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$g_{V,S_{\gamma}}$ coupling constant in light cone QCD

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We recalculated the coupling constants $g_{\phi\sigma\gamma}$, $g_{\phi a_0\gamma}$, $g_{\omega\sigma\gamma}$, $g_{a_0\omega\gamma}$, $g_{\rho\sigma\gamma}$, and $g_{a_0\rho\gamma}$ by taking into account the contributions of the three-particle up to twist-4 distribution amplitudes of the photon involving quark-gluon and quark-anti-quark-photon fields in the light-cone sum-rule framework.

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I. INTRODUCTION I. INTRODUCTION

It is known that the quark model describes hadrons successfully. In its simplest version, mesons are interpreted as pure $\bar{q}q$ states. Scalar mesons might constitute an exception to this scheme and indeed their nature is not well established yet.

In order to perform experiments about hadrons, some physical information is required at large distance, and this information can be obtained from accurate estimations of form factors and other matrix elements. There are some nonperturbative approaches in order to obtain such information. The QCD sum rules are one of the most powerful methods among these approaches [[1](#page-6-0)[,2\]](#page-6-1). They were firstly proposed by Shifman, Vainshtein, and Zakharov. Then, many studies and some extensions have been performed on this method in recent years. The main idea of the method is to calculate the correlator with the help of operator product expansion (OPE) in the framework of QCD and then connect them with the phenomenological part. The interested physical quantities are determined by matching these two representations of the correlator.

A coupling constant, usually denoted g, is a number that determines the strength of an interaction. The Lagrangian or the Hamiltonian of a system is generally separated into a kinetic part and an interaction part. The coupling constant determines the strength of the interaction part with respect to the kinetic part, or between two sectors of the interaction part. These coupling constants can be calculated theoretically using some known methods. The light-cone sum rule is the most popular nonperturbative method for calculating the coupling constants.

Vector meson (V)-scalar meson (S) photon $VS\gamma$ vertex plays a role in photon production reactions of vector mesons on nucleons. Especially, the studies of ϕ meson and, in particular, its radiative decays have been important sources of information in hadron physics. Among the processes involving the vector and scalar mesons $\phi \sigma \gamma$, $\phi a_0 \gamma$, $\omega \sigma \gamma$, $\omega a_0 \gamma$, $\rho \sigma \gamma$, $\rho a_0 \gamma$ vertices are interesting and important for several reasons [[3](#page-6-2)[,4\]](#page-6-3). For example, the $\phi a_0 \gamma$ vertex plays a role in the study of the radiative $\phi \rightarrow \pi^0 \eta \gamma$ decay, and the knowledge of the $\phi \sigma \gamma$ vertex
is needed in the analysis of the decay mechanism of the is needed in the analysis of the decay mechanism of the $\phi \rightarrow \pi^0 \pi^0 \gamma$. Also $\omega \sigma \gamma$ vertex, which compensates the lastic dependence of the $\omega \sigma \gamma$ vertex plays a role in the elastic large effect of the $\rho \sigma \gamma$ vertex, plays a role in the elastic electron-deuteron scattering, and therefore the knowledge of the coupling constant $g_{\omega\sigma\gamma}$ is essential.

In this study, we write the physical ω and ϕ meson states as

$$
|\omega\rangle = \cos\theta |\omega_0\rangle - \sin\theta |\phi_0\rangle
$$

$$
|\phi\rangle = \sin\theta |\omega_0\rangle + \cos\theta |\phi_0\rangle,
$$
 (1)

where $|\omega_0\rangle = \frac{1}{\sqrt{2}} |\bar{u}u|$ + $d\bar{d}$ and $|\phi_0\rangle = |s\bar{s}\rangle$ are the nonstrange and the strange basis states. The mixing angle has been determined from the available experimental data by Bramon *et al.* as $\theta = (3.4 \pm 0.2)^{\circ}$ [[5](#page-6-4)]. But, this mixing
angle has been also determined from the available experiangle has been also determined from the available experimental data as $\theta = 3.18^{\circ}$ in [[6](#page-6-5)]. Therefore, we choose the interpolating currents ρ , ω , and ϕ mesons defined in the quark flavor basis as

$$
J^{\rho}_{\mu} = \frac{1}{2} (\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d),
$$

\n
$$
J^{\omega}_{\mu} = \cos\theta J^{\omega_0}_{\mu} - \sin\theta J^{\phi_0}_{\mu},
$$

