Lifshitz-like systems and AdS null deformations

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Following K. Balasubramanian and K. Narayan [J. High Energy Phys. 08 (2010) 014], we discuss certain lightlike deformations of $\mathrm{AdS}_5 \times X^5$ in type IIB string theory sourced by a lightlike dilaton $\Phi(x^+)$ dual to the $\mathcal{N} = 4$ super Yang-Mills theory with a lightlike varying gauge coupling. We argue that, in the case where the x^+ direction is noncompact, these solutions describe anisotropic 3 + 1-dim Lifshitz-like systems with a potential in the x^+ direction generated by the lightlike dilaton. We then describe solutions of this sort with a linear dilaton. This enables a detailed calculation of two-point correlation functions of operators dual to bulk scalars and helps illustrate the spatial structure of these theories. Following this, we discuss a nongeometric string construction involving a compactification along the x^+ direction of this linear dilaton system. We also point out similar IIB axionic solutions. Similar bulk arguments for x^+-noncompact can be carried out for deformations of $\mathrm{AdS}_4 \times X^7$ in M theory.

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I. INTRODUCTION

It is interesting to explore generalizations of holographic duality to physical systems with nonrelativistic symmetries, with a view towards possible interfaces with condensed matter systems (see e.g. [1–3] for reviews). In this paper, we discuss Lifshitz-like fixed points from a holographic point of view.

Lifshitz points arise in various condensed matter systems, e.g. magnetic systems with antiferromagnetic interactions, dimer models, liquid crystals, and so on (see e.g. [4,5] for lucid descriptions). They exhibit the anisotropic scaling $t \to \lambda^z t$, $x_i \to \lambda x_i$, with z the dynamical exponent. A Landau-Ginzburg description for such theories with $z = 2$ has the effective action $S = \int dt d^2x ((\partial_t \varphi)^2 - \kappa(\nabla^2 \varphi)^2)$. Aspects of correlation functions in these theories have been discussed in e.g. [6,7].

Holographic dual gravitational models of such theories were described in [8]; these spacetimes $ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dx_i^2 + dr^2}{r^2}$, $i = 1, 2$, arise as solutions to 4-dim Einstein gravity with a cosmological constant coupled to a massive Abelian gauge field. The symmetries exhibited by this spacetime are time translations, spatial translations/rotations, as well as the anisotropic scaling above; these are smaller than the Galilean symmetries explored holographically in [9–24]. Previous attempts at string or supergravity constructions of such spacetimes Lif_4 include e.g. [16,25–29].

In [30], certain lightlike deformations of $\mathrm{AdS}_5 \times X^5$ sourced by a lightlike dilaton in type IIB string theory (as well as those of $\mathrm{AdS}_4 \times X^7$ in M theory) were argued to give rise, upon dimensional reduction, to $z = 2$ Lifshitz spacetimes in 3 + 1 and 2 + 1 dimensions. For the AdS_5 case, these are dual to a discrete light cone quantization (DLCQ) of the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory with a gauge coupling that varies along the compact direction. Some of the supporting evidence includes symmetry arguments from both the bulk and dual field theory points of view, as well as matching of certain equal-time two-point correlation functions with those found in [8]. These constructions were found to nicely generalize [31] to a large family of similar $z = 2$ solutions with various other fields incorporated. Lifshitz-like solutions with more general values of the dynamical exponent z have been constructed in [32] (see also related work in [33–38]).

In this paper, we continue to explore the apparently simpler systems in [30], but now without any compactification. This is a lightlike deformation of $\mathrm{AdS}_5 \times X^5$ sourced by a null dilaton, dual to the $\mathcal{N} = 4$ SYM with a gauge coupling varying along one lightlike direction. This system turns out to be interesting and exhibits spatially anisotropic 3 + 1-dim Lifshitz-like symmetries with dynamical exponents $z = 2$ in the x_i, x^- directions and $z = \infty$ in the $x^\pm$ directions. The metric and dilaton respect the scaling symmetry but break x^+-translation invariance. In addition, the lightlike dilaton configuration gives rise to a potential in the x^+ direction.

For the particular case of a dilaton that is linear in the lightlike x^+ direction, the bulk Einstein metric becomes independent of x^+ and the spacetime simplifies. In particular, this enables a detailed calculation of the two-point correlation function for operators dual to bulk scalar modes. The resulting structure obtained from this AdS/CFT calculation bears some similarity to that found in the effective gravity Lifshitz hologram of [8]. However, there is further structure in this case due to the linear dilaton configuration, which is reminiscent of Liouville-like walls in theories in $c = 1$ string theory. The linear dilaton x^+ potential has some reflection in the two-point momentum space correlation function for massless scalars (dual to dimension-4 operators) which contains some structure resembling a mass gap in the x^+ direction. This suggests that solutions of this form will, in general, exhibit features reflecting the spatial x^+ potential generated by the dilaton.

Much of this bulk discussion for x^+-noncompact can be carried out for similar lightlike deformations of $AdS_4 \times X^7$ in M theory, giving insight into 2 + 1-dim Lifshitz-like field theories possibly dual to lightlike deformations of Chern-Simons theories arising holographically on M2-branes [39].

We then discuss a possible dimensional reduction of a linear dilaton system involving a nongeometric string construction using the S-duality of the IIB theory. We also point out very similar Lifshitz-like solutions sourced by the axion in IIB string theory. We finally close with a discussion including comments on some specific solutions.

II. REVIEWING $z = 2$ LIFSHITZ SPACETIMES FROM ADS NULL DEFORMATIONS WITH A NULL DILATON

In [30], we studied null deformations of $\mathrm{AdS} \times X$ sourced by a lightlike dilaton in 10- or 11-dim supergravity or string (or M) theory, and argued that upon dimensional reduction (DLCQ) they represent gravity duals of $z = 2$ Lifshitz fixed points in 2 + 1 or 1 + 1 dimensions. The 10- or 11-dim bulk system with x^+-compact is

$$\begin{aligned} ds^2 &= \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + \gamma w^2(\Phi')^2(dx^+)^2] \\ &\quad + \frac{dw^2}{w^2} + d\Omega_S^2, \\ \Phi &= \Phi(x^+), \end{aligned} \tag{1}$$

with a corresponding 5- or 4-form field strength. The constant γ is $\gamma = \frac{1}{4}$ for AdS_5 and $\gamma = \frac{1}{2}$ for AdS_4 (the $d\Omega_S^2$ is the metric for S^5 or X^7, respectively, with X^7 being some Sasaki-Einstein 7-manifold).

