

Bulk viscosity of anisotropically expanding hot QCD plasma

Vinod Chandra*

Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai-400005, India
(Received 20 July 2011; published 22 November 2011)

The bulk viscosity, ζ and its ratio with the shear viscosity, ζ/η have been studied in an anisotropically expanding pure glue plasma in the presence of turbulent color fields. It has been shown that the anisotropy in the momentum distribution function of gluons, which has been determined from a linearized transport equation eventually leads to the bulk viscosity. For the isotropic (equilibrium) state, a recently proposed quasiparticle model of pure $SU(3)$ lattice QCD equation of state has been employed where the interactions are encoded in the effective fugacity. It has been argued that the interactions present in the equation of state, significantly contribute to the bulk viscosity. Its ratio with the shear viscosity is significant even at $1.5T_c$. Thus, one needs to take in account the effects of the bulk viscosity while studying the hydrodynamic expansion of quark-gluon plasma in the Relativistic Heavy Ion Collider and the Large Hadron Collider.

DOI: 10.1103/PhysRevD.84.094025

PACS numbers: 25.75.-q, 05.20.Dd, 24.85.+p, 12.38.Mh

I. INTRODUCTION

It is by now well established that quark-gluon plasma (QGP) has been created in Relativistic Heavy Ion Collider (RHIC) experiments, and is a strongly coupled fluid [1]. There have been first few reports of QGP in Pb-Pb collisions @2.76 TeV in the Large Hadron Collider (LHC) [2], which reconfirm the formation of strongly coupled fluid. QGP at the RHIC has shown a robust collective phenomenon, viz., the elliptic flow [3]. In the heavy-ion collisions at the LHC, there are other interesting flows, viz., the dipolar and the triangular flow, which are sensitive to the initial collision geometry [4]. In this concern, we refer the reader to the very recent interesting studies [5,6], where these new kinds of flows at the LHC have been investigated.

The shear and bulk viscosities (η and ζ) characterize dissipative processes in the hydrodynamic evolution of a fluid. The former accounts for the entropy production due to the transformation of the shape of hydrodynamic system at a constant volume. On the other hand, the latter accounts for the entropy production at the constant rate of change of the volume of the system (in the context of the RHIC the system stands for the fireball). These transport parameters serve as the inputs for the hydrodynamic evolution of the fluid. Their determination has to be done separately from a microscopic theory (either from a transport equation with appropriate force, collision, and source terms or from the field theoretic approach using the Green-Kubo formula). It has been found that QGP possesses a very tiny value of the shear viscosity to entropy density ratio, η/s [7]. On the other hand, bulk viscosity has achieved considerable attention in the context of QGP in the RHIC after the interesting reports on its rising value close to the QCD transition temperature [8,9]. In the recent investigations, these transport coefficients are found to be sensitive to the

interactions [10,11], and the nature of the phase transition in QCD [12].

The computation of transport coefficients in lattice QCD is a very nontrivial exercise, due to several uncertainties and inadequacy in their determination. Despite this, there are a few first results computed from lattice QCD for bulk and shear viscosities [13,14] which have observed a small value of η/s , and a large value for ζ/s at the RHIC. While determining the behavior of the spectral function in [13], a contribution coming from a δ function has not been taken in to account. This issue has been discussed extensively in [15]. The spectral density has been modified by incorporating the contributions from the δ function by Meyer in [16]. However, more refined lattice studies on η and ζ are awaited in the near future with less dependence on the lattice artifacts and uncertainties. Subsequently, the possible impact of the large bulk viscosity of QGP in the RHIC has been studied by several authors; Song and Heinz [17] have studied, in detail, the interplay of shear and bulk viscosities in the context of collective flow in heavy-ion collisions. Their study revealed that one cannot simply ignore the bulk viscosity while modeling QGP in heavy-ion collisions. In this context, there are other interesting studies reported in the literature [18–24]. The role of bulk viscosity in freeze out phenomenon has been reported in [20,25]. Effects of bulk viscosity in hadronic phase, and in the hadron emission have been reported in [26]. There has been a wealth of recent literature on the computations of bulk viscosity in the context of cosmology [27], strange quark matter [28], and neutron stars [29].

The noteworthy point is that most of the works devoted to study the hydrodynamic evolution of QGP, employ the constant value of η/s [30] and ζ/s [31]. This may not be desirable in the light of experimental and phenomenological observation for QGP at the RHIC. The work presented in this paper is an attempt to achieve, (i) temperature dependence of transport coefficients, in particular, ζ ,

*vinodc@theory.tifr.res.in

(ii) to understand the large bulk viscosity of QGP. In this study, we shall take inputs from the computations of bulk viscosity in quasiparticle models [32,33], and combine the understanding with a transport theory determination of ζ in the presence of chromo-Weibel instabilities [34,35]. In this context the shear viscosity of QGP has already been addressed [10,11,34,36], and we find very interesting results. As it is well emphasized by Pratt [37] that there may be a variety of physical phenomena that can lead to viscous effects in QGP. Among them, in this paper, we are particularly interested in the viscous effects that get contributions from the classical chromofields.

The idea adopted here is based on the mechanism, proposed earlier to explain the small viscosity of a weakly coupled, but expanding hot QCD plasma [34,36]. This mechanism is based on the particle transport theory in turbulent plasmas [38], which are characterized by strongly excited random field modes in the certain regimes of instability, which coherently scatter the charged particles and thus reduce the rate of momentum transport. This eventually leads to the suppression of the transport coefficients in plasmas. This phenomenon in electromagnetic (EM) plasmas has been studied in [39], and generalized by Asakawa, Bass, and Müller [34] to the non-Abelian plasma (QCD), and further employed for the realistic QGP equation of state (EOS) in [10,11]. As it is emphasized in [40], the sufficient condition for the spontaneous formation of turbulent, partially coherent fields is the presence of instabilities in the gauge fields due to the presence of charged particles. This condition is met in both EM plasmas with an anisotropic momentum distribution [41] of charged particles and in QGP with an anisotropic distribution of thermal partons [42]. Here, we shall argue that the similar mechanism can lead to a large bulk viscosity for the hot QCD plasma for the temperatures relevant at the RHIC and heavy-ion collisions at the LHC.

The paper is organized as follows. In Sec. II, we present the general formalism to determine the transport parameters from a transport equation with a Vlasov term. We have neglected the collision and source term, while obtaining bulk viscosity. In Sec. III, we discuss the temperature dependence of bulk viscosity and its comparison with the shear viscosity. Finally, in Sec. III, we present the conclusions and outlook.

