Spontaneous breakdown of Lorentz symmetry in scalar QED with higher order derivatives

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Scalar QED is studied with higher order derivatives for the scalar-field kinetic energy. A local potential is generated for the gauge field due to the covariant derivatives and the vacuum with nonvanishing expectation value for the scalar field, and the vector potential is constructed in the leading-order saddle-point expansion. This vacuum breaks the global gauge and Lorentz symmetry spontaneously. The unitarity of time evolution is assured in the physical, positive norm subspace, and the linearized equations of motion are calculated. The Goldstone theorem always keeps the radiation field massless. A particular model is constructed where the full set of standard Maxwell equations is recovered on the tree level, thereby relegating the effects of broken Lorentz symmetry to the level of radiative corrections.

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I. INTRODUCTION

We know no exact equation of motion in physics; all laws are inferred by ignoring some loosely attached part of the system being considered. As a result, the equations of motion should be tested for the stability of solutions against adding small correction terms to the equations. Such an analysis is usually performed in the framework of the renormalization group [1], and perturbation expansion can be used to establish that perturbing terms with higher mass dimension (we use units $c = \hbar = 1$) are less important at short distances.

Nevertheless, it is an important difference whether the higher mass dimension arises from field amplitude or from space-time derivative because the latter may modify the tree-level, normal mode structure and generate new degrees of freedom. In fact, a field theory with a real, single-component scalar field characterized by a Lagrangian containing n_d space-time derivatives of the field contains n_d degrees of freedom. Once the new propagating degrees of freedom are present, their interactions might well be nonnegligible due to IR or UV divergences even if the coupling constants in the bare Lagrangian are weak. Another more fundamental change is generated by these terms: spontaneous symmetry breaking of space-time symmetries due to an inhomogeneous condensate, the subject of this work. The point is that the renormalization-group equations usually include only quantum fluctuations. The higher order derivative terms may generate new relevant operators in the IR on the tree level which lead to a vacuum with an inhomogeneous condensate. We do not embark on a general renormalization-group study here; rather, we present a simple analysis of the symmetry and the quasi-particle content of an Abelian gauge model in the leading-order saddle-point expansion.

If the condensate consists of bosons with nonvanishing momentum, filling up the whole quantization volume, then the "wavy vacuum" breaks the space-time symmetries in a manner similar to a solid, where the infinite inertia of the solid prevents the zero modes from restoring the broken external symmetries. The result, expected from solid-state physics, is the appearance of several branches of the dispersion relations, different elementary excitations in the theory. Note that, if translation invariance is broken at a length scale sufficiently short to remain undetectable for the class of observables one uses, then the vacuum appears homogeneous. We shall see that, in models with gauge symmetry where the covariant derivative is supposed to acquire nonvanishing value in the condensate, the inhomogeneity of the vacuum may be gauged away, and we find a homogeneous condensate, which simplifies the model enormously. The result is some kind of extension of the Higgs mechanism where the nonvanishing expectation value for the gauge field breaks Lorentz symmetry. The resulting Goldstone modes remain in the gauge field sector and protect some components of the gauge field against mass generation.

The model studied in this work is scalar QED where higher order (covariant) derivative terms are introduced for the charged scalar field. The higher order terms of this model can be imagined either as a smooth cutoff in defining an UV finite theory or as originating from the elimination of some heavy particle and approximating the self-energy of a scalar charged particle by a polynomial of finite order in the momentum. The Goldstone theorem protects the electromagnetic field against becoming massive, and Maxwell equations are recovered in the linearized equation of motion, turning the Lorentz symmetry-breaking effects into radiative corrections. The rather technical problem of proving unitarity of the model within the physical, positive norm subspace is solved within perturbation expansion by assuring a real energy spectrum for normal modes and preserving the physical subspace, consisting of states of positive norm during the time evolution.

