PHYSICAL REVIEW D **84,** 096013 (2011)

Triple-product asymmetries in K, $D_{(s)}$, and $B_{(s)}$ decays

Michael Gronau

Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

(Received 11 July 2011; published 22 November 2011)

One distinguishes between "true" CP-violating triple-product (TP) asymmetries which require no strong phases and "fake" asymmetries which are due to strong phases but require no CP violation. So far a single true TP asymmetry has been measured in $K_L \to \pi^+\pi^- e^+ e^-$. A general discussion is presented for T-odd TP asymmetries in four-body decays. It is shown that TP asymmetries vanish for two identical and kinematically indistinguishable particles in the final state. Two examples are $D^0 \to K^-\pi^+\pi^-\pi^+$ and $D^+ \to K^-\pi^+\pi^+\pi^0$. A nonzero TP asymmetry can be expected when nontrivial kinematic correlations exist, as in the decay $K_L \to e^+e^-e^+e^-$. Triple-product asymmetries measured in charmed particle decays indicate an interesting pattern of final-state interactions. We reiterate a discussion of TP asymmetries in B meson decays to two vector mesons each decaying to a pseudoscalar pair, extending results to decays where one vector meson decays into a lepton pair. We derive expressions for time-dependent TP asymmetries for neutral B decays to flavorless states in terms of the neutral B mass difference Δm and the width-difference $\Delta\Gamma$. Time-integrated true CP-violating asymmetries, measurable for untagged B_s decays, are shown to be suppressed by neither $\Gamma_s/\Delta m_s$ nor $\Delta\Gamma_s/\Gamma_s$ if transversity amplitudes for CP-even and CP-odd states involve different weak phases. In contrast, fake asymmetries require flavor tagging and are suppressed by the former ratio when time integrated. We apply our results to $B \to K^*\phi$ and $B_s \to \phi\phi$ data and suggest an application for $B_s \to J/\psi\phi$.

DOI: 10.1103/PhysRevD.84.096013 PACS numbers: 11.30.Er, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

A powerful tool for displaying CP violation in weak decays is the investigation of triple-product asymmetries [1–4]. A four-body decay gives rise to three independent final momenta in the rest frame of the decaying particle, and one can form a T-odd expectation value out of (e.g.) $\vec{p}_1 \times \vec{p}_2 \cdot \vec{p}_3$. Under certain circumstances a nonzero value of this triple product can also signify CP violation. A famous example is the CP-odd asymmetry of $(13.6 \pm 1.4 \pm 1.5)\%$ reported by the KTeV Collaboration [5]. Here we present a general discussion for T-odd triple-product (TP) asymmetries in four-body decays of strange, charmed, and beauty mesons, focusing on genuine CP-violating asymmetries. While these asymmetries are generally expected to be small in the standard model, larger values can signify new physics, and their observation (in contrast to direct CP asymmetries in decay rates) does not depend on the presence of large (but generally incalculable) strong final-state phases.

Charmed meson decays are expected to exhibit very small CP-violating effects in the standard model [6]. Triple-product asymmetries in four body D and D_s decays are expected to reflect final-state interactions. Comparing triple-product asymmetries in charmed meson decays and in CP-conjugate processes provides CP-violating observables which could serve as potential probes for new physics.

Focusing on B meson decays, four-particle final states are obtained through two vector meson intermediate states. Studying CP-violating TP asymmetries is of particular interest in a class of decays which are induced by $b \to s$ transitions. These CKM (Cabibbo-Kobayashi-Maskawa) and loop-suppressed processes are sensitive to new decay amplitudes [7]. B_s decays to two vector mesons induced by $b \to c\bar{c}s$ involve in the standard model a very small weak phase occurring in the interference of B_s–$\bar{B}_s$ mixing and decay amplitudes. This phase may be affected by new contributions to B_s–$\bar{B}_s$ mixing. The question is whether such new contributions could show up in TP asymmetries.

In Sec. II we lay the foundation for a discussion of triple-product asymmetries in four-body decays. We specialize to an example of neutral kaon decays in Sec. III. Recently measured triple-product asymmetries and CP-violating asymmetries in charmed particle decays are discussed in Sec. IV, drawing some conclusions about final-state interactions. A discussion of T-odd asymmetries is presented in Sec. V for decays of a B meson to a pair of vector mesons, which decay either to two pseudoscalar pairs or to a pseudoscalar pair and a lepton pair. The corresponding CP-violating TP asymmetries are then treated in Sec. VI, studying time dependence for asymmetries in neutral B decays in terms of a mass difference Δm and a width difference $\Delta\Gamma$. We discuss triple products for specific B decays to two vector mesons in Sec. VII and present a short conclusion in Sec. VIII.

II. TRIPLE PRODUCTS IN FOUR-BODY DECAYS

Scalar triple products of three-momentum or spin vectors occurring in particle decays are interesting because they are odd under time-reversal T. This may be due to a T-violating (and CP-violating) phase or caused by a CP-conserving phase from final-state interactions. A nontrivial triple product requires at least four particles in the final state if only momenta are measured. Consider a four-body decay of a particle P, $P \to abcd$, in which one measures the four particles' momenta in the P rest frame. The momenta of the two pairs of particles, ab and cd, form two decay planes intersecting at a straight line given by the momentum vector $\vec{p}_a + \vec{p}_b = -\vec{p}_c - \vec{p}_d$. We define z to be the direction of $\vec{p}_a + \vec{p}_b$ and denote by $\hat{z}$ a unit vector in this direction. Unit vectors normal to the two decay planes and to their line of intersection $\hat{z}$ are denoted by $\hat{n}_{ab}$, $\hat{n}_{cd}$. The angle ϕ between these two normal vectors is conventionally defined to be the angle between the two decay planes.

Thus we have

$$\hat{n}_{ab} \cdot \hat{n}_{cd} = \cos\phi, \qquad \hat{n}_{ab} \times \hat{n}_{cd} = \sin\phi\, \hat{z}, \tag{1}$$

implying a T-odd scalar triple product

$$(\hat{n}_{ab} \times \hat{n}_{cd}) \cdot \hat{z} = \sin\phi, \tag{2}$$

and

$$\sin 2\phi = 2(\hat{n}_{ab} \cdot \hat{n}_{cd})(\hat{n}_{ab} \times \hat{n}_{cd}) \cdot \hat{z}, \tag{3}$$

which is also odd under time-reversal because $\hat{n}_{ab} \cdot \hat{n}_{cd}$ is even under this transformation. A T-odd asymmetry in the decay can be defined by an asymmetry between the number of events N with positive and negative values of $\sin\phi$ or $\sin 2\phi$, for example,

$$A_T(\sin 2\phi) \equiv \frac{N(\sin 2\phi > 0) - N(\sin 2\phi < 0)}{N(\sin 2\phi > 0) + N(\sin 2\phi < 0)}. \tag{4}$$

A special example of this kind of asymmetry has been studied several years ago by the KTeV and NA48 Collaborations in $K_L \to \pi^+\pi^- e^+ e^-$, measuring values $A_T(\sin 2\phi) = (13.6 \pm 1.4 \pm 1.5)\%$ [5] and $A_T(\sin 2\phi) = (14.2 \pm 3.6)\%$ [8], respectively. Here ϕ is the angle between vectors $\hat{n}_\pi$ and $\hat{n}_e$ which are normal to the $\pi^+\pi^-$ and e^+e^- planes, $\sin 2\phi = 2(\hat{n}_\pi \cdot \hat{n}_e)(\hat{n}_\pi \times \hat{n}_e) \cdot \hat{z}$, $\hat{z} \equiv [\vec{p}(\pi^+) + \vec{p}(\pi^-)]/|\vec{p}(\pi^+) + \vec{p}(\pi^-)|$. In this particular decay, which involves two particle-antiparticle pairs, the quantity $\sin 2\phi$ changes sign under both T and CP [9]. The latter property can be seen by noting that under C, $\vec{p}(\pi^\pm) \to \vec{p}(\pi^\mp)$, $\vec{p}(e^\pm) \to \vec{p}(e^\mp)$ while under P, $\vec{p}(\pi^\pm) \to -\vec{p}(\pi^\pm)$, $\vec{p}(e^\pm) \to -\vec{p}(e^\pm)$. CP invariance would imply that the expectation value of this CP-odd observable vanishes for an initial CP eigenstate. Thus, *this measurement provides the largest CP-nonconserving effect observed in kaon decays*.

A particular case, in which the expectation value of a T-odd scalar triple product of three momenta vanishes (irrespective of CP invariance), occurs when two of the four final decay particles are identical, assuming that these particles are kinematically indistinguishable. This happens when one does not include a constraint on the final particle momenta. Two useful examples, which will be discussed in Sec. IV with other charm decays, are $D^0 \to K^-\pi^+\pi^-\pi^+$ and $D^+ \to K^-\pi^+\pi^+\pi^0$ both of which involve two identical π^+ mesons in the final state.

A general proof of this property is based on the covariant form of a triple-product observable in $P \to abcd$ expressed as $\epsilon_{\mu\nu\rho\sigma} p_a^\mu p_b^\nu p_c^\rho p_d^\sigma$ in terms of the four outgoing particle four-momenta. We are assuming that the final particles a and b are identical and are kinematically indistinguishable. Using energy-momentum conservation ($p_d = p_B - p_a - p_b - p_c$), the above expression becomes proportional to $\epsilon_{ijk} p_a^i p_b^j p_c^k = (\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c = -(\vec{p}_b \times \vec{p}_a) \cdot \vec{p}_c$ in the B rest frame. Because of its antisymmetry in $\vec{p}_a$ and $\vec{p}_b$, the expectation value of this triple product vanishes, $\langle(\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c\rangle = 0$, when summing over the indistinguishable momenta of the two identical particles.