II. TRANSPORT PARAMETERS WITHIN A QUASIPARTICLE MODEL

The determination of transport coefficients requires modeling beyond the equilibrium properties, in terms of the collision terms and other transport parameters, and also the nature of perturbation to the equilibrium distribution. In particular, their determination within linearized transport theory needs knowledge of EOS and the equilibrium momentum distribution functions of particles, which constitute the plasma. We shall first discuss the modeling of

the EOS within a quasiparticle model. The EOS chosen here is the pure $SU(3)$ gauge theory EOS [43]. We subsequently discuss the setting up of the transport equation and the determination of ζ .

A. The quasiparticle model

Lattice QCD is the best, and most powerful technique to extract nonperturbative information on the equation of state for QGP [44,45]. Recently, we have proposed a quasiparticle model to describe the lattice data on pure $SU(3)$ gauge theory pressure (LEOS), and studied the bulk and transport properties of QGP [11], which is utilized in obtaining the temperature dependence of bulk viscosity here. In this description, the quasigluon distribution function extracted from LEOS possesses the following form:

$$f_{eq} = \frac{z_g \exp(-\beta p)}{(1 - z_g \exp(-\beta p))}. \quad (1)$$

It has further been argued [11] that the model is in the spirit of Landau theory of Fermi liquids. The connection with Landau's theory is apparent from the single quasigluon energy, which gets nontrivial contributions from the quasiparticle excitations. The dispersion relation (single particle energy) came out to be

$$E_p = p + T^2 \partial_T \ln(z_g). \quad (2)$$

The main feature of the description is the mapping of strongly interacting LEOS in to a system of noninteracting/weakly interacting quasigluons [free up to the temperature dependent fugacity, z_g which encodes all the interactions, and the dispersion relation in Eq. (2)]. This enables us to tackle highly nontrivial strong interaction in QCD in a very simplified manner while studying the properties of QGP. Interestingly, Eq. (2), which is obtained from the thermodynamic definition of the energy density in terms of grand-canonical QCD partition functions, ensures the thermodynamic consistency in hot QCD, and reproduces the lattice results on the trace anomaly correctly. This is also true for the recently proposed quasiparticle model, which describes the (2 + 1)-flavor lattice QCD [46].

This quasiparticle understanding of hot QCD has been quite successful in describing the realistic QGP equations of state, and in investigating the bulk and transport properties of QGP [10,11,47,48]. We shall utilize Eqs. (1) and (2) to determine the bulk and shear viscosities within the transport theory framework here. Note that there are other quasiparticle approaches to describe lattice QCD EOS based on effective thermal masses for quasipartons [49–53], approaches based on Polyakov loop [54], and quasiparticle models with gluon condensate [55,56]. Recently, transport coefficients for QGP within the effective mass models in the relaxation time approximation have been reported in [57,58]. As argued in [48], our model is distinct from all these approaches, but equally successful in describing the thermodynamics of QGP.

B. Determination of the transport coefficients

We now consider the important physical quantities, the bulk viscosity, ζ , its ratio with entropy density, ζ/s . For the entropy density, we again utilize the lattice results quoted in [11]. These quantities are very crucial in understanding the QGP in the RHIC. Their determination requires knowledge of the collisional properties of the medium when it is perturbed away from equilibrium. To determine these quantities, we adopt approach of [11,34,36]. The shear viscosity had been determined in [11], which we shall utilize to study the ratio ζ/η in the later part. Here, we consider ζ and determine it from a transport equation.

The determination of bulk viscosity has been done in a multifold way. First, we need an appropriate modeling of distribution function for the equilibrium state. Second, we need to set up an appropriate transport equation to determine the form of the perturbation to the distribution function. These two steps eventually determine the bulk viscosity. For the former step, we employ the quasiparticle model for LEOS discussed earlier. We shall leave the analysis in the case of full QCD for future investigations.

The bulk viscosity has two contributions same as the shear viscosity in [34], (i) from the Vlasov term which captures the long range component of the interactions, and (ii) the collision term, which models the short range component of the interaction. Here, we shall only concentrate on the former case. The determination of shear and bulk viscosities from an appropriate collision term will be a matter of future investigations. Importantly, the analysis adopted here is based on weak coupling limit in QCD, therefore, the results are shown beyond $1.3T_c$ assuming the validity of weak coupling results for QGP there.

1. Formalism

Let us first briefly outline the standard procedure of determining transport coefficients in transport theory [34,59]. The bulk and shear viscosities, ζ and η of QGP in terms of equilibrium parton distribution functions are obtained by comparing the microscopic definition of the stress tensor with the macroscopic definition of the viscous stress tensor. The microscopic definition of the stress tensor is

$$T_{ik} = \int \frac{d^3 p}{(2\pi)^3 E_p} p_i p_k f(\vec{p}, \vec{r}). \quad (3)$$

On the other hand, macroscopic expression for the viscous stress tensor is given by

$$T_{ik} = P \delta_{ik} + \epsilon u_i u_k - 2\eta (\nabla u)_{ik} - \zeta \delta_{ik} \nabla \cdot \vec{u}, \quad (4)$$

where $(\nabla u)_{ik}$ is the traceless, symmetrized velocity gradient, and $\nabla \cdot \vec{u}$ is the divergence of the fluid velocity field. E_p accounts for the dispersion relation. To determine ζ and η , one writes the gluon distribution function as

$$f(\vec{p}, \vec{r}) = \frac{1}{z_g^{-1} \exp(\beta \mathbf{u} \cdot \mathbf{p} - f_1(\vec{p}, \vec{r})) - 1}. \quad (5)$$

Assuming that $f_1(\vec{p}, \vec{r})$ is a small perturbation to the equilibrium distribution, we expand $f(\vec{p}, \vec{r})$ and keep the linear order term in f_1 ; this leads to

$$\begin{aligned} f(\vec{p}, \vec{r}) &= f_0(\mathbf{p}) + \delta f(\vec{p}, \vec{r}) \\ &= f_0(\mathbf{p})(1 + f_1(\vec{p}, \vec{r})(1 + f_0(\mathbf{p}))), \end{aligned} \quad (6)$$

where $f_0(\mathbf{p})$ is the isotropic distribution function, as we shall see that this will be the same as the equilibrium thermal distribution function of the quasiglons, in the rest frame of the fluid. As discussed in [11], ζ and η are determined by taking the following form of the perturbation f_1 :

$$f_1(\vec{p}, \vec{r}) = -\frac{1}{E_p T^2} p_i p_j \left(\Delta_1(p) \nabla u_{ij} + \Delta_2(\vec{p}) (\nabla \cdot \mathbf{u}) \delta_{ij} \right), \quad (7)$$

where the dimensionless functions $\Delta_1(p)$, $\Delta_2(\vec{p})$ measure the deviation from the equilibrium configuration. $\Delta_1(p)$, $\Delta_2(\vec{p})$, lead to η and ζ , respectively. Note that $\Delta_1(p)$ is an isotropic function of the momentum in contrast to $\Delta_2(\vec{p})$, which is an anisotropic in momentum \vec{p} .

