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Non-Fermi liquid behavior of the drag and diffusion coefficients in QED plasma

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We calculate the drag and diffusion coefficients in low temperature QED plasma and go beyond the leading order approximation. The non-Fermi-liquid behavior of these coefficients are clearly revealed. We observe that the subleading contributions due to the exchange of soft transverse photon in both cases are larger than the leading order terms coming from the longitudinal sector. The results are presented in closed form at zero and low temperature.

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I. INTRODUCTION

It has been known for quite some time now that a fermionic system interacting via the exchange of transverse gauge bosons exhibit deviations from the normal Fermi-liquid behavior. Such a characteristic feature, in presence of transverse or magnetic interactions, for the first time was reported in [1] where the specific heat of a degenerate electron gas was shown to contain correction terms involving $\alpha_s T \ln T^{-1}$. This was interpreted to be a consequence of the long range behavior of the magnetic interaction due to the absence of magnetostatic screening. Initially, such corrections were considered to be of little practical importance, as such a tiny effect was not likely to be detected experimentally. A decade later, however, the scenario changed and such investigations started attracting attention in the context of strongly correlated electron system in which the gauge coupling is not the fine structure constant (1/137) but of order unity [2]. Further impetus to these studies now comes from another domain involving relativistic quark (or electron) gas at high density and zero or low temperature where the specific heat also contains such anomalous terms. This seems to have serious implications in determining the thermodynamic and transport properties of the quark component of neutron or protoneutron stars, *viz.* entropy, pressure, specific heat, viscosity, etc. [3–6]. Similar non-Fermi-liquid terms, for ungapped quark matter, also appear in the calculation of neutrino emissivity and its mean free path [7,8]. For quarks in a color superconducting state also, the chromomagnetic interaction strongly influences the magnitude of the gap as shown in [9,10].

It is well known that the magnetic interaction in nonrelativistic systems is suppressed in powers of $(v/c)^2$. The scenario, however, changes as one enters into the relativistic domain, where it becomes important. Hence, in dealing with relativistic plasma one has to retain both electric and magnetic interactions mediated by the exchange of longitudinal and transverse gauge bosons like photons or gluons. More interestingly, it is observed that for ultradegenerate case, both in QCD and QED, the transverse interactions not only become important but it dominates over its longitudinal counterpart; a characteristic behavior having a nontrivial origin residing in the analytical structure of the Fermion self-energy close to the Fermi surface. This has been beautifully exposed in [11], where the authors calculate Fermionic dispersion relations in ultradegenerate relativistic plasmas and show how such non-Fermi-liquid behavior emerges from the vanishing of the Fermion propagator near the Fermi surface by calculating the group velocity of the corresponding quasiparticle excitations. One can also see, how the fractional power appears there [11], due to the exchange of soft transverse gauge boson in the small temperature expansion of the fermion self-energy in ultradegenerate plasma similar to what one encounters in the expansion of the thermodynamic potential or C_v [3–5]. Actually, the fermion self-energy close to the Fermi surface receives a logarithmic enhancement due to the exchange of magnetic gluons [12]; this, in turn, leads to such dominance. A more rigorous discussion on how and why the dynamics change near the Fermi surface leading to the break down of Fermi-liquid behavior or vanishing of the step discontinuity can be found in [13]. Departure from the Fermi-liquid behavior has also been witnessed in the calculation of quasiparticle damping rate in ultradegenerate relativistic plasma [14–16].

The low temperature, high density region, commonly known as ultradegenerate plasma, is much less explored in comparison with the high temperature low density domain. In particular, in the present work, we calculate fermionic drag (η) and longitudinal diffusion coefficients ($\mathcal{B}_\parallel$) in this regime, and eventually extend it to the limiting case of zero temperature. The salient feature here has been the inclusion of the higher order terms both in the transverse and longitudinal sector with implications to be discussed later. The evaluation of drag (diffusion) coefficient is very similar to the damping rate calculation with one difference, i.e., here we weight the imaginary part of the self energy with

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the energy (square momentum) transfer per scattering to obtain the desired result. Such calculations, as is well known, are plagued with infrared divergences. There are well established techniques to handle such divergences both at finite and zero temperature where one divides the interactions into two regions one involving the exchange of soft gauge bosons while the other involves hard momentum transfer [17]. For the former, one uses the bare photon (gluon) propagator and for the latter the hard thermal/density loop (HTL/HDL) resummed propagator is used. One interesting departure from the high temperature that is observed in dealing with plasma close to zero temperature is the following: in a hot plasma both the hard and the soft part of the electric and magnetic interactions contribute at same order of the coupling parameter. In the ultradegenerate plasma, or when the temperature is much smaller compared to the chemical potential, it is seen that the hard sector contribution come with higher order coupling parameters than the soft sector. Even within the soft sector, for the longitudinal and transverse part, the coupling parameter appears with different powers [18].

