i_{jk} is the three-dimensional Levi-Civita tensor, and $T^{\mu\nu}_{\rm eff}$ is the RHS of the field Eqs. (1) [41]. The gravitomagnetic field causes the precession of the orbital angular momentum of a free falling test particle. The angular velocity of this precession is [41]

$$\vec{\Omega} = -\frac{\vec{B}}{2}. \tag{16}$$

If aether is irrotational, $T^{0k}_{\rm eff} = T^{0k}$ to within the accuracy of linearized theory and since $G' = \frac{4}{3}G_N$, we have

$$\vec{\Omega}_{\rm aether} = \frac{4}{3}\vec{\Omega}_{\rm GR}. \tag{17}$$

Gravity Probe B is an experiment that measures the precession rate $<\Omega>$ of four gyroscopes orbiting the Earth. Recently, Gravity Probe B reported a frame-dragging drift rate of -37.2 ± 7.2 mas/yr, to be compared with the GR prediction of -39.2 mas/yr ("mas" is milliarc-second) [42]. Laser ranging to the LAGEOS and LAGEOS II satellites also provides a measurement of the frame-dragging effect. The total uncertainty in this case is still being debated; with optimistic estimates of 10%–15% (e.g., [43]), and more conservative estimates as large as 20%–30% (e.g., [44]).

Therefore, we conclude that even though perfect corotation of aether by matter is preferred by current tests of intrinsic gravitomagnetic effect, an irrotational aether is still consistent with present constraints at 2σ level.

IV. CONCLUSIONS AND DISCUSSIONS

In the current work, we studied the phenomenological implications of the gravitational aether theory, a modification of Einstein's gravity that solves the old cosmological constant problem at a semiclassical level. We showed that the deviations from general relativity can only be significant in situations with relativistic pressure, or (potentially) relativistic vorticity. The most prominent prediction of this theory is then that gravity should be 33% stronger in the cosmological radiation era than GR predictions. We showed that many cosmological observations, including CMB (with the exception of SPT), Ly-α forest, and ^{7}Li primordial abundance prefer this prediction, while deuterium may prefer GR values. We then examined the implications for precision tests of gravity using the PPN formalism, and showed that the only PPN parameter that deviates from its GR value is ζ_4, the anomalous coupling to pressure, that has never been tested experimentally. Finally, we argued that current tests of Earth's gravitomagnetic effect mildly prefer a corotation of aether with matter, although they are consistent with an irrotational aether at 2σ level.

In our opinion, the fact that gravitational aether has *the same number of free parameters as GR*, and is yet (to our knowledge) consistent with all cosmological and precision tests of gravity at 2σ level, indicates that this theory could be a strong contender for Einstein's theory of gravity.

Future observations are expected to sharpen these distinctions. In particular, the clearest test will come from the Planck CMB anisotropy power spectrum that is expected to be released in early 2013. Judging by the predictions for constraints on the effective number of neutrinos, Planck should be able to distinguish GR and aether at close to 10σ level [28].

Another interesting implication of this theory is for the cosmic baryon fraction. As we increase the gravity due to radiation (or effective number of neutrinos), we need to increase the dark matter density to keep the redshift of equality constant, since it is well constrained by CMB power spectrum (see, e.g., [26]). This implies that the total matter density should be bigger by a factor of $4/3$ (Fig. 2). Given that baryon density is insensitive to this change, the cosmic baryon fraction will decrease by a factor of $3/4$, i.e., from 17% [26] to 13%. This could potentially resolve the missing baryon problem in galaxy clusters [45], as well as the deficit in observed Sunyaev-Zel'dovich power spectra, in comparison with theoretical predictions [33,46].

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APPENDIX A: AETHER PERTURBATIONS THROUGH EQUALITY

Here we present a consistent treatment of cosmological scalar perturbation theory for gravitational aether (GA). As we argued in Sec. II, when matter is approximated by a perfect fluid with density ρ, pressure $p = w\rho$ (w constant), and four-velocity $u^\mu = \frac{dx^\mu}{\sqrt{-ds^2}}$ (i.e., $T_{\mu\nu} = (1+w)\rho u_\mu u_\nu + w\rho g_{\mu\nu}$), $u'_\mu = u_\mu$ and $p' = \frac{(1+w)(3w-1)}{4}\rho$ solve (4) and (5) and the GA field Eq. (1) becomes