The dynamical breakdown of space-time symmetries by higher order derivatives has already been studied in two- [2], three- [3], and four-dimensional [4,5] Euclidean models where periodically modulated condensate has been observed and several particle modes have been found corresponding to a single quantum field [5]. The present work can be considered a continuation of such inquiries for models defined in Minkowski space-time and equipped with gauge symmetry. The spontaneous breakdown of relativistic symmetries has been considered within the scheme of emerging photons [6] and the bumblebee models [7] where an external Mexican-hat potential is assumed for the vector bosons. Our plan is less ambitious and starts with photons as elementary particles.

Our results can best be summarized by comparing them with the conventional Higgs mechanism where a Goldstone mode arising from the spontaneous breakdown of global gauge invariance appears in the gauge field, which becomes massive. In our case, the relativistic space-time symmetry is broken spontaneously as well, leaving behind three more Goldstone modes. Two of them are nonvanishing helicity components of the gauge field and restore the conventional, massless radiation field of electrodynamics. The third soft mode resides in a certain combination of the vanishing helicity component of the photon and the scalar field and is responsible for the preservation of the usual, long-range Coulomb propagator for the temporal component of the gauge field. Therefore, despite the spontaneous breakdown of internal and external symmetries, the free propagator and the normal modes of the electromagnetic field are equivalent to those of conventional electrodynamics. The symmetry breaking influences only radiative corrections and the dynamics of the charged scalar field.

In Sec. II, we start by listing a few salient features of scalar models with higher order derivatives. The issue of unitarity and the way that it can be recovered by proving reflecting positivity in Euclidean space-time are discussed in Sec. III. Our model, scalar electrodynamics with a higher order derivative for the charged scalar field, is introduced in Sec. IV. The dynamics is discussed in the static temporal gauge where the exceptional features of the time component of the gauge field can be dealt with in the easiest manner. Section V covers the construction of the vacuum in the leading, tree-level order of the saddle-point expansion. The stability of the vacuum and the unitarity within the physical subspace are shown in Sec. V C. The particle content of the theory is defined by the quadratic part of the Lagrangian which is explored in Sec. VI. Finally, Sec. VII is reserved for our summary. The Appendix contains the details of calculating the quadratic part of the action.

II. UNITARITY AND HIGHER ORDER DERIVATIVES

Effective theories may or may not be unitary. In fact, the unitarity is lost when a particle, retained in an effective theory, can lower its energy by the emission of other particles which have been eliminated in deriving the effective theory. Nevertheless, nonunitary effective theory remains a powerful approximation scheme when these decay processes are kinematically suppressed and we make the lifetime sufficiently long. But one would still prefer to recover in effective theories, which tend to be rather complicated, the simplicity following from unitarity. For instance, processes whose energy remains below the mass M of the particle eliminated should reflect unitary dynamics when considered for sufficiently long time. Nevertheless, the UV divergences and quantum anomalies of the underlying theory mix the high-energy effects into the low-energy sector. The most natural way of recovering a unitary effective theory is to place the UV cutoff below the eliminated particle mass, $\Lambda < M$. But this solution is not so simple as it seems. On the one hand, smooth cutoff allows decay processes with small but nonvanishing probability, and, on the other hand, sharp cutoff leads to artificial nonlocal, acausal dynamics at the length scale Λ^{-1}, which is observable in this case.

The hallmark of effective theories is the appearance of higher order derivatives in the Lagrangian, reflecting momentum-dependent self-energies of quasiparticles or form factors. The latter appear in vertices and have mainly perturbative effects. But the self-energies of quasiparticles are in the quadratic part of the action in the fields, modify the structure of quasiparticles, and are sometimes used as a kind of Pauli-Villars regulator which renders the effective theory UV finite [8–10]. Even though the scale of this smooth cutoff is M, the nonunitary processes are not fully suppressed. Once the effective theory is rendered UV finite, we may consider it as an extension of the class of potentially interesting, consistent, microscopic models because its UV dynamics is well defined. Motivated by the search of possible fundamental theories, one naturally expects the complete suppression of nonphysical, nonunitary processes.