It is natural to interpret x^- as the time variable since a constant-x^- surface is spacelike (since $g^{--} < 0$), while a constant-x^+ surface is null. The spacetime (1) exhibits the following symmetries: translations and rotations in x_i, translations in $x^- \equiv t$ (time), and a $z = 2$ scaling $x^- \to \lambda^2 x^-$, $x_i \to \lambda x_i$, $w \to \lambda w$ (x^+ being a compact direction does not scale). Possible Galilean boosts $x_i \to x_i - v_i x^-$, $x^+ \to x^+ - \frac{1}{2}(2v_i x_i - v_i^2 x^-)$ are broken by the $g_{++} \sim (\Phi')^2$ term. (If $g_{++} = 0$, this is essentially AdS in light-cone coordinates and the system upon DLCQ has a Schrodinger symmetry, as discussed in e.g. [11,14,15]).

In the AdS_5-deformed case, the gauge theory dual to these systems can be identified to be the DLCQ of the $\mathcal{N} = 4$ super Yang-Mills theory with a gauge coupling varying along the x^+ direction as $g_{\mathrm{YM}}^2 = e^{\Phi(x^+)}$. The boundary metric $\lim_{w\to 0} ds_4^2$ is flat, so the gauge theory lives on flat spacetime. From the point of view of the dual gauge theory, the symmetry structure is intuitively clear: noting that the DLCQ of a relativistic field theory gives a nonrelativistic (Galilean) system, we see that the gauge coupling varying along the x^+ direction then breaks the x^+-shift symmetry, reducing the Galilean symmetry down to a Lifshitz one.

It can be checked directly that these spacetimes (1) along with the scalar Φ and appropriate 5-form (or 4-form) field strength are solutions to the 10-dim (or 11-dim) supergravity equations. For instance, there is no S^5 or X^7 dependence, and the resulting 5- or 4-dim system, with an effective cosmological constant from the flux, solves the equation $R_{MN} = -dg_{MN} + \frac{1}{2}\partial_M\Phi\partial_N\Phi$, with $d = 4$, 3, for AdS_{d+1}, being the 5- or 4-dim effective cosmological constant. Finally, the lightlike nature ensures that the scalar equation of motion is automatically satisfied. However, it is worth mentioning that the coordinate transformation $w = re^{-f/2}$, $x^- = y^- - \frac{w^2 f'}{4}$ recasts these spacetimes (1) into the form (we set the AdS radius $R = 1$)

$$\begin{aligned} ds^2 &= \frac{1}{r^2}[e^{f(x^+)}(-2dx^+dy^- + dx_i^2) + dr^2] + d\Omega_5^2, \\ \Phi &= \Phi(x^+), \end{aligned} \tag{2}$$

with the 4-dim part being conformal to flat space, and the boundary metric becoming $e^f\eta_{\mu\nu}$; indeed, this is where the AdS_5-deformed systems were originally found [40–43] (see also [44,45]).[1] These are Penrose-Brown-Henneaux transformations, a subset of bulk diffeomorphisms leaving the metric invariant (in Fefferman-Graham form), and acting as a Weyl transformation on the boundary. We will refer to the coordinate system in (2) as conformal coordinates in what follows. The only nonzero Ricci component is R_{++}, giving $R_{++} = \frac{1}{2}(\partial_+\Phi)^2$, i.e. $R_{++} = \frac{1}{2}(f')^2 - f'' = 2\gamma(\Phi')^2$, with $\Phi' \equiv \frac{d\Phi}{dx^+}$, $f' = \frac{df}{dx^+}$.

The AdS_5-deformed solutions were shown to preserve half (light-cone) supersymmetry in the form (2) in [40]. Supersymmetry was also shown for various solutions in the more general family in [31] which are generalizations of the metric form (1).

An argument for the dimensional reduction of the spacetime (1) along the compact x^+ direction was given in [30]. This suggests that the system has the right structure, although a clear Wilsonian separation-of-scales argument allowing for a standard Kaluza-Klein reduction of this metric is difficult due to the x^+ dependence of g_{++} in (1) for generic $\Phi(x^+)$.

Further checks involve the equal-time two-point correlation function of operators dual to bulk scalar modes; the bulk calculation in this spacetime agrees with the spatial power-law behavior noted by [8], as we review briefly now. In the conformal coordinates (2), the holographic two-point correlation function for operators dual to scalars can be found in closed form [41]; this was used in [30] to show agreement of the equal-time expression $\frac{1}{[(\Delta x_i)^2]^\Delta}$

[1] After this paper appeared on the arXiv, we were informed of earlier solutions representing null deformations of AdS_3 in the context of Wess Zumino Witten models [46].

with that in [8]. The conformally flat boundary metric is $e^{f(x^+)}\eta_{\mu\nu}$, and the boundary coupling of the bulk mode φ is $\int d^3x dx^+ e^{2f(x^+)}\mathcal{O}\varphi$. Then the holographic boundary action is

$$S = C\int d^4x d^4x' e^{3f(x^+)/2}e^{3f(x^{+\prime})/2}\varphi(x^+,\vec{x})\varphi(x^{+\prime},\vec{x}') \times \left(\frac{\Delta\lambda}{\Delta x^+}\right)^{1-\Delta}\frac{1}{[(\Delta\vec{x})^2]^\Delta},$$
$$(\Delta\vec{x})^2 = -2(\Delta x^+)(\Delta x^-) + \sum_{i=1,2}(\Delta x^i)^2,$$
$$\Delta = 2+\nu = 2+\sqrt{4+m^2}, \tag{3}$$

where C is a constant, $\lambda = \int e^{f(x^+)}dx^+$ is the affine parameter along null geodesics stretched solely along x^+, and $(\Delta\vec{x})^2$ is the four-dimensional distance element. As it stands, this is a 4-dim field theory boundary action; to obtain an action for an effective 3-dim boundary field theory, we need to dimensionally reduce over the x^+ direction. In the compactified limit $\Delta x^+ \ll \Delta x^-$, Δx_i, it is consistent to approximate $\frac{\Delta\lambda}{\Delta x^+}\sim\frac{d\lambda}{dx^+} = e^f$, and $e^{f(x^+)}\sim 1$, essentially smearing the x^+ dependence relative to the uncompactified dimensions. In this approximation, we can read off the equal-time two-point function as $\langle\mathcal{O}(x_i)\mathcal{O}(x_i')\rangle \sim \frac{1}{[\sum_i(\Delta x^i)^2]^\Delta}$. Likewise, for two points at essentially the same spatial location (small Δx_i), we have $\langle\mathcal{O}(t)\mathcal{O}(t')\rangle\sim\frac{1}{(\Delta x^-)^\Delta}$, where we have suppressed the x^+ integrals. The additional x^+ dependences distinguish this from a Galilean theory arising from a DLCQ. This power-law spatial and temporal falloff behavior and the associated scaling again vindicate the $z=2$ Lifshitz scaling; similar power-law behavior was exhibited for some simple operators in the free Lifshitz field theory in [6].