$$(8\pi)^{-1} G_{\mu\nu} = G_N(1+w)T_{\mu\nu}. \tag{A1}$$

In cosmology, therefore, if the constituents of the universe are matter and radiation *and they are separately conserved*, the GA field equations become

$$(8\pi)^{-1} G_{\mu\nu} = G_N T^m_{\mu\nu} + \frac{4}{3} G_N T^r_{\mu\nu}, \tag{A2}$$

where m and r stand for matter and radiation, respectively. This approximation, of course, is false when inhomogeneities are considered since baryons and photons interact strongly. Therefore, we shall perturb about this exact solution and do a consistent treatment of cosmological scalar perturbation theory.

In what follows, b, dm, m, and r stand for baryon, dark matter, matter, and radiation, respectively, and all barred quantities are unperturbed. Following [47], we will use the conformal Newtonian gauge

$$ds^2 = a^2(\tau)\{-[1+2\psi(\tau,\vec{x})]d\tau^2 + [1-2\phi(\tau,\vec{x})]dx^i dx_i\}. \tag{A3}$$

To linear order in perturbation theory, the matter energy-momentum tensor takes the form

$$T^0{}_0 = -(\bar{\rho} + \delta\rho) \tag{A4}$$

$$T^0{}_i = (\bar{\rho} + \bar{p})\frac{\delta u_i}{a} \tag{A5}$$

$$T^i{}_j = (\bar{p} + \delta p)\delta_{ij} + \Sigma^i{}_j, \tag{A6}$$

where $\Sigma^i{}_j$ is the traceless anisotropic shear stress perturbation and

$$\delta\rho = \rho - \bar{\rho}; \qquad \delta p = p_i - \bar{p}; \qquad \delta u^i_\mu = u^i_\mu - \bar{u}_\mu, \tag{A7}$$

where $i = \{dm, b, r\}$. In our coordinate system $\bar{u}^0 = \frac{1}{a}$, $\bar{u}_0 = -a$, and $\bar{u}_i = \bar{u}^i = 0$. The aether pressure and four-velocity perturbations are defined as follows:

$$p' = -\frac{\rho_m}{4} + \delta p', \qquad u'_\mu = u^{dm}_\mu + \delta u_\mu. \tag{A8}$$

Dark matter only interacts gravitationally and is separately conserved. We assume that there is negligible energy transfer between baryons and relativistic particles (i.e., $\nabla^\mu(\rho_b u^b_\mu) = 0$). Then, to first order in perturbation theory (4) and (5) give

$$3\frac{\dot{a}}{a^2}\delta p' = \frac{\bar{\rho}_m}{4}\partial_i\left(\delta u^i + \frac{\bar{\rho}_b}{\bar{\rho}_m}\delta w^i\right) \tag{A9}$$

$$\partial_i \delta p' = \frac{a\bar{\rho}_m}{4}\left(\delta\dot{u}^i + 2\frac{\dot{a}}{a}\delta u^i\right), \qquad (A10)$$

where $\delta w^i = \delta u^i_{dm} - \delta u^i_b = a^{-2}(\delta u_i^{dm} - \delta u_i^b)$ and $\delta u^i = a^{-2}\delta u_i$. Taking the comoving divergence of (A10) and applying the comoving Laplacian to (A9), we can eliminate $\delta p'$ and get an equation for $\Omega \equiv \partial_i \delta u^i$

$$3\frac{\dot{a}}{a^3}\partial_\tau(a^2\Omega) - \nabla^2\Omega = \frac{\bar{\rho}_b}{a\bar{\rho}_m}\nabla^2(\dot{\delta}_b - \dot{\delta}_{dm}), \qquad (A11)$$

where $\delta_{dm} = \frac{\delta\rho_{dm}}{\bar{\rho}_{dm}}$, $\delta_b = \frac{\delta\rho_b}{\bar{\rho}_b}$, and we have used the fact that $\partial_i \delta w^i = \frac{1}{a}(\dot{\delta}_b - \dot{\delta}_{dm})$. In Fourier space, this equation can be numerically integrated for modes of different wavelength, given the equations that govern δ_{dm} and δ_b. Once Ω is known, (A9) can be used to find $\delta p'$. In the conformal Newtonian gauge, only scalar perturbations are treated and we can ignore the rotational part of δu^i. This can also be physically motivated: let $\delta u^i = \partial_i u_S + \partial \delta u^i_V$ where $\partial_i \delta u^i_V = 0$. Taking the curl of (A10), it follows that $\nabla \times \delta\vec{u}_V \propto \frac{1}{a^2}$. As a result, the rotational part of the aether fluid decays and it does not play a major role in cosmology. As a result, given Ω we can find δu^i in Fourier space ($\partial_j \to ik_j$)