We consider in this section a model for a neutral scalar particle described by the field $\phi(x)$. The interaction vertices will be kept in a momentum-independent fashion only and the Lagrangian

$$\mathcal{L} = \frac{1}{2}\phi(x)L(-\Box)\phi(x) - V(\phi^2(x)), \tag{1}$$

where the real function $L(p^2)$ represents the sum of the kinetic term and a momentum-dependent self-energy and is supposed to be a polynomial of order $(p^2)^{n_d}$ which assumes the form

$$L(p^2) - V'(\bar{\phi}^2) = Z^{-1}\prod_{n=1}^{n_d}(p^2 - m_n^2), \tag{2}$$

where Z is real, the potential has a minimum at $\phi = \bar{\phi}$, and the poles might appear in complex pairs. The role of the poles $p^2 = m_n^2$ can be seen more clearly by means of partial fraction decomposition [11],

$$\frac{Z}{L(p^2) - V'(\bar{\phi}^2)} = \sum_n \frac{z_n}{p^2 - m_n^2}, \tag{3}$$

where $z_n = Z/\partial L(m_j^2)/\partial p^2$. We assumed single roots in this equation. In the case that a root $p^2 = m_n^2$ is of ℓ-th order, then the right-hand side may contain terms $z_n^k/(p^2 - m_n^2)^k$ with $1 \le k \le \ell$. Complex roots produce complex contributions to the loop integrals and lead to exponentially damped or increasing amplitudes in time, and unitarity can be saved only by a graph-by-graph modification of the theory [12]. Another problem is seen for real roots when the kinetic term $L(p^2)$ is a real function, and it displays slope with alternating sign at its roots. Thus, approximately half of the contributions to the kinetic energy have the wrong sign, indicating that the Hamiltonian is unbounded from below. This instability can be cured by introducing negative norm states [10], but the unitarity within the physical, positive norm subspace could not be established in a nonperturbative manner [13]. Therefore, a careless truncation of the self-energy may spoil unitarity and the stability of the effective theory.

III. UNITARITY AND REFLECTION POSITIVITY

A proposition to preserve the desired properties was put forward by starting with an effective theory in Euclidean space-time [14], where it is usually derived perturbatively. The effective theory (1) should have a well-defined Euclidean path integral representation, a condition assured by imposing the constraint $L(-p_E^{02} - \mathbf{p}^2) > 0$. The safest is to use lattice regularization in Euclidean space-time where higher order derivatives can be represented as higher order finite differences. It is easy to see in lattice regularization that we need new variables to regain the usual description for theories with higher order derivatives [15]. The Kolmogorov-Chapman equation expresses the group structure of the time evolution in the Fock space and can be written as

$$e^{-S_{t_3-t_1}[\phi^{(3)},\phi^{(1)}]} = \int D[\phi^{(2)}] e^{-S_{t_3-t_2}[\phi^{(3)},\phi^{(2)}] - S_{t_2-t_1}[\phi^{(2)},\phi^{(1)}]}, \tag{4}$$

where the configurations $\phi^{(j)}$ specify states in the field diagonal representation at time t_j, and the $\exp(-S_{t-t'}[\phi, \phi'])$ denotes the matrix element of the Euclidean time-evolution operator during the time interval $t - t'$. This equation can obviously be derived for any theory with nearest-neighbor coupling in time. New variables $\phi(x) \to \phi^a(x)$, $a = 1, \ldots, n_d$, which allow us to rewrite the action with higher derivative in a form with nearest-neighbor coupling in time, can be introduced in the following manner. Start with a hypercubic lattice with lattice spacing $a = 1$ in each direction and construct an anisotropic lattice where the lattice spacing in the time direction is increased to n_d by regrouping n_d time slices of the original lattice. A natural choice is $\phi^a(x) = \partial_0^a \phi(x)$, the a-th order finite-difference operator in time acting on the original field, where the finite difference is calculated from the center of the blocked time slice in a time-reversal covariant manner assuming odd n_d. The map $\phi(x) \to \phi^a(x)$ of the Euclidean field variables is an invertible linear transformation which preserves the lattice regulated action, $S_E[\phi] = S_E[\phi^a]$, and the generator functional,

$$\begin{aligned} Z_E[j] &= \int D[\phi] e^{-S_E[\phi] + \int dx j\phi} \\ &= \prod_a \int D[\phi^a] e^{-S_E[\phi^a] + \sum_x j\phi^0}, \end{aligned} \tag{5}$$

as long as the source is placed at the center of the blocked time slices. The transformation preserves its form in Minkowski space-time and provides the mapping whose inverse can be used after the Wick rotation of the blocked time-slice theory to real time.

