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Minimal 3-3-1 model with only two Higgs triplets

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The simplest non-Abelian gauge extension of the electroweak standard model, the $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$, known as the 3-3-1 model, has a minimal version which demands the least possible fermionic content to account for the whole established phenomenology for the well-known particles and interactions. Nevertheless, in its original form the minimal 3-3-1 model was proposed with a set of three scalar triplets and one sextet in order to yield the spontaneous breaking of the gauge symmetry and generate the observed fermion masses. Such a huge scalar sector turns the task of clearly identifying the physical scalar spectrum into a clumsy labor. It not only adds an obstacle for the development of its phenomenology, but implies a scalar potential plagued with new free coupling constants. In this work, we show that the framework of the minimal 3-3-1 model can be built with only two scalar triplets, but still triggering the desired pattern of spontaneous symmetry breaking and generating the correct fermion masses. We present the exact physical spectrum and also show all the interactions involving the scalars, obtaining a neat minimal 3-3-1 model far more suited for phenomenological studies at the current Large Hadron Collider.

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I. INTRODUCTION

One of the main goals of the Large Hadron Collider (LHC) is to find the Higgs boson and probe new physics at TeV scale. This opens a timely window to review the scalar sector of extensions of the electroweak standard model (SM) at the particular range of energy covered by the LHC. Here we specialize in an simple non-Abelian gauge extension of SM, the $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$, the so-called 3-3-1 model, whose characteristic scale lies at TeV scale. Among the available versions, the minimal 3-3-1 model [1] may be considered the most phenomenologically interesting one because of its reduced leptonic content and the presence of bileptons (fields carrying two units of lepton number): a singly charged gauge boson and a doubly charged gauge boson, $V^{\pm}$ and $U^{\pm\pm}$, and new quarks with exotic electric charges $\frac{5}{3}e$ and $\frac{4}{3}e$, besides scalar bileptons. In a lepton number-conserving framework, these new particles are constrained to involve a pair of sibling leptons in their decay branch, posing a singular signature to be searched for in colliders. From the theoretical side, the model predicts the existence of three fermion generations, solving the so-called family replication problem, explains why the electric charge is quantized [2], and allows in its structure the realization of the Peccei-Quinn symmetry which may provide a solution to the strong-CP problem [3]. These are some of the features that make the minimal 3-3-1 model an excellent proposal of physics beyond the SM, justifying further efforts to make it phenomenologically more appealing. In this way, we should look for some room for improvement without jeopardizing any of the nice features of the model.

It happens that one of the unpleasant parts of the model is exactly the one that offers the opportune window to ameliorate its framework. It concerns its huge scalar sector [4], composed by three triplets and one sextet,[1] totalizing 30 degrees of freedom. The spontaneous breaking of the 3-3-1 model gauge symmetry requires 8 degrees of freedom in the form of Goldstones, meaning that after the breaking there remains 22 degrees of freedom in the spectrum as physical scalars. It would be understandable that some prejudice could be raised against this model relying on such a prolific scalar sector, since we are still waiting for the first fundamental scalar to be detected. We mention that besides having a large amount of additional scalar degrees of freedom compared to the SM, the gauge symmetry also allows for a scalar potential that implies several unknown coupling constants among these scalars. Finally, such a scalar potential leads to intricate mass matrices that are very awkward to deal with [6]. In other words, we have to rely upon several suppositions concerning the couplings and the energy scales of the model in order to obtain a simpler and treatable physical scalar spectrum (mass eigenstates). However, as the LHC is focused on the Higgs boson search, it is imperative that a testable model have a

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[1]If one intends to explain the correct mass for all leptons, including neutrinos, an extra sextet should be added [5] and the number of degrees of freedom are really much bigger.

neat and manageable scalar spectrum, with the lowest number of free parameters and all the couplings well defined, so as to better contrast the LHC findings with the model predictions. Then, if we are able to reduce the number of scalars in this model it would be a huge gain, in number and simplicity.

In this work, we argue that such an approach is possible for the minimal 3-3-1 model. Next, we present the reduced minimal 3-3-1 (RM331) model and implement this scheme by showing that only two scalar triplets are sufficient to break correctly the $SU(3)_C \times SU(3)_L \times U(1)_N$ symmetry to the $SU(3)_C \times U(1)_{\rm QED}$ and geenrate the correct masses of all fermions including neutrinos.

II. THE REDUCED MINIMAL 3-3-1 MODEL

As in the original version [1], each family of leptons comes in a triplet representation of $SU(3)_L$,

$$f_L = \begin{pmatrix} \nu_l \\ l \\ l^c \end{pmatrix}_L \sim (1, 3, 0), \qquad (1)$$

where $l = e, \mu, \tau$. The numbers in parenthesis represent the fields' transformation properties under the gauge group $SU(3)_C \times SU(3)_L \times U(1)_N$.

In the quark sector, anomaly cancellation requires that one generation comes in a $SU(3)_L$ triplet and the other two come in antitriplet representation with the following content:

$$\begin{aligned} Q_{1L} &= \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim \left(3, 3, +\frac{2}{3}\right), \\ Q_{iL} &= \begin{pmatrix} d_i \\ -u_i \\ J_i \end{pmatrix}_L \sim \left(3, 3^*, -\frac{1}{3}\right), \\ u_{iR} &\sim \left(3, 1, +\frac{2}{3}\right); \\ d_{iR} &\sim \left(3, 1, -\frac{1}{3}\right); \\ J_{iR} &\sim \left(3, 1, -\frac{4}{3}\right), \\ u_{1R} &\sim \left(3, 1, +\frac{2}{3}\right); \\ d_{1R} &\sim \left(3, 1, -\frac{1}{3}\right); \\ J_{1R} &\sim \left(3, 1, +\frac{5}{3}\right), \end{aligned} \qquad (2)$$

where $i = 2, 3$.

We now introduce the main point of our proposal. Instead of three triplets and one sextet of scalars as in the original version, we propose a RM331 model with only two scalar triplets,[2]

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, 1); \qquad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1, 3, -1). \qquad (3)$$

Before going into technical details, we first argue that such short scalar content is sufficient to engender the correct pattern of gauge symmetry breaking and generate masses for all fermions including neutrinos.

Let us first analyze the spontaneous symmetry breaking sequence. It is easy to see that when χ^0 develops a nontrivial vacuum expectation value (VEV), v_χ, the $SU(3)_C \times SU(3)_L \times U(1)_N$ symmetry breaks spontaneously to the $SU(3)_C \times SU(2)_L \times U(1)_Y$, which in turn breaks to $SU(3)_C \times U(1)_{QED}$, when ρ^0 develops a VEV, v_ρ.