\n
$$
J^{\phi}_{\mu} = \sin\theta J^{\omega_0}_{\mu} + \cos\theta J^{\phi_0}_{\mu},
$$
\n(2)

where $J_{\mu}^{\omega_0} = \frac{1}{6} (\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d)$ and $J_{\mu}^{\phi_0} = -\frac{1}{3} (\bar{s}\gamma_{\mu}s)$ [[1\]](#page-6-0).
The coupling of the S meson to scalar current I^S can be

The coupling of the S meson to scalar current J^S can be parameterized in terms of a constant λ_s as

$$
\langle 0|J^S|S\rangle = \lambda_S. \tag{3}
$$

In order to determine the coupling constant $g_{VS\gamma}$ in the frame work of the QCD sum rules, we consider the following two point correlation function

$$
\Pi_{\mu} = i \int d^4x e^{ip_2x} \langle 0|T\{J^S(x)J^V_{\mu}(0)\}|0\rangle_{\gamma}.
$$
 (4)

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The interpolating currents of the vector meson are chosen as presented in Eq. [\(2\)](#page-0-5) and the scalar meson current as $J^S = \frac{1}{2} [\bar{u}u]$
isoscalar mes $+(-1)^{l} \overline{d}d$, where $I = 0$ corresponds to isoscalar meson and $I = 1$ corresponds to isovector, respectively.

The physical part of the sum rules can be attained by inserting a complete set of one meson states in the correlator. Therefore, we get

$$
\Pi_{\mu} = \sum \frac{\langle 0|J^{S}(x)|S(p_{2})\rangle}{p_{2}^{2} - m_{S}^{2}} \langle S(p_{2})|V(p_{1})\rangle_{\gamma} \frac{\langle V(p_{1})|J_{\mu}^{V}(0)|0\rangle}{p_{1}^{2} - m_{V}^{2}},
$$
\n(5)

where $p_2 = p_1 + q$, q is the photon momentum. The matrix element $\langle V(p_1) | J_{\mu}^V(0) | 0 \rangle$ is defined as

$$
\langle 0|J_{\mu}^V|V\rangle = m_V f_V \varepsilon_{\mu}^V,\tag{6}
$$

where ε^V is the V meson polarization vector, q is the photon momentum, and $m_V f_V = \lambda_V$. In general, the $\langle S|V\rangle$ _v matrix can be parameterized as

$$
\langle S(p_2)|V(p_1)\rangle_{\gamma} = e\{F_1(q^2)(p_1q)\varepsilon_{\mu}^V + F_2(q^2)(\varepsilon^Vq)p_{1\mu}\}\varepsilon^{\mu},\tag{7}
$$

where ε^{μ} is the photon polarization vector. We have also

$$
q^{\mu}\{F_1(p_1q)\varepsilon_{\mu}^V + F_2(\varepsilon^Vq)p_{1\mu}\} = 0, \tag{8}
$$

then

$$
F_2(0) = -F_1(0).
$$

So, the matrix element $\langle S|V\rangle_{\gamma}$ takes the following form:

$$
\langle S|V\rangle_{\gamma} = eF(0)\{(p_1q)\varepsilon_{\mu}^V - (q\varepsilon^V)p_{1\mu}\}\varepsilon^{\mu}.
$$
 (9)

We can use an alternative parametrization for the $VS\gamma$ vertex as

$$
\mathcal{L}_{\text{int}} = -\frac{e}{m_V} g_{V S \gamma} \varepsilon_{\mu \nu \alpha \beta} (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu) (\partial_\alpha A_\beta - \partial_\beta A_\alpha) S. \tag{10}
$$

Comparing Eqs. (8) – (10) (10) (10) we see that

$$
F(q^2 = 0) \equiv \frac{g_{VS\gamma}}{m_V}.\tag{11}
$$

Having used Eqs. [\(5\)](#page-1-2)–([10](#page-1-1)), for the phenomenological part of the sum rules, we get

$$
\Pi_{\mu}^{\text{phen}} = g_{V S \gamma} \frac{\lambda_S}{p_2^2 - m_S^2} \frac{f_V \varepsilon^{\nu}}{p_1^2 - m_V^2} [-(p_1 q)g_{\mu\nu} + p_{1\nu} q_{\mu}].
$$
\n(12)