The signature of the norm of the states created by the operator $\phi^a(x)$ turns out to be $\sigma[\phi^a] = (-1)^a$. In order to preserve the orthogonality of field eigenvectors, $\langle \phi | \phi' \rangle = 0$ for $\phi(\mathbf{x}) \neq \phi'(\mathbf{x})$, we have to use skew-adjoint field operators, which possess imaginary eigenvalues, in the negative norm sector and $\sigma[\phi] = \pm 1$ for self- and skew-adjoint variables. It is useful to introduce fields with well-defined time reversal parity, $T\phi(t) = \tau[\phi]\phi(-t)$, giving $\tau[\partial_0^a \phi] = (-1)^a \tau[\phi]$. This relation suggests the equivalence of the internal Euclidean time-reversal parity and the signature of the state created by acting on the time-reversal-invariant vacuum by any time-reversal-invariant combination ψ of elementary fields ϕ^a,

$$\sigma[\psi] = \tau[\psi]. \tag{6}$$

One has to make sure that unitarity holds within the physical, positive norm subspace, too. This can be achieved by the reconstruction theorem of axiomatic quantum field theory, in particular, by showing that the main nontrivial condition of the theorem, reflection positivity, holds in the linear space generated by the action of local operators with positive time parity on the vacuum as long as both dynamics and vacuum respect time-reversal invariance and the boundary conditions $\phi^a(t_f, \mathbf{x}) = (-1)^a \phi^a(t_i, \mathbf{x})$ are imposed, where t_i and t_f denote the initial and final times. An important result of the argument [14] is the direct verification of Eq. (6). This relation indicates, as well, that the trajectory of ϕ^a in the path integral is real or imaginary for a even or odd, respectively. The vacuum may contain condensate as long as it is invariant under time reversal.

This construction gives, at first glance, more than expected: it eliminates nonunitarity altogether for theories (1) and (2). But the tacit assumptions the argument relies upon are the convergence of the Euclidean path integral and the possibility of its analytic extension, Wick rotation, back to real time. The former condition imposes $\Re m_n^2 > 0$. The latter assumption requires that the rotation of the frequency contour in the loop integrals is carried out without passing singularities in the integrals. This condition excludes poles from the quadrant $\Im m_n^2 \cdot \Re m_n^2 > 0$ of the complex energy plane. Since poles come in complex conjugate pairs, the remaining complex poles break time-reversal invariance and generate acausality known from the attempts of removing self-acceleration of point charges in classical electrodynamics [16]. Thus, time-reversal invariance restricts the argument to theories where the roots of L are real.

Note that the exclusion of complex poles from the kinetic term restricts the space-time dependence of the perturbative Green functions to the sum of oscillatory terms $e^{i\omega t}$ excluding monotonic terms like $e^{\omega t}$. The functional space in which the expectation values are constructed is tailored in this manner, and the runaway solutions characteristic of unstable theories are excluded. This is in contrast to classical physics, where the integration of the equations of motion is performed in an unlimited functional space of trajectories. Therefore, the classical and the quantum, loop-expansion-based stability analysis disagree as far as the time-dependent instabilities are concerned. This eliminates the notorious instability problem of theories with higher order kinetic term [17].