The drag-coefficient, as we know, is related to the energy loss suffered by the propagating particle in a plasma. This has been studied extensively in a series of works for the last two decades [19–28]. There also exist many calculations for the diffusion-coefficients both for quantum electro and chromomagnetic plasma [19,23,24,28]. All these calculations are performed in situations where the temperature is high but the chemical potential is zero, except in [29,30], where numerical estimates of the energy loss or drag and diffusion-coefficients at nonzero chemical potential have been presented. There, to the best of our knowledge, exists only one calculation so far [18], where the analytical results for η and ($\mathcal{B}_{\|}$) for ultradegenerate relativistic plasma have been presented. There, we have restricted ourselves only to the leading-order results and have shown that the drag and diffusion coefficients are dominated by the soft transverse photon exchanges while the longitudinal terms are subleading. Here, we go beyond the leading order and reveal the importance of the subleading terms in the transverse sector. The approach we adopt in this work is, however, different from the previous one and more in line with [11]. The connections, nevertheless, are made at appropriate places. Here, we probably should mention that the dominance of next-to-leading order (NLO) transverse term over the longitudinal one does not imply breakdown of the perturbation series, because the next-to-leading order terms in transverse or longitudinal sector individually are smaller than the corresponding leading parts.

The plan of this paper is as follows: in Sec. II, the formalism is set forth. In Sec. II A, we evaluate the drag-coefficient in the domain low temperature and eventually arrive at the zero temperature results by taking the appropriate limit. In the next subsection (Sec.II B), we present the results for the diffusion=coefficient both at zero and small temperature. In Sec. III, we conclude.

II. FORMALISM

The drag-coefficient of a quasiparticle having energy (E) is incidentally related to the energy loss of the propagating particle, which undergoes collisions with the constituents of the plasma *viz.* the electrons:

$$\eta = \frac{1}{E}\int d\Gamma\omega, \tag{1}$$

and $d\Gamma$ is the differential interaction rate [31]. This expression is quite general and can be used to calculate collisional energy loss both for the finite temperature and/or density. The phase space will be different due to the modifications of the distribution functions depending upon the values of μ and T. The imaginary part of the fermion self-energy diagram basically gives the damping rate of a hard fermion. This damping mechanism is equivalent to elastic scattering off the thermal electrons via the exchange of a collective photon,

$$\Gamma(E) = -\frac{1}{2E}\,\mathrm{Tr}[\mathrm{Im}\Sigma(p_0 + i\epsilon, \mathbf{p})\not{P}]|_{p_0=E}. \tag{2}$$

The full fermion self-energy represented in Fig. 1 can be written explicitly as:

$$\Sigma(P) = e^2 T\sum_s \int \frac{d^3q}{(2\pi)^3}\gamma_\mu S_f(i(\omega_n - \omega_s), \mathbf{p} - \mathbf{q})\gamma_\nu \Delta_{\mu\nu} \times (i\omega_s, \mathbf{q}), \tag{3}$$

where, $p_0 = i\omega_n + \mu$, $q_0 = i\omega_s$. $\omega_n = \pi(2n + 1)T$ and $\omega_s = 2\pi sT$ are the Matsubara frequencies for fermion and boson, respectively, with integers n and s. After performing the sum over Matsubara frequency in Eq. (3), $i\omega_n + \mu$ is analytically continued to the Minkowski space $i\omega_n + \mu \rightarrow p_0 + i\epsilon$, with $\epsilon \rightarrow 0$. The blob in the wavy line of Fig. 1 represents HTL/HDL corrected photon propagator, which in the Coulomb gauge is given by [31,32]

$$\Delta_{\mu\nu}(Q) = \delta_{\mu 0}\delta_{\nu 0}\Delta_l(Q) + P^t_{\mu\nu}\Delta_t(Q), \tag{4}$$

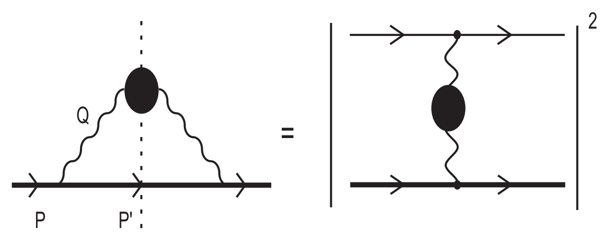

FIG. 1. Fermion-fermion scattering with screened interaction.

with $P^t_{ij} = (\delta_{ij} - \hat{q}_i\hat{q}_j)$, $\hat{q}^i = \mathbf{q}^i/|\mathbf{q}|$, $P^t_{i0} = P^t_{0i} = P^t_{00} = 0$, and Δ_l, Δ_t are given by [31,32]

$$\Delta_l(q_0, q) = \frac{-1}{q^2 + \Pi_l}, \tag{5}$$

$$\Delta_t(q_0, q) = \frac{-1}{q_0^2 - q^2 - \Pi_t}. \tag{6}$$

Here, we introduce the spectral functions $\rho_{l,t}$ as [31,32]:

$$\frac{\rho_{l,t}(q_0, q)}{2\pi} = Z_{l,t}[\delta(q_0 - \omega_{l,t}(q)) - \delta(q_0 + \omega_{l,t}(q))] + \beta_{l,t}(q_0, q). \tag{7}$$