An alternative proof of this theorem for identical particles a and b may be presented by showing that $A_T(\sin\phi) = 0$ or $\langle\sin\phi\rangle = 0$, where $\sin\phi$ is defined in Eq. (2). Writing

$$\sin\phi = \hat{n}_{ab} \cdot (\hat{n}_{cd} \times \hat{z}), \tag{5}$$

one has $\hat{n}_{ab} = (\vec{p}_a \times \vec{p}_b)/|\vec{p}_a \times \vec{p}_b|$ while $\hat{n}_{cd} \times \hat{z}$ is a vector in the plane of $\vec{p}_c$ and $\vec{p}_d$ perpendicular to $\vec{p}_c + \vec{p}_d$. Using momentum conservation, $\vec{p}_d = -\vec{p}_a - \vec{p}_b - \vec{p}_c$, the vector $\hat{n}_{cd} \times \hat{z}$ may be replaced by $\vec{p}_c$ while $\vec{p}_a$ and $\vec{p}_b$ do not contribute to (5). Thus

$$\langle\sin\phi\rangle \propto \langle[(\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c]/|\vec{p}_a \times \vec{p}_b|\rangle, \tag{6}$$

which vanishes when summing symmetrically over the momenta $\vec{p}_a$ and $\vec{p}_b$.

A nonzero triple-product asymmetry may occur when at least one of the two identical particles forms a resonance, or favors a low invariant mass, with a third particle (c or d), in which case one does not sum symmetrically over $\vec{p}_a$ and $\vec{p}_b$ in $\langle(\vec{p}_a \times \vec{p}_b) \cdot \vec{p}_c\rangle$. In four-body decays, where two pairs of final particles are associated with two vector mesons in an intermediate state, the triple-product asymmetry depends also on the vector meson polarization and does not have to vanish for two identical particles. This situation occurs in B and B_s decays to two vector mesons, for instance in $B^0 \to K^{*0}(\to K^+\pi^-)\phi(\to K^+K^-)$ and $B_s \to \phi(\to K^+K^-)\phi(\to K^+K^-)$.

III. THE DECAYS $K_L \to e^+e^-e^+e^-$ AND $K_L \to e^+e^-\mu^+\mu^-$

A simple example demonstrates the above circumstances permitting a CP- or T-violating expectation value in a four-body decay even when two pairs of final-state particles are equal. This is in the decay $K_L \to e^+e^-e^+e^-$ for which 441 and 200 events were observed by the KTeV

[10] and NA48 [11] collaborations. (The decay $K_L \to e^+e^-\mu^+\mu^-$ also has been observed by KTeV [12].) Consider first of all only very low-mass e^+e^- pairs produced by photons very near their mass shell.

Define the CP-even and CP-odd combinations of K^0 and $\bar{K}^0$ to be K_1 and K_2, respectively. We have $K_L \simeq K_2 + \epsilon K_1$, where $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$, $\mathrm{Arg}(\epsilon) = (43.51 \pm 0.05)°$ [13]. Since the K_L is mainly CP odd, its decay to two photons is dominated by the effective Lagrangian $\mathcal{L}_- \propto K_2 F_{\mu\nu}\tilde{F}^{\mu\nu}$, but the small CP-even admixture decays via an effective Lagrangian $\mathcal{L}_+ \propto K_1 F_{\mu\nu}F^{\mu\nu}$. Here

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}, \qquad \tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3 & E_2 \\ B_2 & E_3 & 0 & -E_1 \\ B_3 & -E_2 & E_1 & 0 \end{bmatrix}, \tag{7}$$

so that $\mathcal{L}_+ \propto K_1(\vec{B}^2 - \vec{E}^2)$, $\mathcal{L}_- \propto 2K_2\vec{E}\cdot\vec{B}$. Let one photon be emitted along the $+\hat{z}$ axis with polarization $\epsilon_1 = \hat{x}$, and measure the polarization of a second photon along the $-\hat{z}$ axis with a polarizer oriented in the direction $\epsilon_2 = \hat{x}\cos\phi + \hat{y}\sin\phi$. For the decay of a CP-(even, odd) state, the amplitudes for observing this photon are then proportional to $\cos\phi$, $\sin\phi$, respectively [14]. The decay of a CP admixture such as K_L then will give rise to interference between these two amplitudes and hence an amplitude proportional to $\sin(\phi - \delta)$, where $\delta \neq (0, \pi/2)$.

In the case of $K_L \to e^+e^-e^+e^-$, the virtual photons giving rise to e^+e^- pairs are not exclusively transversely polarized, and the e^+e^- planes do not analyze photon polarizations perfectly, so that the signal for even or odd CP will be diluted. For example, in the case of $\pi^0 \to e^+e^-e^+e^-$ [15], the angular distribution of the decay rate is

$$\pi\frac{1}{\Gamma}\frac{d\Gamma}{d\phi} = (0.59\sin^2\phi + 0.41\cos^2\phi), \tag{8}$$

whereas an argument based on transversely polarized photons would have given $\sin^2\phi$ for the right-hand side. For $K_L \to e^+e^-e^+e^-$ one finds assuming no direct CP violation [15,16]

$$2\pi\frac{1}{\Gamma}\frac{d\Gamma}{d\phi} = 1 + \beta_{CP}\cos(2\phi) + \gamma_{CP}\sin(2\phi), \tag{9}$$

$$\beta_{CP} \equiv \frac{1 - |\epsilon r|^2}{1 + |\epsilon r|^2}B, \qquad \gamma_{CP} \equiv \frac{2\,\mathrm{Re}(\epsilon r)}{1 + |\epsilon r|^2}C, \tag{10}$$

where $r \equiv |A(K_1 \to e^+e^-e^+e^-)/A(K_2 \to e^+e^-e^+e^-)|$ is of order unity, $B \simeq -0.2$ (it would be $+0.2$ for $K_S \to e^+e^-e^+e^-$), and C has not yet been calculated. One would expect C to be of the same order as B as it represents a "dilution" of the interference between CP-even and CP-odd decays as analyzed by the electron-positron pairs.

TABLE I. Measured values of β_{CP} and γ_{CP} [Eqs. (9) and (10)] in $K_L \to e^+e^-e^+e^-$.

Collaboration	KTeV [10]	NA48 [11]
Events	441	200
β_{CP}	$-0.23 \pm 0.09 \pm 0.02$	$-0.13 \pm 0.10 \pm 0.03$
γ_{CP}	$-0.09 \pm 0.09 \pm 0.02$	$0.13 \pm 0.10 \pm 0.03$

The term γ_{CP} is directly related to the T-odd observable in Eq. (4),

$$A_T(\sin 2\phi) = (2/\pi)\gamma_{CP}, \tag{11}$$

which in this case of two particle-antiparticle pairs in the final state is also CP odd. Measured values of β_{CP} and γ_{CP} are shown in Table I. They are consistent with theoretical predictions, although improvement of accuracy by at least a factor of 100 will be needed to see nonzero γ_{CP} at the predicted level. We thus show that in order to form a T and CP-odd observable it is not necessary to have four distinct particles as long as they exhibit nontrivial kinematic correlations.

IV. TP AND CP-VIOLATING ASYMMETRIES IN $D_{(s)}$ DECAYS

Four-body Cabibbo-favored D and D_s decays involve sizable branching ratios. For instance, a few years ago the CLEO Collaboration reported measurements [17] $\mathcal{B}(D^0 \to K^-\pi^+\pi^-\pi^+) = (8.30 \pm 0.07 \pm 0.20)\%$, $\mathcal{B}(D^+ \to K^-\pi^+\pi^+\pi^0) = (5.98 \pm 0.08 \pm 0.18)\%$ and [18] $\mathcal{B}(D_s \to K^+K^-\pi^+\pi^0) = (5.65 \pm 0.29 \pm 0.40)\%$. As we have shown in Sec. II, triple-product asymmetries are expected to vanish in the first two processes both of which involve two identical π^+ mesons which are kinematically indistinguishable.

Triple-product correlations have been studied by the FOCUS and *BABAR* collaborations in Cabibbo-suppressed decays $D^0 \to K^+K^-\pi^+\pi^-$ [19,20] and very recently by the *BABAR* Collaboration in both Cabibbo-favored and Cabibbo-suppressed decays, $D_s^+ \to K^+K_S\pi^+\pi^-$ and $D^+ \to K^+K_S\pi^+\pi^-$, respectively [21]. Denoting a scalar triple-product for momenta of three final particles in the charmed meson rest frame, $C_T \equiv \vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$, one defines a triple-product asymmetry for D or D_s decay [6]

$$A_T \equiv \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)}. \tag{12}$$

This T-odd asymmetry is expected to be nonzero as a result of final-state interactions. In order to test for CP violation

TABLE II. Triple-product asymmetries A_T, $\bar{A}_T$, $\mathcal{A}_T$, and Σ_T (defined in the text) for Cabibbo-suppressed decays $D^0 \to K^+K^-\pi^+\pi^-$ [20], $D^+ \to K^+K_S\pi^+\pi^-$ [21] and Cabibbo-favored decays $D_s^+ \to K^+K_S\pi^+\pi^-$ [21]. Values are quoted in units of 10^{-3}.

