

Chirally symmetric and confining dense matter with a diffused quark Fermi surface

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It is possible that at low temperatures and large density there exists a confining matter with restored chiral symmetry, just after the dense nuclear matter with broken chiral symmetry. Such a phase has, so far, been studied within a confining and chirally symmetric model, assuming a rigid quark Fermi surface. In the confining quarkyonic matter, however, near the Fermi surface, quarks group into color-singlet baryons. Interaction between quarks leads to a diffusion of the quark Fermi surface. Here, we study effects of such diffusion and verify that it does not destroy a possible existence of a confining but chirally symmetric matter at low temperatures.

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I. INTRODUCTION

The QCD phase diagram is an old and very intriguing question. What will happen with the strongly interacting matter at large temperatures and/or densities? The most interesting question is the interconnection of the deconfinement and the chiral restoration phase transitions (cross-overs). For many years, it was considered as almost self-evident that the deconfinement and chiral restoration lines on the QCD phase diagram coincide. The reason for such expectation was the belief that, in the confining mode, mass of hadrons is directly related to the quark condensate of the vacuum. Consequently, beyond the chiral restoration line, hadrons cannot exist, and the QCD matter should be in a deconfined plasma form. At low densities, it is indeed established on the lattice that both chiral restoration and deconfinement crossovers coincide or are rather close to each other [1,2]. What happens with these lines at larger densities is unknown.

In the large N_c world at low temperatures, confinement persists up to arbitrary large densities [3]. This is because both the quark-antiquark and quark-quark hole loops are suppressed at large N_c . Consequently, there is no back-reaction of quarks on gluonic dynamics (no Debye screening of the confining gluonic field) and confinement persists like in vacuum. In such system, the only allowed excitation modes are of the color-singlet hadronic type. The uncorrelated single quark excitations are not allowed. In this case, it is possible to define a quarkyonic matter as a dense confining matter with baryonic excitation modes [3].

In the real $N_c = 3$ world at some large critical density, the confining gluonic field might be screened, and a deconfining transition (crossover) would appear. Then a key question is how big is this critical density? Lattice simulations for the $N_c = 2$ QCD suggest that at low temperatures, the deconfinement transition happens at densities ~ 100 times bigger than the normal nuclear matter density [4]. Since the $N_c = 3$ world is between the two known limiting cases ($N_c = 2$; $N_c = \infty$), we expect that at $N_c = 3$, the deconfinement at low temperatures happens at the

very high densities, much larger than can be achieved in our laboratories or in the neutron stars.

Note, by definition, the quarkyonic matter is a dense cold matter with confinement. Nothing can be *a priori* concluded about existence or nonexistence of the chiral restoration phase transition within such a dense matter. If the chiral restoration transition does exist within the quarkyonic matter, then it would imply that at some conditions, there exists a QCD matter with confinement and with unbroken chiral symmetry (we imply for simplicity the chiral limit). The mass origin in such a matter is obviously not related to dynamical chiral symmetry breaking in the vacuum.

As mentioned above, such a possibility had not even been considered in the past on *a priori* grounds. Indeed, the old Casher argument [5] claims that the chiral symmetry breaking is required for quarks to be confined. In addition, it is understood that chiral symmetry breaking in the vacuum is important for the mass generation of hadrons such as N , ρ , or π . Then, naively, hadrons with nonzero mass cannot exist in a world with unbroken chiral symmetry.

However, the Casher argument is not general and can be easily bypassed [6]. Recent lattice simulations have convincingly demonstrated [7] that in the world without the low-lying eigenmodes of the Dirac operator (*i.e.* with the artificially restored chiral symmetry), hadrons still exist, and confinement persists. Finally, if effective chiral restoration in highly excited hadrons is correct [8–11], then it is possible to have hadrons with the mass that is not directly related to the quark condensate of the vacuum.