The poles $\omega_{l,t}$ are the solutions of the dispersion relations. The δ function corresponds to the (timelike) poles of the resumed propagator and $\beta_{l,t}$ represent cuts. The latter terms i.e. Landau damping pieces of the spectral functions are nonvanishing only for $q_0^2 \leq q^2$, and are given by

$$\begin{aligned}\beta_l(q_0, q) &= \frac{m_D^2 x\Theta(1 - x^2)}{2[q^2 + m_D^2(1 - \frac{x}{2}\ln|\frac{x+1}{x-1}|)]^2 + \frac{m_D^4\pi^2x^2}{4}},\\ \beta_t(q_0, q) &= \frac{m_D^2 x(1 - x^2)\Theta(1 - x^2)}{[2q^2(x^2 - 1) - m_D^2x^2(1 + \frac{(1-x^2)}{2x}\ln|\frac{x+1}{x-1}|)]^2 + \frac{m_D^4\pi^2x^2(1-x^2)^2}{4}},\end{aligned} \tag{8}$$

where $x = q_0/q$. The Debye mass is $m_D^2 = \frac{e^2}{\pi^2}(\mu^2 + \frac{\pi^2T^2}{3})$.

At the leading order, these are derived from the one-loop photon self-energy where the loop momenta are assumed to be hard in comparison to the photon momentum [31,32]. In the literature, the formalism is known as the HTL/HDL approximation as discussed in [15,31–34].

In Eq. (3), fermion propagator has the following spectral representation with the notation $\mathbf{k} = (\mathbf{p} - \mathbf{q})$ [31],

$$S_f(i\omega_n, \mathbf{k}) = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\frac{\not{K}\rho_f(K)}{k_0 - i\omega_n - \mu}. \tag{9}$$

Taking the imaginary part of Eq. (3), the scattering rate with the help of Eq. (2) can be calculated. One then inserts the energy exchange ω in the expression of Γ and calculates η from Eq. (1) to obtain

$$\begin{aligned}\eta = \frac{\pi e^2}{E^2}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\rho_f(k_0)\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}q_0(1 + n(q_0)\\ - \bar{n}(k_0))\delta(E - k_0 - q_0)[p_0k_0 + \mathbf{p}\cdot\mathbf{k}]\rho_l(q_0, q)\\ + 2[p_0k_0 - (\mathbf{p}\cdot\hat{\mathbf{q}})(\mathbf{k}\cdot\hat{\mathbf{q}})]\rho_t(q_0, q).\end{aligned} \tag{10}$$

The energy conserving delta function in the last equation deletes the contribution from the delta function and therefore η receives contribution only from the cuts. The same holds true for diffusion coefficients $\mathcal{B}_{\parallel,\perp}$ as well. In the above equation, n and $\bar{n}$ are the Bose-Einstein and the Fermi-Dirac distribution functions:

$$n(q_0) = \frac{1}{e^{\beta q_0} - 1}, \qquad \bar{n}(k_0) = \frac{1}{e^{\beta(k_0-\mu)} + 1}, \tag{11}$$

where $\beta = \frac{1}{T}$. Equation (15) is the general expression of drag coefficient.

Apart from η, the quantity momentum diffusion-coefficient (B_{ij}) could be of importance in the study of fermion propagating in the plasma [19,23,24,28]. It can be defined as follows [19,23,24,28],

$$B_{ij} = \int d\Gamma q_i q_j. \tag{12}$$

Decomposing B_{ij} into longitudinal ($B_\parallel$) and transverse components ($B_\perp$) we get the following expression,

$$B_{ij} = B_\perp\left(\delta_{ij} - \frac{p_ip_j}{p^2}\right) + B_\parallel\frac{p_ip_j}{p^2}. \tag{13}$$

The imaginary part of Eq. (3) multiplied by the square of the longitudinal momentum transfer in the fermion scattering gives the expression for $\mathcal{B}_\parallel$. Using Eqs. (12) and (13), longitudinal momentum diffusion coefficient ($B_\parallel = \mathcal{B}$) can be written as follows,

$$\begin{aligned}\mathcal{B} = \frac{\pi e^2}{E}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\rho_f(k_0)\\ \times\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}q_\parallel^2(1 + n(q_0) - \bar{n}(k_0))\delta(E - k_0 - q_0)\\ \times[p_0k_0 + \mathbf{p}\cdot\mathbf{k}]\rho_l(q_0, q)\\ + 2[p_0k_0 - (\mathbf{p}\cdot\hat{\mathbf{q}})(\mathbf{k}\cdot\hat{\mathbf{q}})]\rho_t(q_0, q).\end{aligned} \tag{14}$$

Here, $q_\parallel = q\cos\theta$ i.e the longitudinal momentum transfer.