Since ζ and η are Lorentz scalars; they may be evaluated conveniently in the local rest frame. In the local rest frame of the fluid $f_0 \equiv f_{eq}$. Considering the a boost invariant longitudinal flow, $\nabla \cdot \mathbf{u} = \frac{1}{\tau}$ and, $(\nabla u)_{ij} = \frac{1}{3\tau} \text{diag}(-1, -1, 2)$, in the local rest frame, we find that $f_1(p)$ takes the form

$$f_1(\vec{p}) = -\frac{\Delta_1(p)}{E_p T^2 \tau} \left(p_z^2 - \frac{p^2}{3} \right) - \frac{\Delta_2(\vec{p})}{E_p T^2 \tau} p^2, \quad (8)$$

where τ is the proper time ($\tau = \sqrt{t^2 - z^2}$). The shear and bulk viscosities are obtained in terms of entirely unknown function $\Delta_1(p)$ and $\Delta_2(\vec{p})$ as

$$\eta = \frac{\nu_g}{15T^2} \int \frac{d^3 p}{8\pi^3} \frac{p^4}{E_p^2} \Delta_1(p) f_{eq}(1 + f_{eq}), \quad (9)$$

$$\zeta = \frac{\nu_g}{3T^2} \int \frac{d^3 p}{8\pi^3} \frac{p^2}{E_p^2} (p^2 - 3c_s^2 E_p^2) \Delta_2(\vec{p}) f_{eq}(1 + f_{eq}). \quad (10)$$

In these expressions, $\nu_g \equiv 2(N_c^2 - 1)$ is the degrees of freedom. Notice that while obtaining the expression for the bulk viscosity, we have exploited the Landau-Lifshitz condition for the stress energy tensor. The factor $-(3c_s^2 E_p^2)$ in the right-hand side of Eq. (10) is coming only because of that. For details, we refer the reader to [33].

The determinations of $\Delta_1(p)$ and η have already been done in [10,11]. We shall utilize these results to fix the temperature dependence of ζ in the later part of the analysis. Now, we shall focus on the determination of the unknown function $\Delta_2(\vec{p})$ and ζ .

2. Determination of $\Delta_2(\vec{p})$

For simplicity, we consider the purely chromomagnetic plasma for our analysis. The modeling of transport equation for full chromo-electromagnetic plasma is straight forward [10,11,34] and differs by simple factors. Here, we only quote the mathematical form of the drift term and the Vlasov term. (For details see [10,34].)

The drift term in the transport equation for the full chromo-electromagnetic plasma for LEOS is obtained as

$$(\mathbf{v} \cdot \partial) f_{eq}(p) = f_{eq}(1 + f_{eq}) \left[\frac{p_i p_j}{E_p T} (\nabla u)_{ij} - \frac{m_D^2 \langle E^2 \rangle \tau_m^{el} E_p}{3T^2 \partial \mathcal{E} / \partial T} + \left(\frac{p^2}{3E_p^2} - c_s^2 \right) \frac{E_p}{T} (\nabla \cdot \vec{u}) \right], \quad (11)$$

where c_s^2 is the speed of sound. The other notations are kept same as in [11]. Note that $\langle E^2 \rangle$ stands for the chromo-electric field, τ_{el} relaxation time associated with the instability [34]. In Eq. (11), the first term contributes to the shear viscosity, the second term contributes to the thermal conductivity, and the third term contributes to the bulk viscosity. Since we are considering the purely chromomagnetic plasma, the second term will not be present.

On the other hand, the force term (Vlasov term) which we denote as \mathbf{V}_A , is obtained in [11,34] as follows:

$$\mathbf{V}_A = \frac{g^2 C_2}{2(N_c^2 - 1)E_p^2} \langle B^2 \rangle \tau_m \mathbf{L}^2, \quad (12)$$

where $C_2 = N_c$, $\langle B^2 \rangle$ denotes chromomagnetic field, τ_m is the time scale associated with instability in the field, and the operator \mathbf{L}^2 is

$$\mathbf{L}^2 = -(\vec{p} \times \partial_{\vec{p}})^2 + (\vec{p} \times \partial_{\vec{p}})|_z^2 \equiv -(L^p)^2 + (L_z^p)^2. \quad (13)$$

Since \mathbf{L}^2 contains angular momentum operator L^p , it therefore gives a nonvanishing contribution while operating on an anisotropic function of \vec{p} . It will always lead to the vanishing contribution while operating on an isotropic function of \vec{p} . Therefore, $\mathbf{V}_A f_{eq} \equiv 0$. Now, we write the transport equation containing only those terms which contribute to bulk viscosity ζ as

$$\begin{aligned} & \left(\frac{p^2}{3E_p^2} - c_s^2 \right) \frac{E_p}{T} (\nabla \cdot \vec{u}) f_{eq}(1 + f_{eq}) \\ &= \frac{g^2 C_2}{3(N_c^2 - 1)E_p^2} \langle B^2 \rangle \tau_m \mathbf{L}^2 f_1(\vec{p}, \vec{r}) f_{eq}(1 + f_{eq}). \end{aligned} \quad (14)$$

Substituting for f_1 in term of the unknown function $\Delta_2(\vec{p})$ and rearranging above equation, we obtain a differential equation for $\Delta_2(\vec{p})$ as

$$\mathbf{L}^2 \Delta_2(\vec{p}) = \frac{2(N_c^2 - 1)TE_p^2}{N_c g^2 \langle B^2 \rangle \tau_m p^2} \left(\frac{p^2}{3} - c_s^2 E_p^2 \right). \quad (15)$$

