

Unbreaking chiral symmetryC. B. Lang^{1,*} and Mario Schröck^{1,†}¹*Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria*

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In quantum chromodynamics (QCD) the eigenmodes of the Dirac operator with small absolute eigenvalues have a close relationship to the dynamical breaking of the chiral symmetry. In a simulation with two dynamical quarks, we study the behavior of meson propagators when removing increasingly more of those modes in the valence sector, thus partially removing effects of chiral symmetry breaking. We find that some of the symmetry aspects are restored (e.g., the masses of ρ and a_1 approach each other) while confining properties persist.

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I. MOTIVATION AND INTRODUCTION

Dynamical chiral symmetry breaking in QCD is associated with the low lying spectral modes of the Dirac operator D [1]. They affect the path integral weight of the gauge configurations through the determinant of D . As indicated by the Atiyah-Singer index theorem [2], the exact zero modes are related to topological excitations, the instantons. For Dirac operators violating chiral symmetry these are real eigenmodes. The nearby nonreal modes are also thought to be related to composed structure of, e.g., overlapping instantons [3].¹

In a series of papers [4–6] it was emphasized that low modes saturate the pseudoscalar and axial vector correlators at large distances and do not affect the part where high lying states appear. In [6,7] low mode saturation and also effects of low mode removal for mesons were studied for quenched configurations with the overlap Dirac operator [8,9].

Subsequently low modes were utilized to improve the convergence of the determination of hadron propagators [6,7,10,11] (see also the recent study [12,13] comparing the efficiency when using the low modes of the Dirac operator or the Hermitian Dirac operator, where strong dependence on the parity of the hadron states was presented).

Associating the low mode sector with the nonperturbative chiral symmetry breaking and the condensate [1], a complementary question is how important it is for confinement and mass generation of hadrons. Here we study what happens if one removes up to 512 low lying modes from the valence quark sector. We compute propagators of the pion and other mesons and determine the effect of this removal on the mass spectrum. This way we want to shed

light on the role of the condensate related to the spectral part of the Dirac operator in confinement and chiral symmetry breaking. Our analysis is done for configurations generated for two light, mass degenerate dynamical quark flavors. The removal of the low lying modes is effective only in the valence quarks sector. However, as will be seen, this already has significant impact on the meson mass spectrum.

In [14,15] it has been conjectured that chiral symmetry is “effectively restored” for highly excited hadrons, in the sense that valence quarks become less affected by the quark condensate. This situation is similar to ours, where we artificially suppress the condensate as seen by the valence quarks. In the context of effective restoration such an approach has been discussed already in [6,16].

II. REDUCED DIRAC OPERATOR

Lattice Wilson Dirac operators and approximate Ginsparg-Wilson Dirac operators are γ_5 -Hermitian, $\gamma_5 D \gamma_5 = D^\dagger$, but non-normal, thus their spectral representation has real and complex eigenvalues and the left and right eigenvectors are bi-orthogonal, i.e. $\langle L_i | R_j \rangle = \delta_{ij}$. The so-called Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ has real eigenvalues μ_i and the eigenvectors are orthogonal.

We want to construct meson correlators from valence quark propagators that exclude the lowest part of the Dirac spectrum. There are two alternative definitions of reduction: based on eigenmodes of D or based on eigenmodes of the Hermitian Dirac operator. We introduce the reduced quark propagator via the spectral representation of D_5 ,

$$S_{\text{red5}(k)} = S - S_{\text{lm5}(k)} \equiv S - \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5. \quad (1)$$

Another alternative works with the bi-orthogonal eigensystem of D . The two types of truncation are not equivalent. We first tested the convergence of the low mode approximation and, as has been observed in [13], find a clearly slower convergence rate for the standard non-Hermitian as compared to the Hermitian Dirac operator. In our study we

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¹Even when studying the low lying modes in quenched gauge ensembles one observes nonvanishing density and also the Gell-Mann–Oakes–Renner relation works down to small values of the valence quark mass until quenched chiral logs destroy the leading chiral symmetry breaking behavior.

therefore concentrate on our results from truncating the Hermitian Dirac operator.

III. CHIRAL SYMMETRY AND ITS BREAKING

The nonvanishing quark masses of the two lightest quark flavors are relatively small in comparison to the typical QCD scale. Neglecting the masses of the u and d quarks the QCD Lagrangian is invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A. \quad (2)$$

The chiral symmetry $SU(2)_L \times SU(2)_R$ consists of independent transformations in the isospin space for the left- and right-handed quark fields and can be represented equivalently by independent isospin and axial rotations for the combined quark fields.