The photon wave functions up to twist 4 are defined as [[7](#page-6-6),[8\]](#page-6-7)

$$
\langle \gamma(p)|\bar{q}\sigma_{\mu\nu}q|0\rangle
$$

= $-ie_q\langle \bar{q}q\rangle(\varepsilon_{\mu}p_{\nu} - \varepsilon_{\nu}p_{\mu}) \int_0^1 du e^{i\bar{u}p x} \Big(\chi \varphi_{\gamma}(u)$
+ $\frac{x^2}{16} \mathcal{A}(u) \Big) - \frac{i}{2px} e_q \langle \bar{q}q\rangle \Big[x_{\nu} \Big(\varepsilon_{\mu} - p_{\mu} \frac{\varepsilon x}{px}\Big)$
- $x_{\mu} \Big(\varepsilon_{\nu} - p_{\nu} \frac{\varepsilon x}{px}\Big) \Big] \int_0^1 du e^{i\bar{u}p x} h_{\gamma}(u).$ (13)

It can be seen from expression of the correlation function in [Eq. ([4\)](#page-0-6)] that only light u and d quarks give contributions and s quark part is not participated in this study.

After evaluating the Fourier transform, performing the double Borel transformation with respect to the variables $p_2^2 = p^2$ and $p_1^2 = (p + q)^2$ on both sides of the correlator
suppress the contributions of the higher states. It also suppress the contributions of the higher states. It also removes the subtraction terms in the dispersion relation. Finally, after a long and straightforward calculation, also involving quark-gluon field and quark-antiquark-photon field [[7](#page-6-6)[,8\]](#page-6-7), we obtain the following sum rules for the $g_{V S_{\gamma}}$ coupling constant

$$
g_{VS\gamma} = C \frac{m_V (e_u + (-1)^I e_d) \langle \bar{u}u \rangle}{\lambda_V \lambda_S}
$$

$$
\times e^{(m_V^2/M_1^2 + m_S^2/M_2^2)} \Big\{ M^2 \chi(\varphi(u_0) E_0(s_0/M^2)) - \frac{\mathcal{A}(u_0)}{4}
$$

+
$$
2 \int_0^u dt \int_0^t d\tau h_\gamma(\tau) - I_G(u_0) - I_F(u_0) \Big\} y(\theta),
$$
 (14)

where the values of constant C and $y(\theta)$ are given in Table [I.](#page-1-3) The functions $I_G(u_0)$ and $I_F(u_0)$ are defined as

TABLE I. The constant values in Eq. [\(14\)](#page-1-4) for the calculated coupling constants.

Values	$\phi \sigma \gamma$			$\rho \sigma \gamma$ $\omega \sigma \gamma$ $\phi a_0 \gamma$ $\rho a_0 \gamma$ $\omega a_0 \gamma$		
$e_u + (-1)^l e_d$	1/3		1/3		1/3	
	$1/\sqrt{2}$	1/2	1/6	$1/\sqrt{2}$	1/2	/6
$v(\theta)$	$sin\theta$			$sin\theta$		

 $g_{V Sv}$ COUPLING CONSTANT IN LIGHT CONE QCD PHYSICAL REVIEW D 84, 096010 (2011)

$$
I_G(u_0) = \int_0^{u_0} d\alpha_u \int_0^{1-u_0} d\alpha_{\bar{u}} \frac{1}{1 - \alpha_u - \alpha_{\bar{u}}} [\mathcal{T}_1(\alpha_i) - \mathcal{T}_2(\alpha_i) + \mathcal{T}_3(\alpha_i) - \mathcal{T}_4(\alpha_i) - \mathcal{S}(\alpha_i) - \tilde{\mathcal{S}}(\alpha_i)]
$$

+
$$
2 \int_0^{u_0} d\alpha_u \int_0^{1-u_0} d\alpha_{\bar{u}} \frac{1 - u_0 - \alpha_{\bar{u}}}{(1 - \alpha_u - \alpha_{\bar{u}})^2} [\mathcal{T}_2(\alpha_i) - \mathcal{T}_3(\alpha_i) + \mathcal{S}(\alpha_i)]
$$
(15)

and

$$
I_F(u_0)
$$

= $-\int_0^{u_0} d\alpha_u \int_0^{1-u_0} d\alpha_{\bar{u}} \frac{1}{1 - \alpha_u - \alpha_{\bar{u}}} [\mathcal{S}_{\gamma}(\alpha_i) + \mathcal{T}_{4}^{\gamma}(\alpha_i)]$
+ $2 \int_0^{u_0} d\alpha_u \int_0^{1-u_0} d\alpha_{\bar{u}} \frac{1 - u_0 - \alpha_{\bar{u}}}{(1 - \alpha_u - \alpha_{\bar{u}})^2} [\mathcal{S}_{\gamma}(\alpha_i)],$ (16)

where s_0 is the continuum threshold and

$$
u_0 = \frac{M_2^2}{M_1^2 + M_2^2}, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2},
$$

where M_1^2 and M_2^2 are the Borel parameters in V and S channels and $E_0(s_0/M^2) = 1 - e^{-s_0/M^2}$ is the continuum
substraction substraction.