Although the breaking occurs as desired, we have to check that there are enough degrees of freedom for supporting this spontaneous symmetry breaking pattern. To illustrate that this is the case, we recall that the charged scalars $\chi^\pm$ are the only ones to carry two units of lepton number (bileptons) [1], meaning that they are the Goldstone bosons absorbed by the $V^\pm$ gauge bosons (also bileptons), while $\rho^\pm$ are the unique singly charged scalars that do not carry lepton number, thus they should be the Goldstones eaten by the standard $W^\pm$. The doubly charged scalars, $\rho^{\pm\pm}$ and $\chi^{\pm\pm}$, are both bileptons, and one linear combination of them should give the longitudinal component of the vector bilepton $U^{\pm\pm}$, while the orthogonal combination remains in the physical spectrum. Regarding the neutral sector, I_ρ is the Goldstone boson eaten by Z_μ and I_χ is the Goldstone eaten by Z'_μ. So, from this short analysis, we conclude that the two scalar triplets possess the right number of degrees of freedom to engender the expected pattern of spontaneous symmetry breaking in the RM331 model. As a consequence, from the initial 12 scalar degrees of freedom, only four of them survive in the physical spectrum, two doubly charged, $h^{\pm\pm}$, and two neutral ones, h^1 and h_2. One of the neutral scalars, the lightest, is going to be identified with the Higgs boson. All this makes it clear that the RM331 model keeps the main ingredients to become a successful gauge extension of SM. Next, we explore the scalar sector in a little more detail.

[2]It is important to stress that our proposal would work also for the alternative choice of keeping the scalar triplet $\eta = (\eta^0, \eta_1^-, \eta_2^+) \sim (1, 3, 0)$ instead of ρ. However, in this case neutrino masses would arise through the dimension-five effective operator, $\frac{h}{\Lambda}(\bar{L}^C \eta^*)(\eta^\dagger L)$, leading to a neutrino mass scale two orders of magnitude higher than the dimension-seven operator we are going to obtain in this work. Thus, our choice for ρ is based in obtaining the higher effective operator for neutrino masses, demanding less fine-tuning in the respective coupling.

III. SCALAR SECTOR

Considering the scalar content in Eq. (3), we can write down the most general renormalizable, gauge and Lorentz invariant scalar potential,

$$V(\chi,\rho)=\mu_1^2\rho^\dagger\rho+\mu_2^2\chi^\dagger\chi+\lambda_1(\rho^\dagger\rho)^2+\lambda_2(\chi^\dagger\chi)^2+\lambda_3(\rho^\dagger\rho)(\chi^\dagger\chi)+\lambda_4(\rho^\dagger\chi)(\chi^\dagger\rho), \tag{4}$$

whose manifest simplicity turns the RM331 model into a real gain as compared to the original minimal 3-3-1 model since the number of free parameters is reduced from at least 13 to only 6.

Let us expand ρ^0 and χ^0 around their VEVs in the usual way,

$$\rho^0,\chi^0\to\frac{1}{\sqrt{2}}(v_{\rho,\chi}+R_{\rho,\chi}+iI_{\rho,\chi}). \tag{5}$$

On substituting this expansion in the above potential, we obtain the following set of minimum constraint equations:

$$\mu_1^2+\lambda_1v_\rho^2+\frac{\lambda_3v_\chi^2}{2}=0,\qquad \mu_2^2+\lambda_2v_\chi^2+\frac{\lambda_3v_\rho^2}{2}=0. \tag{6}$$

Let us first consider the doubly charged scalars. Their mass matrix in the basis (χ^{++},ρ^{++}) is given by

$$m_{++}^2=\frac{\lambda_4v_\chi^2}{2}\begin{pmatrix}t^2 & t\\ t & 1\end{pmatrix}, \tag{7}$$

where $t=\frac{v_\rho}{v_\chi}$. Diagonalizing this matrix, we obtain the following squared mass eigenvalues:

$$m_{\tilde{h}^{++}}^2=0\quad\text{and}\quad m_{h^{++}}^2=\frac{\lambda_4}{2}(v_\chi^2+v_\rho^2), \tag{8}$$

where the corresponding eigenstates are

$$\begin{pmatrix}\tilde{h}^{++}\\ h^{++}\end{pmatrix}=\begin{pmatrix}c_\alpha & -s_\alpha\\ s_\alpha & c_\alpha\end{pmatrix}\begin{pmatrix}\chi^{++}\\ \rho^{++}\end{pmatrix}, \tag{9}$$

with

$$c_\alpha=\frac{v_\chi}{\sqrt{v_\chi^2+v_\rho^2}},\qquad s_\alpha=\frac{v_\rho}{\sqrt{v_\chi^2+v_\rho^2}}. \tag{10}$$

It is easy to see that when $v_\chi\gg v_\rho$, we have that $\tilde{h}^{++}\approx\chi^{++}$ and $h^{++}\approx\rho^{++}$. Notice that $\tilde{h}^{\pm\pm}$ are the Goldstones eaten by the gauge bosons $U^{\pm\pm}$, while $h^{\pm\pm}$ remains a physical scalar in the spectrum.

Regarding the neutral scalars, their mass matrix takes the following form in the basis (R_χ,R_ρ):

$$m_0^2=\frac{v_\chi^2}{2}\begin{pmatrix}2\lambda_2 & \lambda_3t\\ \lambda_3t & 2\lambda_1t^2\end{pmatrix}. \tag{11}$$

On diagonalizing this mass matrix, we obtain the following eigenvalues:

$$m_{h_1}^2=\left(\lambda_1-\frac{\lambda_3^2}{4\lambda_2}\right)v_\rho^2,\qquad m_{h_2}^2=\lambda_2v_\chi^2+\frac{\lambda_3^2}{4\lambda_2}v_\rho^2, \tag{12}$$

with their corresponding neutral physical eigenstates

$$h_1=c_\beta R_\rho-s_\beta R_\chi,\qquad h_2=c_\beta R_\chi+s_\beta R_\rho, \tag{13}$$

where $c_\beta\approx1-\frac{\lambda_3^2}{8\lambda_2^2}\frac{v_\rho^2}{v_\chi^2}$ and $s_\beta\approx\frac{\lambda_3}{2\lambda_2}\frac{v_\rho}{v_\chi}$.

As for the pseudoscalars, I_ρ and I_χ, they do not mix among themselves and are massless, meaning that they are Goldstone bosons eaten by the gauge bosons Z_μ and Z'_μ, respectively.