The corresponding calculation in the metric (1) appears relatively difficult to do since the g_{++} piece is x^+ dependent for general $\Phi(x^+)$ and ruins a simple separation of variables approach to solve the scalar wave equation.

The 11-dim $z=2$ Lifshitz-like solutions in (1) involve a dimensional reduction of null deformations of $AdS_4\times X^7$, with X^7 being some Sasaki-Einstein 7-manifold. In this case, the scalar does not have any natural interpretation in the 11-dim theory directly; it arises instead from the 4-form flux after compactification on X^7. We expect that these deformations are dual to the DLCQ of appropriate lightlike deformations of Chern-Simons theories arising holographically on M2-branes [39] (and various generalizations).

We mention in passing that there also exist time-dependent deformations of AdS_5 [40,42,43] and AdS_4; in particular, the asymmetric Kasner-like solutions exhibit interesting (anisotropic) Lifshitz scaling symmetries. These solutions are qualitatively different from the null ones above.

Finally, [30] also discussed solutions of 5-dimensional gravity with a negative cosmological constant and a massless complex scalar, that are similar to the null solutions (1) above; these, upon dimensional reduction, give rise to 2 + 1-dim Lifshitz spacetimes. This 5-dim solution can be uplifted to 11-dimensional supergravity.

III. x^+-NONCOMPACT AND ANISOTROPIC LIFSHITZ SYSTEMS

We want to now study the IIB null-deformed system (1),

$$ds^2 = \frac{1}{w^2}\left[-2dx^+dx^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2\right] + \frac{dw^2}{w^2},$$
$$\Phi = \Phi(x^+), \tag{4}$$

but with x^+ treated as a noncompact direction. First, it is worth noting that the functional dependence of the dilaton is really $\Phi = \Phi(Qx^+)$, the constant Q being a parameter of mass dimension 1. Lightlike boosts $x^+\to\lambda x^+$, $x^-\to\lambda^{-1}x^-$ were symmetries in the original system with $Q=0$: these are broken in the present case. These boosts rescale Q as $Q\to\frac{Q}{\lambda}$ so that theories with different values of Q are related by these boosts.

In this case, the symmetries include time x^- translations and spatial x_i translations/rotations; translations in the x^+ direction are broken by the nontrivial x^+ dependence. In addition, the metric (4) exhibits the scaling

$$w\to\lambda w, \qquad x_i\to\lambda x_i,$$
$$x^-\to\lambda^2 x^-, \qquad x^+\to\lambda^0 x^+. \tag{5}$$

The system of course contains the $z=2$ Lifshitz scaling symmetry $x_i\to\lambda x_i$, $x^-\to\lambda^2x^-$ in the 2 + 1 dimensions x^-, x_i (induced by the associated scaling of w). However, in addition, note that since x^+ does not scale, we effectively have $z=\infty$ Lifshitz scaling in the x^+, x^- directions [reading off the dynamical exponent as the ratio of the scaling of time (x^-) to the spatial one (x^+)]. Thus it appears best to interpret this system as a spatially anisotropic Lifshitz system, with $z=2$ scaling for the 2 + 1-dimensional x_i, x^- plane and $z=\infty$ scaling for the x^+, x^- directions. This is reminiscent of the anisotropic scaling[2] observed in the D3–D7 construction of [28] and the scalings in the dilatonic black brane solutions in [47]; however, in this case, the scaling (5) is an actual symmetry of (4), respected by the dilaton $\Phi(x^+)$ as well. This sort of anisotropic scaling would seem to also hold for some of the more general solutions in [31]. The dilaton, however, breaks x^+-translation invariance, and in fact gives rise to a spatial potential in the x^+ direction; this gives rise to additional structure in observables in this system which reflect this effective x^+ potential. In the next subsection,

[2] We recall that radial Kasner-like solutions of the form $ds^2 = \frac{1}{r^2}[dr^2 - r^{2p_0}dt^2 + \sum_i r^{2p_i}(dx^i)^2]$ exist, sourced by several massive vector fields [16].

we will analyze the case of a linear dilaton potential in some detail, illustrating some of this structure.

The system (4) also exhibits the symmetry $x_i \to x_i - v_i x^+$, with x^+ unchanged and a corresponding shift in x^-; this is broken if x^+ is compact. This symmetry, however, is not a Galilean boost since x^+ cannot be interpreted as time; x^- is the natural time coordinate here, consistent with constant-x^- surfaces being spacelike [$g^{--} \sim -w^4(\Phi')^2 < 0$]. This sign of g^{--} appears crucial for this interpretation and the $z = \infty$ scaling we have mentioned above. Some of the more general solutions in [31] have $g_{++} < 0$, and are more akin to $z = 0$ Schrodinger systems $ds^2 = -dt^2 + \frac{dx_i^2 + dt d\xi + dr^2}{r^2}$. Indeed, transforming $t \to ix^+$, $\xi \to ix^-$ recasts this solution into the form in (4) with $\Phi' = \text{const}$. Thus its symmetries formally exist for (4) too. In the present case, it appears best to interpret (4) as an anisotropic Lifshitz-like system with a spatial x^+ potential stemming from the dilaton. This is corroborated by our discussion on linear dilatonic systems in the next subsection, where we also calculate some correlation functions which, in part, exhibit structure similar to those in [8]. There, we will also make further comments on this point.

As we have mentioned, these can be recast in conformal coordinates (2), with a conformally flat boundary metric $e^{f(x^+)}\eta_{\mu\nu}$. In these variables, the holographic two-point function for operators $\mathcal{O}(x)$ with boundary coupling $\int d^4x e^{2f(x^+)}\mathcal{O}(x)\varphi(x)$ to the bulk mode $\varphi(x)$ can be read off from (3) as [41]

$$\langle O(x)O(x')\rangle = e^{-f(x^+)/2} e^{-f(x'^+)/2}\left(\frac{\Delta\lambda}{\Delta x^+}\right)^{1-\Delta}\frac{1}{[(\Delta\vec{x})^2]^\Delta}, \tag{6}$$

with $\Delta = 2 + \sqrt{4 + m^2}$, $\lambda = \int e^{f(x^+)}dx^+$. For two points with $\Delta x^+ \ll \Delta x^-$, Δx_i, i.e. that are essentially on a constant-x^+ slice, we can approximate $\frac{\Delta\lambda}{\Delta x^+} \sim \frac{d\lambda}{dx^+} = e^f$, and $(\Delta\vec{x})^2 \sim (\Delta x_i)^2$. This gives $\langle O(x)O(x')\rangle \sim \frac{e^{-f(x^+)\Delta}}{[\sum_i(\Delta x_i)^2]^\Delta}$. The factor $e^{-f\Delta}$ here is a reflection of the fact that the conformally dressed operators $e^{f(x^+)\Delta/2}\mathcal{O}(x)$ in this conformally flat background $e^f\eta_{\mu\nu}$ behave like undressed operators in the flat space background, possessing a flat space two-point function. We also see that the equal-time correlator ($\Delta x^- = 0$) is $(\frac{\Delta\lambda}{\Delta x^+})^{1-\Delta}\frac{e^{-f(x^+)\Delta/2}e^{-f(x'^+)\Delta/2}}{[\sum_i(\Delta x_i)^2]^\Delta}$, exhibiting spatial power-law behavior in the x_i, similar to the equal-time two-point correlator in [8], but also possessing additional x^+ dependence. Note that this calculation has been done at the boundary $r = \epsilon$; recalling that the radial coordinates are related as $w = re^{-f(x^+)/2}$, this boundary differs from the corresponding boundary $w = \epsilon$ in the metric (4), although they are in the same conformal class.