A key question is to clarify whether existence of confining but chirally symmetric dense cold matter is possible. We cannot solve QCD and answer this question from first principles. In this situation, an important step would be to construct a model for such a matter and see what mechanism could be at work. Clearly, the model must be manifestly confining, chirally symmetric, and provide dynamical breaking of chiral symmetry. Constructions that are based on the Nambu–Jona-Lasinio model or the linear sigma model and their extensions are not suited because of lack of confinement of quarks.

The simplest possible model that satisfies all required criteria is the model of Refs. [12,13]. This 3 + 1-dimensional model can be considered as a straightforward generalization of the QCD in 1 + 1 dimension in the large N_c limit—the 't Hooft model [14]. It is assumed within the model that the only gluonic interaction between quarks is an instantaneous linear potential of Coulomb type. Such a potential is indeed observed in variational [15] as well as lattice Coulomb gauge simulations [16]. An important aspect of this model is that it exhibits effective chiral restoration in hadrons with large spins J [11,17,18].

This model was used in Ref. [19] to answer the question about a possible mechanism that could be responsible for the confining but chirally symmetric dense cold matter. Assuming a liquid phase, *i.e.* that the rotational and translational symmetries are not broken (as it is in the nuclear matter), the Pauli blocking of the positive energy levels by valence quarks prevents the gap equation to generate a nontrivial solution with broken chiral symmetry above some critical quark Fermi momentum. At the same time, confinement still persists in the system, so that the uncorrelated colored single quark excitations are not possible.

In the latter work, a rigid quark Fermi surface was assumed, like for the noninteracting fermions at $T = 0$. If such a phase exists, however, relevant degrees of freedom near the Fermi surface are baryons. Quarks interact inside baryons. Consequently, the quark distribution function near the Fermi surface must be smooth. Here, we study effects of such diffusion of the quark Fermi surface and solve the corresponding gap and Bethe-Salpeter equations. We conclude that, for any reasonable diffusion, there always exists such a critical “Fermi momentum” of quarks at which the chiral restoration phase transition persists, and, hence, elementary excitations above this critical Fermi momentum are the color-singlet chirally symmetric hadron modes.

II. OVERVIEW OF CONFINEMENT AND CHIRAL SYMMETRY BREAKING IN A VACUUM

The $SU(2)_L \times SU(2)_R \times U(1)_A \times U(1)_V$ symmetric Hamiltonian,

$$\hat{H} = \int d^3x \bar{\psi}(\vec{x}, t) (-i\vec{\gamma} \cdot \vec{\nabla}) \psi(\vec{x}, t) + \frac{1}{2} \times \int d^3x d^3y J_\mu^a(\vec{x}, t) K_{\mu\nu}^{ab}(\vec{x} - \vec{y}) J_\nu^b(\vec{y}, t), \quad (1)$$

relies on the quark current-current $J_\mu^a(\vec{x}, t) = \bar{\psi}(\vec{x}, t) \gamma_\mu \frac{\lambda^a}{2} \psi(\vec{x}, t)$ interaction via the instantaneous linear interquark potential of the Coulomb type,

$$K_{\mu\nu}^{ab}(\vec{x} - \vec{y}) = g_{\mu 0} g_{\nu 0} \delta^{ab} V(|\vec{x} - \vec{y}|); \quad (2)$$

$$\frac{\lambda^a \lambda^a}{4} V(r) = \sigma r,$$

where a, b are color indices. This model was intensively used in the past to study chiral symmetry breaking, chiral properties of hadrons, etc. (see, for example, Refs. [12,13,20]). In the following, we redefine for simplicity the total confining potential that includes all required color factors to be $V(r) = \sigma r$.

The Fourier transform of the linear potential and any loop integral are not defined in the infrared region, $p \sim 0$. Hence, an infrared regularization is required. Physical color-singlet observables, such as hadron masses, chiral condensate, etc., must be independent of the infrared regulator μ_{IR} in the infrared limit $\mu_{\text{IR}} \rightarrow 0$.