At the end of the day, the physical scalar spectrum of the RM331 model is composed by a doubly charged scalar, h^{++}, and two neutral scalars, h_1 and h_2. Since the lightest neutral field, h_1, is basically a $SU(2)_L$ component in the linear combination Eq. (13), we identify it as the standard Higgs boson in this model.

Note that for $\lambda_2,\lambda_3<1$, and $\lambda_2>\lambda_3$ we obtain from the first expression in Eq. (12) that $m_{h_1}^2\approx\lambda_1v_\rho^2$, which recovers the standard model case. The value of v_ρ is fixed in the next section. Thus, only when LHC finds out the Higgs and determine its mass, we then can adjust λ_1 to generate the correct Higgs mass.

IV. GAUGE BOSON SPECTRUM

In order to obtain the expression for the masses of the massive gauge bosons of the model, we have to substitute the expansion of the Eq. (5) in the Lagrangian,

$$\mathcal{L}=(\mathcal{D}_\mu\chi)^\dagger(\mathcal{D}^\mu\chi)+(\mathcal{D}_\mu\rho)^\dagger(\mathcal{D}^\mu\rho), \tag{14}$$

where

$$D_\mu=\partial_\mu-igW_\mu^a\frac{\lambda^a}{2}-ig_NNW_\mu^N, \tag{15}$$

with $a=1,\ldots,8$ and λ^a being the Gell-Mann matrices.

On doing this, the eigenstates of the charged gauge bosons and their respective masses are given by

$$\begin{aligned}W^\pm&=\frac{W^1\mp iW^2}{\sqrt{2}}\to M_{W^\pm}^2=\frac{g^2v_\rho^2}{4},\\ V^\pm&=\frac{W^4\pm iW^5}{\sqrt{2}}\to M_{V^\pm}^2=\frac{g^2v_\chi^2}{4},\\ U^{\pm\pm}&=\frac{W^6\pm iW^7}{\sqrt{2}}\to M_{U^{\pm\pm}}^2=\frac{g^2(v_\rho^2+v_\chi^2)}{4}.\end{aligned} \tag{16}$$

We call attention to the interesting fact that a hierarchy arises among the charged gauge bosons. Namely, the mass expressions in Eq. (16) provide $M_U^2 - M_V^2 = M_W^2$, which is a direct consequence of the shortened scalar sector in the RM331 model.

From the three neutral gauge bosons, one is identified as the photon, A_μ, which is massless, and the other two are the standard Z_μ and a new Z'_μ, whose masses are given by

$$M_Z^2 = \frac{g^2}{4c_W^2}v_\rho^2, \qquad M_{Z'}^2 = \frac{g^2 c_W^2}{3(1-4s_W^2)}v_\chi^2. \tag{17}$$

We then write the massless gauge fields in terms of the mass eigenstates as follows:

$$\begin{aligned} W_\mu^N &= -t_W\sqrt{h_W}Z_\mu + \sqrt{3}t_W Z'_\mu + \sqrt{h_W}A_\mu, \\ W_\mu^8 &= \sqrt{3}t_W s_W Z_\mu + \frac{h_W}{c_W}Z'_\mu - \sqrt{3}s_W A_\mu, \\ W_\mu^3 &= c_W Z_\mu + s_W A_\mu, \end{aligned} \tag{18}$$

where $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $t_W = \tan\theta_W$, $h_W = 1\text{–}4s_W^2$, with θ_W being the Weinberg mixing angle.

Perceive that the mass expressions for the standard gauge bosons W and Z are exactly the same ones of the standard model. Thus, for $M_W = 80.4$ GeV we obtain $v_\rho = 246$ GeV for $G_F = 1.166 \times 10^{-5}$ GeV^{-2}. Consequently, we obtain $M_Z = 91.2$ GeV for $\sin^2\theta_W = 0.22$.

Hence, we have four singly charged gauge bosons, the standard $W^\pm$ and the heavy one $V^\pm$, two doubly charged gauge bosons, $U^{\pm\pm}$, and three neutral gauge bosons, the photon A_μ, the standard Z_μ, and the heavy Z'_μ. Their trilinear and quartic interactions are the same as in Ref. [7], their neutral and charged current interactions with the fermions are given in Appendix A while their couplings with the physical scalars are presented in Tables I and II.

In the couplings presented in Tables I and II we used

$$\begin{aligned} k_1 &= (3+2h_W)c_\beta v_\rho - (3+4h_W)s_\beta v_\chi \\ &\quad - 3(c_\beta v_\rho - s_\beta v_\chi)c_{2W}, \\ k_2 &= (s_\beta v_\rho + 2c_\beta v_\rho)h_W + 3(s_\beta v_\rho + c_\beta v_\chi)s_W^2, \\ k_3 &= h_W^2(c_\beta^2 + 4s_\beta^2)\sec_W^2, \\ k_4 &= h_W^2(4c_\beta^2 + s_\beta^2)\sec_W^2, \\ k_5 &= -6(c_\alpha^2(-3+4h_W) + (-3+2h_W)s_\alpha^2 + 3c_{2W})t_W^2, \\ k_6 &= -6s_W^2(-s_\alpha^2 + (4c_\alpha^2 + 3s_\alpha^2)t_W^2). \end{aligned} \tag{19}$$

TABLE I. Trilinear couplings among gauge bosons and scalars in the minimal 3-3-1 model with two Higgs triplets.