In the next subsection, we discuss the case of a linear dilaton, in which case the holographic correlator can be calculated in the metric (4), giving some detailed insight into the structure of this system.

Linear-dilaton-like deformations

As we have seen, the AdS null-deformed solutions (4) have $g_{++} = \frac{(\Phi')^2}{4}$. For $\Phi' = \text{const}$, we see that the Einstein metric is independent of x^+. This is the case of a dilaton that is linear.

Consider $\Phi' = \text{const}$; this gives $\Phi = \Phi_0 + 2Qx^+$, which is a linear dilaton profile, the constant Q being a parameter (we have chosen the constant $2Q$ for convenience). Then the bulk spacetime and dilaton are

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2] + \frac{dw^2}{w^2},$$
$$\Phi = \Phi_0 + 2Qx^+. \tag{7}$$

The symmetries in this case, besides those mentioned above (5), also include translations in x^+ in the metric (7); however, there is a spatial x^+ potential stemming from the linear dilaton.

The action for a massless scalar $S = \frac{1}{G_5}\int d^5x\sqrt{-g}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ on restricting to modes propagating on a constant-x^+ surface, i.e. with no x^+ dependence ($\partial_+\varphi = 0$),

$$S = \frac{1}{G_5}\int\frac{d^4x dx^+}{w^5}\bigg[-\frac{w^4(\Phi')^2}{4}(\partial_-\varphi)^2 - 2w^2(\partial_-\varphi) \times(\partial_+\varphi) + w^2(\partial_i\varphi)^2 + w^2(\partial_w\varphi)^2\bigg]$$
$$= \frac{1}{G_5}\int dx_+\frac{d^4x}{w^5}[-Q^2w^4(\partial_-\varphi)^2 + w^2(\partial_i\varphi)^2 + w^2(\partial_w\varphi)^2]. \tag{8}$$

Such scalar modes see an effective $z = 2$ Lifshitz geometry in the 3 + 1-dim (x^-, x_i, w) part of the bulk. Modes propagating only in the 2 + 1-dim $(x^\pm, w)$ part of the bulk see $z = \infty$ Lifshitz scaling.

It is interesting to ask if a constant-x^+ hypersurface has an induced metric resembling that of a 4-dim $z = 2$ Lifshitz spacetime. Consider a static D5-brane probe stretched along x^-, x_i, w and an $S^2 \in S^5$. This is effectively a domain wall in the x^+ direction of the AdS_5 part of the bulk spacetime. Since a constant-x^+ hypersurface is null, it is difficult to explicitly realize a $z = 2$ $d = 4$ Lifshitz spacetime as the induced metric on the 4-dim part of the D5-brane probe. Consider the bulk metric $ds^2 = \frac{1}{w^2}[-2dx^+dx^- + w^2Q^2(dx^+)^2 + dx_i^2] + \frac{dw^2}{w^2} = (Qdx^+ - \frac{dx^-}{Qw^2})^2 - \frac{(dx^-)^2}{Q^2w^4} + \frac{dx_i^2}{w^2} + \frac{dw^2}{w^2}$. We see that on the slice $dx^+ = \frac{dx^-}{Qw^2}$, the induced metric is precisely $\text{Lif}_4^{z=2}$ times a compact space; however, this is not a well-defined hypersurface.

The dual gauge theory is the 4-dim $\mathcal{N} = 4$ SYM theory living on flat spacetime, the boundary metric being flat, with the gauge coupling lightlike-deformed as $g_{\rm YM}^2(x^+) = e^{\Phi(x^+)} \equiv g_s e^{2Qx^+}$, with $g_s = e^{\Phi_0}$. The linear-dilaton-like coupling gives a strong coupling Liouville-like wall at one x^+ end, while for $x^+ \ll 0$, the gauge theory becomes weakly coupled and arbitrarily calculable perturbatively. Correspondingly, the spacetime ceases to be reliable in this regime, where the string coupling becomes small. The string frame metric is $ds_{\rm str}^2 = e^{\Phi/2}ds^2$. This degenerates for $x^+ \to -\infty$, where the curvatures become large.

The fact that the dilaton has a non-normalizable deformation turned on means that the operator ${\rm Tr}F^2$ is sourced. Since the deformation is lightlike, there exist no nonzero contractions involving $\partial_+ g_{\rm YM}^2$ since there are no tensors with multiple upper + indices. Thus ${\rm Tr}F^2$ continues to be a marginal dim-4 operator.[3]

We will now calculate the two-point correlation function in this linear dilaton case (7); possible mode functions $\varphi(x) = e^{ik_-x^- + ik_ix^i}e^{g(x^+)}R(w)$ reduce the scalar wave equation $\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu\varphi) - m^2\varphi = 0$ to $-2ik_-g' + \frac{w^3}{R(w)}\partial_w(\frac{1}{w^3}\partial_w R(w)) - k_i^2 - \frac{m^2}{w^2} + w^2Q^2k_-^2 = 0$. With $g = \frac{-i\chi^2x^+}{2k_-}$, the radial equation becomes

$$w^3\partial_w\left(\frac{1}{w^3}\partial_w R(w)\right) - \left(k_i^2 + \chi^2 + \frac{m^2}{w^2} - w^2Q^2k_-^2\right)R(w) = 0. \tag{9}$$

Before we see this in detail, note that the boundary asymptotics of this equation near $w = 0$ show that the last term $w^2Q^2k_-^2$ in the equation is subdominant and the mode functions approach those of AdS in light-cone coordinates. Thus the two-point function in this leading approximation is $\frac{1}{[(\Delta\vec{x})^2]^\Delta}$; in particular, for points that are essentially on the same x^+ plane, i.e. $\Delta x^+ \ll \Delta x_i$, the two-point function exhibits spatial power-law behavior $\frac{1}{[\sum_i(\Delta x^i)^2]^\Delta}$.