There are several physically equivalent ways to perform this infrared regularization. Here, we follow Ref. [21] and define the potential in momentum space as

$$V(p) = \frac{8\pi\sigma}{(p^2 + \mu_{\text{IR}}^2)^2}. \quad (3)$$

This potential in the configuration space contains the required σr term, the infrared divergent term $-\sigma/\mu_{\text{IR}}$, as well as terms that vanish in the infrared limit.

The self-energy operator

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma} \hat{p})(B_p - p) \quad (4)$$

consists of the Lorentz-scalar chiral symmetry-breaking part A_p and chirally symmetric part $(\vec{\gamma} \hat{p})(B_p - p)$. The unknown functions A_p and B_p are to be determined from the gap equation for the chiral angle φ_p ,

$$A_p \cos \varphi_p - B_p \sin \varphi_p = 0, \quad (5)$$

where

$$A_p = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} V(\vec{p} - \vec{k}) \sin \varphi_k, \quad (6)$$

$$B_p = p + \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} (\hat{p} \hat{k}) V(\vec{p} - \vec{k}) \cos \varphi_k. \quad (7)$$

These integrals contain both the infrared-divergent and the infrared-finite parts

$$A_p = \frac{\sigma}{2\mu_{\text{IR}}} \sin \varphi_p + A_p^f, \quad (8)$$

$$B_p = \frac{\sigma}{2\mu_{\text{IR}}} \cos \varphi_p + B_p^f. \quad (9)$$

The same is true for the single quark energy:

$$\omega_p = \sqrt{A_p^2 + B_p^2} = \frac{\sigma}{2\mu_{\text{IR}}} + \omega_p^f. \quad (10)$$

Consequently, the single-quark Green function is divergent and the single-quark energy is infinite in the infrared limit. Actually, energy of any color-nonsinglet state is infinite. At the same time, the infrared divergence exactly cancels out in any color-singlet quantity, and these quantities are

finite and well-defined [19]. This is a manifestation of confinement within this model.

Similarly, the infrared divergence in A_p and B_p cancels in the gap equation, and this equation can be solved directly in the infrared limit. The gap equation can be solved numerically and a nontrivial solution with the chiral (Bogoliubov) angle $\varphi_p \neq 0$ signals dynamical breaking of chiral symmetry. Hence, the single quark Green function is not chirally symmetric, $A_p \neq 0$; there appears the quark condensate and dynamical mass of quarks

$$\langle \bar{q}q \rangle = -\frac{N_C}{\pi^2} \int_0^\infty dp p^2 \sin \varphi_p, \quad M(p) = p \tan \varphi_p. \quad (11)$$

III. CHIRAL SYMMETRY BREAKING AND CONFINEMENT IN A DENSE MATTER AT $T = 0$ WITH A RIGID QUARK FERMİ SURFACE

It is practically impossible to solve exactly the model in a dense matter. Indeed, that would imply to solve it first for a single baryon; then to obtain a baryon-baryon interaction; given this interaction to construct a nuclear matter and then slowly to increase its density. Obviously, it is a formidable problem. In order to proceed and get some insight, one needs justifiable simplifications.

In the large N_c limit, the nucleon is infinitely heavy, translational invariance is broken and a many-nucleon system is certainly in a crystal phase. Whether a (dense) nuclear matter will be a liquid or a crystal at $N_c = 3$ is subject to dynamical calculations. Such microscopical calculations cannot be pursued for any ‘‘realistic’’ model in $3 + 1$ dimensions with confinement and (broken) chiral symmetry. However, in the real world $N_c = 3$, we do know that the nuclear matter is in a liquid phase; both translational and rotational invariances are intact. We then assume a liquid phase with manifest translational and rotational invariances in a dense quarkyonic matter.