A. Drag coefficient when $|E - \mu| \sim T$

In this section, we calculate the drag coefficient (η) when $T \sim |E - \mu| \ll e\mu \ll \mu$; this is the region that is relevant for the astrophysical applications. It has been mentioned already that evaluation of η is plagued with infrared divergences. To circumvent this problem, as mentioned in the introduction, the region of integration as it appears below has to be divided into two regions distinguished by the scale of the momentum transfer, i.e. the soft and the hard sector. For the former, we use the one-loop resummed propagator with a finite upper limit on the momentum, which is designated as q^*, and for the latter we use the bare photon propagator. Following this prescription, for the soft part, one writes:

$$\eta|^{\rm soft}(E) \simeq \frac{e^2}{8\pi^2 E}\int_0^{q^*} dq q^3 \int_{-1}^{1} dx x(1+n(qx) - \bar{n}(E-\mu-qx))\{\rho_l(qx,q)+(1-x^2)\rho_t(qx,q)\}. \tag{15}$$

From the expression after subtracting the energy independent part, we have [11],

$$\eta|^{\rm soft}(E) - \eta|^{\rm soft}_{E=\mu} = -\frac{e^2}{8\pi^2 E}\int_0^{q^*} dq q^3 \int_{-1}^{1} dx x(\bar{n}(E-\mu-qx) - \bar{n}(-qx))[(1-x^2)\rho_t(qx,q)+\rho_l(qx,q)]. \tag{16}$$

First, we calculate the transverse photon contribution then the longitudinal one. For this, in Eq. (16) we substitute q and q_0 by introducing dimensionless variables z and v,

$$q = 2q_s z/(\pi v)^{1/3}, \qquad q_0 = Tv, \tag{17}$$

where q_s is the screening distance in the magnetic sector, and we take $a = \frac{T}{m_D} \ll 1$. From the above substitutions, it immediately follows that

$$q = m_D a^{1/3} z, \qquad x = a^{2/3} v/z. \tag{18}$$

After expanding the integrand with respect to a, we find for the transverse contribution of η,

$$\eta|^{\rm soft}_t(E) - \eta|^{\rm soft}_{t,E=\mu} = -\frac{e^2 m_D^2 a^2}{2\pi E}\int_{-((q^*)/(am_D))}^{((q^*)/(am_D))} dv v \int_{a^{2/3}|v|}^{((q^*)/(a^{1/3}m_D))} dz \times \frac{e^\alpha - 1}{(1+e^v)(1+e^{\alpha-v})} \times\Big[-\frac{z^2 v}{v^2\pi^2+4z^6} + \frac{16v^3 z^4}{(v^2\pi^2+4z^6)^2}a^{2/3} + \frac{16v^5(v^2\pi^2-12z^6)}{(v^2\pi^2+4z^6)^3}a^{4/3}+\cdots\Big], \tag{19}$$

where, $\alpha = \frac{|E-\mu|}{T} \sim O(1)$. Here, we neglect the terms, which are more than $a^{((10)/(3))}$ and $(\frac{m_D}{q^*})^4$. After z integration, we obtain

$$\eta|^{\rm soft}_t(E) - \eta|^{\rm soft}_{t,E=\mu} = \frac{e^2 m_D^2}{E}\int_{-\infty}^{\infty} dv \frac{e^\alpha - 1}{(1+e^v)(1+e^{\alpha-v})} \times\Big(\frac{va^2}{24\pi} - \frac{2^{(1/3)}v^{(5/3)}a^{(8/3)}}{9\pi^{(7/3)}} - \frac{20\times 2^{(2/3)}v^{(7/3)}a^{(10/3)}}{27\pi^{(11/3)}}+\cdots\Big). \tag{20}$$

Now, we use the formula for v integration sending the integration limits to $\pm\infty$,

$$\int_{-\infty}^{\infty} dv \frac{e^\alpha - 1}{(1+e^v)(1+e^{\alpha-v})}|v|^\lambda = \Gamma(\lambda+1)[{\rm Li}_{\lambda+1}(-e^{-\alpha}) - {\rm Li}_{\lambda+1}(-e^{\alpha})]. \quad \forall\ \lambda \geq 0 \tag{21}$$

Clearly, the expression for $\eta|^{\rm soft}_t$ is Polylogarithmic in nature,

$$\eta|^{\rm soft}_t(E) - \eta|^{\rm soft}_{t,E=\mu} = \frac{e^2 m_D^2}{E}\Big\{\frac{a^2}{24\pi}[\Gamma(2)({\rm Li}_2(-e^{-\alpha})+{\rm Li}_2(-e^{\alpha}))] - \frac{2^{1/3}a^{8/3}}{9\pi^{7/3}} \times\Big[\Gamma\Big(\frac{8}{3}\Big)({\rm Li}_{8/3}(-e^{-\alpha})+{\rm Li}_{8/3}(-e^{\alpha}))\Big] - \frac{20\times 2^{2/3}a^{10/3}}{9\pi^{11/3}} \times\Big[\Gamma\Big(\frac{10}{3}\Big)({\rm Li}_{10/3}(-e^{-\alpha})+{\rm Li}_{10/3}(-e^{\alpha}))\Big]+\cdots\Big\}. \tag{22}$$