IV. SCALAR ELECTRODYNAMICS

An important step toward more realistic models is the extension of the previous discussion for gauge models. We now turn to scalar electrodynamics, defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\int dx F_{\mu\nu}F^{\mu\nu} + \int dx[\phi^* L(-D^2)\phi - V(\phi^*\phi)], \tag{7}$$

with $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $D_\mu = \partial_\mu - ieB_\mu$, $L(z)$ being a polynomial of finite order and supposed to possess separate time- and space-inversion invariance. In relativistically covariant canonical quantization procedure, one adds a gauge fixing term, $\mathcal{L} \to \mathcal{L} - \xi(\partial A)^2/2$, and imposes the canonical commutation relations

$$[A_\mu(t, \mathbf{x}), \Pi_\nu(t, \mathbf{y})] = -ig_{\mu\nu}\delta(\mathbf{x} - \mathbf{y}), \tag{8}$$

where $\Pi^\mu = \partial\mathcal{L}/\partial\partial_0 A_\mu$ and $g_{\mu\nu} = (1, -1, -1, -1)$. The wrong sign on the right-hand side for $\mu = \nu = 0$ indicates that temporal photon states have negative norm. The Gupta-Bleuler quantization procedure or BRST symmetry can be used to prove that the usual QED, without a higher order derivative, is unitary in the physical subspace, spanned by states with positive norm.

With an A_0 field represented by a self-adjoint field operator, the field eigenstates are not orthogonal. Orthogonality is assured if the operator A_0 is skew-adjoint only [14]. The complication, induced by the use of the traditional self-adjoint representation, is that nonorthogonality renders the path integral expression for the transition amplitudes rather complicated. How then to recover the standard path integral representation for gauge theories in Minkowski space-time? The usual path integral over real field configurations $A_\mu(x)$ can easily be found by treating A_0 as an auxiliary, nondynamical field either in the static temporal or Coulomb gauge. The former will be imposed to establish unitarity in the physical subspace because the impact of a nonvanishing vacuum expectation value for A_0 on the dynamics and the similarity with spontaneous symmetry breaking can better be seen in the static temporal gauge. The latter gauge will be used to clarify the physical content of the theory since the dynamical degrees of freedom can be traced more easily in the Coulomb gauge.

We start with fields defined without initial or final conditions in time for $-\infty < t < \infty$ and carry out the gauge transformation $A_\mu \to A_\mu + \partial_\mu\alpha$, $\phi \to e^{ie\alpha}\phi$, and $\phi^* \to e^{-ie\alpha}\phi^*$, with

$$\alpha(t, \mathbf{x}) = -\int dt' A_0(t', \mathbf{x}), \tag{9}$$

to arrive at the temporal gauge $A_0 = 0$, where the functional Schrödinger representation is constructed, using $\mathbf{A}(\mathbf{x})$ as coordinates. The canonical momentum $\Pi = \partial_0\mathbf{A} = -\mathbf{E}$ satisfies the canonical commutation relations $[A_j(\mathbf{x}), \Pi_k(\mathbf{y})] = i\delta_{jk}\delta(\mathbf{x} - \mathbf{y})$. Under Gauss's law, $\nabla\mathbf{E} = \rho$, where ρ is the electric charge density, the equation of motion for A_0 is lost in this gauge but can be regained as a constraint. In fact, it can easily be shown by the help of the canonical commutation relations that

$$G[\alpha] = \int d^3x[\nabla\alpha(\mathbf{x})\mathbf{E}(\mathbf{x}) + \alpha(\mathbf{x})\rho(\mathbf{x})] \tag{10}$$

generates static gauge transformations; hence, it commutes with the gauge-invariant Hamiltonian H, $[G(\mathbf{x}), H] = 0$. The average over static gauge transformations,

$$\mathcal{P} = \int D[\alpha]e^{i\int d^3x[\nabla\alpha(\mathbf{x})\mathbf{E} + \alpha(\mathbf{x})\rho(\mathbf{x})]} \tag{11}$$

projects into the subspace, satisfying Gauss's law for a given static charge distribution $\rho(\mathbf{x})$.