Asymmetry	$D^0/\bar{D}^0$	D^+/D^-	D_s^+/D_s^-
A_T	$-68.5 \pm 7.3 \pm 5.8$	$11.2 \pm 14.1 \pm 5.7$	$-99.2 \pm 10.7 \pm 8.3$
$\bar{A}_T$	$-70.5 \pm 7.3 \pm 3.9$	$35.1 \pm 14.3 \pm 7.2$	$-72.1 \pm 10.9 \pm 10.7$
$\mathcal{A}_T$	$1.0 \pm 5.1 \pm 4.4$	$-12.0 \pm 10.0 \pm 4.6$	$-13.6 \pm 7.7 \pm 3.4$
Σ_T	-69.5 ± 6.2	23.1 ± 11.0	85.6 ± 10.2

one compares this asymmetry with a corresponding asymmetry in the CP conjugate process involving $\bar{D}$ or $\bar{D}_s$,

$$\bar{A}_T \equiv \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)}. \tag{13}$$

Here $\bar{C}_T$ denotes a triple product of momenta for charge-conjugate particles while the minus sign in front of $\bar{C}_T$ follows by applying parity.

The difference

$$\mathcal{A}_T \equiv \frac{1}{2}(A_T - \bar{A}_T) \tag{14}$$

provides a measure for CP violation. A nonzero asymmetry $\mathcal{A}_T$ may follow from a CP asymmetry in partial rates. In the absence of such asymmetry [assuming $\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0) = \Gamma(C_T > 0) + \Gamma(C_T < 0)$] $\mathcal{A}_T \neq 0$ may be the result of a CP asymmetry in triple-product correlations, $\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0) \neq \Gamma(C_T > 0) - \Gamma(C_T < 0)$.

Table II quotes values of A_T, $\bar{A}_T$, and $\mathcal{A}_T$ from Refs. [20,21] for Cabibbo-suppressed $D^0 \to K^+K^-\pi^+\pi^-$, $D^+ \to K^+K_S\pi^+\pi^-$ and Cabibbo-favored $D_s^+ \to K^+K_S\pi^+\pi^-$. For completeness we also include in the table values calculated for a quantity

$$\Sigma_T \equiv \frac{1}{2}(A_T + \bar{A}_T). \tag{15}$$

This average of triple-product asymmetries in a charmed meson decay and its CP conjugate is not CP violating. Rather, being T odd, it may provide information on final-state interaction.

While all three values of $\mathcal{A}_T$ in Table II are consistent with zero, the values of Σ_T are considerably more significant for D^0 and D_s^+ decays than for D^+ decays. This pattern seems to indicate a difference among final-state interactions in the three decays. Final-state interactions in Cabibbo-favored D decays could in part be responsible for the hierarchy of lifetimes $\tau(D^+) > \tau(D_s^+) \gtrsim \tau(D^0)$. The final states in Cabibbo-favored D^+ decays are "exotic" involving $I = 3/2$ with quantum numbers of $su\bar{d}\bar{d}$ and do not correspond to any known resonances, whereas Cabibbo-favored D^0 and D_s^+ decays populate $I = 1/2$ and $I = 1$ states with quantum numbers of $s\bar{d}$ and $u\bar{d}$, respectively. The measured longer D^+ lifetime could thus be associated with the lack of resonances contributing to its decays [22,23].

One may perhaps expect an enhancement pattern similar to the one observed in the total hadronic decay rate of D^0 relative to that of D^+ also in Cabibbo-suppressed decays. The total hadronic enhancement is given by [13]

$$\frac{\Gamma_h(D^0)}{\Gamma_h(D^+)} = \frac{\tau(D^+)}{\tau(D^0)}\left(\frac{1 - \mathcal{B}_{\rm sl}(D^0)}{1 - \mathcal{B}_{\rm sl}(D^+)}\right) = \frac{1040 \pm 7}{410.1 \pm 1.5}\left(\frac{0.868 \pm 0.006}{0.66 \pm 0.03}\right) = 3.34 \pm 0.15. \tag{16}$$

Here $\mathcal{B}_{\rm sl} \equiv \mathcal{B}_{{\rm sl},e} + \mathcal{B}_{{\rm sl},\mu}$ are semileptonic branching ratios, $\mathcal{B}_{{\rm sl},e}(D^0) = (6.49 \pm 0.11)\%$, $\mathcal{B}_{{\rm sl},\mu}(D^0) = (6.7 \pm 0.6)\%$, $\mathcal{B}_{{\rm sl},e}(D^+) = (16.07 \pm 0.30)\%$, $\mathcal{B}_{{\rm sl},\mu}(D^+) = (17.6 \pm 3.2)\%$. Using [13] $\mathcal{B}(D^0 \to K^+K^-\pi^+\pi^-) = (2.42 \pm 0.12) \times 10^{-3}$, $\mathcal{B}(D^+ \to K^+K_S\pi^+\pi^-) = (1.75 \pm 0.18) \times 10^{-3}$, one calculates the ratio of Cabibbo-suppressed decay rates,

$$\frac{\Gamma(D^0 \to K^+K^-\pi^+\pi^-)}{\Gamma(D^+ \to K^+\bar{K}^0\pi^+\pi^-)} = \frac{\tau(D^+)}{\tau(D^0)}\frac{\mathcal{B}(D^0 \to K^+K^-\pi^+\pi^-)}{2\mathcal{B}(D^+ \to K^+K_S\pi^+\pi^-)} = 1.75 \pm 0.20. \tag{17}$$

Thus we conclude that some enhancement of Cabibbo-suppressed $D^0 \to K^+K^-\pi^+\pi^-$ relative to $D^+ \to K^+\bar{K}^0\pi^+\pi^-$ occurs, although it is less than in Cabibbo-favored decays.

This partial enhancement may account for the pattern of measured values of Σ_T quoted for these two Cabibbo-suppressed processes in Table II. The large value of Σ_T measured for $D_s^+ \to K^+K_S\pi^+\pi^-$ reflects an enhancement in D_s^+ Cabibbo-favored decay rates. A total hadronic enhancement factor for D_s^+ similar to (16), $\Gamma_h(D_s^+)/\Gamma_h(D^+) \simeq 2.6$, is calculated including in the numerator a subtraction of $\mathcal{B}(D_s^+ \to \tau^+\nu_\tau) = (5.43 \pm 0.31)\%$ [13].

V. T-ODD ASYMMETRIES IN $B_{(s)} \to V_1V_2$

Consider $B_{(s)}$ decays into two vector mesons V_1 and V_2, each decaying to a pair of pseudoscalars, P_1P_1' and P_2P_2'. The decay amplitude for $B_{(s)}(p) \to V_1(k_1, \epsilon_1) + V_2(k_2, \epsilon_2)$

may be written in terms of angular momentum amplitudes [1] (we use normalization as in [3]),

$$M = \mathbf{a}\epsilon_1^* \cdot \epsilon_2^* + \frac{\mathbf{b}}{m_B^2}(p\cdot\epsilon_1^*)(p\cdot\epsilon_2^*) + i\frac{\mathbf{c}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \epsilon_1^{*\rho}\epsilon_2^{*\sigma}, \tag{18}$$

where $q \equiv k_1 - k_2$. The amplitudes $\mathbf{a}$ and $\mathbf{b}$ are linear combinations of S and D wave amplitudes while $\mathbf{c}$ corresponds to P wave. It is customary to use transversity amplitudes [24], which are related to the angular momentum amplitudes through the following relations [3] (see also [25] for relations involving helicity amplitudes):

$$A_\parallel = \sqrt{2}\mathbf{a}, \qquad A_0 = -\mathbf{a}x - \frac{m_1 m_2}{m_B^2}\mathbf{b}(x^2-1), \qquad A_\perp = 2\sqrt{2}\frac{m_1 m_2}{m_B^2}\mathbf{c}\sqrt{x^2-1}. \tag{19}$$

Here $x \equiv (k_1\cdot k_2)/(m_1 m_2)$; m_1 and m_2 are the masses of V_1 and V_2.

A. $V_1 \to P_1 P_1'$, $V_2 \to P_2 P_2'$

Let us consider decays in which each of the two vector mesons in $B_{(s)} \to V_1 V_2$ decays into two pseudoscalar mesons. This class of decays consists of charmless decays of B and B_s mesons including $B \to \phi(\to K^+K^-)K^*(\to K\pi)$ and $B_s \to \phi(\to K^+K^-)\phi(\to K^+K^-)$. We denote by θ_1 (θ_2) the angle between the directions of motion of P_1 (P_2) in the V_1 (V_2) rest frame and V_1 (V_2) in the B rest frame. The angle between the planes defined by P_1P_1' and P_2P_2' in the $B_{(s)}$ rest frame will be denoted by ϕ as in Sec. II. The decay angular distribution in these three angles is given in terms of the three transversity amplitudes A_0, $A_\parallel$, $A_\perp$ [26] (see also [25]):

$$\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = N\bigg(|A_0|^2\cos^2\theta_1\cos^2\theta_2 + \frac{|A_\parallel|^2}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\phi + \frac{|A_\perp|^2}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\phi + \frac{\mathrm{Re}(A_0A_\parallel^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\cos\phi - \frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\sin\phi - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\phi\bigg). \tag{20}$$

Integrating over θ_1 and θ_2 and using

$$\int_{-1}^1 \cos^2\theta d\cos\theta = \frac{2}{3}, \quad \int_{-1}^1 \sin^2\theta d\cos\theta = \frac{4}{3}, \quad \int_{-1}^1 \sin 2\theta d\cos\theta = 0, \tag{21}$$

one obtains the following distribution in ϕ:

$$\frac{d\Gamma}{d\phi} = \frac{4}{9}N(|A_0|^2 + 2|A_\perp|^2\sin^2\phi + 2|A_\parallel|^2\cos^2\phi - 2\,\mathrm{Im}(A_\perp A_\parallel^*)\sin 2\phi). \tag{22}$$

The last term in this angular distribution provides a potential T-odd asymmetry. Note that the term involving $\mathrm{Im}(A_\perp A_0^*)$ does not contribute to a T-odd asymmetry when integrating over the angle θ_1 or θ_2.