The isospin axial transformation mixes states with opposite parity but the same spin. Depending on quantum numbers the chiral partners can have the same or different isospin. The nondegenerate masses of parity partners indicate the dynamical (spontaneous) breaking of this chiral symmetry with the order parameter $\langle \bar{\psi} \psi \rangle$, the chiral condensate. Spontaneous breaking of the chiral symmetry leads to the appearance of the pseudoscalar Goldstone bosons, the pions.

The flavor singlet axial transformation symmetry $U(1)_A$ is broken explicitly due to the noninvariance of the fermion integration measure, the so-called axial anomaly. It is not a symmetry of the quantized QCD. Consequently no isosinglet Goldstone boson exists within the two-flavor QCD and the η meson(s) are heavier than the pion, attributed to the anomaly. In addition to the anomaly also the chiral condensate breaks this symmetry.

Both symmetry breaking signals are related to low lying modes of the Dirac operator. The axial anomaly involves the topological charge of the gauge configuration, which is proportional to the net number of exactly chiral (zero) modes via the Atiyah-Singer index theorem [2]. The chiral condensate is associated with the density of the Dirac operator's low lying (but nonzero) modes [1]. The nonvanishing quark condensate indicates breaking of both symmetries.

IV. GAUGE CONFIGURATIONS

For our analysis we used 161 gauge field configurations [17,18] of lattice size $16^3 \times 32$; with the lattice spacing $a = 0.144(1)$ fm this corresponds to a spatial size of 2.3 fm. The simulation includes two degenerate flavors of light fermions and a corresponding pion mass of $m_\pi = 322(5)$ MeV. For the dynamical quarks of the configurations as well as for the valence quarks the so-called chirally improved Dirac operator [19,20] has been used. This operator is an approximate solution to the Ginsparg–Wilson equation and therefore exhibits better chiral properties than

the simpler Wilson Dirac operator while being less expensive by an order of magnitude—in terms of computation time—in comparison to the chirally exact overlap operator.

We calculated up to the lowest 256 eigenmodes of the Dirac operator D and up to lowest the 512 eigenmodes of the Hermitian operator D_5 using ARPACK, which is an implementation of the Arnoldi method to calculate part of the spectrum of arbitrary matrices [21].

The quark propagator S is determined by inverting the Dirac operator for a given source. Instead of using point sources we use Jacobi smeared sources [22,23] that are approximately of Gaussian shape. Their shape was adjusted to a width of about 0.27 fm [17]. The low mode contribution $S_{\text{Im}5(k)}$ to the quark propagator, see (1), has to be multiplied with the same sources as the full propagator S in order to achieve the correct reduced propagators $S_{\text{red}5(k)}$.

V. MESONS

We restrict ourselves to the study of isovectors, in particular, the chiral partners:

- (i) The vector mesons ρ ($J^{PC} = 1^{--}$) with interpolating fields $\bar{u}(x)\gamma_i d(x)$ and $\bar{u}(x)\gamma_4\gamma_i d(x)$ and a_1 ($J^{PC} = 1^{++}$) with interpolating field $\bar{u}(x)\gamma_i\gamma_5 d(x)$; in a chirally symmetric world the vector and the axial vector interpolator get mixed via the isospin axial transformations.
- (ii) The pseudoscalar π ($J^{PC} = 0^{-+}$) with interpolating fields $\bar{u}(x)\gamma_5 d(x)$ and $\bar{u}(x)\gamma_4\gamma_5 d(x)$. We also study the scalar a_0 ($J^{PC} = 0^{++}$), $\bar{u}(x)d(x)$, which would get mixed with $\bar{u}(x)\gamma_5 d(x)$ via the $U(1)_A$ transformation.

(In the interpolators γ_4 denotes the Dirac matrix in Euclidean time direction.)

We compute from the quark propagators meson propagators, projected to vanishing momentum and determine the hadron masses from a range of Euclidean time values where the correlation function exhibits exponential decay. The final errors are statistical only and obtained by standard jackknife elimination sampling.