In greater detail, the radial equation (9) is exactly solvable in terms of confluent hypergeometric functions [48]. Taking $R(w) = w^\Delta e^{\alpha w^2}f(w)$, and redefining

$$\begin{aligned} k^2 &= \chi^2 + k_i^2 \equiv -2k_+k_- + k_i^2, \\ \alpha &= -\frac{iQk_-}{2}, \\ \Delta &= 2 + \sqrt{4+m^2} = 2 + \nu, \end{aligned} \tag{10}$$

the radial equation becomes $z\frac{d^2f}{dz^2} + (\Delta - 1 - z)\frac{df}{dz} - (\frac{\Delta-1}{2} - \frac{k^2}{8\alpha})f = 0$ (where $z = -2\alpha w^2$), which is the confluent hypergeometric equation. This gives the momentum space bulk-to-boundary propagator [we have $\varphi(x^\mu, w) = \int d^4k\varphi(k)G(k,w)$]

$$\begin{aligned} G(k_i, k_+, k_-, w) &= \mathcal{N}(k)e^{ik_ix^i + ik_-x^- + ik_+x^+}w^\Delta e^{\alpha w^2} \\ &\quad \times U(a, c, -2\alpha w^2), \\ a &= \frac{\Delta - 1}{2} - \frac{k^2}{8\alpha} = \frac{\nu+1}{2} + \frac{k^2}{4iQk_-}, \\ c &= \Delta - 1 = \nu + 1, \end{aligned} \tag{11}$$

where $\mathcal{N}(k)$ is a normalization factor which we will choose so as to set $G(k, \epsilon) = e^{ik_\mu x^\mu}$ on a cutoff surface $w = \epsilon$. We have chosen the confluent hypergeometric function $U(a, c, z)$ in accordance with the requirement of regularity in the interior ($w \to \infty$) and the expectation that for $Q = 0$, this should reduce to the standard Bessel functions $w^2K_\nu(kw)$ (radial part) for AdS_5 in light-cone coordinates. Now from the near-boundary ($w \sim 0$) asymptotic form of the bulk-to-boundary propagator, we can identify the momentum space two-point correlation function as a ratio of the growing and decaying (non-normalizable and normalizable) pieces. For nonintegral ν (i.e. c), we have the confluent hypergeometric function asymptotics $U(a,c,z) \sim \frac{\pi}{\sin\pi c}(\frac{1}{\Gamma(1+a-c)\Gamma(c)} - \frac{z^{1-c}}{\Gamma(a)\Gamma(2-c)} + \ldots)$. Then the momentum space two-point correlation function can be read off as

$$\begin{aligned} \langle\mathcal{O}(k)\mathcal{O}(-k)\rangle &= -\nu 2^\nu\alpha^\nu\frac{\Gamma(-\nu)}{\Gamma(\nu)}\frac{\Gamma(a)}{\Gamma(a-\nu)} \\ &= -\nu 2^\nu\alpha^\nu\frac{\Gamma(-\nu)}{\Gamma(\nu)}\frac{\Gamma(\frac{1+\nu}{2} + \frac{k^2}{4iQk_-})}{\Gamma(\frac{1-\nu}{2} + \frac{k^2}{4iQk_-})}. \end{aligned} \tag{12}$$

For the ν integral, there are additional terms in the asymptotics of $U(a,c,z)$; the small-w expansion of the radial part of the bulk-to-boundary propagator, after appending an overall normalizing factor so as to make $G(k_i, k_+, k_-, w = \epsilon) = e^{ik_\mu x^\mu}$, is

$$\begin{aligned} &(-2\alpha)^2\Gamma(a)w^4e^{\alpha w^2}U(a, \nu+1, -2\alpha w^2) \\ &= \left(1 + \alpha w^2 + \frac{\alpha^2w^4}{2} + \ldots\right)\left(1 + w^4\frac{(-1)^{\nu+1}(-2\alpha)^\nu}{\nu(\Gamma(\nu))^2}\right. \\ &\quad \left.\times\frac{\Gamma(a)}{\Gamma(a-2)}(\log(-2\alpha w^2) + \psi(a)) + \ldots\right), \end{aligned} \tag{13}$$

where $\psi(a)$ is the digamma function. After removing unimportant terms—those removable by local counterterms and contact terms—the momentum space two-point correlation function (noting that G vanishes in the interior $w \to \infty$) can be read off from the boundary action $\int d^4k\varphi(k)\varphi(-k)(\sqrt{-g}g^{ww}G(-k,w)\partial_wG(k,w))|_{w=\epsilon}$; it is basically the coefficient of the w^4 term in the second bracket (higher order terms vanish as $\epsilon \to 0$). In particular, for the case of massless scalars with $\Delta = 4$, $\nu = 2$, this can be read as

[3] After this paper appeared on the arXiv, we were informed of [35], where some aspects of lightlike, or chiral, deformations of AdS/CFT have been discussed.

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim 4(\alpha a-\alpha)(\alpha a-2\alpha)(\log(2\alpha) + \psi(a)) \sim ((k_i^2-2k_+k_-)^2+4Q^2k_-^2) \times \left(\log(iQk_-)+\psi\left(\frac{3}{2}+\frac{k_i^2-2k_+k_-}{4iQk_-}\right)\right). \tag{14}$$

In the limit $Q\to 0$, we have $\alpha\to 0$, $a\to\infty$, so that using the asymptotics $\psi(a)\sim \log a$ and (10), we recover the familiar AdS_5 correlator $k^4\log k^2$ as a check. For other integral ν values, we obtain a correlator of the form $2^\nu(\alpha a-\alpha)\ldots(\alpha a-\nu\alpha)(\log(-2\alpha)+\psi(a))$ which asymptotes to $k^{2\nu}\log k^2$ in the $Q\to 0$ (AdS) limit.

This calculation, beginning with (9), and the resulting momentum space expressions (12) and (14) are structurally similar to those in [8]. In fact, for $k_+\sim 0$, we see that the radial scalar equation (9), using (10) reduces precisely to the equation for a scalar in the 4-dim $z=2$ Lifshitz background $ds^2=-\frac{dt^2}{r^4}+\frac{dx_i^2+dr^2}{r^2}$. Thus the corresponding momentum space correlation functions (12) and (14) (continued to Euclidean signature) are in fact identical to those discussed (in the Euclidean calculation) in [8], as expected for the dimensional reduction of the 5-dim AdS lightlike deformation to the 4-dim Lifshitz one argued in [30,31] (which we expect to correspond to the $k_+\sim 0$ sector here).