Now, using the fact that \mathbf{L}^2 only operates on the anisotropic function of \vec{p} , we can write

$$\Delta_2(\vec{p}) = \frac{2(N_c^2 - 1)TE_p^2}{N_c g^2 \langle B^2 \rangle \tau_m p^2} \left(\frac{p^2}{3} - c_s^2 E_p^2 \right) \times g(\vec{p}), \quad (16)$$

where $g(\vec{p})$ can be determined from the following condition:

$$\mathbf{L}^2 g(\vec{p}) = 1, \quad (17)$$

which leads to

$$g(\vec{p}) = \frac{1}{2} \ln \left(\frac{p_x^2 + p_y^2}{p_0^2} \right) \equiv \ln \left(\frac{p_T}{p_0} \right). \quad (18)$$

Since, at high temperature average value of the energy is $3T$. Employing equipartition theorem for relativistic massless gas, we obtain $p_0^2 = 6T^2$. Substituting Eq. (18) in Eq. (16), we obtain

$$\Delta_2(\vec{p}) = \frac{2(N_c^2 - 1)TE_p^2}{N_c g^2 \langle B^2 \rangle \tau_m p^2} \left(\frac{p^2}{3} - c_s^2 E_p^2 \right) \ln \left(\frac{p_T}{p_0} \right). \quad (19)$$

The determination of bulk viscosity is incomplete unless we know not only the temperature dependence of the speed of sound square, c_s^2 , and the collective contributions of quasiparticle to the single particle energy, $T^2 \partial_T \ln(z_g)$ but also the quantity $g^2 \langle B^2 \rangle \tau_m$.

We determine first two quantities using the quasiparticle model. As from Ref. [11], the trace anomaly in terms of effective quasiparticle number density and effective gluon fugacity reads

$$\frac{(\epsilon - 3P)}{T^4} = \frac{\mathcal{N}_g}{T^3} \{ T \partial_T \ln(z_g) \}. \quad (20)$$

The thermodynamic quantities can be obtained using the well-known thermodynamic relations. In particular, the energy density and the entropy density were shown to be in almost perfect agreement with the lattice data [11]. We determine, c_s^2 by employing a method reported in [60]. The temperature dependence is shown in Fig. 1.

To relate the denominator of Eq. (19) to the gluon quenching parameter, \hat{q} we go to the light cone frame. In this frame, Eq. (19) can be rewritten as

$$\Delta_2(\vec{p}) = \frac{4(N_c^2 - 1)TE_p^2}{N_c g^2 \langle E^2 + B^2 \rangle \tau_m p^2} \left(\frac{p^2}{3} - c_s^2 E_p^2 \right) \ln \left(\frac{p_T}{p_0} \right). \quad (21)$$

The gluon quenching parameter, \hat{q} is related with the denominator of right-hand side of the above equation as [36]

$$\hat{q} = \frac{2g^2 N_c}{3(N_c^2 - 1)} \langle E^2 + B^2 \rangle \tau_m. \quad (22)$$

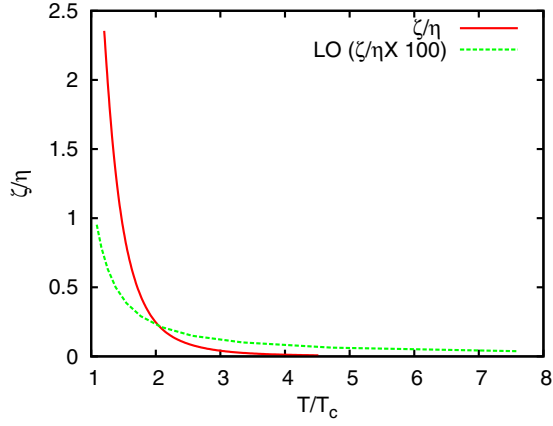


FIG. 1 (color online). The ratio of bulk viscosity, ζ to the shear viscosity, η as a function of temperature. The leading order result of ζ/η has been obtained from the data taken from Refs. [62,63], and shown as dashed line. For the sake of comparison, we have multiplied the leading order ζ/η by a factor of hundred.

Now, employing Eq. (21) in Eq. (8), we obtain the ζ as

$$\zeta = \frac{(N_c^2 - 1)}{3T\pi^2\hat{q}} \int_0^\infty \int_{-\infty}^\infty p_T dp_T dp_z \left(\frac{p^2}{3} - c_s^2 E_p^2 \right)^2 \times \ln\left(\frac{p_T}{p_0}\right) \times f_{eq}(1 + f_{eq}). \quad (23)$$

On the other hand, if we employ the results of [11] for $\Delta_1(p)$ in Eq. (10) for η , we obtain

$$\eta = \frac{T^6}{\hat{q}} \frac{64(N_c^2 - 1)}{3\pi^2} \text{PolyLog}[6, z_g], \quad (24)$$

where $N_c = 3$ and $\text{PolyLog}[6, z_g] = \sum_{k=1}^\infty \frac{z_g^k}{k^6}$.

Now scaling, all the quantities in the integrand in Eq. (23) by T , and rewriting Eq. (24) in the form given below, we obtain

$$\zeta = \frac{T^6}{\hat{q}} I_1(T/T_c); \quad \eta = \frac{T^6}{\hat{q}} I_2(T/T_c), \quad (25)$$

where $I_1(T/T_c)$, is evaluated by integrating the right-hand side of Eqs. (23) numerically, and $I_2(T/T_c) \equiv \frac{8^3}{3\pi^3} \text{PolyLog}[6, z_g]$. The T/T_c scaling of these quantities is coming from the temperature dependence of the effective gluon fugacity, z_g . Here, T_c is taken to be 0.27 GeV [61]. Clearly, the quantity which can be determined unambiguously in our approach is the ratio $\zeta/\eta \equiv I_1(T/T_c)/I_2(T/T_c)$.

In the recent past, Chen *et al.* [62,63] computed the leading order shear and bulk viscosities for purely gluonic plasma. This is nothing but the collisional contribution to these transport parameters for a gluonic plasma. It is to be instructive to compare the results on ζ/η obtained in the present work with those reported in [62]. This has been shown in Fig. 1, where both the results on ζ/η are plotted

as a function of T/T_c . Note that while obtaining the temperature dependence of the ratio ζ/η , we have employed the two-loop expression for the running coupling constant at finite temperature quoted in [62]. Quantitatively the ratio ζ/η is much smaller than what we have obtained from the diffusive Vlasov term. If we compare the two curves on the ratio ζ/η shown in Fig. 1, we find that in contrast to our prediction on ζ/η , the leading order result suggests the near conformal picture of hot QCD even at lower temperatures.