We also assume that confinement persists up to large densities at $N_c = 3$. At $T = 0$, deconfinement could happen through the Debye screening of the confining gluon propagator: A gluon creates the quark-quark hole pair that again annihilates into a gluon. If this vacuum polarization diagram is finite, then at some density, there should happen a complete screening of the confining gluon field. However, in the confining mode, energy of the colored quark-quark hole pair is infinite [22]. The allowed excitations in the confining mode are the color-singlet excitations like baryon-baryon hole pairs, etc. These excitation modes cannot screen the confining colored gluon propagator. In this sense, the $T = 0$ physics is rather different from the deconfinement at zero density and large temperature. In the latter case, a screening proceeds via the incoherent thermal gluon loops.

One could expect that the deconfinement should happen in a dense medium at $T = 0$ due to perlocation of baryons.

Such a reasoning is too naive, however, because the perlocation does not yet imply screening of the confining gluon field. For example, deconfinement never happens at $T = 0$ in QCD at large N_c or in the ’t Hooft model. In both cases, baryons ‘‘sit’’ on top of each other in a very dense medium, but it is still a system with confinement.

We want to address the chiral symmetry-breaking properties of a dense matter with confinement. In the vacuum, dynamical chiral symmetry breaking happens because there is an attractive interaction between the left quarks and the right antiquarks and vice versa. This attractive interaction shifts the energy of the vacuum with broken chiral symmetry below the energy of the perturbative Dirac vacuum.

Consequently, in a dense matter at $T = 0$, the most important physics that leads to the restoration of chiral symmetry is the Pauli blocking (by the valence quarks) of the positive energy levels required for the very existence of the quark condensate.

In our previous work [19], the effect of the Pauli blocking was studied assuming a rigid valence quark Fermi sphere, like for the ideal Fermi gas at $T = 0$ (see Fig. 1(a)). In this case, all quark positive energy levels below the Fermi momentum p_f are occupied by the valence quarks, and one has to replace the vacuum density matrix $v(\vec{p})v^\dagger(\vec{p})$ by the density matrix in the medium:

$$\rho(\vec{p}) = \Theta(p_f - p)u(\vec{p})u^\dagger(\vec{p}) + v(\vec{p})v^\dagger(\vec{p}). \quad (12)$$

Hence, within the mean field approximation, one has to remove from the integration in the gap equation all quark momenta below p_f since they are Pauli blocked (see Fig. 1(b)). The modified gap equation is then the same as in Eqs. (5)–(7), but the integration starts not from $k = 0$, but from $k = p_f$.

At the critical Fermi momentum $p_f^{cr} \sim 0.109\sqrt{\sigma}$, the chiral phase transition is observed (see Fig. 2) because the nontrivial solution with broken chiral symmetry disappears. Above this phase transition, the chiral angle φ_p , the quark condensate $\langle \bar{q}q \rangle$, the dynamical mass of quarks $M(p)$, as well as the chiral symmetry-breaking part A_p of the quark Green function identically vanish. At the same time, the chirally symmetric part B_p of the quark Green function does not vanish and is still, in fact, infrared divergent. The single quark energy is infinite,

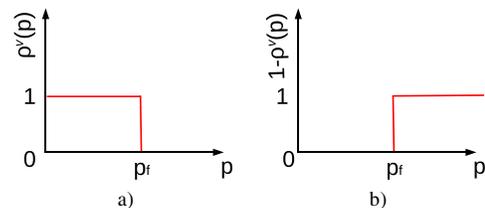


FIG. 1 (color online). Valence quark distribution and the corresponding integration weight.