For the case with the x^+ direction treated as noncompact, there are more features here, due to the linear dilaton configuration which acts like a potential in the x^+ direction. [We note as an aside that this calculation also holds if we change $Q^2\to -Q^2$, changing the sign of g^{--}: In this case x^+ becomes the natural time variable appropriate for this $z=0$ Schrodinger system, and k_- (coupling to x^-) is then a spatial momentum.] Some insight into the structure here is obtained by noting that

$$(k_i^2-2k_+k_-)^2+4Q^2k_-^2 = (k_i^2-2(k_+-iQ)k_-)(k_i^2-2(k_++iQ)k_-). \tag{15}$$

Thus the effective x^+ momentum is shifted as $k_+\to k_+\pm iQ$, and $e^{ik_+x^+}\to e^{ik_+x^+}e^{\pm Qx^+}$. This is reminiscent of the Liouville-like wall in $c=1$ string theory [49,50]. Also note that for $k_i=0$, we have $(k_i^2-2k_+k_-)^2+4Q^2k_-^2\to(k_+^2+Q^2)k_-^2$, which gives an effective mass gap in the x^+ direction. Another corroboration of this interpretation comes from looking at solutions to the wave equation in the SYM theory. The gauge field sector has the free action $S=\int d^4x\frac{1}{g_{YM}^2(Qx^+)}\mathrm{Tr}F^2$, which, for the transverse components A_i that are physical degrees of freedom in e.g. light-cone gauge, becomes $S_{A_i}=\int d^4xe^{-\Phi(x^+)}[(\partial_jA_i)^2-2(\partial_+A_i)(\partial_-A_i)]$, which is essentially two copies of a scalar moving in an x^+-dependent background. Then the wave equation $e^{\Phi(x^+)}\partial_+(e^{-\Phi(x^+)}\partial_-A_i)+\partial_-\partial_+A_i+\partial_j^2A_i=0$ for modes of the form $e^{ik_+x^++ik_-x^-+ik_ix^i}$ gives $k_i^2+2(k_++iQ)k_-=0$, i.e. $k_+=-\frac{k_i^2}{2k_-}-iQ$. The wave modes then become $e^{ik_ix^i+ik_-x^-+i(k_i^2/2k_-)x^+}e^{Qx^+}$, which are damped as $x^+\to-\infty$. Furthermore, for generic k_i, k_-, we see that the x^+ momentum k_+ is nonzero; i.e. generic waves will be forced to move along the x^+ direction due to the dilaton x^+ potential. Admittedly, this is in the free gauge theory, while our calculation in the weakly coupled bulk geometry applies to a strongly coupled gauge theory; however, the basic structure of the wave modes seems suggestive.

The scaling of the coefficient $(k_i^2-2k_+k_-)^2+4Q^2k_-^2$ in (14) is consistent with that of the x_i, $x^\pm$ in (5). To gain some insight into the $z=\infty$ scaling of the $x^\pm$ directions, note that the dispersion relation $\omega\sim k^z$ with $z\to\infty$ can be rewritten as $k\sim\omega^{1/z}\sim\omega^0$. In the $x^\pm$ directions ($k_i=0$), we see from the coefficient in (14) that k_+ has a damping piece independent of k_-, reflecting the x^+ potential given by the dilaton.

The momentum space correlator can now be used to evaluate position space two-point functions in certain limiting cases. For instance, as in [8], in the limit where the digamma function $\psi(a)$ has a dominant contribution and can be approximated by its leading constant term [for the argument of $\psi(a)$ being small], we can Fourier transform the coefficient $(k_i^2-2k_+k_-)^2+4Q^2k_-^2$ using Schwinger parameters and find $\frac{e^{-Q\Delta x^+}}{[\Delta x_i^2-2\Delta x^+\Delta x^-]^4}\times\frac{1}{\Gamma(-1-\delta)}\int_0^1dx\frac{e^{-2Q\Delta x^+x}}{x^2(1-x)^2}$, after regulating it (the pole near $x=0$ cancels with corresponding terms in the Γ prefactor). For $Q=0$, this is in fact just the correlator for AdS_5 in light-cone coordinates. For $Q\neq 0$, the last integral can be expressed in terms of incomplete Γ functions.

Finally, we note that we have calculated this two-point correlation function by imposing certain boundary conditions on the solution to the bulk scalar wave equation, which are natural generalizations of those in AdS_5. Presumably one can find interesting real-time structure by studying various other boundary conditions on this system as discussed in e.g. [1–3].

The bulk arguments here for x^+-noncompact can also be made for linear scalar deformations of $AdS_4\times X^7$ in M theory, the scalar arising from the flux components. The correlator then gives some insight into the spatial structure of 2 + 1-dim Lifshitz-like field theories. It would be interesting to explore this further.

IV. ON (NONGEOMETRIC) DLCQ OF A LINEAR DILATON LIFSHITZ SYSTEM

We will now revert to compactifying the x^+ direction as in [30]. The basic point there from the dual gauge theory point of view is that a DLCQ of the $\mathcal{N}=4$ SYM theory gives a nonrelativistic theory with Galilean symmetries with dynamical exponent $z=2$; then a varying coupling $g_{\mathrm{YM}}^2=e^{\Phi(x^+)}$ along the compact x^+ direction breaks the shift symmetry, giving Lifshitz symmetries. While a similar symmetries-level argument holds in the bulk geometry too, a more detailed dimensional reduction appears difficult, especially in terms of understanding if a Wilsonian separation-of-scales argument holds. For instance, since $g_{++} \sim (\Phi')^2$ contains x^+ dependence in general, the bulk 5-dim geometry does not admit a standard Kaluza-Klein reduction. We note, however, the exception $\Phi' = \text{const}$; this is a linear dilaton system that we have discussed in the previous section with the x^+ direction noncompact.

In this section, we would like to investigate whether we can compactify the x^+ direction with such a linear dilaton configuration, with a view to finding AdS null deformations (1) with $\Phi' = \text{const}$, admitting conventional Kaluza-Klein reduction along the x^+ circle.