Next, we discuss the interplay of the two contributions to the bulk viscosity, viz., the anomalous, and the leading order (collisional). As it is emphasized in [34], these two contributions for η are inverse additive. Their inverse additivity has been argued from the additivity of various rates in the hot QCD medium. In the case of weak coupling, the former is predominant. It seems that a similar additivity of the inverse of two contributions to ζ (denoted as ζ_a and ζ_c , respectively) may perhaps be valid. This could be understood as follows: since ζ_a is inversely proportional to the \hat{q} (transport rate), on the other hand collisional ζ_c will be inversely proportional to the collision rate. Following the argument previously mentioned, one may write, $\zeta_T^{-1} = \zeta_a^{-1} + \zeta_c^{-1}$, where ζ_T denotes the total bulk viscosity. This inverse additivity of ζ_a and ζ_c at weak coupling, suggests that the collisional bulk viscosity (leading order) will dominate over the anomalous one, since the former is quantitatively much smaller than the latter. However, one has to be very cautious while comparing these two contributions for the temperature ranges relevant for QGP at the RHIC. This is because of the strongly coupled fluid like picture of QGP. At this moment, we do not know whether the inverse additivity of ζ will be followed at the temperatures which are closer to T_c or not. This is a very crucial issue, and will require much deeper investigations, which is beyond the scope of the present work. Henceforth, we denote the anomalous bulk viscosity as ζ dropping the subscript, a .

We now proceed to discuss the temperature dependence of ζ/η and ζ/s .

C. Temperature dependence of ζ/η and ζ/s

In our analysis, determination of the temperature dependence of ζ and η is incomplete, without the knowledge of the temperature dependence of \hat{q} in QGP. This issue was addressed by fixing the temperature dependence of \hat{q} by calculating the soft part of the energy density and the relaxation time associated with the instability of chromo-fields [11]. To do that we take inputs from the phenomenological values of \hat{q} , which is known at a particular temperature [64]. Here, we have utilized the same transport equation and quasiparticle model developed for pure $SU(3)$ lattice QCD EOS, as in [11]. Therefore, we employ the temperature dependence of η/s to obtain the temperature dependence of ζ/s . This is quite easier to do, since the ratio, ζ/η can easily be obtained from Eq. (25).

The temperature dependence of ζ/η is shown in Fig. 1, ζ/η relative to perturbative QCD prediction [65], and strongly coupled near conformal gauge theories [66] is shown in Fig. 2. On the other hand, ζ/s and η/s are shown together in Fig. 3. Let us discuss their behavior one by one. From Fig. 1, it is clear that ζ/η is equally significant while studying the hydrodynamic evolution of hot QCD matter until we reach $T = 2T_c$. As we go to the higher temperatures the ratio further decreases and eventually vanishes when $c_s^2 = \frac{1}{3}$ and the dispersion relation $E_p = p$. Quantitatively, $\zeta/\eta \sim 2.3$ at $1.3T_c$; 1.0 at $1.5T_c$; 0.2 at $2.0T_c$. Therefore, for $T \geq 2.5T_c$, one can ignore ζ over η . In other words, the hot QCD becomes almost conformal there.

The ratio, ζ/η decreases as we increase the temperature. The decrease is quite steeper until we reach $T = 2.0T_c$. For

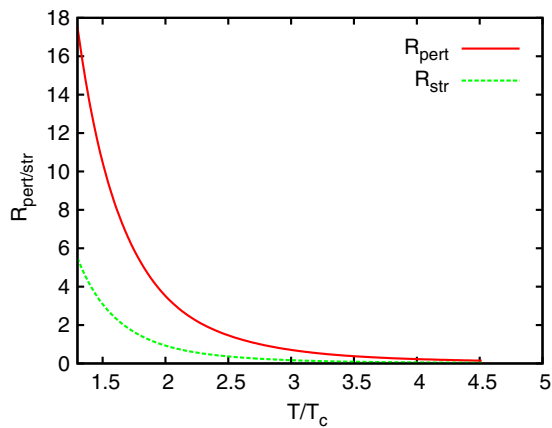


FIG. 2 (color online). Comparison of the ratio ζ/η with the perturbative QCD, and strongly coupled theories. The quantities R_{pert} , and R_{str} are defined in terms of the ratios $\frac{\zeta}{\eta(c_s^2 - \frac{1}{3})^2}$, and $\frac{\zeta}{\eta(c_s^2 - \frac{1}{3})}$ respectively. Here, *pert* stands for perturbative QCD, and *str* stands for the strongly coupled near conformal theories.

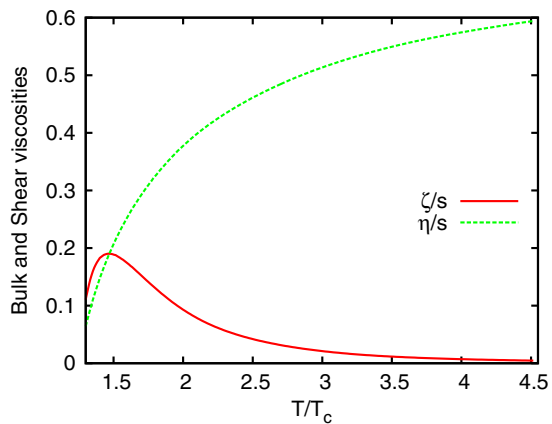


FIG. 3 (color online). ζ/s and η/s as a function of temperature. The dashed (green) line denotes η/s and solid (red) line denotes the ζ/s .

higher values of T it is much slower. It is hard to make clear cut statement in regard to the behavior of ζ/η with temperature, since by looking at Eqs. (23) and (24), it is clear that the behavior of ζ/η as a function of temperature is mainly governed by the temperature dependence of trace anomaly (through quasigluon dispersion relation), speed of sound, c_s^2 , and temperature dependence of z_g and gluon quenching parameter, \hat{q} .