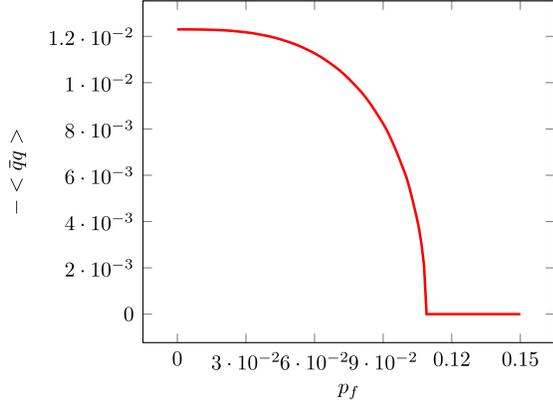


FIG. 2 (color online). Quark condensate in units of $\sigma^{3/2}$ as a function of the Fermi momentum, which is in units of $\sqrt{\sigma}$.

and confinement persists even in the chirally symmetric phase. The color-singlet hadronic excitations have finite and well-defined energy.

IV. EFFECTS OF A DIFFUSION OF THE QUARK FERMION SURFACE

In reality, valence quarks near the Fermi surface interact and cluster into the color-singlet baryons. This interaction, in general, would lead to a diffusion of the rigid Fermi surface for quarks. Some levels above the Fermi momentum must be occupied with some probability, as well as some levels below the Fermi momentum with some probability must be empty.

In principle, the quark distribution function near the diffused Fermi surface could be obtained self-consistently from the full solution of the problem. It is a formidable task, and such a program cannot be pursued. With the present state of the arts, it is difficult to obtain a microscopic insight into the dynamics of the diffusion. However, it is clear that the realistic distribution function will be smooth, of the form on Fig. 3(a). A goal of the present study is to clarify the effect of the diffusion on the very existence of the chiral phase transition. Then, for our present goal, it will suffice to parameterize such a smooth distribution in a simple form and study effect of the diffusion on the solution of the gap equation.

We parameterize a smooth valence quark distribution function by

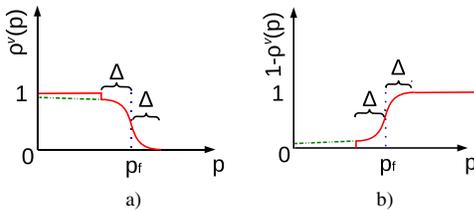


FIG. 3 (color online). Diffused step valence quark distribution and the corresponding integration weight.

$$\rho^v(p) = \Theta(-p + p_f - \Delta) + \Theta(p - p_f + \Delta) \frac{1}{e^{(p-p_f)/\Delta} + 1}. \quad (13)$$

Given the valence distribution function we multiply the integrands in Eqs. (6) and (7) by the weight function $1 - \rho^v(k)$ and solve the gap equation for different p_f and diffusion width Δ .

If the diffusion width is much smaller than the critical Fermi momentum for a rigid Fermi surface, $\Delta \ll p_f^{cr}(\Delta = 0)$ (what should be considered as a realistic situation), then the evolution of the chiral condensate with p_f is similar to the case of the rigid quark Fermi surface. This situation is represented by the curve $\Delta = 0.02$ on Fig. 4. The phase transition happens at the ‘‘Fermi momenta’’ that are rather close to the critical Fermi momentum, $p_f^{cr}(\Delta = 0) = 0.109$ from Fig. 2 (see also the curve $\Delta = 0$ on Fig. 4). This can be easily understood. At all momenta $p \ll p_f$, the Pauli blocking on Fig. 3 is the same as for the rigid quark Fermi surface. At momenta just below the p_f , the effect of the Pauli blocking is weaker than for the rigid Fermi surface. However, this is compensated by additional Pauli blocking of the levels that are just above the Fermi momentum for the rigid quark distribution. Consequently, with small diffusion widths, the ‘‘critical Fermi momentum’’, $p_f^{cr}(\Delta)$, at which the phase transition happens, is shifted to slightly lower values of p_f .

However, if a diffusion width becomes larger and eventually comparable with the Fermi momentum, then the critical Fermi momentum, at which the phase transition happens, increases.