II. NUMERICAL RESULTS

For the numerical evaluation of the sum rules, we use the value (ū*u*
condensai $\phi = -(0.240 \pm 0.010)^3$ GeV³ [\[9\]](#page-6-8) for the vacuum
te and $y = (-3.15 + 0.3)$ GeV⁻² [10] for the condensate and $\chi = (-3.15 \pm 0.3) \text{ GeV}^{-2}$ [[10](#page-6-9)] for the magnetic susceptibility of the quark condensate magnetic susceptibility of the quark condensate.

The theoretical framework for the photon distribution amplitudes (DA) is based on an expansion in terms of the matrix element of conformal operators [[7](#page-6-6),[8,](#page-6-7)[11](#page-6-10)]. The photon DA's entering the sum-rules are expressed as [[7](#page-6-6),[12](#page-6-11)]

$$
\mathcal{A}(u) = 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) \n+ 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})],
$$
\n
$$
\varphi(u) = 6u\bar{u}(1 + \varphi_2 c_2^{3/2}(u - \bar{u})),
$$
\n
$$
h_{\gamma}(u) = -10(1 + 2\kappa^+)c_2^{1/2}(u - \bar{u}),
$$
\n
$$
\mathcal{T}_1(\alpha_i) = -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g,
$$
\n
$$
\mathcal{T}_2(\alpha_i) = 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)),
$$
\n
$$
\mathcal{T}_3(\alpha_i) = -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g,
$$
\n
$$
\mathcal{T}_4(\alpha_i) = 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)),
$$
\n
$$
\mathcal{S}(\alpha_i) = 30\alpha_g^2\{(\kappa + \kappa^+) (1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\},
$$
\n
$$
\tilde{\mathcal{S}}(\alpha_i) = -30\alpha_g^2\{(\kappa - \kappa^+) (1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha
$$

The parameters appearing in the above DA's are given as $\varphi_2 = 0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$,
 $\zeta_1^+ = 0$ at the scale $\mu = 1$ GeV and $\alpha = (1 - \alpha_2 - \alpha_1)$ $\zeta_2^+ = 0$ at the scale $\mu = 1$ GeV and $\alpha_g = (1 - \alpha_{\bar{q}} - \alpha_q)$
[13] Numerical values of mass [14] and λ for different [\[13\]](#page-6-12). Numerical values of mass [[14](#page-6-13)] and λ for different types of meson are selected as in the Table [II](#page-2-0).

Then, we have calculated the $g_{VS\gamma}$ coupling constant numerically using the values in Table [I](#page-1-3) for the above meson types. In the following sections, the coupling constant is

TABLE II. Numerical values of mass and λ .

Meson type	Mass (GeV)	λ (GeV ²)	
ϕ	1.020	0.250 [15]	
ρ	0.775	0.156 [14]	
ω	0.782	0.046 [14]	
a_0	0.980	0.210 [15]	
σ	0.500	0.120 [15]	

displayed in terms of a function of the Borel parameter M_2^2 for different values of the threshold parameters s_0 and for $M_1 = 2.0 \text{ GeV}^2$.

We present the dependence of the coupling constant $g_{\phi\sigma\gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [1\(a\),](#page-3-0) and Borel platform of the coupling g_{ℓ} for the 1.2 GeV² $\leq M^2 M^2 \leq 2.8 \text{ GeV}^2$ in coupling $g_{\phi\sigma\gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV² in
Fig. 1(b). We obtain the range of the coupling constant as Fig. [1\(b\).](#page-3-0) We obtain the range of the coupling constant as

$$
(0.054 < g_{\phi\sigma\gamma} < 0.082).
$$

The previous results are $(0.055 < g_{\phi\sigma\gamma} < 0.076)$ ob-tained by Du and Zhou [[15](#page-6-14)] and $g_{\phi\sigma\gamma} = (0.036 \pm 0.008)$
obtained by Gokaln and Vilmaz [3] obtained by Gokalp and Yilmaz [[3\]](#page-6-2).