The above expression can be written in the following form,

$$\eta|^{\rm soft}_t(E) - \eta|^{\rm soft}_{t,E=\mu} = \frac{e^2 m_D^2}{E}\Big\{\frac{1}{48\pi}\Big(\frac{T}{m_D}h_1\Big(\frac{(E-\mu)}{T}\Big)\Big)^2 - \frac{3\times 2^{1/3}}{72\pi^{7/3}}\Big(\frac{T}{m_D}h_2\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(8/3)} - \frac{6\times 2^{2/3}}{9\pi^{11/3}}\Big(\frac{T}{m_D}h_3\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(10/3)}\Big\}, \tag{23}$$

where

$$h_1\Big(\frac{(E-\mu)}{T}\Big) = [\Gamma(3)({\rm Li}_2(-e^{-\alpha}) - {\rm Li}_2(-e^{\alpha})]^{(1/2)},$$
$$h_2\Big(\frac{(E-\mu)}{T}\Big) = \Big[\Gamma\Big(\frac{11}{3}\Big)({\rm Li}_{8/3}(-e^{-\alpha}) - {\rm Li}_{8/3}(-e^{\alpha}))\Big]^{3/8},$$
$$h_3\Big(\frac{(E-\mu)}{T}\Big) = \Big[\Gamma\Big(\frac{13}{3}\Big)({\rm Li}_{10/3}(-e^{-\alpha}) - {\rm Li}_{10/3}(-e^{\alpha}))\Big]^{3/10}. \tag{24}$$

From the expression (22), it is evident that the expression contains fractional powers in $(E-\mu)$. This nature is basically a non-Fermi-liquid behavior of ultradegenerate relativistic plasma. After the magnetic part, we derive the expression of the electric part.

In case of the electric part, we substitute $q = q_s y$ and $q_0 = Tu/y$, or $q = m_D y$ and $x = au/y$. Though the substitutions in electric and magnetic sectors look different, the nature of substitutions can be seen from the structure of $\beta_{l,t}$ (Eq. (8)). As screening length is different in electric and magnetic sectors, the substitutions therefore involve different coefficients of m_D and T for the transverse and the longitudinal case [11]. The longitudinal term after simplification like the transverse one becomes,

$$\eta|_l^{\rm soft}(E) - \eta|_{l,E=\mu}^{\rm soft} = \frac{e^2 m_D^2 a^3}{32E}[\Gamma(3)({\rm Li}_3(-e^{-\alpha}) - {\rm Li}_3(-e^{\alpha}))] + O(a^4). \quad (25)$$

Again, for the leading term, we can write it in the following form:

$$\eta|_l^{\rm soft}(E) - \eta|_{l,E=\mu}^{\rm soft} = \frac{e^2 m_D^2}{96E}\left(\frac{T}{m_D} g_1\left(\frac{(E-\mu)}{T}\right)\right)^3, \quad (26)$$

where

$$g_1\left(\frac{(E-\mu)}{T}\right) = [\Gamma(4)({\rm Li}_3(-e^{-\alpha}) - {\rm Li}_3(-e^{\alpha}))]^{(1/3)}. \quad (27)$$

The final expression for drag-coefficient then becomes

$$\eta = \frac{e^2 m_D^2}{E}\left\{\frac{1}{48\pi}\left(\frac{T}{m_D} h_1\left(\frac{(E-\mu)}{T}\right)\right)^2 - \frac{3\times 2^{1/3}}{72\pi^{7/3}}\left(\frac{T}{m_D} h_2\left(\frac{(E-\mu)}{T}\right)\right)^{(8/3)} - \frac{6\times 2^{2/3}}{9\pi^{11/3}}\left(\frac{T}{m_D} h_3\left(\frac{(E-\mu)}{T}\right)\right)^{(10/3)}\right\} + \frac{e^2 m_D^2}{96E}\left(\frac{T}{m_D} g_1\left(\frac{(E-\mu)}{T}\right)\right)^3. \quad (28)$$

In the zero temperature limit, the functions behave as $h_i(\alpha) \rightarrow |\alpha|$ and $g_i(\alpha) \rightarrow |\alpha|$. Hence, η in the extreme zero temperature limit becomes,

$$\eta = \frac{e^2|E-\mu|^2}{48\pi E} - \frac{3\times 2^{1/3} e^2 m_D^2}{72\pi^{7/3} E}\left(\frac{|E-\mu|}{m_D}\right)^{(8/3)} + \frac{e^2|E-\mu|^3}{96 m_D E} + \cdots. \quad (29)$$

This is the result for zero temperature plasma. Both the first and the second term here come from the transverse sector, while the last piece emanates from the longitudinal interactions. The appearance of the second term with fractional power both in Eqs. (28) and (29) clearly shows that full contributions to η cannot be obtained by adding leading order contributions of the transverse and longitudinal photon exchange as the subleading terms of the former is larger than the leading order contribution of the latter. This observation, in connection to the evaluation of fermion self-energy was first noted in [11] and was overlooked in [14–16,18]. The zero temperature leading order contributions for l and t part are, however, consistent with our previous calculation reported in [18]. It is needless to mention here that such a characteristic feature, also known as non-Fermi-liquid behavior, can be attributed to the absence of the magnetostatic screening as noted in the introduction.