For each fixed diffusion width Δ , there always exist such Fermi momenta where the Wigner-Weyl mode of chiral symmetry is realized. This can be seen from Fig. 5, where a

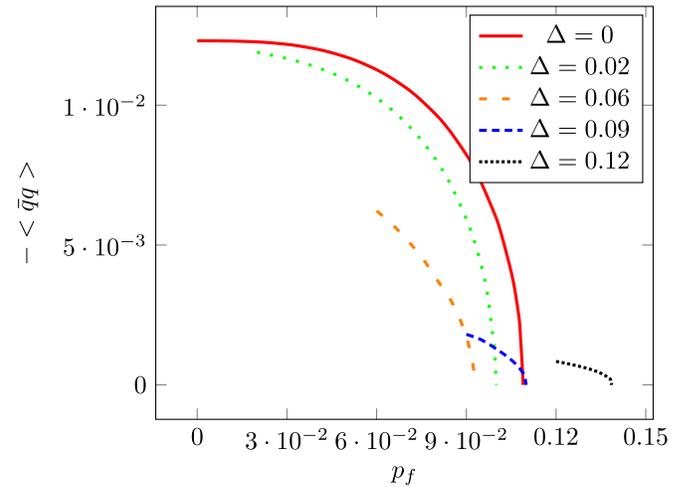


FIG. 4 (color online). Quark condensate (in units of $\sigma^{3/2}$) as a function of the Fermi momentum (in units of $\sqrt{\sigma}$) and the diffusion width Δ (in units of $\sqrt{\sigma}$).

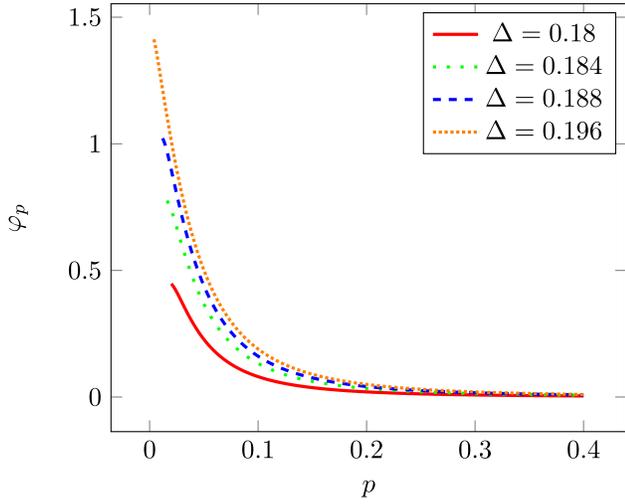


FIG. 6 (color online). Chiral angle φ_p as a function of the momentum p for a Fermi momentum $p_f = 0.2$ at different fixed values of smoothing Δ . p and Δ are in units of $\sqrt{\sigma}$.

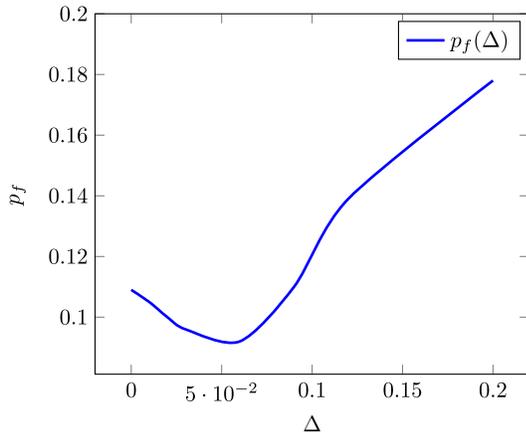


FIG. 5 (color online). Critical line that separates the quarkyonic matter with broken and restored chiral symmetry.