With $\Phi' = \text{const}$, we have $\Phi = \Phi_0 + 2Qx^+$; i.e. we have a linear dilaton profile. *A priori*, this is in contradiction with the proposed compactification of the x^+ direction; a dilaton linear for all x^+ does not respect this. However, let us consider a piecewise linear dilaton configuration

$$\begin{aligned}\Phi &= \Phi_0 + 2Qx^+, && x^+ \in [0, L],\\ &= \Phi(L) - 2Q(x^+ - L), && x^+ \in [L, 2L], \ldots . \end{aligned} \tag{16}$$

This is a ‘‘sawtooth’’-like (piecewise) linear dilaton configuration $\Phi(x^+)$, plotted in Fig. 1. $\Phi(x^+)$ is a continuous but not smooth function on the x^+ line; at the locations $x^+ = L, 2L, \ldots$, we see that Φ' has a jump discontinuity from $+2Q$ to $-2Q$ (to be precise, define $\Phi' = +2Q$, $0 < x^+ \le L$, and $\Phi' = -2Q$ $L < x^+ \le 2L$, and so on). However, we see that the Einstein metric is smooth since the dilaton Φ appears in the metric as $g_{++} \sim (\Phi')^2$. Thus all metric properties (curvature, geodesics, and so on) are smooth also. The bulk Einstein metric is

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2 + dw^2]. \tag{17}$$

It is desirable to demand that $\Phi(x^+)$ be a continuous function on the unwrapped x^+ circle, i.e. on the x^+ line; this is true for the dilaton configuration (16) above. However, the dilaton $\Phi(x^+)$ is not periodic and does not

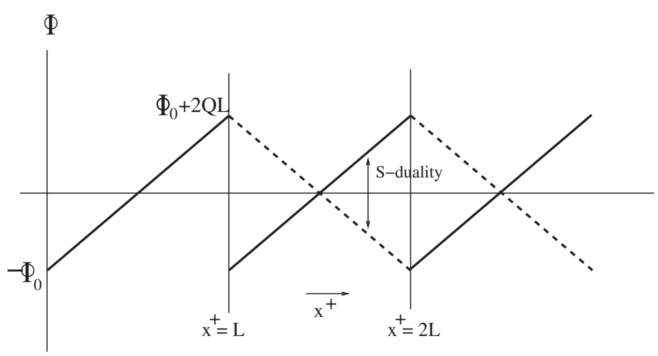

FIG. 1. A linear dilaton configuration and compactification up to S-duality.

therefore respect the compactification along the x^+ direction. We would like to understand if this dilaton configuration can somehow be made to respect the compactification, i.e. $\Phi(x^+ + kL) = \Phi(x^+)$, for any $k \in \mathbb{Z}$.

This does not appear to be possible in conventional supergravity *per se*. However, note that we have, at our disposal, the exact S-duality symmetry of the IIB string theory. It is therefore interesting to ask if we can use this to construct a solution with the dilaton $\Phi(x^+)$ periodic on the x^+ circle, possibly along the lines of nongeometric string constructions. Supergravity solutions involving nongeometric constructions with nontrivial winding around U-duality orbits have been studied previously in e.g. [51–55]. The S-duality in question here makes our construction nonperturbative, in contrast to some of these constructions that involve T-duality. However, the backgrounds in question here are considerably simpler in some ways: the absence of a nontrivial axion means that there are no nontrivial 7-brane sources of the sort that arise in F-theory constructions, nor are there singularities from degenerations of the fiber.

S-duality is the symmetry $\tau \to -\frac{1}{\tau}$, with $\tau = c_0 + ie^{-\Phi}$ the complexified axion-dilaton coupling. Since the axion is trivial in this background, the S-duality symmetry reduces to strong $\to$ weak coupling duality, i.e. $\Phi \to -\Phi$.

Using this, we see that it suffices to require e.g. $\Phi(x^+ + L) = -\Phi(x^+)$. From (16), at the end of the interval $x^+ \in [0, L]$, we have $\Phi(L) = \Phi_0 + 2QL$, which cannot equal $\Phi(0) = \Phi_0$. However, since $\Phi(0) \equiv -\Phi(0)$ up to S-duality, consider requiring $\Phi(L) = -\Phi(0)$; this is consistent since

$$\begin{aligned}&\Phi(L) = -\Phi(0) \Rightarrow 2\Phi_0 = -2QL,\\ &\text{i.e. } g_s = e^{\Phi_0} = e^{-QL}.\end{aligned} \tag{18}$$

This gives $\Phi(x^+) = 2Q(x^+ - \frac{L}{2})$ for $x^+ \in [0, L]$. We note that $\Phi = 0$ at $x^+ = \frac{L}{2}$, with $\Phi(x^+)$ being antisymmetric about this midpoint.

Now consider the interval $x^+ \in [L, 2L]$: from (16), we have $\Phi(x^+) = \Phi(L) - 2Q(x^+ - L) = -\Phi_0 - 2Q(x^+ - L)$. We see that on this interval, $\Phi(x^+) \equiv -\Phi(x^+) = \Phi_0 + 2Q(x^+ - L)$. Thus the S-duality symmetry $\Phi \to -\Phi$ amounts to flipping a local linear piece about the x^+ axis, i.e. flipping a downward sloping linear piece to an upward sloping one. Thus comparing the dilaton values at $x^+ = a \in [0, L]$ and $x^+ = L + a \in [L, 2L]$ using (16), we see

$$\begin{aligned}\Phi(L) = -\Phi_0 \Rightarrow (\Phi_0 + 2Qx^+)|_a\\ = -(\Phi(L) - 2Q(x^+ - L))|_{L+a}.\end{aligned} \tag{19}$$

In other words, up to S-duality, the piecewise linear dilaton configuration is x^+ periodic if the linear pieces are antisymmetric about the x^+ axis, i.e. if $\Phi_0 = \log g_s = -QL$. This is easy to see pictorially (Fig. 1).

Since the dilaton is periodic up to S-duality in this manner, the string frame metric appears to be well defined. It is unclear to us at this point if there is a geometric way to interpret this resulting nongeometric construction along the lines of e.g. [52,53].

Since the asymptotic value of the string coupling is not arbitrary but fixed by this nongeometric construction as $g_s = e^{\Phi_0} = e^{-QL}$, one could potentially worry if our solution is reliable and whether stringy corrections to this geometry are becoming important. In this regard, note first that the dilaton is always bounded so that there are no apparent singularities that wreck the solution anywhere (in contrast, the points on moduli space where the elliptic fiber degenerates correspond to the locations of the 7-brane singularities in F theory). Furthermore, note that the solution preserves half supersymmetry, suggesting the absence of various corrections. The supersymmetry of these solutions is closely related to the lightlike nature of these solutions, which suggests that higher derivative corrections to the solution in fact vanish, i.e. that these are exact string backgrounds (perhaps similar to $\mathrm{AdS}_5 \times S^5$ [56]). We have seen that a way to uplift the Lifshitz-like symmetries of the 4-dim Lifshitz system to the 5-dim system is to turn on lightlike deformations of the form (17) which reflect these Lifshitz-like symmetries. Thus the requirement of Lifshitz-like symmetries is effectively captured by the lightlike nature of these solutions, which would seem to preclude corrections to the higher dimensional solution. These arguments suggest that this nongeometric solution is reliable, insofar as standard dimensional reduction is concerned; the low energy or Einstein metric is now x^+ independent and admits a conventional Kaluza-Klein reduction, giving rise to the $z = 2$ Lifshitz 4-dim spacetime. It would be interesting to investigate these arguments further with a view to understanding how robust they are.