VI. RESULTS

A. Low mode sector

Figure 1 shows the integral over the distribution $H(|\mu|)$ of the (real) eigenvalues of D_5 . The scale is set by the lattice spacing. There is a transition region up to roughly twice the size of the quark mass (for this simulation the unrenormalized mass calculated from the axial Ward identity is 15 MeV [18]) corresponding to $\mathcal{O}(16)$ eigenmodes, as also observed in, e.g., [24–27]. As will be seen below, this is in accordance with the behavior observed for the meson propagators.

For the overlap operator the real eigenvalues correspond to exact chiral modes, the zero modes. This is no longer true for Wilson-type operators. There one may associate

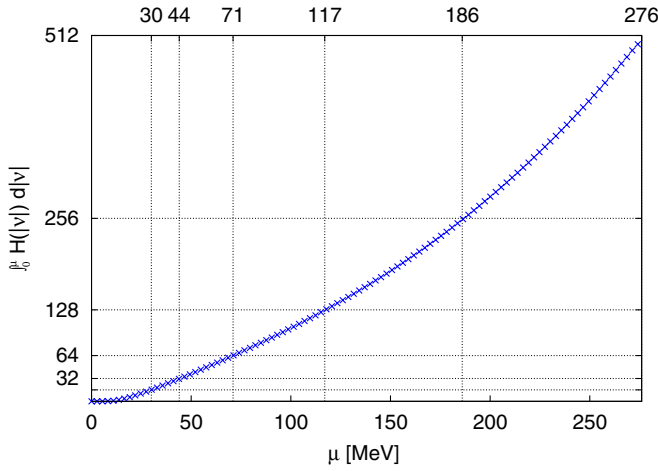


FIG. 1 (color online). The integrated eigenvalue density for the lowest 512 (absolute) eigenvalues of D_5 . The eigenvalues are scaled according to the lattice spacing. The number on the upper axis indicates the values of μ where there are 16, 32, 64, 128, 256, and 512 eigenvalues below that value.

zero modes with real eigenvalues, although their chirality is not unity. For the Hermitian Dirac operator there is no simple method to identify these would-be zero modes, and thus all we can say is that the lowest eight modes include a significant number (if not all) of the would-be zero modes (instantons).

Before we construct meson correlators out of reduced quark propagators, let us first consider meson correlators approximated by the lowest k modes only, using propagators $S_{\text{lm}5(k)}$, see (1).

In Fig. 2 we compare the pseudoscalar correlator using standard full propagators to the correlators using only the lowest modes of the Hermitian Dirac operator D_5 . For the two pseudoscalar operators the exponential pion decay behavior sets in much earlier (at lower numbers of eigenmodes) for the interpolator $\bar{u}\gamma_5 d$ than for the other interpolator $\bar{u}\gamma_4\gamma_5 d$. Clearly the first one is stronger, dominated by the low lying modes whereas the short time region—where excited states dominate—gets

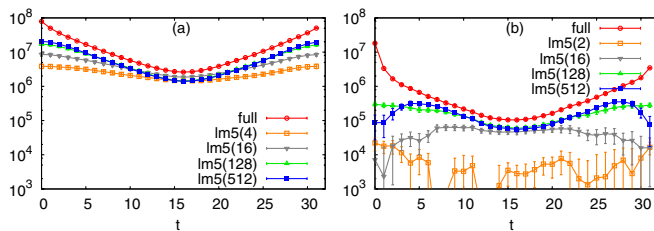


FIG. 2 (color online). Low mode contribution to the correlators for the $J^{PC} = 0^{-+}$ sector in comparison to the correlators from full propagators with interpolators (a) $\bar{u}\gamma_5 d$ and (b) $\bar{u}\gamma_4\gamma_5 d$. The number of included modes is shown in the legend.

saturated much more slowly. Comparing with the result for an equivalent approximation for the non-Hermitian Dirac operator (not shown here), we find that less eigenmodes of D_5 are needed to obtain a similar quality of approximation of the correlators with full propagators. These results agree with the observations in [6,7,13].

B. Removing the low mode sector

Figure 3 shows the meson propagators for various stages of low mode *removal*, always in comparison with the full propagator, and Fig. 5 combines the corresponding mass fits to the regions of exponential behavior.

All mass values (except for the ρ) exhibit a strong dependence on the truncation of the lowest eigenmodes; from truncations levels of ~ 16 modes upwards (corresponding to quark masses of approximately 30 MeV) all mass values then follow a roughly parallel, rising behavior. The range of exponential behavior of the correlators shrinks, as can be seen in the log plots in Fig. 3.