One is usually interested in the transition amplitude between gauge invariant states, the latter constructed from a gauge-noninvariant representative like a field eigenstate,

$$|\mathbf{A}, \phi, \phi^*\rangle_{\rm sym} = \mathcal{P}|\mathbf{A}, \phi, \phi^*\rangle. \tag{12}$$

It is enough to insert the projection operator $\mathcal{P}$ only once in the matrix element,

$$\langle \mathbf{A}_f, \phi_f, \phi_f^* | e^{-itH} | \mathbf{A}_i, \phi_i, \phi_i^* \rangle_{\rm sym} = \langle \mathbf{A}_f, \phi_f, \phi_f^* | \mathcal{P} e^{-itH} | \mathbf{A}_i, \phi_i, \phi_i^* \rangle = \int D[\alpha] \langle \mathbf{A}_f, \phi_f, \phi_f^* | e^{i\int d^3x [\nabla\alpha(\mathbf{x})\mathbf{E} + \alpha(\mathbf{x})\rho(\mathbf{x})]} e^{-itH} | \mathbf{A}_i, \phi_i, \phi_i^* \rangle, \tag{13}$$

and one finds the path integral representation

$$\langle \mathbf{A}_f, \phi_f, \phi_f^* | e^{-itH} | \mathbf{A}_i, \phi_i, \phi_i^* \rangle_{\rm sym} = \int D[A] D[\phi] D[\phi^*] e^{iS_{st}[A,\phi,\phi^*]}, \tag{14}$$

where $S_{st}[A, \phi, \phi^*]$ is the usual action in the static temporal gauge,

$$\partial_0 A_0(x) = 0, \tag{15}$$

and $tA_0(\mathbf{x}) = \alpha(\mathbf{x})$ denotes the time-independent integral parameter of the projector. The integration is over configurations $\mathbf{A}(t_i, \mathbf{x}) = \mathbf{A}_i(\mathbf{x})$, $\phi(t_i, \mathbf{x}) = \phi_i(\mathbf{x})$, $\phi^*(t_i, \mathbf{x}) = \phi_i^*(\mathbf{x})$, $\mathbf{A}(t_f, \mathbf{x}) = \mathbf{A}_f(\mathbf{x})$, $\phi(t_f, \mathbf{x}) = \phi_f(\mathbf{x})$, and $\phi^*(t_f, \mathbf{x}) = \phi_f^*(\mathbf{x})$. If the projector is inserted at each time slice of the path integral expression for transition amplitude, then the gauge-invariant action is recovered:

$$\langle \mathbf{A}_f, \phi_f, \phi_f^* | e^{-i\Delta tH} \mathcal{P} \cdots \mathcal{P} e^{-i\Delta tH} | \mathbf{A}_i, \phi_i, \phi_i^* \rangle_{\rm sym} = \int D[A] D[\phi] D[\phi^*] e^{iS[A,\phi,\phi^*]}. \tag{16}$$

$\Delta t A_0(t, \mathbf{x})$ playing the role of parameter $\alpha(\mathbf{x})$ in the projector inserted at time t.

The temporal gauge, used in the Hamiltonian formalism after Eq. (9), is usually not accessible when boundary conditions are imposed in time, as done in path integral expressions. Actually, the field component $A_0(x)$ represents a true physical variable. We can see this by noting that $A_0(x)$ cannot be transformed away from the path integral by gauge transformation. In fact, setting $A_0 = 0$ instead of integrating over $A_0(x)$ on the right-hand side of Eq. (16) removes the projector $\mathcal{P}$ on the left-hand side, and the matrix element is changed, $\langle \cdots \rangle_{\rm sym} \to \langle \cdots \rangle$.