Vertex	Coupling
$W^+W^-h^1$	$\frac{1}{2}c_\beta g^2 v_\rho$
$V^+V^-h^1$	$-\frac{1}{2}s_\beta g^2 v_\chi$
$U^{++}U^{--}h^1$	$\frac{1}{2}(c_\beta v_\rho - s_\beta v_\chi)g^2$
ZZh_1	$\frac{1}{4}g^2 v_\rho c_\beta c_\theta^2 \sec_W^2$
$Z'Z'h_1$	$\frac{g^2c_\theta^2}{12h_W}[h_W^2(c_\beta v_\rho - 4s_\beta v_\chi)\sec_W^2 + 6k_1t_W^2]$
$ZZ'h_1$	$-\frac{1}{2\sqrt{3}\sqrt{h_W}}g^2c_\beta(3+h_W-3c_{2W})c_\theta^2\sec_W^2$
$W^+W^-h^2$	$\frac{1}{2}s_\beta g^2 v_\rho$
$V^+V^-h^2$	$\frac{1}{2}c_\beta g^2 v_\chi$
$U^{++}U^{--}h^2$	$\frac{1}{2}(s_\beta v_\rho + c_\beta v_\chi)g^2$
ZZh_2	$\frac{1}{4}g^2 v_\rho s_\beta c_\theta^2 \sec_W^2$
ZZh_2	$\frac{g^2c_\theta^2}{12h_W}[h_W^2(s_\beta v_\rho + 4c_\beta v_\chi)\sec_W^2 + 12k_2t_W^2]$
$Z'Z'h_2$	$-\frac{1}{2\sqrt{3}\sqrt{h_W}}g^2s_\beta(3+h_W-3c_{2W})c_\theta^2\sec_W^2$
$W^\pm V^\pm h^{\pm\pm}$	$\frac{1}{2\sqrt{2}}(c_\alpha v_\rho + s_\alpha v_\chi)g^2$
$U^{\pm\pm}Ah^{\mp\mp}$	$g^2(c_\alpha v_\rho - s_\alpha v_\chi)s_W$
$U^{\pm\pm}Zh^{\mp\mp}$	$\frac{1}{4}g^2c_\theta[-3c_\alpha v_\rho + s_\alpha v_\chi + 2(c_\alpha v_\rho - s_\alpha v_\chi)c_{2W}]\sec_W$
$A_\mu h^{++}h^{--}$	$-\frac{3}{2}igs_W(p_1-p_2)_\mu$
$Z_\mu h^{++}h^{--}$	$-\frac{1}{4}igc_\theta[-3c_\alpha^2 - s_\alpha^2 + 3c_{2W}]\sec_W(p_1-p_2)_\mu$
$U_\mu^{\pm\pm}h_1h^{\mp\mp}$	$\pm ig\cos(\alpha+\beta)(p_1-p_2)_\mu$
$U_\mu^{\pm\pm}h_2h^{\mp\mp}$	$\pm ig\sin(\alpha+\beta)(p_1-p_2)_\mu$

TABLE II. Quartic couplings among gauge bosons and scalars in the minimal 3-3-1 model with two Higgs triplets.

Vertex	Coupling
$W^+W^-h^1h^1$	$\frac{1}{4}c_\beta^2 g^2$
$V^+V^-h^1h^1$	$\frac{1}{4}s_\beta^2 g^2$
$U^{++}U^{--}h^1h^1$	$\frac{1}{4}g^2$
ZZh_1h_1	$\frac{1}{8}g^2c_\beta^2c_\theta^2\sec_W^2$
$Z'Z'h_1h_1$	$\frac{1}{24h_W}g^2c_\theta^2[k_3+12(h_W(c_\beta^2+2s_\beta^2)+3s_W^2)t_W^2]$
$ZZ'h_1h_1$	$-\frac{1}{4\sqrt{3}\sqrt{h_W}}g^2c_\beta^2(3+h_W-3c_{2W})c_\theta^2\sec_W^2$
$W^+W^-h^2h^2$	$\frac{1}{4}s_\beta^2 g^2$
$V^+V^-h^2h^2$	$\frac{1}{4}c_\beta^2 g^2$
$U^{++}U^{--}h^2h^2$	$\frac{1}{4}g^2$
ZZh_2h_2	$\frac{1}{8}g^2s_\beta^2c_\theta^2\sec_W^2$
$Z'Z'h_2h_2$	$\frac{1}{24h_W}g^2c_\theta^2[k_4+12(h_W(2c_\beta^2+s_\beta^2)+3s_W^2)t_W^2]$
$ZZ'h_2h_2$	$-\frac{1}{12h_W}g^2s_\beta^2c_\theta(\sqrt{3}(3+h_W-3c_{2W})c_\theta^2\sec_W^2-6\sqrt{h_W}s_W^2s_\theta)$
$W^+W^-h^1h^2$	$\frac{1}{2}c_\beta s_\beta g^2$
$V^+V^-h^1h^2$	$-\frac{1}{2}c_\beta s_\beta g^2$
ZZh_1h_2	$\frac{1}{8}g^2s_{2\beta}c_\theta^2\sec_W^2$
$Z'Z'h_1h_2$	$-\frac{1}{8}g^2s_{2\beta}c_\theta^2[h_W\sec_W^2+4t_W^2]$
$ZZ'h_1h_2$	$-\frac{1}{4\sqrt{3}\sqrt{h_W}}g^2c_\theta^2(-6+h_W-3c_{2W})c_\theta^2\sec_W^2$
$W^+W^-h^{++}h^{--}$	$\frac{1}{2}s_\alpha^2 g^2$
$V^+V^-h^{++}h^{--}$	$\frac{1}{2}c_\alpha^2 g^2$
$ZZh^{++}h^{--}$	$\frac{1}{4}g^2[3(2c_\alpha^2-s_\alpha^2)-4(2c_\alpha^2+s_\alpha^2)c_{2W}+2c_{2W}]c_\theta^2\sec_W^2$
$Z'Z'h^{++}h^{--}$	$\frac{g^2c_\theta^2}{12h_W}[k_5+h_W^2(4c_\alpha^2+s_\alpha^2)\sec_W^2]$
$ZZ'h^{++}h^{--}$	$\frac{1}{2\sqrt{3}h_W}g^2c_\theta^2[k_6+h_W(-s_\alpha^2+(8c_\alpha^2+3s_\alpha^2)t_W^2)]$
$U^{++}U^{--}h^{++}h^{--}$	$\frac{1}{2}g^2$
$AAh^{++}h^{--}$	$4g^2s_W^2$
$W^\pm V^\pm h^1h^{\mp\mp}$	$\frac{1}{2\sqrt{2}}g^2\cos(\alpha+\beta)$
$U^{\pm\pm}Ah^1h^{\mp\mp}$	$g^2s_w^2\cos(\alpha-\beta)$
$W^\pm V^\pm h^2h^{\mp\mp}$	$\frac{1}{2\sqrt{2}}g^2\sin(\alpha+\beta)$
$U^{\pm\pm}Ah^{\mp\mp}h^2$	$g^2s_w^2\sin(\beta-\alpha)$

V. FERMION MASSES AND PROTON DECAY

The greatest impact of a shortened scalar content is that part of the fermion masses originates from Yukawa couplings and another part is due to effective operators. Next, we list the appropriate sources of mass for each fermion in the model.

The new quarks (exotic) get mass from the following Yukawa couplings:

$$\lambda_{11}^J\bar{Q}_{1L}\chi J_{1R}+\lambda_{ij}^J\bar{Q}_{iL}\chi^*J_{jR}+\text{H.c.} \tag{20}$$

where $i, j = 2, 3$. When the χ field develops its VEV, these couplings lead to the mass matrix in the basis (J_1, J_2, J_3),

$$M_J=\begin{pmatrix}\lambda_{11}^J & 0 & 0\\ 0 & \lambda_{22}^J & \lambda_{23}^J\\ 0 & \lambda_{32}^J & \lambda_{33}^J\end{pmatrix}v_\chi \tag{21}$$

which, after diagonalization, should lead to mass eigenvalues around few TeV's scale.