$$\delta u^j = -i\frac{k_j}{k^2}\Omega, \qquad (A12)$$

where $k^2 = \delta_{ij}k_i k_j$. Similarly,

$$\delta w^j = \delta u^j_{dm} - \delta u^j_b = i\frac{k_j}{ak^2}(\dot{\delta}_{dm} - \dot{\delta}_b). \qquad (A13)$$

To first order in perturbation theory, the GA field equations now take the form

$$(8\pi G_N)^{-1}G_{\mu\nu} = T^m_{\mu\nu} + \frac{4}{3}T^r_{\mu\nu} + \epsilon_{\mu\nu} \qquad (A14)$$

with $\epsilon_{00} = 0$, $\epsilon_{0i} = \frac{a\bar{\rho}_m}{3}(\delta u_i + \frac{\bar{\rho}_b}{\bar{\rho}_m}\delta w_i)$, and $\epsilon_{ij} = \frac{4}{3}a^2\delta p'\bar{g}_{ij}$. Having both the left-and right-hand sides of this equation, we can now solve for the scalar perturbations. However, this does not provide an obvious way of checking the prediction of this theory, namely $\frac{G_R}{G_N} = \frac{4}{3}$. This can be easily accommodated for by having field equations that contain G_R as a constant, and reduce to general relativity and GA for $G_R = G_N$ and $G_R = \frac{4}{3}G_N$ respectively. Consider the field equations (which we will refer to as the generalized gravitational aether (GGA) field equations)

$$(8\pi)^{-1}G_{\mu\nu}[g_{\mu\nu}] = G_R T_{\mu\nu} + (G_N - G_R)T^\alpha_\alpha g_{\mu\nu} + \tilde{T}_{\mu\nu}$$
$$\tilde{T}_{\mu\nu} = \tilde{p}(\tilde{u}_\mu\tilde{u}_\nu + g_{\mu\nu}). \qquad (A15)$$

Conservation of $G_{\mu\nu}$ and $T_{\mu\nu}$ implies

$$\nabla^\mu\tilde{T}_{\mu\nu} = (G_R - G_N)\nabla_\nu T. \qquad (A16)$$

Defining $p' = \frac{\tilde{p}}{4(G_R - G_N)}$ and making the obvious identification $\tilde{u}_\mu = u'_\mu$, we see that this equation becomes exactly (3). Therefore, all of our solutions before can be used here after a trivial rescaling of the pressure. For example, if $T_{\mu\nu}$ is a perfect fluid with equation of state w, exact solutions are obtained by $\tilde{u}_\mu = u_\mu$ and $\tilde{p} = (G_R - G_N)(1 + w)(3w - 1)\rho$, which again just renormalizes Newton's constant

$$G_N \to G_{\rm eff}(w) = G_N\left\{3w\frac{G_R}{G_N} + (1 - 3w)\right\}. \qquad (A17)$$

Note that $G_{\rm eff}(w = 0) = G_N$ and $G_{\rm eff}(w = 1/3) = G_R$. Again, if matter and radiation are separately conserved in a cosmological setting, (A15) becomes

$$(8\pi)^{-1}G_{\mu\nu} = G_N T^m_{\mu\nu} + G_R T^r_{\mu\nu}. \qquad (A18)$$

More importantly, when $G_R = G_N$, these field equations reduce to those of general relativity (GR) (this is true in the cosmological case because $\nabla_\mu\tilde{u}^\mu \neq 0$, which means that the conservation of aether implies that its pressure vanishes identically). Also when $G_R = \frac{4}{3}G_N$, the GGA field equations reduce to those of GA, with the appropriate rescaling $T'_{\mu\nu} = \frac{3}{4G_N}\tilde{T}_{\mu\nu}$. Therefore, fitting this theory to data, we will be able to make a likelihood plot of $\frac{G_R}{G_N}$ and see how far away the best fit is from the GA and GR predictions.