One has now, in analogy with Eqs. (2) and (3),

$$\sin\phi = (\hat{n}_{V_1}\times\hat{n}_{V_2})\cdot\hat{p}_{V_1}, \qquad \sin 2\phi = 2(\hat{n}_{V_1}\cdot\hat{n}_{V_2})(\hat{n}_{V_1}\times\hat{n}_{V_2})\cdot\hat{p}_{V_1}, \tag{23}$$

where $\hat{n}_{V_i}(i = 1, 2)$ is a unit vector perpendicular to the V_i decay plane and $\hat{p}_{V_1}$ is a unit vector in the direction of V_1 in the $B_{(s)}$ rest frame. A triple-product (or more precisely a T-odd) asymmetry is now defined similarly to Eq. (4) as an asymmetry between the number of decays involving positive and negative values of $\sin 2\phi$ [3]:

$$A_T^{(2)} \equiv \frac{\Gamma(\sin 2\phi > 0) - \Gamma(\sin 2\phi < 0)}{\Gamma(\sin 2\phi > 0) + \Gamma(\sin 2\phi < 0)} = \frac{[\int_0^{\pi/2} + \int_\pi^{3\pi/2}](d\Gamma/d\phi)d\phi - [\int_{\pi/2}^{\pi} + \int_{3\pi/2}^{2\pi}](d\Gamma/d\phi)d\phi}{\int_0^{2\pi}(d\Gamma/d\phi)d\phi}. \tag{24}$$

Using (22) one obtains

$$A_T^{(2)} = -\frac{4}{\pi}\frac{\mathrm{Im}(A_\perp A_\parallel^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2}. \tag{25}$$

The dependence of the angular distribution (20) on θ_1 and θ_2 permits considering a second triple-product asymmetry [3] (or, more precisely, a T-odd asymmetry) $A_T^{(1)}$ involving the ratio $\mathrm{Im}(A_\perp A_0^*)/(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2)$. One defines an asymmetry with respect to values of $\sin\phi$ (a triple product), assigning it the sign of $\cos\theta_1\cos\theta_2$ (a T-even quantity) and integrating over all angles,

$$A_T^{(1)} \equiv \frac{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\phi > 0] - \Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\phi < 0]}{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\phi > 0] + \Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\phi < 0]}. \tag{26}$$

A straightforward calculation using Eq. (20) gives

$$A_T^{(1)} = -\frac{2\sqrt{2}}{\pi}\frac{\mathrm{Im}(A_\perp A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2}. \tag{27}$$

The two triple-product asymmetries, defined in Eqs. (24) and (26) and given in (25) and (27) in terms of transversity amplitudes, are odd under time reversal; however, they are not genuine CP-violating or T-violating observables. Rather, they may be nonzero due to a CP-conserving phase difference between two corresponding transversity amplitudes while the weak phase difference of these amplitudes vanishes.

B. $V_1 \to P_1 P_1'$, $V_2 \to \ell^+\ell^-$

We now consider a second class of decays into two vector mesons of which one meson decays into a pair of pseudoscalars while the other decays into a lepton pair $\ell^+\ell^-$, $\ell = e$, μ. This class of processes involving charmonium in the final state includes the decays $B \to K^*(\to K\pi)J/\psi(\to \mu^+\mu^-)$ and $B_s \to \phi(\to K^+K^-)J/\psi(\to \mu^+\mu^-)$. As in decays into four pseudoscalars, we denote by θ_1 the angle between the directions of motion of P_1 in the V_1 rest frame and V_1 in the $B_{(s)}$ rest frame, while θ_ℓ is the corresponding angle of ℓ^+ in the V_2 rest frame. The angle between the planes defined by P_1P_1' and $\ell^+\ell^-$ in the $B_{(s)}$ rest frame will be denoted here by ϕ. One is interested in triple products which are functions of this angle.

The complete decay angular distribution for this class of decays is given by [24] (see also [25]):

$$\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_\ell d\phi} = N\Big(|A_0^\ell|^2\cos^2\theta_1\sin^2\theta_\ell + \frac{|A_\parallel^\ell|^2}{2}\sin^2\theta_1(\sin^2\phi + \cos^2\theta_\ell\cos^2\phi) + \frac{|A_\perp^\ell|^2}{2}\sin^2\theta_1(\cos^2\phi + \cos^2\theta_\ell\sin^2\phi) \tag{28}$$

$$+ \frac{1}{2\sqrt{2}}\mathrm{Im}(A_\perp^\ell A_0^{\ell*})\sin2\theta_1\sin2\theta_\ell\sin\phi - \frac{\mathrm{Re}(A_0^\ell A_\parallel^{\ell*})}{2\sqrt{2}}\sin2\theta_1\sin2\theta_\ell\cos\phi + \frac{1}{2}\mathrm{Im}(A_\perp^\ell A_\parallel^{\ell*})\sin^2\theta_1\sin^2\theta_2\sin2\phi\Big). \tag{29}$$

Integrating over the angles θ_1 and θ_ℓ one obtains

$$\frac{d\Gamma}{d\phi} = \frac{4}{9}N(2|A_0^\ell|^2 + |A_\parallel^\ell|^2(1 + 2\sin^2\phi) + |A_\perp^\ell|^2(1 + 2\cos^2\phi) + 2\,\mathrm{Im}(A_\perp^\ell A_\parallel^{\ell*})\sin2\phi). \tag{30}$$

The last term is a source of one of two triple-product asymmetries. A T-odd asymmetry defined for $\sin2\phi$ in analogy with (24) obtains a similar expression (but different sign and normalization) in terms of transversity amplitudes,

$$A_T^{(2)\ell} \equiv \frac{\Gamma(\sin2\phi > 0) - \Gamma(\sin2\phi < 0)}{\Gamma(\sin2\phi > 0) + \Gamma(\sin2\phi < 0)} = \frac{2}{\pi}\frac{\mathrm{Im}(A_\perp^\ell A_\parallel^{\ell*})}{|A_0^\ell|^2 + |A_\perp^\ell|^2 + |A_\parallel^\ell|^2}. \tag{31}$$

A second asymmetry can be defined for values of the triple product $\sin\phi$, in the same manner as Eq. (26). One obtains

$$A_T^{(1)\ell} = \frac{\sqrt{2}}{\pi}\frac{\mathrm{Im}(A_\perp^\ell A_0^{\ell*})}{|A_0|^2 + |A_\perp^\ell|^2 + |A_\parallel^\ell|^2}. \tag{32}$$

VI. CP-VIOLATING TP ASYMMETRIES IN $B_{(s)} \to V_1V_2$

A. Self-tagged decays of charged and neutral B mesons

In this subsection we consider B and B_s decays to states with specific flavor, e.g. $B^{(+,0)} \to K^{*(+,0)}\phi$ and $B^{(+,0)} \to K^{*(+,0)}J/\psi$ belonging to the two classes considered in Secs. V A and V B, respectively. We denote by $\bar{A}_0$, $\bar{A}_\parallel$, and $\bar{A}_\perp$ transversity amplitudes for the CP-conjugate decay $\bar{B}_{(s)} \to \bar{V}_1\bar{V}_2$. The corresponding three angles describing the two vector meson decays into pairs of pseudoscalar mesons will be denoted by $\bar{\theta}_1$, $\bar{\theta}_2$, and $\bar{\phi}$. The decay angular distribution for $\bar{B}_{(s)}$ decays has an expression similar to $B_{(s)}$ decays. The two terms linear in the parity-odd amplitude $\bar{A}_\perp$ change sign relative to the corresponding two terms in Eq. (20). Thus, for decays in which both vector mesons $\bar{V}_1$ and $\bar{V}_2$ decay to a pseudoscalar pair one has

$$\frac{d\bar{\Gamma}}{d\cos\bar{\theta}_1 d\cos\bar{\theta}_2 d\bar{\phi}} = N\Big(|\bar{A}_0|^2\cos^2\bar{\theta}_1\cos^2\bar{\theta}_2 + \frac{|\bar{A}_\perp|^2}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\sin^2\bar{\phi} + \frac{|\bar{A}_\parallel|^2}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\cos^2\bar{\phi} + \frac{\mathrm{Re}(\bar{A}_0\bar{A}_\parallel^*)}{2\sqrt{2}}\sin 2\bar{\theta}_1\sin 2\bar{\theta}_2\cos\bar{\phi} + \frac{\mathrm{Im}(\bar{A}_\perp\bar{A}_0^*)}{2\sqrt{2}}\sin 2\bar{\theta}_1\sin 2\bar{\theta}_2\sin\bar{\phi} + \frac{\mathrm{Im}(\bar{A}_\perp\bar{A}_\parallel^*)}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\sin 2\bar{\phi}\Big). \tag{33}$$

It has been pointed out [1,3] that the two quantities $\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*)$ and $\mathrm{Im}(A_\perp A_\parallel^* - \bar{A}_\perp\bar{A}_\parallel^*)$, *occurring in the sum (rather than the difference) of decay distributions* (20) and (33) *for $B_{(s)}$ and $\bar{B}_{(s)}$ for $\bar{\theta}_1 = \theta_1$, $\bar{\theta}_2 = \theta_2$, $\bar{\phi} = \phi$, are genuinely CP-violating* and do not require nonzero CP conserving phases. For instance, assuming that each of the transversity amplitudes is dominated by a magnitude, $|A_\lambda|$, a single CP-conserving phase, δ_λ, and a single CP-violating phase, ϕ_λ (which amounts to assuming no direct CP violation),

$$A_\lambda = |A_\lambda|e^{i\delta_\lambda}e^{i\phi_\lambda}, \qquad \bar{A}_\lambda = |A_\lambda|e^{i\delta_\lambda}e^{-i\phi_\lambda}(\lambda = 0, \parallel, \perp), \tag{34}$$

implies

$$\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*) = 2|A_\perp||A_0|\cos(\delta_\perp - \delta_0) \times \sin(\phi_\perp - \phi_0). \tag{35}$$

This true CP-violating quantity is nonzero also when the CP-conserving phase difference $\delta_\perp - \delta_0$ vanishes, provided that the CP-violating phase difference $\phi_\perp - \phi_0$ between the two transversity amplitudes $A_\perp$ and A_0 is nonzero. In contrast, a quantity *occurring in the difference of rates for $B_{(s)}$ and $\bar{B}_{(s)}$*,

$$\mathrm{Im}(A_\perp A_0^* + \bar{A}_\perp\bar{A}_0^*) = 2|A_\perp||A_0|\sin(\delta_\perp - \delta_0) \times \cos(\phi_\perp - \phi_0), \tag{36}$$

is not CP-violating as it is nonzero also when CP-violating phases vanish. Such a quantity will sometimes be referred to as a fake asymmetry.