As for the standard quarks, their masses come from a combination of renormalizable Yukawa interactions and specific effective dimension-five operators given by

$$\lambda^d_{1a}\bar{Q}_{1L}\rho d_{aR} + \frac{\lambda^d_{ia}}{\Lambda}\varepsilon_{nmp}(\bar{Q}_{iLn}\rho_m\chi_p)d_{aR} + \lambda^u_{ia}\bar{Q}_{iL}\rho^* u_{aR} + \frac{\lambda^u_{1a}}{\Lambda}\varepsilon_{nmp}(\bar{Q}_{1Ln}\rho^*_m\chi^*_p)u_{aR} + \text{H.c.} \quad (22)$$

where $a = 1, 2, 3$. We remember that the highest energy scale where the model is found to be perturbatively reliable is about $\Lambda = 4$–5 TeV [8]. In this model, the up-type quarks mass matrix takes the following form in the basis (u_1, u_2, u_3),

$$m_u = \begin{pmatrix} \lambda^u_{11}\frac{v_\rho v_\chi}{2\Lambda} & \Lambda^u_{12}\frac{v_\rho v_\chi}{2\Lambda} & \lambda^u_{13}\frac{v_\rho v_\chi}{2\lambda} \\ -\lambda^u_{21}\frac{v_\rho}{\sqrt{2}} & -\lambda^u_{22}\frac{v_\rho}{\sqrt{2}} & -\lambda^u_{23}\frac{v_\rho}{\sqrt{2}} \\ -\lambda^u_{31}\frac{v_\rho}{\sqrt{2}} & -\lambda^u_{32}\frac{v_\rho}{\sqrt{2}} & -\lambda^u_{33}\frac{v_\rho}{\sqrt{2}} \end{pmatrix}, \quad (23)$$

while the mass matrix for the down-type quarks in the basis (d_1, d_2, d_3) is

$$m_d = \begin{pmatrix} \lambda^d_{11}\frac{v_\rho}{\sqrt{2}} & \lambda^d_{12}\frac{v_\rho}{\sqrt{2}} & \lambda^d_{13}\frac{v_\rho}{\sqrt{2}} \\ \lambda^d_{21}\frac{v_\rho v_\chi}{2\Lambda} & \lambda^d_{22}\frac{v_\rho v_\chi}{2\Lambda} & \lambda^d_{23}\frac{v_\rho v_\chi}{2\Lambda} \\ \lambda^d_{31}\frac{v_\rho v_\chi}{2\Lambda} & \lambda^d_{32}\frac{v_\rho v_\chi}{2\Lambda} & \lambda^d_{33}\frac{v_\rho v_\chi}{2\Lambda} \end{pmatrix}. \quad (24)$$

Considering these two mass matrices, note that we have 18 free Yukawa couplings to generate mass for 6 quarks only. Thus we have enough free parameters. For a naive analysis, it is just necessary to consider the diagonal case where

$$m_u \approx \lambda^u_{11}\frac{v_\rho v_\chi}{2\Lambda}, \qquad m_d \approx \frac{\lambda^d_{11}v_\rho}{\sqrt{2}}, \qquad m_s \approx \lambda^d_{22}\frac{v_\rho v_\chi}{2\Lambda}$$
$$m_c \approx -\frac{\lambda^u_{22}v_\rho}{\sqrt{2}}, \qquad m_b \approx \lambda^d_{33}\frac{v_\rho v_\chi}{2\Lambda}, \qquad m_t \approx -\frac{\lambda^u_{33}v_\rho}{\sqrt{2}}, \quad (25)$$

where m_u, m_d, m_c, m_s, m_b, m_t represent the masses of the quarks up, down, charm, strange, bottom, and top, respectively. For $\Lambda = 5$ TeV, $v_\chi = 1$ TeV, $m_u = 2.5$ MeV, $m_d = 4.95$ MeV, $m_s = 105$ MeV, $m_c = 1.26$ GeV, $m_b = 4.25$ GeV, and $m_t = 179$ GeV, we need $\lambda^u_{11} \approx \times 10^{-4}$, $\lambda^d_{11} \approx 2.8 \times 10^{-5}$, $\lambda^d_{22} \approx 4.2 \times 10^{-3}$, $\lambda^u_{22} \approx -7.24 \times 10^{-3}$, $\lambda^d_{33} \approx 1.7 \times 10^{-1}$, $\lambda^u_{33} \approx -1.03$.

Finally, the charged lepton masses arise from the effective dimension-five operator

$$\frac{\kappa}{\Lambda}(\overline{f^c_L}\rho^*)(\chi^\dagger f_L) + \text{H.c.} \quad (26)$$

This nonrenormalizable operator generates the following mass term for the charged leptons $m_l \approx \frac{1}{2}\kappa\frac{v_\rho v_\chi}{\Lambda}$ where $l = e, \mu, \tau$. Thus, for $m_e = 0.5$ MeV, $m_\mu = 105.7$ MeV and $m_\tau = 1.77$ GeV, we obtain $\kappa_e = 2 \times 10^{-5}$, $\kappa_\mu = 4.3 \times 10^{-3}$ and $\kappa_\tau = 7.2 \times 10^{-2}$.

It is important to remark that, although some fermion masses have their origin through nonrenormalizable operators, the standard fermions develop the same mass pattern as in the SM. In other words, we have the same fine-tuning in their couplings as we had in the SM in order to obtain their observed masses.

Concerning the neutrinos, we first call the attention to the fact that, whatever the particle content of the minimal 3-3-1 model, as the highest energy scale of the model is about $\Lambda = 4$–5 TeV, it generally faces difficulties in generating neutrino masses at eV scale. For example, in the original version of the minimal 3-3-1 model, effective dimension-five operators will lead to neutrino masses at GeV scale. In the RM331 proposed here, the lowest order effective operator that generates neutrino mass is a dimension-seven one,

$$\frac{\kappa'}{\Lambda^3}\epsilon_{ijk}\epsilon_{lmn}(\bar{f}^C_{Li}\rho_j\chi_k)(f_{Ll}\rho_m\chi_n) + \text{H.c.} \quad (27)$$

When the neutral scalars of the triplets ρ and χ develop their VEVs, as in Eq. (5), this effective operator yields the following expression for the Majorana neutrino mass:

$$m_\nu = \frac{\kappa' v^2_\rho v^2_\chi}{2\Lambda^3}. \quad (28)$$

For a naive analysis, let us take $v_\chi = 1$ TeV and $\Lambda = 5$ TeV. For $v_\rho = 246$ GeV, we obtain $m_\nu \approx \kappa' \times 10^{-1}$ GeV, which requires $\kappa' \approx 10^{-8}$ in order to generate neutrino masses at eV scale. This scenario is not sufficient to generate neutrino masses at eV scale, but represents a little improvement in the fine-tuning in the coupling when compared to the original version.