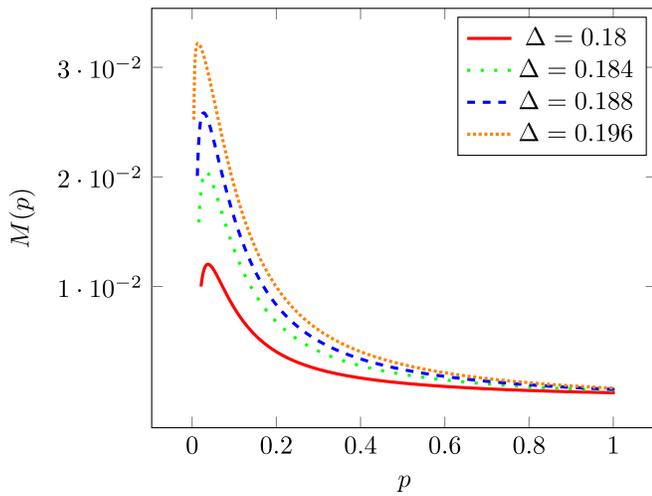


FIG. 7 (color online). Dynamical mass $M(p)$ as a function of the momentum p for a Fermi momentum $p_f = 0.2$ at different fixed values of smoothing Δ . $M(p)$, p , and Δ are in units of $\sqrt{\sigma}$.

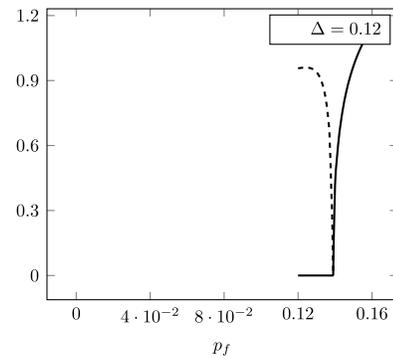
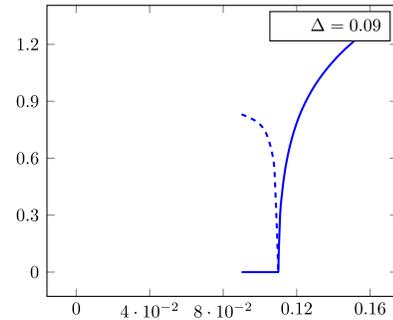
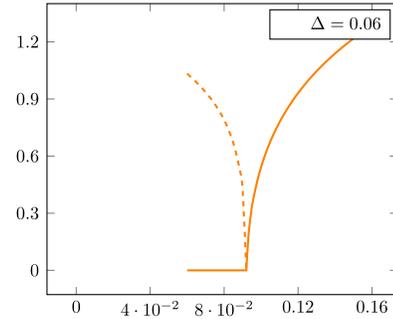
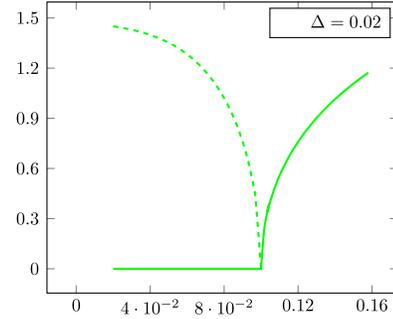
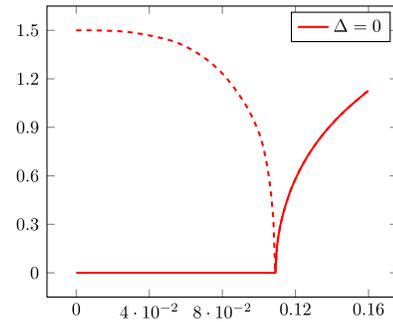


FIG. 8 (color online). Masses of the pseudoscalar (solid) and scalar (dashed) mesons in units of $\sqrt{\sigma}$ as functions of the “Fermi momentum” p_f and of the diffusion width Δ (in units of $\sqrt{\sigma}$).

line of critical Fermi momenta is depicted. The area above this critical line corresponds to the chirally symmetric phase, while all points below the critical line represent a matter with broken chiral symmetry.