The above expressions for the quantities $\mathrm{Im}(A_\perp A_0^* \pm \bar{A}_\perp\bar{A}_0^*)$ may be generalized to the case of direct CP violation, in which transversity amplitudes involve each several contributions with distinct weak and strong phases,

$$A_\lambda = \Sigma_l|A_\lambda^l|e^{i\delta_\lambda^l}e^{i\phi_\lambda^l}. \tag{37}$$

One finds

$$\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*) = 2\Sigma_{l,m}|A_\perp^l||A_0^m|\cos(\delta_\perp^l - \delta_0^m) \times \sin(\phi_\perp^l - \phi_0^m), \tag{38}$$

$$\mathrm{Im}(A_\perp A_0^* + \bar{A}_\perp\bar{A}_0^*) = 2\Sigma_{l,m}|A_\perp^l||A_0^m|\sin(\delta_\perp^l - \delta_0^m) \times \cos(\phi_\perp^l - \phi_0^m). \tag{39}$$

It is interesting to note that the CP-violating quantities $\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*)$ and $\mathrm{Im}(A_\perp A_\parallel^* - \bar{A}_\perp\bar{A}_\parallel^*)$ occur in *triple-product asymmetries for CP-averaged decay rates*. We denote partial decay rates for $B_{(s)} \to f$ and $\bar{B}_{(s)} \to \bar{f}$ by $\Gamma(B_{(s)} \to f)$ and $\bar{\Gamma}(\bar{B}_{(s)} \to \bar{f})$, respectively. The charge-averaged decay rate is $[\Gamma(B_{(s)} \to f) + \bar{\Gamma}(\bar{B}_{(s)} \to \bar{f})]/2$, and a triple-product asymmetry defined for this rate is given by

$$\mathcal{A}_T^{(2)\mathrm{chg\text{-}avg}} \equiv \frac{[\Gamma(\sin 2\phi > 0) + \bar{\Gamma}(\sin 2\bar{\phi} > 0)] - [\Gamma(\sin 2\phi < 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0)]}{[\Gamma(\sin 2\phi > 0) + \bar{\Gamma}(\sin 2\bar{\phi} > 0)] + [\Gamma(\sin 2\phi < 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0)]} = -\frac{4}{\pi}\frac{\mathrm{Im}(A_\perp A_\parallel^* - \bar{A}_\perp\bar{A}_\parallel^*)}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2)}. \tag{40}$$

As noted above the numerator is genuinely CP-violating. A second charge-averaged asymmetry, defined with respect to the variables $S \equiv \mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\phi$ for $B_{(s)}$ and $\bar{S} \equiv \mathrm{sign}(\cos\bar{\theta}_1\cos\bar{\theta}_2)\sin\bar{\phi}$ for $\bar{B}_{(s)}$, is proportional to $\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*)$:

$$\mathcal{A}_T^{(1)\mathrm{chg\text{-}avg}} \equiv \frac{[\Gamma(S > 0) + \bar{\Gamma}(\bar{S} > 0)] - [\Gamma(S < 0) + \bar{\Gamma}(\bar{S} < 0)]}{[\Gamma(S > 0) + \bar{\Gamma}(\bar{S} > 0)] + [\Gamma(S < 0) + \bar{\Gamma}(\bar{S} < 0)]} = -\frac{2\sqrt{2}}{\pi}\frac{\mathrm{Im}(A_\perp A_0^* - \bar{A}_\perp\bar{A}_0^*)}{(|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2) + (|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2)}. \tag{41}$$

Similarly, one may define charge-averaged triple-product asymmetries for decays in which one vector meson decays to a pseudoscalar pair while the other meson decays into a lepton pair. (Corresponding CP-violating observables in angular distributions for $B \to J/\psi K^*$ have been discussed in Ref. [27].) For these decays one finds

$$\mathcal{A}_T^{(2)\ell,\text{chg-avg}} = \frac{2}{\pi}\frac{\text{Im}(A_\perp^\ell A_\parallel^{\ell *} - \bar{A}_\perp^\ell \bar{A}_\parallel^{\ell *})}{(|A_0^\ell|^2 + |A_\perp^\ell|^2 + |A_\parallel^\ell|^2) + (|\bar{A}_0^\ell|^2 + |\bar{A}_\perp^\ell|^2 + |\bar{A}_\parallel^\ell|^2)},$$

$$\mathcal{A}_T^{(1)\ell,\text{chg-avg}} = \frac{\sqrt{2}}{\pi}\frac{\text{Im}(A_\perp^\ell A_0^{\ell *} - \bar{A}_\perp^\ell \bar{A}_0^{\ell *})}{(|A_0^\ell|^2 + |A_\perp^\ell|^2 + |A_\parallel^\ell|^2) + (|\bar{A}_0^\ell|^2 + |\bar{A}_\perp^\ell|^2 + |\bar{A}_\parallel^\ell|^2)}. \tag{42}$$

The two asymmetries $\mathcal{A}_T^{(i)\text{chg-avg}}$ ($i = 1, 2$) should be distinguished from somewhat different quantities discussed in Refs. [1,3], the average of the asymmetries $A_T^{(i)}$ and their charge-conjugates $\bar{A}_T^{(i)}$. For instance,

$$\frac{1}{2}(A_T^{(2)} + \bar{A}_T^{(2)}) \equiv \frac{1}{2}\left[\frac{\Gamma(\sin 2\phi > 0) - \Gamma(\sin 2\phi < 0)}{\Gamma(\sin 2\phi > 0) + \Gamma(\sin 2\phi < 0)} + \frac{\bar{\Gamma}(\sin 2\bar{\phi} > 0) - \bar{\Gamma}(\sin 2\bar{\phi} < 0)}{\bar{\Gamma}(\sin 2\bar{\phi} > 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0)}\right]$$
$$= -\frac{2}{\pi}\left(\frac{\text{Im}(A_\perp A_\parallel^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2} - \frac{\text{Im}(\bar{A}_\perp \bar{A}_\parallel^*)}{|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2}\right). \tag{43}$$

In general this quantity is not proportional to $\text{Im}(A_\perp A_\parallel^* - \bar{A}_\perp \bar{A}_\parallel^*)$. That is, the two asymmetries defined in Eqs. (40) and (43) are different in the most general case. They become equal when no direct *CP* asymmetry occurs in the total decay rate,

$$\Gamma(\sin 2\phi \geq 0) + \Gamma(\sin 2\phi < 0) = \bar{\Gamma}(\sin 2\bar{\phi} \geq 0) + \bar{\Gamma}(\sin 2\bar{\phi} < 0), \tag{44}$$

namely, when

$$|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2 = |\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2. \tag{45}$$

CP may be violated in decay rates for individual transversity amplitudes, $|A_k|^2 \neq |\bar{A}_k|^2$ ($k = 0, \parallel, \perp$). This implies nonzero *CP* asymmetries in these channels and a potential violation of (45) leading to $(A_T^{(1,2)} + \bar{A}_T^{(1,2)})/2 \neq \mathcal{A}_T^{(1,2)\text{chg-avg}}$. This happens when a given transversity amplitude obtains contributions involving at least two different weak phases and two different strong phases. [See Eq. (37)]. This is to be contrasted with a very special case of no direct *CP* violation in which $A_\perp$, A_0, and $A_\parallel$ each involve a single weak phase.