In Fig. 2, we have plotted η versus energy of the incoming fermion in the small temperature ($T/T_f \ll 1$) region where $T_f = \mu/k_B$ is the Fermi temperature. From the figure, it is evident that with increasing T/T_f, η decreases. This trend is consistent with what one finds for the fermionic damping rate at small temperature [11].

So far, we have not discussed the hard sector and tacitly assumed that the entire contribution to η in the relevant domains i.e. for small and zero temperature, come from the soft photon exchange. This, for degenerate plasma, is indeed so, as demonstrated explicitly in [18]. In [18], it was shown that the leading order of the hard sector fails to contribute to η at least up to $O(e^2)$. As in the present work, on the other hand, we go beyond the leading order; in principle, one should calculate the NLO part for the hard sector as well and see if the hard sector contributes to the drag and diffusion coefficients in this case. But such an explicit calculation has not been done here. One can justify this omission on the ground that, for the soft sector, we see even after the inclusion of the NLO corrections no intermediate cutoff (q^*) dependent term appear up to $O(e^2)$. Therefore, in the spirit of our previous work [18], we conclude up to this order the entire contribution comes from the soft sector providing indirect justification of this omission. The finite temperature NLO calculation can shed further light on this issue [35–37].

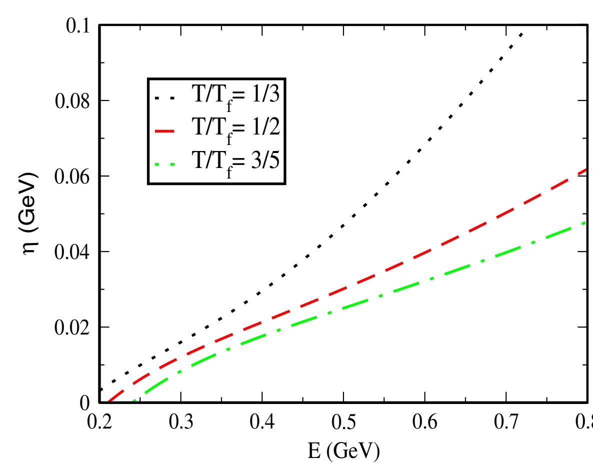

FIG. 2 (color online). The next-to-leading order drag coefficient at $T/T_f = 1/3$ (dotted curve), $T/T_f = 1/2$ (dashed curve), $T/T_f = 3/5$ (dash dotted curve).

B. Diffusion coefficient when $|E - \mu| \simeq T$

Along with η, momentum diffusion-coefficient (B_{ij}), [19,23,24,28] is another relevant quantity to study the equilibration of a fermion propagating in the plasma. For Coulomb plasma η and the longitudinal momentum diffusion coefficient ($\mathcal{B}$) are related *via* Einstein's Relation (ER). In this section, we study the nature of longitudinal diffusion coefficient in the low temperature region. In the soft region the expression looks like

$$\mathcal{B}|^{\rm soft}(E) \simeq \frac{e^2}{8\pi^2}\int_0^{q^*} dq q^4 \times \int_{-1}^{1} dx x^2 (1 + n(qx) - \bar{n}(E - \mu - qx)) \times \{\rho_l(qx, q) + (1 - x^2)\rho_t(qx, q)\}. \tag{30}$$

First, we calculate the transverse photon contribution, then the longitudinal one. For the transverse photon propagator, we proceed along the same line of the previous subsection and find,

$$\mathcal{B}|_t^{\rm soft}(E) - \mathcal{B}|_{t,E=\mu}^{\rm soft} = e^2 m_D^3 \Big\{ \frac{a^3}{24\pi}[\Gamma(3)(\mathrm{Li}_3(-e^{-\alpha}) - \mathrm{Li}_3(-e^{\alpha}))] - \frac{2^{1/3} a^{11/3}}{9\pi^{7/3}}\Big(\Gamma\Big(\frac{11}{3}\Big)(\mathrm{Li}_{11/3}(-e^{-\alpha}) - \mathrm{Li}_{11/3}(-e^{\alpha})\Big) - \frac{20 \times 2^{2/3} a^{13/3}}{9\pi^{11/3}}\Big(\Gamma\Big(\frac{13}{3}\Big)(\mathrm{Li}_{13/3}(-e^{-\alpha}) - \mathrm{Li}_{13/3}(-e^{\alpha})\Big) + \cdots \Big\}. \tag{31}$$