The chiral angle φ_p and dynamical mass of quarks $M(p)$ in the Nambu—Goldstone mode of chiral symmetry at some Fermi momentum and different diffusion widths Δ are shown on Figs. 6 and 7.

V. CHIRAL RESTORATION IN MESON SPECTRA

Another explicit illustration of chiral symmetry of a dense matter above the chiral restoration phase transition are properties of hadronic excitations. In the Nambu—Goldstone mode of chiral symmetry, there must be a massless excitation mode that is associated with the massless pion. At the same time, energies of all other mesons must be finite. In particular, there must be a finite splitting of the excitations with quantum numbers $I, J^{PC} = 1, 0^{-+}$ and $I, J^{PC} = 0, 0^{++}$, that will be referred to as the pion and the σ meson, respectively, according to the standard nomenclature. In contrast, these excitations must be exactly degenerate in the Wigner-Weyl mode of chiral symmetry and form the $(1/2, 1/2)_a$ representation of the $SU(2)_L \times SU(2)_R$ chiral group [11].

To obtain the quark-antiquark bound states, we solve the homogeneous Bethe-Salpeter equation in the rest frame

$$\begin{aligned} \chi(m, \vec{p}) = & -i \int \frac{d^4 q}{(2\pi)^4} V(|\vec{p} - \vec{q}|) \gamma_0 \\ & \times S(q_0 + m/2, \vec{p} - \vec{q}) \chi(m, \vec{q}) \\ & \times S(q_0 - m/2, \vec{p} - \vec{q}) \gamma_0 (1 - \rho^\nu(q)). \end{aligned} \quad (14)$$

Here, S is the dressed single quark propagator, that is the solution of the gap equation, m is the meson mass, and \vec{p} is the relative momentum. The Bethe-Salpeter equation is solved by means of expansion of the vertex function $\chi(m, \vec{p})$ into a set of all possible independent amplitudes consistent with I, J^{PC} , and it transforms into a system of coupled equations. The infrared divergence cancels exactly in these equations, and they can be solved numerically [17]. The Pauli blocking by valence quarks is taken into account via the weight function $1 - \rho^\nu(q)$.

In the Wigner-Weyl mode, *i.e.* when dynamical quark mass and chiral angle vanish, $M(p) = 0$; $\varphi_p = 0$, the Bethe-Salpeter equations for the $1, 0^{-+}$ and $0, 0^{++}$ bound states become identical [17] and, consequently, energies of these states coincide.

On Fig. 8, we show masses of both pseudoscalar and scalar modes for different Fermi momenta p_f and diffusion widths Δ . For each Δ there is a critical $p_f^{cr}(\Delta)$ at which the chiral restoration phase transition takes place. Below this $p_f^{cr}(\Delta)$, there is a massless pion and a massive σ meson. Above the critical p_f , both the pion and the σ meson are massive and exactly degenerate.

VI. CONCLUSIONS

In the confining mode, the valence quarks interact, and near the Fermi surface, cluster into the color-singlet baryons. This implies that there cannot be a rigid quark Fermi surface. The valence quark distribution function near the Fermi surface must be smooth. The valence quark levels above the Fermi momentum are occupied with some probability, as well as the levels below the Fermi momentum must be, with some probability, empty. We assume unbroken translational and rotational invariances, *i. e.* a liquid phase. We parameterize such a diffused “Fermi surface” by a simplest-possible function and solve the corresponding gap and Bethe-Salpeter equations. By this, we verify whether a chiral phase transition, previously observed for a rigid quark Fermi surface, survives or not. It turns out that, for any reasonable diffusion width, there always exists such a Fermi momentum that the chiral restoration phase transition does take place. This confirms our previous conclusions about possible existence of the confining but chirally symmetric phase. Below the phase transition, the elementary excitation modes of a matter are hadrons with broken chiral symmetry, while above the phase transition, such excitations are chirally symmetric hadrons.

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