There is another important issue to be considered in all versions of the minimal 3-3-1 model, which was first pointed out in Ref. [9]. It concerns the proton decay. Since the model loses its perturbative character at about $\Lambda \sim 4$–5 TeV [8], new interactions are supposed to emerge at this new scale, implying the appearance of effective couplings at lower energies, like those leading to fermion masses in Eqs. (22), (26), and (27). Some operators involving quarks can be very dangerous considering that such a scale, $\Lambda \sim 4$–5 TeV, is not sufficient to avoid proton decay [9]. In the RM331, the situation is exactly the same as in the minimal 3-3-1 model, where the most dangerous proton decay operator is dimension-seven,

$$\frac{C_1\epsilon_{ijk}}{\Lambda^3}\overline{(Q_{1i_L})^c}f_{1j_L}\chi_k\overline{(u_{1_R})^c}d_{1_R} + \text{H.c.}, \quad (29)$$

with C_1 a dimensionless coupling and the color indexes are omitted (but contracted through the invariant and completely antisymmetric tensor), i, j, k are $SU(3)_L$ indexes and the lower index 1 means we are taking only the first family into account. This leads to proton decay by, for example, the following interaction:

$$\frac{C_1 v_\chi}{\Lambda^3}\bar{u}^c_L e_L \bar{u}^c_R d_L + \text{H.c.} \quad (30)$$

We choose, as in Ref. [9] to impose a discrete Z_2 symmetry over the quark fields,

$$Q_{aL} \to -Q_{aL}, \qquad q_{aR} \to -q_{aR},$$

that guarantees the proton stability for such effective operators.

VI. CONCLUSIONS

In this work, we have shown that the minimal 3-3-1 model can be implemented with only two scalar triplets, a curious and simplifying feature that was not observed before (at least for the minimal model [10]). We have seen that such a shrunken scalar sector is sufficient to engender the spontaneous breaking of the $SU(3)_C \times SU(3)_L \times U(1)_N$ symmetry to the $SU(3)_C \times U(1)_{\rm QED}$ one and generate the correct masses of all gauge bosons and fermions, including the neutrinos.

With only two scalar triplets, it is not possible to generate mass for all fermions through renormalizable Yukawa interactions. We have to resort to effective operators. This is particularly feasible in this minimal 3-3-1 model because the cutoff energy scale of the model is about $\Lambda = 4$–5 TeV. Because of this, on choosing appropriate dimension-five effective operators, we obtain mass terms for the charged leptons and some quarks at electroweak scale. As for the neutrinos, we got Majorana mass terms from effective dimension-seven operators that, although not leading naturally to the observed eV mass scale, represents a small improvement in the fine-tuning of the scalar-neutrino coupling as compared to the original minimal 3-3-1 model.

The main difference of the RM331 from the original minimal 3-3-1 model is the amount of physical degrees of freedom in their scalar sector. While in the original model we have 22 physical degrees of freedom in the form of scalars, in the RM331 model only four physical degrees of freedom remain in the spectrum. Namely, h_1 and h_2 and a doubly charged bilepton scalar h^{++}. The neutral scalar, h_1, is the lightest scalar and recovers the SM Higgs interactions, while the new scalars, h_2 and h^{++}, have masses around TeV scale. The advantage of this reduced spectrum is that we can easily extract all the scalar-gauge boson and scalar-fermion interactions without making the usual cumbersome assumptions for the couplings in the scalar potential. In this way, we turn possibly a neater treatment of the phenomenology involving the scalar sector of the model, which is going to be essential for a careful analysis of Higgs search in the minimal 3-3-1 model, as well as other interesting possibilities involving the scalars as intermediate states.

It is important to stress that although h_1 can reproduce all the physics of the SM Higgs, it is not exactly the standard one because it presents new interactions with the new quarks and also intermediates flavor-changing neutral processes with the standard quarks, which may also be mediated by the neutral scalar h_2 and the neutral gauge boson Z'. Besides, the new gauge boson degrees of freedom allows some enhancement in the diphoton Higgs decay channel [11] and may give a very different contribution to the $H \to \gamma Z$ channel, since heavy degrees of freedom can possibly be important as they do not decouple in the usual sense [12]. Finally, the doubly charged scalar $h^{\pm\pm}$ can be probed at the LHC through the reaction $p + p \to e^+ + e^+ + e^- + e^-$ or in a future ILC through the reaction $e^- + e^- \to h^{--} \to e^- + e^-$, a phenomenological study we wish to pursue somewhere else.

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APPENDIX A

In this appendix, we present the neutral and charged currents with the gauge bosons of the model. They arise from the following matter Lagrangian:

$$\mathcal{L}_{\rm cin} = \bar{f}_{a_L} i\mathcal{D} f_{a_L} + \bar{Q}_{1_L} i\mathcal{D} Q_{1_L} + \bar{Q}_{i_L} i\mathcal{D} Q_{i_L} + \bar{u}_{a_R} i\mathcal{D} u_{a_R} + \bar{d}_{a_R} i\mathcal{D} d_{a_R} + \bar{J}_{a_R} i\mathcal{D} J_{a_R}, \tag{A1}$$

where $a = 1, 2, 3$ and

$$\mathcal{D}_\mu = \partial_\mu + i g_N N_L W^N_\mu + i\frac{g}{2}\vec{\lambda}\vec{W}_\mu \tag{A2}$$

is the covariant derivative for any left-handed fermion triplet (N_L the corresponding $U(1)_N$ quantum number), and

$$\mathcal{D}_\mu = \partial_\mu + i g_N N_R W^N_\mu \tag{A3}$$

is the covariant derivative for the right-handed fermion singlets (N_R is the corresponding $U(1)_N$ quantum number).