Because of the similarity of the underlying equations, the linear perturbation theory of the GGA field equations is very close to those of GA, which we already described. We treat all matter perturbations as before and perturb $\tilde{T}_{\mu\nu}$ similarly

$$\tilde{p} = (G_N - G_R)\rho_m + \delta\tilde{p}, \tilde{u}_\mu = u^{dm}_\mu + \delta u_\mu. \qquad (A19)$$

The equations of interest are (in Fourier space)

$$3H\partial_\tau(a^2\Omega) + a(\tau)k^2\Omega = k^2\frac{\bar{\rho}_{b_0}}{\bar{\rho}_{m_0}}(\dot{\delta}_{dm} - \dot{\delta}_b) \qquad (A20)$$

$$\delta\tilde{p} = \frac{(G_R - G_N)\bar{\rho}_m}{3H}\left[\Omega + \frac{\bar{\rho}_{b_0}}{a\bar{\rho}_{m_0}}(\dot{\delta}_b - \dot{\delta}_{dm})\right] \qquad (A21)$$

$$\delta\tilde{u}^j = -i\frac{k_j}{k^2}\Omega, \qquad (A22)$$

where $H = \frac{\dot{a}}{a^2}$ and we have recognized that $\frac{\bar{\rho}_b}{\bar{\rho}_m} = \frac{\bar{\rho}_{b0}}{\bar{\rho}_{m0}}$ is fixed by the values at the present time. Once (A20) is solved for Ω, $\delta\tilde{p}$ and $\delta\tilde{u}^j$ are determined by (A21) and (A22), respectively. At long last, to linear order in perturbation theory, the GGA field equations read

$$(8\pi)^{-1}G_{\mu\nu} = G_N T^m_{\mu\nu} + G_R T^r_{\mu\nu} + \tilde{\epsilon}_{\mu\nu}, \qquad (A23)$$

where

$$\tilde{\epsilon}_{00} = 0 \qquad (A24)$$

$$\tilde{\epsilon}_{0j} = i\frac{k_j}{k^2}(G_N - G_R)(a^3\bar{\rho}_m)\left[\Omega + \frac{\bar{\rho}_{b_0}}{a\bar{\rho}_{m_0}}(\dot{\delta}_b - \dot{\delta}_{dm})\right] \tag{A25}$$

$$\tilde{\epsilon}_{ij} = (G_R - G_N)\frac{\bar{\rho}_m a^2}{3H}\left[\Omega + \frac{\bar{\rho}_{b_0}}{a\bar{\rho}_{m_0}}(\dot{\delta}_b - \dot{\delta}_{dm})\right]\delta_{ij}. \tag{A26}$$

Having both the left- and right-hand sides of (A23), the scalar perturbations can now be consistently solved for.

APPENDIX B: PPN NOTATIONS

The metric components are in terms of particular potential functions, thus defining the PPN parameters

$$\begin{aligned} g_{00} = &-1 + 2U - 2\beta U^2 - 2\xi\phi_W \\ &+ (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\phi_1 \\ &+ 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\phi_2 \\ &+ 2(1 + \zeta_3)\phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\phi_4 - (\zeta_1 - 2\xi)A \end{aligned} \tag{B1}$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} \tag{B2}$$

$$\begin{aligned} g_{0i} = &-\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i \\ &-\frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i. \end{aligned} \tag{B3}$$

The potentials are all of the form

$$F(x) = G_N \int d^3y \frac{\rho(y)f}{|x - y|} \tag{B4}$$

and the correspondences F: f are given by

$$\begin{aligned} &U{:}\ 1 \qquad \phi_1{:}\ u_i u_j \qquad \phi_2{:}\ U \qquad \phi_3{:}\ \Pi \qquad \phi_4{:}\ p/\rho \\ &\phi_W{:}\ \int d^3z\rho(z)\frac{(x-y)_j}{|x-y|^2}\left[\frac{(y-z)_j}{|x-z|} - \frac{(x-z)_j}{|y-z|}\right] \\ &A{:}\ \frac{(v_i(x-y)_i)^2}{|x-y|^2} \qquad V_i{:}\ u^i \qquad W_i{:}\ \frac{u_j(x_j - y_j)(x^i - y^i)}{|x-y|^2}. \end{aligned} \tag{B5}$$

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