B. Neutral $B_{(s)}$ decays to flavorless states

We now consider neutral $B_{(s)}$ decays into flavorless states which are accessible to both $B_{(s)}$ and $\bar{B}_{(s)}$ decays. Two examples, belonging to the two classes considered in Secs. V A and V B are $B_s \to \phi\phi$ and $B_s \to J/\psi\phi$. As a result of $B_{(s)}$-$\bar{B}_{(s)}$ oscillation angular decay distributions become time-dependent. Decay distributions for initial $B_{(s)}$ mesons are given for these two classes by Eqs. (20) and (28), where the coefficients $|A_k|^2 (k = 0, \parallel, \perp)$, $\text{Re}(A_0 A_\parallel^*)$, $\text{Im}(A_\perp A_i^*)(i = 0, \parallel)$ are now functions of time. The instantaneous transversity amplitude for a $B_{(s)}$ meson is $A_k \equiv A_k(t = 0)$. Similar expressions, in which $A_k(t)$ are replaced by $\bar{A}_k(t)$, apply to angular distributions for initial $\bar{B}_{(s)}$ mesons with $\bar{A}_k \equiv \bar{A}_k(t = 0)$. Thus, for decays in which each of the two vector mesons decays into a pseudoscalar pair,

$$\frac{d\bar{\Gamma}(t)}{dt d\cos\theta_1 d\cos\theta_2 d\phi} = N\Bigg(|\bar{A}_0(t)|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{|\bar{A}_\perp(t)|^2}{2}\sin^2\theta_1 \sin^2\theta_2 \sin^2\phi + \frac{|\bar{A}_\parallel(t)|^2}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\phi$$
$$+ \frac{\text{Re}(\bar{A}_0(t)\bar{A}_\parallel^*(t))}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \cos\phi - \frac{\text{Im}(\bar{A}_\perp(t)\bar{A}_0^*(t))}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \sin\phi$$
$$- \frac{\text{Im}(\bar{A}_\perp(t)\bar{A}_\parallel^*(t))}{2}\sin^2\theta_1 \sin^2\theta_2 \sin 2\phi\Bigg). \tag{46}$$

In particular, time-dependent terms relevant for triple products involving $\text{Im}[A_\perp(t)A_i^*(t)]$ and $\text{Im}[\bar{A}_\perp(t)\bar{A}_i^*(t)]$ appear with *equal signs* in the distributions for initial $B_{(s)}$ and $\bar{B}_{(s)}$. Thus, time-dependent TP quantities measured in untagged neutral $B_{(s)}$ decays to flavorless states are of the form $\text{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)]$. Note that the corresponding time-independent terms in Eqs. (20) and (33) appear with opposite signs for two distributions written in terms of θ_1, θ_2, ϕ, and $\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{\phi}$. The opposite relative signs in the two cases may be explained by noting that a *CP* transformation in decays to flavorless states corresponds to $\sin\bar{\phi} = -\sin\phi$ while the functions of θ_i and $\bar{\theta}_i$ are equal.

Let us study flavor-untagged decays which involve the time-dependent triple products $\text{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)](i = 0, \parallel)$. Considering their values at $t = 0$,

$\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*)$, we now show that these two quantities are genuinely CP-violating. We use standard notations for $B_{(s)}$–$\bar{B}_{(s)}$ mixing and assume no CP violation in mixing ($|q/p| = 1$). For a moment we will also assume no direct decay CP violation ($|\bar{A}_\lambda| = |A_\lambda|$) so that [28]

$$\frac{q}{p}\frac{\bar{A}_\lambda}{A_\lambda} = \eta_\lambda e^{-2i\phi_\lambda}. \tag{47}$$

Here η_λ is the CP parity for a state of transversity λ ($\eta_0 = \eta_\parallel = -\eta_\perp = +1$), while ϕ_λ is the weak phase involved in an interference between mixing and decay amplitudes. Denoting the CP conserving strong phase of A_λ by δ_λ, $A_\lambda = |A_\lambda| e^{i\delta_\lambda} e^{i\phi_\lambda}$, so $\bar{A}_\lambda = (p/q)\eta_\lambda e^{i\delta_\lambda} e^{-i\phi_\lambda}$, one has for $i = 0, \parallel$:

$$\begin{aligned}\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) &= |A_\perp||A_i| \\ &\quad \times \mathrm{Im}[e^{i(\delta_\perp - \delta_i)}(e^{i(\phi_\perp - \phi_i)} - e^{-i(\phi_\perp - \phi_i)})] \\ &= 2|A_\perp||A_i|\cos(\delta_\perp - \delta_i)\sin(\phi_\perp - \phi_i).\end{aligned} \tag{48}$$

In the case of direct CP violation, when each transversity amplitude obtains contributions with different weak phases, this expression is generalized to a sum as on the right-hand side of (38). As argued above, this true CP-violating quantity is nonzero also when the CP-conserving phase difference vanishes, provided that the CP-violating phase difference between the two transversity amplitudes is nonzero. Note the change of relative sign between terms on the left-hand-side of Eqs. (35) and (48), defining true CP-violating asymmetries in decays into specific flavor states and into flavorless CP states of opposite CP parity, respectively.

Time-dependence of the CP-violating triple products $\mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)]$ ($i = 0, \parallel$) depends on the $B_{(s)}$–$\bar{B}_{(s)}$ oscillation frequency determined by a mass difference Δm and on a width difference $\Delta\Gamma$ affecting the exponential decay. Early studies of time-dependent angular distributions [29], applied, in particular, to $B_s \to J/\psi\phi$, have assumed that a single weak phase, common to all three transversity states, is associated with interference between B_s–$\bar{B}_s$ mixing and decay amplitudes. In this case ($\phi_\perp = \phi_i$) the above two triple products vanish. Refs. [1,3] study some aspects of TP asymmetries induced by B–$\bar{B}$ mixing. We will now generalize the time dependence of the two triple products to the case under consideration, $\phi_\perp \neq \phi_i (i = 0, \parallel)$. Our calculation applies to both strange and nonstrange neutral mesons, $B = B^0$, B_s and their antiparticles, $\bar{B} = \bar{B}^0$, $\bar{B}_s$.

One starts with evolution equations for B and $\bar{B}$ [28]

$$\begin{aligned} B(t) &= g_+(t)B + (q/p)g_-(t)\bar{B}, \\ \bar{B}(t) &= (p/q)g_-(t)B + g_+(t)\bar{B}, \end{aligned} \tag{49}$$

where

$$\begin{aligned} g_+(t) &= e^{-imt}e^{-\Gamma t/2}[\cosh(\Delta\Gamma t/4)\cos(\Delta m t/2) \\ &\quad - i\sinh(\Delta\Gamma t/4)\sin(\Delta m t/2)], \\ g_-(t) &= e^{-imt}e^{-\gamma t/2}[-\sinh(\Delta\Gamma t/4)\cos(\Delta m t/2) \\ &\quad + i\cosh(\Delta\Gamma t/4)\sin(\Delta m t/2)], \end{aligned} \tag{50}$$

$$\begin{aligned} |g_\pm(t)|^2 &= (e^{-\Gamma t}/2)[\cosh(\Delta\Gamma t/2) \pm \cos(\Delta m t)], \\ g_+^*(t)g_-(t) &= (e^{-\Gamma t}/2)[-\sinh(\Delta\Gamma t/2) + i\sin(\Delta m t)]. \end{aligned} \tag{51}$$

Time dependence of transversity amplitudes, $A_k \equiv \langle k|B\rangle$, $\bar{A}_k \equiv \langle k|\bar{B}\rangle$ ($k = 0, \parallel, \perp$), is given by

$$\begin{aligned} A_k(t) &\equiv \langle k|B(t)\rangle = g_+(t)A_k + (q/p)g_-(t)\bar{A}_k, \\ \bar{A}_k(t) &\equiv \langle k|\bar{B}(t)\rangle = (p/q)g_-(t)A_k + g_+(t)\bar{A}_k. \end{aligned} \tag{52}$$

We are interested in interference terms $A_i^*(t)A_k(t)$ and $\bar{A}_i^*(t)\bar{A}_k(t)$. Using Eqs. (47) and (51) one obtains

$$\begin{aligned} A_i^*(t)A_k(t) &= [g_+^*A_i^* + (q/p)^*g_-^*\bar{A}_i^*][g_+A_k + (q/p)g_-\bar{A}_k] \\ &= A_i^*A_k[|g_+|^2 + (q/p)(\bar{A}_k/A_k)g_+^*g_-] + \bar{A}_i^*\bar{A}_k[|g_-|^2 + (p/q)(A_k/\bar{A}_k)g_+g_-^*] \\ &= \frac{e^{-\Gamma t}}{2}[A_i^*A_k(\cosh(\Delta\Gamma t/2) + \cos(\Delta m t) + \eta_k e^{-2i\phi_k}[-\sinh(\Delta\Gamma t/2) + i\sin(\Delta m t)]) \\ &\quad + \bar{A}_i^*\bar{A}_k(\cosh(\Delta\Gamma t/2) - \cos(\Delta m t) + \eta_k e^{2i\phi_k}[-\sinh(\Delta\Gamma t/2) - i\sin(\Delta m t)])]. \end{aligned} \tag{53}$$

Inserting $A_i^*A_k = |A_i||A_k|e^{i(\delta_k - \delta_i)}e^{i(\phi_k - \phi_i)}$, $\bar{A}_i^*\bar{A}_k = \eta_i\eta_k|A_i||A_k|e^{i(\delta_k - \delta_i)}e^{-i(\phi_k - \phi_i)}$ (we assume for a moment no direct CP violation) implies for $i = 0, \parallel$, $k = \perp$,

$$\begin{aligned} A_i^*(t)A_\perp(t) &= e^{-\Gamma t}|A_i||A_\perp|e^{i(\delta_\perp - \delta_i)}[i\sin(\phi_\perp - \phi_i)\cosh(\Delta\Gamma t/2) + \cos(\phi_\perp - \phi_i)\cos(\Delta m t) \\ &\quad - i\sin(\phi_\perp + \phi_i)\sinh(\Delta\Gamma t/2) - i\cos(\phi_\perp + \phi_i)\sin(\Delta m t)], \end{aligned} \tag{54}$$

leading to

$$\begin{aligned} \mathrm{Im}[A_i^*(t)A_\perp(t)] &= e^{-\Gamma t}|A_i||A_\perp|(\cos(\delta_\perp - \delta_i)[\sin(\phi_\perp - \phi_i)\cosh(\Delta\Gamma t/2) - \sin(\phi_\perp + \phi_i)\sinh(\Delta\Gamma t/2) \\ &\quad - \cos(\phi_\perp + \phi_i)\sin(\Delta m t)] + \sin(\delta_\perp - \delta_i)\cos(\phi_\perp - \phi_i)\cos(\Delta m t)). \end{aligned} \tag{55}$$

Similarly one has

$$\mathrm{Im}[\bar{A}_i^*(t)\bar{A}_\perp(t)] = e^{-\Gamma t}|A_i||A_\perp|(\cos(\delta_\perp-\delta_i)[\sin(\phi_\perp-\phi_i)\cosh(\Delta\Gamma t/2)-\sin(\phi_\perp+\phi_i)\sinh(\Delta\Gamma t/2) + \cos(\phi_\perp+\phi_i)\sin(\Delta m t)] - \sin(\delta_\perp-\delta_i)\cos(\phi_\perp-\phi_i)\cos(\Delta m t)). \quad (56)$$

Thus

$$\mathrm{Im}[A_\perp(t)A_i^*(t)+\bar{A}_\perp(t)\bar{A}_i^*(t)] = 2|A_\perp||A_i|e^{-\Gamma t}\cos(\delta_\perp-\delta_i)[\sin(\phi_\perp-\phi_i)\cosh(\Delta\Gamma t/2)-\sin(\phi_\perp+\phi_i)\sinh(\Delta\Gamma t/2)]. \quad (57)$$

This time-dependent result agrees with (48) at $t=0$. It demonstrates for arbitrary time a behavior of a genuine CP-violating quantity which does not vanish for nonzero weak phases and requires no strong phases.