When the covariant derivative acts on the fermion triplets, we have

$$\frac{g}{2}\vec{\lambda}\vec{W}_\mu = \begin{pmatrix} \frac{g}{2}\left(W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu\right) & \frac{g}{\sqrt{2}}W^+_\mu & \frac{g}{\sqrt{2}}V^-_\mu \\ \frac{g}{\sqrt{2}}W^-_\mu & \frac{g}{2}\left(-W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu\right) & \frac{g}{\sqrt{2}}U^{++}_\mu \\ \frac{g}{\sqrt{2}}V^+_\mu & \frac{g}{\sqrt{2}}U^{--}_\mu & -g\frac{1}{\sqrt{3}}W^8_\mu \end{pmatrix}, \tag{A4}$$

while for the antitriplets we have,

$$\frac{g}{2}\vec{\bar{\lambda}}\vec{W}_\mu = \begin{pmatrix} -\frac{g}{2}\left(W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu\right) & -\frac{g}{\sqrt{2}}W^-_\mu & -\frac{g}{\sqrt{2}}V^+_\mu \\ -\frac{g}{\sqrt{2}}W^+_\mu & -\frac{g}{2}\left(-W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu\right) & -\frac{g}{\sqrt{2}}U^{--}_\mu \\ -\frac{g}{\sqrt{2}}V^-_\mu & -\frac{g}{\sqrt{2}}U^{++}_\mu & g\frac{1}{\sqrt{3}}W^8_\mu \end{pmatrix}, \tag{A5}$$

where $\bar{\lambda} = -\lambda^*$.

With all this in hand, we are able to obtain the charged and neutral currents involving fermions and gauge bosons. We first present the charged currents for the leptons. On considering that the charged leptons come in a diagonal basis, we obtain

$$\mathcal{L}^{\rm CC}_l = \frac{g}{\sqrt{2}}\bar{\nu}_{a_L}\gamma^\mu V^l_{\rm PMNS}e_{a_L}W^+_\mu + \frac{g}{\sqrt{2}}\overline{e^c}_{a_L}O^V\gamma^\mu\nu_{a_L}V^+_\mu + \frac{g}{\sqrt{2}}\overline{e^c}_{a_L}\gamma^\mu e_{a_L}U^{++}_\mu + {\rm H.c.}, \tag{A6}$$

where $a = 1, 2, 3$ with $V^l_{\rm PMNS} = V^{\nu\dagger}_L$ being the PMNS mixing matrix and $O^V = V^\nu_L$. The matrix V^ν_L diagonalizes the neutrino mass matrix.

For the quarks, the charged currents take the following form:

$$\mathcal{L}^{\rm cc}_q = \frac{g}{\sqrt{2}}\bar{u}_L V^q_{\rm CKM}\gamma^\mu d_L W^+_\mu + \frac{g}{\sqrt{2}}(\bar{J}_{1_L}\gamma^\mu(V^u_L)_{1a}u_{a_L} - \bar{d}_{l_L}(V^{d\dagger}_L)_{li}\gamma^\mu J_{i_L})V^+_\mu + \frac{g}{\sqrt{2}}(\bar{J}_{1_L}\gamma^\mu(V^d_L)_{1a}d_{1_L} + \bar{u}_{l_L}(V^{u\dagger}_L)_{li}\gamma^\mu J_{i_L})U^{++}_\mu + {\rm H.c.}, \tag{A7}$$

where $a = 1, 2, 3$, while $i, l = 2, 3$. $V^q_{\rm CKM} = V^{u\dagger}_L V^d_L$ is the Cabibbo-Kobayashi-Maskawa mixing matrix. In the charged current above, $V^{u,d}_L$ are the mixing matrices that connect the left-handed up and down quarks symmetry eigenstates with the mass eigenstates. We assume that the new quarks come in a diagonal basis.

Next we present the neutral current among the fermions and the neutral gauge bosons Z_μ and Z'_μ.

The neutral currents for leptons are

$$\mathcal{L}^{\rm NC}_l = -\frac{g}{2c_W}\bar{\nu}_{a_L}\gamma^\mu\nu_{a_L}Z_\mu - \frac{g}{2c_W}\sqrt{\frac{h_W}{3}}\bar{\nu}_{a_L}\gamma^\mu\nu_{a_L}Z'_\mu - \frac{g}{2c_W}\bar{e}_a\gamma^\mu(a_1 - b_1\gamma^5)e_a Z_\mu - \frac{g}{2c_W}\bar{e}_a\gamma^\mu(a_2 - b_2\gamma^5)e_a Z'_\mu, \tag{A8}$$

where

$$a_1 = -\frac{1}{2}h_W, \qquad b_1 = -\frac{1}{2}$$
$$a_2 = \frac{1}{2}\sqrt{3h_W}, \qquad b_2 = -\frac{1}{2}\sqrt{3h_W}. \tag{A9}$$

The neutral currents among the up quarks and the standard neutral gauge boson Z_μ take the form,

$$\mathcal{L}^{\rm NC}_{Z^1,u} = \frac{g}{2C_W}(\bar{u}\gamma_\mu[a_{1L}(u)(1-\gamma_5) + a_{1R}(u)(1+\gamma_5)]u+)Z^1_\mu, \tag{A10}$$

where

$$a_{1L}(u) = \left(-\frac{1}{2} + \frac{2}{3}s^2_W\right), \qquad a_{1R}(u) = \frac{2}{3}s^2_W. \tag{A11}$$

The neutral currents among the down quarks and the standard neutral gauge boson Z_μ are

$$\mathcal{L}^{\rm NC}_{Z^1,d} = \frac{g}{2c_W}(\bar{d}\gamma_\mu[a_{1L}(d)(1-\gamma_5) + a_{1R}(d)(1+\gamma_5)]d)Z_\mu, \tag{A12}$$

where

$$a_{1L}(d) = \left(\frac{1}{2} - \frac{1}{3}s^2_W\right), \qquad a_{1R}(d) = -\frac{1}{3}s^2_W. \tag{A13}$$

The neutral currents among the new (exotic) quarks and the standard neutral gauge boson Z_μ take the form

$$\mathcal{L}^{\rm NC}_{Z^1,J} = \frac{g}{2c_W}(\bar{J}_1\gamma_\mu[a_{1L}(J_1)(1-\gamma_5) + a_{1R}(J_1)(1+\gamma_5)]J_1 + \bar{J}\gamma_\mu[a_{1L}(J)(1-\gamma_5) + a_{1R}(J)(1+\gamma_5)]J)Z^1_\mu, \tag{A14}$$

where

$$a_{1L}(J_1) = \left(\frac{5s^2_W}{3}\right), \qquad a_{1R}(J_1) = \left(\frac{5s^2_W}{3}\right),$$
$$a_{1L}(J) = \left(-\frac{4s^2_W}{3}\right), \qquad a_{1R}(J) = \left(-\frac{4s^2_W}{3}\right). \tag{A15}$$