FIG. 1 (color online). (a) Graph of the coupling constant $g_{\phi\sigma\gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1, 1.6, 2.1 \text{ GeV}^2$ with $M_1^2 = 2.0 \text{ GeV}^2$. (b) The coupling
constant g_{\perp} in the Borel platform $(M_1^2 M_1^2)$. The upper (blue) constant $g_{\phi\sigma\gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{\phi \sigma \gamma}$ with $s_0 = 1.1$ GeV² and with $s_0 = 2.1$ GeV², respectively.

We present the dependence of the coupling constant $g_{\rho\sigma\gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [2\(a\),](#page-3-1) and Borel platform of the coupling $g = \text{for the 1.2 GeV}^2 \le M^2 M^2 \le 2.8 \text{ GeV}^2$ in coupling $g_{\rho\sigma\gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV² in
Fig. 2(b). We obtain the range of coupling constant as Fig. [2\(b\).](#page-3-1) We obtain the range of coupling constant as

$$
(2.587 < g_{\rho\sigma\gamma} < 3.372).
$$

The previous result obtained by Aliev et al. [[9\]](#page-6-8) is $g_{\rho\sigma\gamma} = (2.2 \pm 0.4).$

FIG. 2 (color online). (a) Graph of the coupling constant $g_{\rho\sigma\gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1, 1.6, 2.1 \text{ GeV}^2$ with $M_1^2 = 2.0 \text{ GeV}^2$. (b) The coupling
constant a in the Borel platform $(M_1^2 M_1^2)$. The upper (blue) constant $g_{\rho\sigma\gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{\rho\sigma\gamma}$ with $s_0 = 1.1$ GeV² and with $s_0 = 2.1$ GeV², respectively.

We present the dependence of the coupling constant $g_{\omega\sigma\gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [3\(a\),](#page-4-0) and Borel platform of the coupling e for the 1.2 GeV² $\leq M^2 M^2 \leq 2.8 \text{ GeV}^2$ in coupling $g_{\omega\sigma\gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV² in
Fig. 3(b). We obtain the range of the coupling constant as Fig. [3\(b\).](#page-4-0) We obtain the range of the coupling constant as

$$
(0.970 < g_{\omega\sigma\gamma} < 1.267).
$$

The previous result obtained by Gokalp and Yilmaz [\[10\]](#page-6-9) is $g_{\omega \sigma \gamma} = (0.72 \pm 0.08).$

FIG. 3 (color online). (a) Graph of the coupling constant $g_{\omega\sigma\gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1, 1.6, 2.1 \text{ GeV}^2$ with $M_1^2 = 2.0 \text{ GeV}^2$. (b) The coupling
constant g in the Borel platform $(M_1^2 M_1^2)$. The upper (blue) constant $g_{\omega\sigma\gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{\omega\sigma\gamma}$ with $s_0 = 1.1$ GeV² and with $s_0 = 2.1$ GeV², respectively.

$\cos \varphi a_0 \gamma$ coupling constant

We present the dependence of the coupling constant $g_{\phi a_0 \gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [4\(a\),](#page-4-1) and Borel platform of the coupling g_{tot} for the 1.2 GeV² $\leq M^2$ $M^2 \leq 2.8 \text{ GeV}^2$ coupling $g_{\phi a_0 \gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV²
in Fig. 4(b). We obtain the range of the coupling constant in Fig. [4\(b\)](#page-4-1). We obtain the range of the coupling constant as

$$
(0.132 < g_{\phi a_0 \gamma} < 0.201).
$$

The previous results are $(0.130 < g_{\phi a_0 \gamma} < 0.194)$ ob-tained Du and Zhou [[15](#page-6-14)] $g_{\phi a_0 \gamma} = (0.11 \pm 0.03)$ obtained
by Gokaln and Yilmaz [3] by Gokalp and Yilmaz [\[3\]](#page-6-2).

FIG. 4 (color online). (a) Graph of the coupling constant $g_{\phi a_0 \gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1, 1.6, 2.1 \text{ GeV}^2$ with $M_1^2 = 2.0 \text{ GeV}^2$. (b) The coupling
constant g, in the Borel platform $(M_1^2 M_1^2)$. The upper (blue) constant $g_{\phi a_0 \gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{\phi a_0 \gamma}$ with $s_0 = 1.1$ GeV² and with $s_0 = 2.1$ GeV², respectively.