The gauge theory dual in this case has the gauge coupling varying as $g^2_{\mathrm{YM}} = g_s e^{2Qx^+}$: the coupling is always bounded on the x^+ circle. The gauge theory also has an exact S-duality symmetry, and our bulk construction effectively implies a nongeometric construction of the gauge theory too. The gauge coupling is then periodic up to S-duality, as we have argued for the dilaton. It would be interesting to explore this further.

Solutions with a lightlike axion

It is worth mentioning that there are very similar solutions sourced purely by a lightlike type IIB axion $c_0 = c_0(x^+)$ too; these are of the form

$$\begin{aligned} ds^2 &= \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2(\partial_+ c_0)^2(dx^+)^2 + dw^2] \\ &\quad + ds_5^2, \\ c_0 &= c_0(x^+). \end{aligned} \tag{20}$$

Specializing to a linear axion configuration, we have

$$\begin{aligned} ds^2 &= \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2(dx^+)^2 + dw^2] \\ &\quad + ds_5^2, \\ c_0 &= c_0^0 + 2Qx^+, \end{aligned} \tag{21}$$

which is akin to (7) except with a constant dilaton Φ. The existence of these solutions can be directly seen from the IIB supergravity equations of motion or, alternatively, by restricting to a nontrivial axion alone in the solutions in [31]. The axion equation of motion is automatically satisfied due to the lightlike nature.

With the x^+ direction treated as compact, we expect then that similar nongeometric constructions can be performed on these linear axionic solutions too, with perhaps more similarity to F-theory constructions. In this case, $c_0(x^+) = c_0^0 + 2Qx^+$ would shift as $c_0 \to c_0 + 2QL$ under $x^+ \to x^+ + L$. This is then equivalent to the $\tau \to \tau + 1$ shift if $QL = \frac{1}{2}$. The dilaton in this case is constant. Presumably there exist solutions of this sort with both the axion and dilaton nontrivial.

These axionic solutions are reminiscent of the D3–D7 solutions in [28]. The axion gives rise to a θ-angle term in the dual gauge theory. It would be interesting to explore the interpretation of these solutions further.

V. DISCUSSION

We have discussed certain lightlike deformations of $\mathrm{AdS}_5 \times X^5$ sourced by a lightlike dilaton $\Phi(x^+)$ dual to the $\mathcal{N} = 4$ SYM theory with a lightlike varying gauge coupling, building on [30], and argued that in the case with x^+ noncompact, these solutions describe anisotropic Lifshitz-like systems with $z = 2$ and $z = \infty$ scaling in the x^-, x_{i^-}, and $x^\pm$ directions, respectively, along with a spatial x^+ potential stemming from the dilaton. We have then focused on linear dilatonic systems and studied two-point correlation functions of operators dual to bulk scalar modes in these cases. We then discussed a certain nongeometric string construction to compactify the x^+ direction. We have also pointed out similar axionic solutions.

Our bulk discussion with x^+ noncompact readily generalizes to $\mathrm{AdS}_4 \times X^7$ solutions in 11-dim supergravity, building on the corresponding $z = 2$ solutions in [30]. The dual field theory is not entirely clear to us; it would be interesting to explore this further with a view to identifying possible lightlike deformations of Chern-Simons theories arising on M2-brane stacks.

The linear dilaton system we have discussed is reminiscent of $c = 1$ string theory and also NS5-branes where a linear dilaton arises; perhaps there are interesting nonrelativistic systems involving these.

The linear dilaton example illustrates the spatial structure of these sorts of theories as we have seen, with the dilaton acting as a potential in a sense. We expect that more general solutions (4) will exhibit similar features too, although studying observables such as correlation functions might be more intricate. However, it is interesting to note some aspects of certain specific solutions; since the spacetimes (4) are essentially a family of solutions for any Φ, distinct solutions exhibit various interesting features. An interesting solution is obtained by taking $\Phi' = 2Q\tanh(Qx^+)$, giving

$$ds^2 = \frac{1}{w^2}[-2dx^+dx^- + dx_i^2 + w^2Q^2\tanh^2(Qx^+) \times (dx^+)^2 + dw^2],$$
$$\Phi = \Phi_0 + 2\,\mathrm{logcosh}(Qx^+). \tag{22}$$

Note that $\Phi' \to \pm Q$ as $x^+ \to \pm\infty$. Also, as $x^+ \to 0$, we have $\Phi' \to 0$. Thus the bulk spacetime is Lifshitz-like away from $x^+ = 0$, a $\frac{1}{Q}$-sized region near which the spacetime is approximately AdS-Schrodinger with $g_{++} \to 0$. This suggests that this solution constitutes a "junction" of two $z = 2$ Lifshitz-like systems joined together with an AdS-Schrodinger-like core about $x^+ = 0$. For Q large, the size of this core shrinks and the junction becomes sharper. The gauge coupling for the dual $\mathcal{N} = 4$ SYM theory in this case is $g_{\mathrm{YM}}^2 = e^\Phi = g_s\cosh^2(Qx^+)$. The fact that the string (or gauge) coupling runs along x^+ implies a varying D-probe tension along that direction; i.e. there is a potential for D-brane probes (i.e. charged dyonic states in the gauge theory) along the x^+ direction, with a maximum for a D-brane probe tension $\frac{1}{g_s}$ at $x^+ = 0$.

Another interesting system arises for $\Phi = \Phi_0 + \tanh Qx^+$. Then the metric becomes

$$ds^2 = \frac{1}{w^2}\left[-2dx^+dx^- + dx_i^2 + w^2\frac{Q^2(dx^+)^2}{\cosh^4(Qx^+)} + dw^2\right],$$
$$\Phi = \Phi_0 + \tanh(Qx^+). \tag{23}$$

Now for large x^+, we see that $g_{++} \to 0$ and we have a Schrodinger system there, while in the vicinity of $x^+ = 0$, this resembles the linear dilatonic system discussed previously, with Lifshitz-like behavior. The dual field theory reflects this, with an interpolation between Galilean and Lifshitz-like regimes.

In a sense, these spacetimes exhibit holographic interpolations in the x^+ direction between asymptotic $z = 2$ Lifshitz-like and Schrodinger-like regions. It would be interesting to find solutions generalizing these, where there is additional radial dependence, perhaps along the lines of [36], and obtain a more detailed understanding of holographic renormalization group flows between these nonrelativistic systems.

It is tempting to speculate that solutions with x^+-noncompact, where the coupling $g_{\mathrm{YM}}^2 = e^{\Phi(x^+)}$ is periodic in x^+, would appear effectively latticelike with a periodic potential for charged dyon states. Likewise, solutions with $e^{\Phi(x^+)}$ possessing some randomness might simulate disorder along the x^+ direction. It might be interesting to explore these solutions further.

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