To compare the perturbative QCD prediction of the ratio ζ/η , we consider $R_{\text{pert}} \equiv \frac{\zeta}{\eta(c_s^2 - \frac{1}{3})^2}$, where $(c_s^2 - \frac{1}{3})$ can be thought of as the measure of conformal symmetry, which we call conformal measure. For scalar field theories, $\zeta/\eta = 15(c_s^2 - \frac{1}{3})^2$ [67], and this has been found to be true for a photon gas coupled with hot matter by Weinberg [68]. The prefactor 15 is not fixed for perturbative QCD but the scaling $\zeta/\eta \sim (c_s^2 - \frac{1}{3})^2$ is valid [65]. Note that in certain strongly coupled near conformal theories with gravity dual the ratio ζ/η shows linear dependence on the conformal measure [66]. To compare with the latter, we consider the ratio $R_{\text{str}} \equiv \frac{\zeta}{\eta(c_s^2 - \frac{1}{3})}$. We have shown the behavior of R_{pert} and R_{str} as a function of temperature in Fig. 2. Clearly, none of these two scalings are respected by the ratio ζ/η in Fig. 2 even at $2.5T_c$. It is safer to say that ζ/η for LEOS which is obtained from transport equation with Vlasov-Dupree term [11,34] neither shows linear nor the quadratic dependence with the conformal measure, $(c_s^2 - \frac{1}{3})$. However, one can realize the quadratic scaling of ζ/η with the conformal measure in a certain limiting case. It is easy to say from Eqs. (24) and (25) that for $E_p = p$ ($p \ll T^2 \partial_T (\ln(z_g))$), if the thermal distribution of quasigluons shows near ideal behavior, and with constant value of \hat{q}/T^3 , the quadratic scaling can be achieved. Moreover, this may perhaps be realized at higher temperatures which are not relevant for QGP in the RHIC and the LHC. If we compare qualitatively our prediction of R_{pert} , and R_{str} with the leading order result of [62] (see Fig. 4 in Ref. [62]), we find opposite trend of these quantities at very high temperature. The former decreases, although slowly, in contrast to the latter, as a function of temperature. This could perhaps be because of their origin from the distinct physical processes in hot QCD medium. The slow decreases of the former at higher temperatures, could be understood as the effect of thermal distribution function of quasigluons (through z_g , since z_g increases very slowly as a function of temperature, and will asymptotically approach to unity).

Finally, in Fig. 3, we have shown the temperature dependence of ζ/s and η/s . The ζ/s decreases as with increasing temperature for $T \geq 1.5$, in contrast to η/s . As mentioned earlier, ζ/s and η/s become equal around $1.5T_c$ (below which ζ/s is higher, and lower for higher temperatures). Again the behavior is predominantly controlled by the behavior of c_s^2 , and the trace anomaly through the modified dispersion relation with temperature.

III. CONCLUSIONS AND FUTURE PROSPECTS

In conclusion, we have estimated the temperature dependence of bulk viscosity to entropy density ratio (ζ/s), and bulk viscosity relative to shear viscosity, ζ/η within a quasiparticle model for pure glue QCD at high temperature by employing transport theory. We have determined ζ/η , exactly and unambiguously. In our analysis, these quantities get contributions from the instabilities in the chromo-electromagnetic fields due to the anisotropic thermal distribution of the partons in QGP. The mechanism has succeeded in explaining the small η/s and large value of the ratio ζ/η . In fact, ζ/η is around 2.3 at $1.3T_c$, of the order of unity at $1.5T_c$, and 0.2 at $2T_c$. This tells us that the breaking of conformal symmetry in hot QCD plays crucial role even at $2T_c$. In consequence, shear and bulk viscosities are equally important while studying the hydrodynamic evolution of QGP at the RHIC and the LHC. One cannot simply ignore bulk viscosity even at $2.0T_c$ while modeling the heavy-ion collisions. Moreover, η/s increases as a function of temperature, in contrast to ζ/s beyond $1.5T_c$. As expected ζ/s and ζ/η are vanishingly small beyond $2.5T_c$. This may be due to the fact that conformal measure is very small there, and the speed of sound is closer to $1/3$. We have compared our predictions on ζ/η to the leading order result on the same quantity obtained by [62]. Interestingly, in the perturbative region (temperatures beyond $1.5T_c$), our study also agree with the near conformal picture of hot QCD similar to leading order results of Chen *et al.* [62]. On the other hand the predictions are in contrast

at lower temperatures. However, this may not be thought of as the complete story, an adequate analysis on the interplay of our predictions on ζ , and leading order prediction is very much desired, and will be a matter of future investigations.

We have addressed the temperature dependence of the bulk and shear viscosities of pure glue sector of hot QCD only. An extension to full QCD including collision term, employing the understanding of [46], will be a matter of future investigations. We strongly believe that a similar analysis will also be valid in the case of full QCD. The most interesting study would be to include the temperature dependence of η/s and ζ/s in the existing hydro codes to model QGP, and see how various observables get modifications. Moreover, future directions may include exploration on the effects of η and ζ on the quarkonia suppression in heavy-ion collisions along the lines of [69,70]. Finally, it would be of interest to include the baryon chemical potential utilizing the very recent lattice studies [71,72], and determine the transport coefficients.

ACKNOWLEDGMENTS

V. C is thankful to F. Karsch, and S. Datta for providing the lattice QCD data, R. Bhalerao and V. Ravishankar for invaluable suggestions and encouragement, and S. Biswal for the numerical help. V. C would like to thank U. A. Wiedemann for invaluable discussions at the CERN-Theory Division, and sincerely acknowledge the hospitality of the CERN-Theory Division, CERN Geneva through the CERN visitor program.