In the case of direct CP violation, in which each transversity amplitude involves contributions with different CP-violating phases, one has

$$\mathrm{Im}[A_\perp(t)A_i^*(t)+\bar{A}_\perp(t)\bar{A}_i^*(t)] = 2\Sigma_{l,m}|A_\perp^l||A_i^m|e^{-\Gamma t}\cos(\delta_\perp^l-\delta_i^m)[\sin(\phi_\perp^l-\phi_i^m)\cosh(\Delta\Gamma t/2) - \sin(\phi_\perp^l+\phi_i^m)\sinh(\Delta\Gamma t/2)]. \quad (58)$$

The two true CP-violating time-integrated triple-product asymmetries ($i=0,\|$) for untagged decays are proportional to

$$\Gamma\int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t)+\bar{A}_\perp(t)\bar{A}_i^*(t)]dt = 2\Sigma_{l,m}|A_\perp^l||A_i^m|\cos(\delta_\perp^l-\delta_i^m)(\sin(\phi_\perp^l-\phi_i^m)-\sin(\phi_\perp^l+\phi_i^m)(\Delta\Gamma/2\Gamma) + \mathcal{O}[(\Delta\Gamma/2\Gamma)^2]). \quad (59)$$

We conclude that sizable CP-violating TP asymmetries do not require direct CP violation. They do require however that weak phases ϕ_i^m and $\phi_\perp^l$ occurring in $A_i (i=0,\|)$ and $A_\perp$ respectively differ from one another.

Assuming that the first term in the sum (59) is dominated by amplitudes $A_\perp^l$ and A_i^m one finds

$$\mathcal{A}_T^{(1)\mathrm{untagged}} = -\frac{4\sqrt{2}}{\pi}\frac{|A_\perp^l||A_0^m|\cos(\delta_\perp^l-\delta_0^m)\sin(\phi_\perp^l-\phi_0^m)}{(|A_0|^2+|A_\perp|^2+|A_\||^2)+(|\bar{A}_0|^2+|\bar{A}_\perp|^2+|\bar{A}_\||^2)} + \mathcal{O}(\Delta\Gamma/2\Gamma), \quad (60)$$

$$\mathcal{A}_T^{(2)\mathrm{untagged}} = -\frac{8}{\pi}\frac{|A_\perp^l||A_\|^m|\cos(\delta_\perp^l-\delta_\|^m)\sin(\phi_\perp^l-\phi_\|^m)}{(|A_0|^2+|A_\perp|^2+|A_\||^2)+(|\bar{A}_0|^2+|\bar{A}_\perp|^2+|\bar{A}_\||^2)} + \mathcal{O}(\Delta\Gamma/2\Gamma). \quad (61)$$

In the special case of a single weak phase $\phi_\perp=\phi_0=\phi_\|$ considered in Ref. [29] (including the standard model) the first terms in (60) and (61) vanish while the remaining terms are suppressed by $\Delta\Gamma/2\Gamma$.

It is interesting (and perhaps surprising) that the time-integrated asymmetries for untagged B_s decays are not suppressed due to fast B_s–$\bar{B}_s$ oscillations by $(\Gamma_s/\Delta m_s)^2$ or by $\Gamma_s/\Delta m_s$, as they would be for time-dependent terms behaving like $\cos(\Delta m t)$ or $\sin(\Delta m t)$. This behavior characterizes the two fake asymmetries which are proportional to

$$\mathrm{Im}[A_\perp(t)A_i^*(t)-\bar{A}_\perp(t)\bar{A}_i^*(t)] = 2\Sigma_{l,m}|A_\perp^l||A_i^m|e^{-\Gamma t}[\sin(\delta_\perp^l-\delta_i^m)\cos(\phi_\perp^l-\phi_i^m)\cos(\Delta m t) - \cos(\delta_\perp^l-\delta_i^m)\cos(\phi_\perp^l+\phi_i^m)\sin(\Delta m t)]. \quad (62)$$

For B_s decays the corresponding time-integrated fake asymmetries are suppressed by powers of $\Gamma_s/\Delta m_s \sim 0.04$ [30]:

$$\Gamma\int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t)-\bar{A}_\perp(t)\bar{A}_i^*(t)]dt \approx 2\Sigma_{l,m}|A_\perp^l||A_i^m|[\sin(\delta_\perp^l-\delta_i^m)\cos(\phi_\perp^l-\phi_i^m)(\Gamma_s/\Delta m_s)^2 - \cos(\delta_\perp^l-\delta_i^m)\cos(\phi_\perp^l+\phi_i^m)(\Gamma_s/\Delta m_s)]. \quad (63)$$

Note that measurements of both time-dependent and time-integrated fake asymmetries do require flavor tagging.

Equations (38), (60), and (61) imply that nonzero CP-violating triple-product asymmetries in self-tagged and flavorless $B_{(s)}$ decays require that transversity amplitudes of opposite parity ($A_\perp$ and A_0 and/or $A_\perp$ and $A_\|$) involve different weak phases. In the standard model the three transversity amplitudes have approximately equal and very small weak phases. Models with right-handed b-quark couplings could involve contributions to transversity amplitudes with substantially larger weak phases [3]. In such models transversity amplitudes of opposite parity obtain contributions with unequal weak phases implying nonzero CP-violating triple-product asymmetries.

VII. TRIPLE PRODUCTS IN SPECIFIC $B_{(s)} \to V_1 V_2$ DECAYS

The first class of decays we shall discuss in this section includes processes dominated by a penguin $b \to s$ amplitude. Before treating asymmetries associated with specific final states it is worth noting polarization properties in such decays. We shall then discuss TP asymmetries in $B \to \phi K^*$ and $B_s \to \phi\phi$.

A. Polarization in penguin-dominated decays

We shall reiterate a discussion given in Ref. [31]. The decays $B \to \phi K^*$ and $B_s \to \phi\phi$ are both dominated by the $b \to s$ penguin diagram. Factorization predicts dominant longitudinal polarization of the vector mesons, in contrast to observations [32–34]. Table III quotes longitudinal and transverse fractions for the above penguin-dominated processes as well as for $B^{(+,0)} \to \rho^0 K^{*(0,+)}$ which belong to the same class. By contrast, the tree-dominated decay $B^0 \to \rho^+\rho^-$ has $f_L = 0.992 \pm 0.024^{+0.026}_{-0.013}$ [35] or nearly 1 as predicted. There is no reason to trust factorization for the penguin amplitude, which may be due to rescattering from charm-anticharm intermediate states. Although $f_L < 1$ in penguin-dominated decays has frequently been quoted as possible evidence for new physics (see, e.g., [4]; however see also [36]), we prefer to reserve judgment on this issue.

B. $B \to \phi K^*$

True and fake TP asymmetries were defined in Sec. VI A as

$$\mathcal{A}_T^{\text{true}} \propto \text{Im}(A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*),$$
$$\mathcal{A}_T^{\text{fake}} \propto \text{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*), \qquad (i = 0, \|), \tag{64}$$

using normalizations for $\mathcal{A}_T^{(1)}$ and $\mathcal{A}_T^{(2)}$ as in the second line of Eqs. (41) and (40). From $B^0 \to \phi K^{*0}$ amplitudes and relative phases quoted in [30] we estimate

$$A_T^{(1)} = -0.117 \pm 0.022; \quad \bar{A}_T^{(1)} = +0.091 \pm 0.023;$$
$$A_T^{(2)} = -0.003 \pm 0.045; \quad \bar{A}_T^{(2)} = -0.006 \pm 0.041. \tag{65}$$

These values imply a large fake $\mathcal{A}_T{}^{(1)}$ (since $A_T^{(1)} - \bar{A}_T^{(1)} \neq 0$), no true $\mathcal{A}_T^{(1)}$ (since $A_T^{(1)} + \bar{A}_T^{(1)}$ is consistent with zero), and no fake *or* true $\mathcal{A}_T^{(2)}$ (since both $A_T^{(2)}$ and $\bar{A}_T^{(2)}$ are consistent with zero). The large fake $\mathcal{A}_T^{(1)}$ simply reflects the importance of strong final-state phases.