The above expression is written in the following form:

$$\mathcal{B}|_t^{\rm soft}(E) - \mathcal{B}|_{t,E=\mu}^{\rm soft} = e^2 m_D^3 \Big\{ \frac{1}{72\pi}\Big(\frac{T}{m_D} h_4\Big(\frac{(E-\mu)}{T}\Big)\Big)^3 - \frac{3 \times 2^{1/3}}{99\pi^{7/3}}\Big(\frac{T}{m_D} h_5\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(11/3)} - \frac{20 \times 2^{2/3}}{39\pi^{11/3}}\Big(\frac{T}{m_D} h_6\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(13/3)} \Big\}, \tag{32}$$

where

$$h_4\Big(\frac{(E-\mu)}{T}\Big) = [\Gamma(4)(\mathrm{Li}_3(-e^{-\alpha}) - \mathrm{Li}_3(-e^{\alpha})]^{(1/3)}$$
$$h_5\Big(\frac{(E-\mu)}{T}\Big) = \Big[\Gamma\Big(\frac{14}{3}\Big)(\mathrm{Li}_{11/3}(-e^{-\alpha}) - \mathrm{Li}_{11/3}(-e^{\alpha})\Big]^{3/11}$$
$$h_6\Big(\frac{(E-\mu)}{T}\Big) = \Big[\Gamma\Big(\frac{16}{3}\Big)(\mathrm{Li}_{13/3}(-e^{-\alpha}) - \mathrm{Li}_{13/3}(-e^{\alpha})\Big]^{3/13}. \tag{33}$$

After the magnetic part, we derive the expression of the electric part. In case of electric term, one finds

$$\mathcal{B}|_l^{\rm soft}(E) - \mathcal{B}|_{l,E=\mu}^{\rm soft} = \frac{e^2 m_D^3}{128}\Big(\frac{T}{m_D} g_2\Big(\frac{(E-\mu)}{T}\Big)\Big)^4 + O(a^5), \tag{34}$$

where

$$g_2\Big(\frac{(E-\mu)}{T}\Big) = [\Gamma(5)(\mathrm{Li}_4(-e^{-\alpha}) - \mathrm{Li}_4(-e^{\alpha}))]^{(1/4)}. \tag{35}$$

Finally, we obtain the expression for longitudinal momentum diffusion-coefficient as

$$\mathcal{B} = e^2 m_D^3 \Big\{ \frac{1}{72\pi}\Big(\frac{T}{m_D} h_4\Big(\frac{(E-\mu)}{T}\Big)\Big)^3 - \frac{3 \times 2^{1/3}}{99\pi^{7/3}} \times \Big(\frac{T}{m_D} h_5\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(11/3)} - \frac{20 \times 2^{2/3}}{39\pi^{11/3}} \times \Big(\frac{T}{m_D} h_6\Big(\frac{(E-\mu)}{T}\Big)\Big)^{(13/3)} \Big\} + \frac{e^2 m_D^3}{128} \times \Big(\frac{T}{m_D} g_2\Big(\frac{(E-\mu)}{T}\Big)\Big)^4. \tag{36}$$

This expression is polylogarithmic in nature and also contains fractional power in $|E - \mu|$. This fractional power indicates the deviation from Fermi-liquid behavior. This departure can also be seen in the zero temperature case. Hence, the final expression for $\mathcal{B}$ in the extreme zero temperature limit becomes

$$\mathcal{B} = \frac{e^2|E-\mu|^3}{72\pi} - \frac{2^{1/3} e^2 m_D^3}{33\pi^{7/3}}\Big(\frac{|E-\mu|}{m_D}\Big)^{(11/3)} + \frac{e^2|E-\mu|^4}{128 m_D} + \cdots. \tag{37}$$

The first two terms in the last equation correspond to the transverse contribution and the remaining third term comes

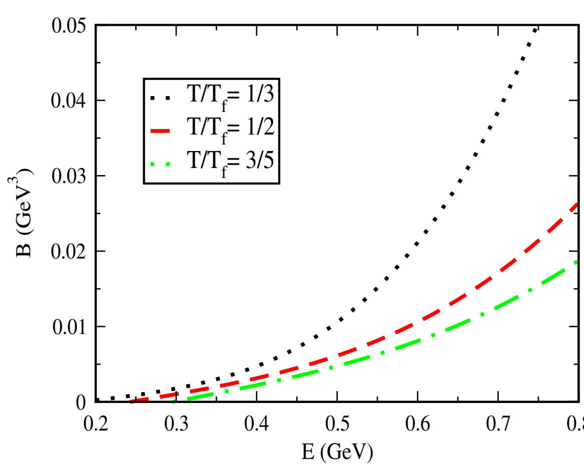

FIG. 3 (color online). The next-to-leading order diffusion-coefficient at $T/T_f = 1/3$ (dotted curve), $T/T_f = 1/2$ (dashed curve), $T/T_f = 3/5$ (dash dotted curve).

from the longitudinal interaction. The expression for longitudinal diffusion coefficient has been already obtained in [18]. Like η, in $\mathcal{B}$ also we find that the subleading transverse part is greater than the leading longitudinal contribution. We see from the Fig. 3 that nature of the curve for the diffusion coefficient is same as that of η as shown in the previous subsection.