-
- [1] J. Adams *et al.* (STAR Collaboration), *Nucl. Phys.* **A757**, 102 (2005); K. Adcox *et al.* (PHENIX Collaboration) *ibid.* **A757**, 184 (2005); B. B. Backus *et al.* (PHOBOS Collaboration) *ibid.* **A757**, 28 (2005); A. Arsene *et al.* (BRAHMS Collaboration) *ibid.* **A757**, 1 (2005).
- [2] K. Aamodt *et al.* (Alice Collaboration), arXiv:1011.3914; K. Aamodt *et al.* (Alice Collaboration) *Phys. Rev. Lett.* **105**, 252301 (2010); K. Aamodt *et al.* (Alice Collaboration) *ibid.* **106**, 032301 (2011).
- [3] J. Adams *et al.* (STAR Collaboration), *Nucl. Phys.* **A757**, 102 (2005).
- [4] D. Teaney and L. Yan, *Phys. Rev. C* **83**, 064904 (2011); B. H. Alver, C. Gombaud, M. Luzum, and J.-Y. Ollitrault, *Phys. Rev. C* **82**, 034913 (2010).
- [5] R. S. Bhalerao, M. Luzum, and J.-Y. Ollitrault, arXiv:1106.4940; arXiv:1104.4740; M. Luzum and Jean-Yves Ollitrault, *Phys. Rev. Lett.* **106**, 102301 (2011).
- [6] M. Krzewicki (for the ALICE Collaboration), QM-2011, arXiv:1107.0080v1.
- [7] H. B. Meyer, *Phys. Rev. D* **76**, 10171 (2007); R. A. Lacey *et al.*, *Phys. Rev. Lett.* **98**, 092301 (2007); Z. Xu and C. Greiner, *Phys. Rev. Lett.* **100**, 172301 (2008); Z. Xu, C. Greiner, and H. Stoecker, *Phys. Rev. Lett.* **101**, 082302 (2008); A. Adare *et al.* (PHENIX Collaboration) *Phys. Rev. Lett.* **98**, 172301 (2007); S. Gavin and M. Abdel-Aziz, *Phys. Rev. Lett.* **97**, 162302 (2006); A. Buchel, *Phys. Lett. B* **663**, 286 (2008); P. Kovtun, D. T. Son, and A. O. Starinets, *Phys. Rev. Lett.* **94**, 111601 (2005).
- [8] D. Kharzeev and K. Tuchin, *J. High Energy Phys.* **09** (2008) 093.
- [9] F. Karsch, D. Kharzeev, and K. Tuchin, *Phys. Lett. B* **663**, 217 (2008).
- [10] V. Chandra and V. Ravishankar, *Eur. Phys. J. C* **59**, 705 (2008).
- [11] V. Chandra and V. Ravishankar, *Eur. Phys. J. C* **64**, 63 (2009).
- [12] G. D. Moore and O. Saremi, *J. High Energy Phys.* **09** (2008) 015.
- [13] H. B. Meyer, *Phys. Rev. Lett.* **100**, 162001 (2008).
- [14] A. Nakamura and Sunao Sakai, *Phys. Rev. Lett.* **94**, 072305 (2005); *Nucl. Phys.* **A774**, 775 (2006).
- [15] D. Teaney, *Phys. Rev. D* **74**, 0450125 (2006); G. D. Moore and O. Saremi, *J. High Energy Phys.* **09** (2008) 015; P. Romatschke and D. T. Son, *Phys. Rev. D* **80**, 065021 (2009).