E. $g_{a_0\rho\gamma}$ coupling constant

We present the dependence of the coupling constant $g_{a_0\rho\gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [5\(a\),](#page-5-0) and Borel platform of the coupling g for the 1.2 GeV² $\leq M_1^2 M_2^2 \leq 2.8 \text{ GeV}^2$ in coupling $g_{a_0 \rho \gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV² in
Fig. 5(b). We obtain the range of the coupling constant as Fig. [5\(b\).](#page-5-0) We obtain the range of the coupling constant as

$$
(0.702 < g_{a_0\rho\gamma} < 0.916).
$$

The previous results are $g_{a_0\rho\gamma} = (1.18 \pm 0.27)$ obtained
[4] and $g = (1.3 \pm 0.3)$ obtained in [16] in [[4\]](#page-6-3) and $g_{a_0 \rho \gamma} = (1.3 \pm 0.3)$ obtained in [\[16\]](#page-6-15).

FIG. 5 (color online). (a) Graph of the coupling constant $g_{a_0 \rho \gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1$,
1.6. 2.1. GeV² with $M^2 = 2.0$ GeV² (b). The coupling constant 1.6, 2.1 GeV² with $M_1^2 = 2.0 \text{ GeV}^2$. (b) The coupling constant g in the Borel platform $(M_1^2 M_1^2)$. The upper (blue) curved $g_{a_0\rho\gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{a_0 \rho \gamma}$ with $s_0 = 1.1$ GeV² and with $s_0 = 2.1$ GeV², respectively.

F. $g_{a_0 \omega \gamma}$ coupling constant

We present the dependence of the coupling constant $g_{a_0\omega\gamma}$ on the Borel parameter M_2^2 at three different values of the continuum threshold: $s_0 = 1.1$, 1.6, and 2.1 GeV² and $M_1^2 = 2.0 \text{ GeV}^2$ in Fig. [6\(a\),](#page-5-1) and Borel platform of the coupling g for the 1.2 GeV² $\leq M^2 M^2 \leq 2.8 \text{ GeV}^2$ in coupling $g_{a_0 \omega \gamma}$ for the 1.2 GeV² $\leq M_1^2$, $M_2^2 \leq 2.8$ GeV² in
Fig. 6(b). We obtain the range of the coupling constant as Fig. [6\(b\).](#page-5-1) We obtain the range of the coupling constant as

$$
(2.373 < g_{a_0\omega\gamma} < 3.100).
$$

The previous result obtained by Aydin and Yilmaz [\[17\]](#page-6-16) is $g_{a_0 \omega \gamma} = (2.57 \pm 0.21).$

III. CONCLUSION

In this study, we recalculate the coupling constants g_{VSy} , $(V = \phi, \rho, \omega; S = a_0, \sigma)$ radiative decays into the

FIG. 6 (color online). (a) Graph of the coupling constant $g_{a_0 \omega \gamma}$ as a function of the Borel parameter M_2^2 for the values of $s_0 = 1.1 \pm 1.6 \pm 2.1$ GeV² with $M^2 = 2.0$ GeV² (b). The coupling 1.1, 1.6, 2.1 GeV² with $M_1^2 = 2.0$ GeV². (b) The coupling constant *a* in the Borel platform $(M^2 \ M^2)$. The upper constant $g_{a_0 \omega \gamma}$ in the Borel platform (M_1^2, M_2^2) . The upper (blue) curved surface and the lower (red) curved surface indicate the coupling $g_{a_0 \omega \gamma}$ with $s_0 = 1.1 \text{ GeV}^2$ and with $s_0 =$ 2.1 GeV², respectively.

scalar meson by taking into account the contributions of the three-particle twist-4 DA's of the photon involving quark-gluon and quark-anti-quark-photon fields in the light-cone sum-rule framework. It is observed that quarkgluon and quark-anti-quark-photon fields are not effective. The main contributions are coming from photon distribution functions.

It is well known that the $g_{V\text{S}\gamma}$ couplings are deeply related to the internal structure of the scalar mesons which is not well established and is still a subject of debate. Additionally, determination of the mass of the σ meson is not an easy task [σ meson is very side mass (400– 1200 MeV)]. Therefore we adopt $m_{\sigma} = 0.5$ GeV in this work. However, obtained results are compatible with exwork. However, obtained results are compatible with existence ones in literature.

ACCEPT TO A CHECK CONTRACTS

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