III. CONCLUSION

In this paper, we have calculated the fermionic drag and diffusion-coefficients in a relativistic plasma both at zero and small temperature by retaining terms beyond the leading contributions. It is seen that the subleading terms of the transverse sector, which appear with fractional power, are larger than the leading terms coming from the exchange of soft longitudinal photons or in other words we show that the leading order contributions to the drag and diffusion-coefficients in ultradegenerate plasma cannot be obtained just by adding the leading order contributions coming from each of these sectors. Both the appearance of the fractional power and dominance of the transverse sector are related to absence of the magnetostatic screening or the singular behavior of the fermion self-energy near the Fermi surface. Furthermore, we find that the contributions coming from the hard sectors are suppressed and the entire physics is dominated by the soft excitations. This is a clear departure from the finite temperature case, where both the hard and the soft part contribute at the same order. As a last remark, we note that here the entire calculation has been done for QED plasma. It would be interesting to extend the present calculation for QCD matter, which might be tricky due to the existence of triple gluon vertex and possible magnetic screening in the QCD sector.

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[1] T. Holstein, R. E. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973).
[2] S. Chakravarty, R. E. Norton, and O. F. Syljuåsen, Phys. Rev. Lett. **74**, 1423 (1995).
[3] D. Boyanovsky and H. J. de Vega, Phys. Rev. D **63**, 114028 (2001).
[4] A. Ipp, A. Gerhold, and A. Rebhan, Phys. Rev. D **69**, R011901 (2004).
[5] A. Gerhold, A. Ipp, and A. Rebhan, Phys. Rev. D **70**, 105015 (2004).
[6] H. Heiselberg and C. J. Pethick, Phys. Rev. D **48**, 2916 (1993).
[7] T. Schäfer and K. Schwenzer, Phys. Rev. D **70**, 114037 (2004).
[8] K. Pal and A. K. Dutt-Mazumder, Phys. Rev. D **84**, 034004 (2011).
[9] W. E. Brown, J. T. Liu, and H.-c. Ren, Phys. Rev. D **61**, 114012 (2000); **62**, 054013 (2000).
[10] Q. Wang and D. H. Rischke, Phys. Rev. D **65**, 054005 (2002).
[11] A. Gerhold and A. Rebhan, Phys. Rev. D **71**, 085010 (2005).
[12] T. Schäfer and K. Schwenzer, Phys. Rev. D **70**, 054007 (2004).
[13] D. Boyanovsky and H. J. de Vega, Phys. Rev. D **63**, 034016 (2001).
[14] M. Le Bellac and C. Manuel, Phys. Rev. D **55**, 3215 (1997).
[15] C. Manuel, Phys. Rev. D **62**, 076009 (2000).
[16] B. Vanderheyden and J. Ollitrault, Phys. Rev. D **56**, 5108 (1997).
[17] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **66**, 2183 (1991).
[18] S. Sarkar and A. K. Dutt-Mazumder, Phys. Rev. D **82**, 056003 (2010).
[19] B. Svetitsky, Phys. Rev. D **37**, 2484 (1988).
[20] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991).
[21] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, R2625 (1991).
[22] A. K. Dutt-Mazumder, Jan-e Alam, P. Roy, and B. Sinha, Phys. Rev. D **71**, 094016 (2005).
[23] G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
[24] P. Roy, A. K. Dutt-Mazumder, and Jan-e Alam, Phys. Rev. C **73**, 044911 (2006).
[25] M. G. Mustafa, Phys. Rev. C **72**, 014905 (2005).
[26] S. Peigne and A. Peshier, Phys. Rev. D **77**, 014015 (2008).
[27] S. Peigne and A. Peshier, Phys. Rev. D **77**, 114017 (2008).
[28] A. Beraudo, A. De Pace, W. M. Alberico, and A. Molinari, Nucl. Phys. A **831**, 59 (2009).
[29] H. Vija and M. H. Thoma, Phys. Lett. B **342**, 212 (1995).
[30] S. K. Das, Jan-e Alam, P. Mohanty, and B. Sinha, Phys. Rev. C **81**, 044912 (2010).
[31] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
[32] J. I. Kapusta and Charles Gale, *Finite Temperature Field Theory: Principles and Applications* (Cambridge University Press, Cambridge, England, 2006).
[33] C. Manuel, Phys. Rev. D **53**, 5866 (1996).
[34] K. Pal and A. K. Dutt-Mazumder, Phys. Rev. C **81**, 054906 (2010).
[35] P. Aurenche, F. Gelis, H. Zaraket, and R. Kobes, Phys. Rev. D **60**, 076002 (1999).
[36] M. E. Carrington, Phys. Rev. D **75**, 045019 (2007).
[37] M. E. Carrington, A. Gynther, and D. Pickering, Phys. Rev. D **78**, 045018 (2008).