- [16] H. B. Mayer, *J. High Energy Phys.* 04 (2010) 099.
- [17] H. Song and U. W. Heinz, *Phys. Rev. C* **81**, 024905 (2010).
- [18] G. S. Denicol, T. Kodama, T. Koide, Ph. Mota, *Phys. Rev. C* **80**, 064901 (2009).
- [19] G. S. Denicol, T. Kodama, T. Koide, and Ph. Mota, *Nucl. Phys.* **A830**, 729c (2009).
- [20] A. Monnai and T. Hirano, *Nucl. Phys.* **A830**, 471c (2009); *Phys. Rev. C* **80**, 054906 (2009).
- [21] K. Rajagopal and N. Tripuraneni, *J. High Energy Phys.* 03 (2010) 018; J. R. Bhatt, H. Mishra, and V. Sreekanth, arXiv:1103.4333.
- [22] M. J. Efaaf, Z.-Q. Su, and W.-N. Zhang, arXiv:1008.1531.
- [23] D. Fernandez-Fraile and A. Gomez Nicola, *Phys. Rev. Lett.* **102**, 121601 (2009).
- [24] R. J. Fries, B. Müller, and A. Schäfer, *Phys. Rev. C* **78**, 034913 (2008).
- [25] G. Torrieri and B. Tomasik, and I. Mishustin *Phys. Rev. C* **77**, 034903 (2008); *Acta Phys. Pol. B* **39**, 1733 (2008).
- [26] P. Bozek, *Phys. Rev. C* **81**, 034909 (2010).
- [27] A. Tawfik, M. Wahba, H. Mansour, and T. Harko, *Ann. Phys. (Leipzig)* **522** 912 (2010); A. Avelino and U. Nucamendi, *J. Cosmol. Astropart. Phys.* 08 (2010) 009.
- [28] X. Wang and I. A. Shovkovy, *Phys. Rev. D* **82**, 085007 (2010); S.-w. Chen, H. Dong, and Q. Wang, *J. Phys. G* **36**, 064050 (2009).
- [29] B. Haskell and N. Andersson, arXiv:1003.5849; M. Mannarelli and C. Manuel, *Phys. Rev. D* **81**, 043002 (2010); X.-G. Huang, M. Huang, D. H. Rischke, and A. Sedrakian, *Phys. Rev. D* **81**, 045015 (2010).
- [30] M. Luzum and P. Romatschke, *Phys. Rev. C* **78**, 034915 (2008).
- [31] H. Song and U. W. Heinz, *Nucl. Phys.* **A830**, 467c (2009).
- [32] C. Sasaki and K. Redlich, *Phys. Rev. C* **79**, 055207 (2009).
- [33] P. Chakraborty and J. I. Kapusta, *Phys. Rev. C* **83**, 014906 (2011).
- [34] M. Asakawa, S. A. Bass, and B. Müller, *Prog. Theor. Phys.* **116**, 725 (2006).
- [35] S. Mrowczynski, *Phys. Rev. C* **49**, 2191 (1994); M. Strickland, *Braz. J. Phys.* **37**, 762 (2007); P. Arnold and G. Moore, *Phys. Rev. D* **73**, 025013 (2006).
- [36] M. Asakawa, S. A. Bass, and B. Müller, *Phys. Rev. Lett.* **96**, 252301 (2006); A. Majumdar, B. Müller, and X.-N. Wang, *Phys. Rev. Lett.* **99**, 192301 (2007).
- [37] Kerstin Paech and Scott Pratt, *Phys. Rev. C* **74**, 014901 (2006).
- [38] T. H. Dupree, *Phys. Fluids* **9**, 1773 (1966); , **11**, 2680 (1968).
- [39] T. Abe and K. Niu, *J. Phys. Soc. Jpn.* **49**, 717 (1980); and , **49**, 725 (1980).
- [40] M. Asakawa, S. A. Bass, and B. Müller, *J. Phys. G* **34**, S839 (2007).
- [41] E. S. Weibel, *Phys. Rev. Lett.* **2**, 83 (1959).
- [42] S. Mrowczyński, *Phys. Lett. B* **214**, 587 (1988); S. Mrowczyński, *ibid.* **314**, 118 (1993); P. Romatschke and M. Strickland, *Phys. Rev. D* **68**, 036004, (2003).
- [43] Our sincere thanks to F. Karsch for providing us the lattice data for pure $SU(3)$ gauge theory in the past which has been quoted in [11], due to which the present analysis became possible.
- [44] G. Boyd *et al.*, *Phys. Rev. Lett.* **75**, 4169 (1995); *Nucl. Phys.* **B469**, 419 (1996); M. Panero, *Phys. Rev. Lett.* **103**, 232001 (2009); F. Karsch, E. Laermann, and A. Peikert, *Phys. Lett. B* **478**, 447 (2000); M. Cheng *et al.*, *Phys. Rev. D* **77**, 014511 (2008); A. Bazavov *et al.*, *Phys. Rev. D* **80**, 014504 (2009); M. Cheng *et al.*, *Phys. Rev. D* **81**, 054504 (2010).
- [45] S. Borsanyi *et al.*, *J. High Energy Phys.* 09 (2010) 073; 11 (2010) 077; Y. Aoki *et al.*, *J. High Energy Phys.* 01 (2006) 089; 06 (2009) 088.
- [46] V. Chandra and V. Ravishankar, *Phys. Rev. D* **84**, 074013 (2011).
- [47] V. Chandra, R. Kumar, and V. Ravishankar, *Phys. Rev. C* **76**, 054909 (2007); *Indian J. Phys. B* **84**, 1789 (2011); , *Eur. Phys. J. A* **40**, 109 (2009);
- [48] V. Chandra and V. Ravishankar, *Nucl. Phys.* **A848**, 330 (2010).
- [49] A. Peshier, B. Kämpfer, and G. Soff, *Phys. Rev. C* **61**, 045203 (2000); *Phys. Rev. D* **66**, 094003 (2002).
- [50] A. Peshier *et al.*, *Phys. Lett. B* **337**, 235 (1994); *Phys. Rev. D* **54**, 2399 (1996).
- [51] V. M. Bannur, *Phys. Rev. C* **75**, 044905 (2007); **78**, 045206 (2008); *J. High Energy Phys.* 09 (2007) 046.
- [52] A. Rebhan and P. Romatschke, *Phys. Rev. D* **68**, 0250022 (2003).
- [53] M. A. Thaler, R. A. Scheider, and W. Weise, *Phys. Rev. C* **69**, 035210 (2004); K. K. Szabò and A. I. Tóth, *J. High Energy Phys.* 06 (2003) 008.
- [54] A. Dumitru and R. D. Pisarski, *Phys. Lett. B* **525**, 95 (2002); K. Fukushima, *Phys. Lett. B* **591**, 277 (2004); S. K. Ghosh *et al.*, *Phys. Rev. D* **73**, 114007 (2006); H. Abuki and K. Fukushima, *Phys. Lett. B* **676**, 57 (2009); H. M. Tsai and B. Müller, *J. Phys. G* **36**, 075101 (2009).
- [55] M. D'Elia, A. Di Giacomo, and E. Meggiolaro, *Phys. Lett. B* **408**, 315 (1997); *Phys. Rev. D* **67**, 114504 (2003).
- [56] P. Castorina and M. Mannarelli, *Phys. Rev. C* **75**, 054901 (2007); *Phys. Lett. B* **644**, 336 (2007).
- [57] M. Bluhm, B. Kämpfer, and K. Redlich, *Phys. Rev. C* **84**, 025201 (2011);
- [58] A. S. Khvorostukhin, V. D. Toneev, and D. N. Voskresersky, *Phys. Rev. C* **83**, 035204 (2011); S. K. Das and J.-e. Alam, *Phys. Rev. D* **83**, 114011 (2011).
- [59] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics (Landau and Lifshitz)*, Vol. 10 (Pergamon Press, New York, 1981).
- [60] R. V. Gavai, S. Gupta, and S. Mukherjee, *Proc. Sci., LAT* (2005), 173.
- [61] O. Kaczmarek, F. Karsch, P. Petreczky, and F. Zantow, *Phys. Lett. B* **543**, 41 (2002); *Phys. Rev. D* **70**, 074505 (2004); O. Kaczmarek and F. Zantow, *Phys. Rev. D* **71**, 114510 (2005).
- [62] J.-W. Chen, J. Deng, H. Dong, and Q. Wang, arXiv:1107.0522.
- [63] J.-W. Chen, J. Deng, H. Dong, and Q. Wang, *Phys. Rev. D* **83**, 034031 (2011); J.-W. Chen, J. Deng, H. Dong, Q. Wang, *ibid.* **84**, 039902(E) (2011).
- [64] H. Zhang, J. F. Owens, E. Wang, and X.-N. Wang, *Phys. Rev. Lett.* **98**, 212301 (2007); A. Majumder, C. Nonaka, and S. A. Bass, *Phys. Rev. C* **76**, 041902 (2007); R. Baier, Yu. L. Dokshitzer, S. Peigné, and D. Schiff, *Nucl. Phys.* **B483**, 291 (1997); N. Armesto, L. Cunqueiro, C. A.

- Salgado, and W.-C. Xiang, *J. High Energy Phys.* **02** (2008) 048.
- [65] P. Arnold, C.D. Guy, and D. Moore, *Phys. Rev. D* **74**, 085021 (2006).
- [66] P. Benincasa, A. Buchal, and A.O. Stralinets, *Nucl. Phys.* **B733**, 160 (2006); A. Buchal, *Phys. Rev. D* **72**, 106002 (2005).
- [67] R. Horsley and W. Schoenmaker, *Nucl. Phys.* **B280**, 716 (1987).
- [68] S. Weinberg, *Astrophys. J.* **168**, 175 (1971).
- [69] V. Agotiya, V. Chandra, and B. K. Patra, *Phys. Rev. C* **80**, 025210 (2009); *Eur. Phys. J. C* **67**, 465 (2010).
- [70] A. Dumitru, Y. Guo, A. Mocsy, and M. Strickland, *Phys. Rev. D* **79**, 054019 (2009).
- [71] F. Karsch, B.-J. Schaefer, M. Wagner, and J. Wambach, *Phys. Lett. B* **698**, 256 (2011).
- [72] P. Huovinen and P. Petreczky, [arXiv:1106.6227](https://arxiv.org/abs/1106.6227).