C. $B_s \to \phi\phi$

True triple-product asymmetries discussed in Sec. VI B with definitions as in the first line of Eqs. (40) and (41) are related to those recently reported by Dorigo on behalf of the CDF Collaboration for the decay $B_s \to \phi\phi$ [37]. The measured values are $\mathcal{A}_u \leftrightarrow \mathcal{A}_T^{(2)} = (-0.7 \pm 6.4 \pm 1.8)\%$; $\mathcal{A}_v \leftrightarrow \mathcal{A}_T{}^{(1)} = (-12.0 \pm 6.4 \pm 1.6)\%$. These observables represent time-integrated and untagged quantities, to which Eqs. (60) and (61) apply. As mentioned, these two triple-product asymmetries require nonzero values of the weak phase differences $\phi_\perp - \phi_\|$ and $\phi_\perp - \phi_0$, respectively, to avoid being suppressed by a factor of $\Delta\Gamma_s/2\Gamma_s < 0.1$ [38].

D. $B_s \to J/\psi\phi$

Angular and time dependence studied for $B_s \to J/\psi\phi$ by the CDF [39] and D0 [40] collaborations provided information on the weak phase occurring in the interference between B_s–$\bar{B}_s$ mixing and $b \to c\bar{c}s$ decay. This phase, expected to be very small in the CKM framework [30], may obtain corrections from new physics contributions to B_s–$\bar{B}_s$ mixing. Here we are interested in lessons to be learned from measuring CP-violating triple-product asymmetries in this process.

Triple-product asymmetries in this class of decays were studied in Sec. V B in terms of transversity amplitudes. Time-dependent CP-violating asymmetries given by Eq. (57) are obtained by adding up events for initial B_s and initial $\bar{B}_s$. The first term, $\propto \sin(\phi_\perp - \phi_i)\cosh(\Delta\Gamma_s t/2)$ $(i = 0, \|)$, vanishes for $\phi_\perp = \phi_i$, while the second term, $\propto -\sin(\phi_\perp + \phi_i)\sinh(\Delta\Gamma_s t/2)$, remains nonzero in this limit. The phases ϕ_k $(k = 0, \|, \perp)$, occurring in the interference of the mixing amplitude with the three transversity amplitudes [see Eq. (47)], are equal in the CKM framework. They are expected to be equal to a very good approximation also in extensions of this framework

TABLE III. Longitudinal and transverse fractions f_L and f_T for some $b \to s$-penguin $B \to VV$ processes.

	$B_s \to \phi\phi$ [32]	$B^+ \to \phi K^{*+}$ [33]	$B^+ \to \rho^0 K^{*+}$ [34]	$B^0 \to \rho^0 K^{*0}$ [34]
f_L	$0.348 \pm 0.041 \pm 0.021$	$0.49 \pm 0.05 \pm 0.03$	$0.52 \pm 0.10 \pm 0.04$	$0.57 \pm 0.09 \pm 0.08$
f_T	$0.652 \pm 0.041 \pm 0.021$	$0.51 \pm 0.05 \pm 0.03$	$0.48 \pm 0.10 \pm 0.04$	$0.43 \pm 0.09 \pm 0.08$

because $b \to c\bar{c}s$ is CKM-favored. The quantity which can potentially be affected in new physics schemes is $\phi_\perp + \phi_i \approx 2\phi_k$ ($k = 0, \parallel, \perp$), which determines the magnitude of the coefficient of the $\sinh(\Delta\Gamma_s t/2)$ term in the CP-violating TP asymmetry. This coefficient is of order a few percent in the CKM framework but may be sizable in the presence of new contributions to B_s–$\bar{B}_s$ mixing. This term is suppressed by $\Delta\Gamma_s/2\Gamma_s$ when time-integrated.

VIII. CONCLUDING REMARKS

We have discussed the differences between true CP-violating TP asymmetries, which require no strong phases, and fake asymmetries, which require nonzero strong phases but no CP violation. We have shown that TP asymmetries vanish for two identical and kinematically indistinguishable particles in the final state, demonstrating this property through two examples of Cabibbo-favored four-body D decays. Such asymmetries need not vanish even when two identical particles are present as long as they have nontrivial kinematic correlations, as in $K_L \to e^+e^-e^+e^-$. We have shown that while triple-product asymmetries in charmed meson decays do not manifest CP violation, they display an interesting pattern of final-state interactions correlated with total decay widths.

We studied TP asymmetries in B and B_s meson decays to two vector mesons each decaying to a pseudoscalar pair, extending results to decays where one vector meson decays into a lepton pair. We derived expressions for time-dependent TP asymmetries for neutral B and B_s decays to flavorless states in terms of the neutral $B_{(s)}$ mass difference Δm and the width difference $\Delta\Gamma$. Time-integrated true CP-violating asymmetries, measurable for untagged B_s decays, were shown to be suppressed by neither $\Gamma_s/\Delta m_s$ nor $\Delta\Gamma_s/\Gamma_s$ but to require two different weak phases in decays to CP-even and CP-odd transversity states. Finally, implications were discussed for TP asymmetries in $B \to K^*\phi$, $B_s \to \phi\phi$, and $B_s \to J/\psi\phi$.

ACKNOWLEDGMENTS

M. G. is grateful to the Enrico Fermi Institute at the University of Chicago and to the CERN Theory Division for their kind hospitality. We thank Mirco Dorigo, Marco Rescigno, and Diego Tonelli for helpful questions, and Alakabha Datta, Boris Kayser, David London, Maurizio Martinelli, and Brian Meadows for useful communications. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560.

[1] G. Valencia, Phys. Rev. D **39**, 3339 (1989).
[2] B. Kayser, Nucl. Phys. B, Proc. Suppl. **13**, 487 (1990).
[3] A. Datta and D. London, Int. J. Mod. Phys. A **19**, 2505 (2004).
[4] A. Datta, M. Duraisamy, and D. London, Phys. Lett. B **701**, 357 (2011).
[5] A. Alavi-Harati *et al.* (KTeV Collaboration), Phys. Rev. Lett. **84**, 408 (2000); E. Abouzaid *et al.* (KTeV Collaboration), Phys. Rev. Lett. **96**, 101801 (2006).
[6] I. I. Y. Bigi, in *Proceedings of Kaon2001: International Conference on CP Violation, Pisa, Italy, 2001*, edited by F. Costantini, G. Isidori, and M. Sozzi (Frascati, Rome, 2001), p. 417;
[7] Y. Grossman and M. P. Worah, Phys. Lett. B **395**, 241 (1997).
[8] A. Lai *et al.* (NA48 Collaboration), Eur. Phys. J. C **30**, 33 (2003).
[9] L. M. Sehgal and M. Wanninger, Phys. Rev. D **46**, 1035 (1992); **46**, 5209(E) (1992).
[10] A. Alavi-Harati *et al.* (KTeV Collaboration), Phys. Rev. Lett. **86**, 5425 (2001).
[11] A. Lai *et al.* (NA48 Collaboration), Phys. Lett. B **615**, 31 (2005).
[12] A. Alavi-Harati *et al.* (KTeV Collaboration), Phys. Rev. Lett. **87**, 111802 (2001); **90**, 141801 (2003).
[13] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[14] C. N. Yang, Phys. Rev. **77**, 242 (1950).
[15] N. M. Kroll and W. Wada, Phys. Rev. **98**, 1355 (1955).
[16] P. Gu *et al.*, Phys. Rev. Lett. **72**, 3000 (1994).
[17] S. Dobbs *et al.* (CLEO Collaboration), Phys. Rev. D **76**, 112001 (2007).
[18] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. Lett. **100**, 161804 (2008).
[19] J. M. Link *et al.* (FOCUS Collaboration), Phys. Lett. B **622**, 239 (2005).
[20] P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. D **81**, 111103 (2010).
[21] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **84**, 031103 (2011).
[22] M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. **47**, 277 (1975).
[23] M. Gronau, Phys. Rev. Lett. **83**, 4005 (1999).
[24] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).
[25] G. Kramer and W. F. Palmer, Phys. Rev. D **45**, 193 (1992).
[26] N. Sinha and R. Sinha, Phys. Rev. Lett. **80**, 3706 (1998).
[27] X.-G. He and W.-S. Hou, Phys. Rev. D **58**, 117502 (1998).
[28] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation*, International Series of Monographs on Physics (Oxford Univerisity Press, Inc., New York, 1999), Vol. 103.
[29] I. Dunietz, Phys. Rev. D **52**, 3048 (1995); R. Fleischer and I. Dunietz, Phys. Rev. D **55**, 259

(1997); A. S. Dighe, I. Dunietz, and R. Fleischer, Eur. Phys. J. C **6**, 647 (1999); I. Dunietz, R. Fleischer, and U. Nierste, Phys. Rev. D **63**, 114015 (2001).

[30] E. Barberio *et al.* (Heavy Flavor Averaging Group (HFAG) Collaboration), arXiv:0808.1297, periodically updated in http://www.slac.stanford.edu/xorg/hfag/

[31] J. L. Rosner and M. Gronau, Proc. Sci., BEAUTY2011 (2011) 045 [arXiv:1105.1923].

[32] T. Aaltonen *et al.* (CDF Collaboration), arXiv:1107.4999 [Phys. Rev. Lett. (to be published)].

[33] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **99**, 201802 (2007).

[34] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 201801 (2006).

[35] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **76**, 052007 (2007).

[36] M. Beneke, J. Rohrer, and D. Yang, Nucl. Phys. B **774**, 64 (2007).

[37] M. Dorigo (CDF Collaboration), Proc. Sci., BEAUTY2011 (2011) 015 [arXiv:1105 .4437].

[38] A. Lenz and U. Nierste, arXiv:1102.4274.

[39] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 161802 (2008).

[40] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **101**, 241801 (2008).