The effective mass plots (the local two-point approximation of the derivative of the logarithm of the correlators) in Fig. 4 indicate the regions, where an exponential fit to the correlators has been done. We find that the fluctuation typically decreases with increasing reduction. This may be related to the relative importance of the noisy low lying modes in the quark propagators.

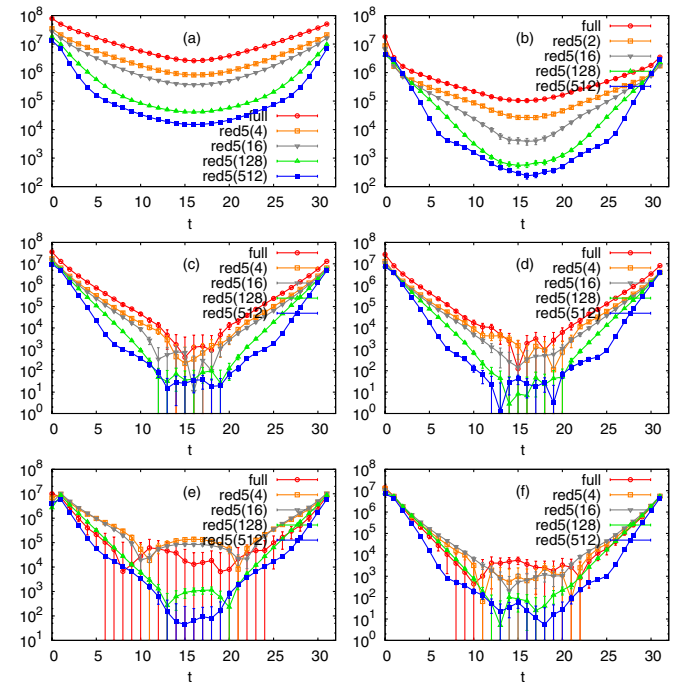


FIG. 3 (color online). Correlation functions for the *reduced* interpolators as compared to the correlators from full propagators. Top: $J^{PC} = 0^{-+}$ with interpolators (a) $\bar{u}\gamma_5 d$, (b) $\bar{u}\gamma_4\gamma_5 d$. Middle: $J^{PC} = 1^{-+}$ with (c) $\bar{u}\gamma_i d$, (d) $\bar{u}\gamma_4\gamma_i d$. Bottom: Reduced (e) $J^{PC} = 0^{++}$ ($\bar{u}d$) and (f) $J^{PC} = 1^{++}$ ($\bar{u}\gamma_i\gamma_5 d$).

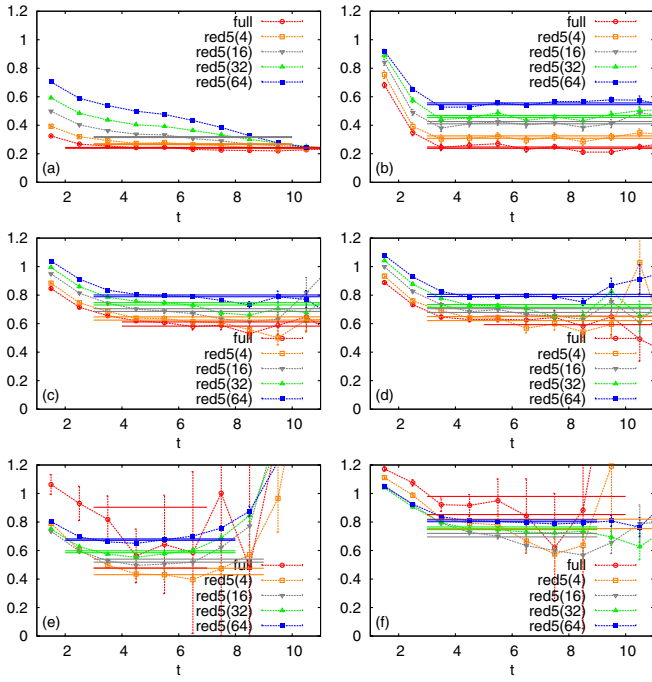


FIG. 4 (color online). Effective mass plots for the *reduced* interpolators as compared to the full propagators. For the notation (a–f) see Fig. 3.