The neutral currents among the up quarks and the new neutral gauge boson Z'_μ take the form

$$\mathcal{L}^{\rm NC}_{Z',u} = \frac{g}{2c_W}(\bar{u}\gamma_\mu[a_{2R}(u)(1+\gamma_5)]u + \bar{u}\gamma_\mu V^{u*}_L Y^{um}_{Z'}V^u_L(1-\gamma_5)u)Z^2_\mu, \tag{A16}$$

where

$$a_{1R}(u) = -\frac{2}{\sqrt{3h_W}}s^2_W,$$
$$Y^{um}_{Z'} = \frac{1}{\sqrt{12h_W}}{\rm diag}(1, 1-2s^2_W, 1-2s^2_W). \tag{A17}$$

The neutral currents among the down quarks and the new neutral gauge boson Z'_μ take the form

$$\mathcal{L}^{NC}_{Z',d} = \frac{g}{2c_W}(\bar{d}\gamma_\mu[a_{2R}(d)(1+\gamma_5)]d + \bar{d}\gamma_\mu V_L^{d*} Y_{Z^2}^{dm} V_L^d (1-\gamma_5)d)Z^2_\mu, \tag{A18}$$

where

$$a_{2R}(d) = \frac{1}{\sqrt{3h_W}} s_W^2, \quad Y_{Z'}^{md} = \frac{1}{\sqrt{12h_W}}\mathrm{diag}(1, 1-2s_W^2, 1-2s_W^2). \tag{A19}$$

The neutral currents among the new (exotic) quarks and the new neutral gauge boson Z'_μ take the form

$$\mathcal{L}^{NC}_{Z',J} = \frac{g}{2c_W}(\bar{J}_1\gamma_\mu[a_{2L}(J_1)(1-\gamma_5) + a_{2R}(J_1)(1+\gamma_5)]J_1 + \bar{J}\gamma_\mu[a_{2L}(J)(1-\gamma_5) + a_{2R}(J)(1+\gamma_5)]J)Z^2_\mu, \tag{A20}$$

where

$$a_{2L}(J_1) = \frac{(1-9s_W^2)}{\sqrt{3h_W}}, \quad a_{2R}(J_1) = -\frac{5s_W^2}{\sqrt{3h_W}}, \quad a_{2L}(J) = -\frac{(1-5s_W^2)}{\sqrt{3h_W}}, \quad a_{2R}(J) = +\frac{4s_W^2}{\sqrt{3h_W}}, \tag{A21}$$

$$h_W = 1 - 4s_W^2.$$

APPENDIX B

In this appendix, we present the interactions among charged fermions and scalars. We start with the leptons. On opening the effective dimension-five operator in Eq. (26), we obtain the following interactions:

$$\begin{aligned}\mathcal{L}_l = {}& \frac{\kappa}{2}\left(\frac{v_\rho}{v_\chi}\cos(\beta) - \sin(\beta)\right)\bar{l}lh_1 + \frac{\kappa}{2}\left(\frac{v_\rho}{v_\chi}\sin(\beta) + \cos(\beta)\right)\bar{l}lh_2 + \frac{\kappa v_\rho \sin(\alpha)}{\sqrt{2}v_\chi}h^{--}\overline{l_L}(l^c)_R + \frac{\kappa\cos(\alpha)}{\sqrt{2}}h^{--}\overline{l_R}(l^c)_L \\ & - \frac{\kappa}{2v_\chi}\sin(\beta)\cos(\beta)\bar{l}lh_1h_1 + \frac{\kappa}{2v_\chi}\sin(\beta)\cos(\beta)\bar{l}lh_2h_2 + \frac{\kappa\sin(\alpha)\cos(\alpha)}{v_\chi}\bar{l}lh^{++}h^{--} + \frac{\kappa}{2v_\chi}\cos(2\beta)\bar{l}lh_1h_2 \\ & + \frac{\kappa}{v_\chi}\left(\frac{-\sin(\alpha)\sin(\beta)}{\sqrt{2}}h^{--}h_1\overline{l_L}(l^c)_R + \frac{\sin(\alpha)\cos(\beta)}{\sqrt{2}}h^{--}h_2\overline{l_L}(l^c)_R\right) \\ & + \frac{\kappa}{v_\chi}\left(\frac{\cos(\alpha)\cos(\beta)}{\sqrt{2}}h^{--}h_1\overline{l_R}(l^c)_L + \frac{\cos(\alpha)\sin(\beta)}{\sqrt{2}}h^{--}h_2\overline{l_R}(l^c)_L\right) + \text{H.c.},\end{aligned} \tag{B1}$$

where $l = e, \mu, \tau$.

The renormalizable interactions among standard quarks and scalars are composed by the terms

$$\mathcal{L} = \bar{u}_L\Gamma_1^u u_R h_1 + \bar{u}_L\Gamma_2^u u_R h_2 + \bar{d}_L\Gamma_1^d d_R h_1 + \bar{d}_L\Gamma_2^d d_R h_2 + \text{H.c.}, \tag{B2}$$

where $u = (u, c, t)$ and $d = (d, s, b)$ and

$$\Gamma_1^u = \frac{\sin(\beta)}{\sqrt{2}}\begin{pmatrix} \frac{\lambda_{11}^u}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{12}^u}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{13}^u}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix}; \tag{B3}$$

$$\Gamma_2^u = \frac{\cos(\beta)}{\sqrt{2}}\left(\lambda_{11}^u\sqrt{2}\right) \tag{B4}$$

$$\Gamma_1^d = \frac{\sin(\beta)}{\sqrt{2}}\begin{pmatrix} -\lambda_{11}^d & -\lambda_{12}^d & -\lambda_{13}^d \\ \frac{\lambda_{21}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{22}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{23}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) \\ \frac{\lambda_{31}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{32}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) & \frac{\lambda_{33}^d}{\sqrt{2}}\left(\frac{v_\rho\cos(\beta)}{v_\chi\sin(\beta)} - 1\right) \end{pmatrix}; \tag{B5}$$

$$\Gamma_2^d = \frac{\cos(\beta)}{\sqrt{2}} \begin{pmatrix} \lambda_{11}^d & \lambda_{12}^d & \lambda_{13}^d \\ \frac{\lambda_{21}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta)} + 1\right) & \frac{\lambda_{22}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta)} + 1\right) & \frac{\lambda_{23}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta)} + 1\right) \\ \frac{\lambda_{31}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta} + 1\right) & \frac{\lambda_{32}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta)} + 1\right) & \frac{\lambda_{33}^d}{\sqrt{2}}\left(\frac{v_\rho \sin(\beta)}{v_\chi \cos(\beta)} + 1\right) \end{pmatrix}. \tag{B6}$$

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