In [28] the parity-chiral group and the effect of symmetry breaking on the meson spectrum is discussed. For example, whereas the $U(1)_A$ breaking lifts the degeneracy between pion and a_0 (and between η and f_0) the breaking of the chiral $SU(2)_L \times SU(2)_R$ symmetry is related to the mass differences of pion and f_0 (and a_0 and η). From Fig. 5 we find drastic sensitivity on low modes for both, the pion interpolator masses and the a_0 mass. At low truncation levels the a_0 mass rapidly drops; it does not drop down

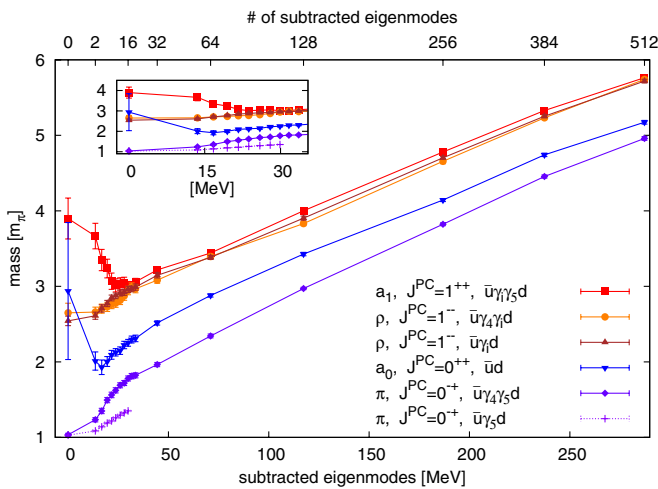


FIG. 5 (color online). The masses of all considered mesons as a function of the reduced spectrum, subtracting the 0–512 lowest modes of D_5 .

to the pion mass value. This might indicate some remnant of the anomaly breaking for the $J = 0$ states.

The pion interpolators exhibit a puzzling behavior. The classical pion interpolator $\bar{u}\gamma_5 d$ quickly loses its exponential behavior at larger (Euclidean) distances; only a more massive decay signal is observed at smaller distances (Fig. 3). From truncation level 16 onwards we therefore do not exhibit mass values in Fig. 5 for that interpolator. A fit to the very small time slices gives a mass approaching the mass value from the second interpolator $\bar{u}\gamma_4\gamma_5 d$ with the pion quantum numbers, which couples due to PCAC (partial conservation of the axial current, proportional to the quark mass).

For the $J^{PC} = 1^{--}$ vector meson ρ there are two chiral representations, which correspond to the vector interpolator $\bar{u}\gamma_i d$ and (Dirac-)tensor interpolator $\bar{u}\gamma_4\gamma_i d$. Their chiral partners [28] are the a_1 and the h_1 mesons, respectively. We did not determine the h_1 mass, since its interpolator includes disconnected graphs (it is an $I = 0$ state). There is no noticeable splitting between the two ρ -interpolators for all stages of truncation. We do find, however, intriguing behavior comparing the ρ mass with the a_1 result. Starting out quite differently for the full quark propagator, the masses approach each other and are compatible with each other from truncation level 8 onwards. This indicates restoration of the $SU(2)_L \times SU(2)_R$ symmetry for $J = 1$ states. The very fact that all three interpolators (vector, tensor, and axial vector) give the same mass hints to the restoration of the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry for $J = 1$ states. The latter could be reliably concluded, however, only after studying of the h_1 meson.

VII. CONCLUSIONS

The low lying eigenvalues of the Dirac operator are usually associated with chiral symmetry breaking. We have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator. This allows us to study their influence on certain hadron masses. Because of the relationship of the low eigensector with chiral symmetry breaking, this amounts to partially restoring chiral symmetry (in the valence quarks).

We find drastic behavior for some meson interpolators when starting to remove low eigenmodes. At truncation level 16 the behavior saturates and then the mass values rise uniformly with roughly parallel slopes. The confinement properties remain intact, i.e., we still observe clear bound states for most of the studied isovector (scalar, axial vector, and vector) mesons. An exception is the pion, where no clear exponential decay of the correlation function is seen in the $\bar{u}\gamma_5 d$ interpolator, but a massive state is seen in the $\bar{u}\gamma_4\gamma_5 d$ interpolator. The mass values of the vector meson chiral partners a_1 and ρ approach each other rapidly when eight or more low modes are removed.

We conclude that essential confinement properties remain intact, even when the low eigenmodes of the Dirac operator are removed in the valence sector. Restoration of chiral symmetry is observed in that approximation.

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