A generally applicable gauge choice is the static temporal gauge, given by Eq. (15). Whatever gauge we use, the Polyakov line

$$\Omega(\mathbf{x}) = e^{-ie\int_{t_i}^{t_f} dt A_0(t,\mathbf{x})} \tag{17}$$

denotes a physical, gauge-invariant quantity which prevents us from reaching the temporal gauge as soon as some boundary conditions are imposed at the initial and final time. But the integrand of the path integral (14) remains unchanged under global gauge transformation of the initial or final state by the phase factor $1 = \exp 2\pi i$, represented by the shift

$$A_0(\mathbf{x}) \to A_0(\mathbf{x}) + \frac{2\pi}{e(t_f - t_i)}. \tag{18}$$

Because of this discrete symmetry, the integrand in Eq. (16) does not depend on the space-time-independent component, $A_0(x) = A_0$, and the variable A_0 decouples in the limit $t_f - t_i \to \infty$. Nevertheless, the homogeneous component A_0 remains a physical parameter when matrix elements among the vacuum are considered because the vacuum state depends on A_0. In fact, eA_0 acts as a chemical potential, and one arrives at a grand canonical ensemble where expectation values of observables are saturated by the total charge sector of the Fock space which minimizes $H - eA_0 \int d^3x \rho$.

V. SEMICLASSICAL VACUUM

Let us suppose that the model given by Eq. (7) is weakly coupled and that saddle-point expansion can be used to explore its phase structure. The case of global symmetry, $e = 0$ in the absence of higher order derivative terms $L(p^2) = p^2$, is well known; the model supports a homogeneous condensate for an appropriately chosen local potential. Higher order derivative terms in the action may induce a condensation of particles with nonvanishing momentum, which is an inhomogeneous coherent state, and a relativistic "band structure" reminiscent of solid state physics is observed. When the interaction with the gauge field is turned on with $L(p^2) = p^2$, then the usual Higgs phase can be found. An interesting variant of the Higgs mechanism can be generated by the higher order derivative's terms. The point is that the partial derivatives are turned into covariant derivatives in the minimal coupling scheme and contain the connection term which can induce a nontrivial local potential for the gauge field. The effective interaction, represented by this potential, may induce a nonvanishing expectation value for the gauge field. We call such a vacuum "condensate," although one should keep in mind that it is actually a coherent state only because our gauge particle, the photon, is neutral and the Bose-Einstein condensation is not possible.

A. Condensate

We follow the strategy of the saddle-point approximation, and to this end we separate the fields into the sum of saddle-point and quantum fluctuations by writing $\phi = \bar\phi + \chi$ and $B_\mu = \bar A_\mu + A_\mu$, the first term in each expression representing the saddle point. When a nonvanishing value of the covariant derivative

$$-D^2 \bar\phi(x) = k^2 \bar\phi(x) \tag{19}$$

is selected for the semiclassical vacuum by the kinetic energy of the charges, then a gauge transformation can always exchange contributions of the partial derivative and the connection term. One possibility is when the eigenvalue k^2 in this equation is provided by the partial derivative alone, $\bar{\phi}(x) = \bar{\phi}e^{-ipx}$, $\bar{A}_\mu = 0$. By a suitable gauge transformation, we may rearrange the semiclassical vacuum into $\bar{\phi}(x) = \bar{\phi}$, $e\bar{A}_\mu = k_\mu$. This is a remarkable simplification offered by gauge invariance; the vacuum consisting of the condensate of particles of nonvanishing momentum can be made homogeneous. We exploit this possibility and assume the homogeneity of the saddle point and the orthogonality of the fluctuations to the saddle point,

$$\int dx \chi(x) = \int dx A_\mu(x) = 0. \tag{20}$$

Note that apart from broken global gauge invariance the gauge-field condensate leads to the spontaneous breakdown of the Lorentz symmetry. When the function $L(p^2)$ generates spacelike gauge-field condensate, $k^2 < 0$, then Lorentz symmetry is reduced to $O(1,2)$ and the excitation spectrum loses rotational invariance. We seek a vacuum with nonrelativistic Galilean $O(3)$ invariance; hence, we restrict our attention to models with timelike gauge-field condensate, $e\bar{A}_\mu = g_{\mu 0}k > 0$. Hence, there will be four combinations of fields playing the role of Goldstone bosons when $\bar{\phi}, \bar{A}_\mu \neq 0$, corresponding to gauge rotations and Lorentz boosts. The number of massless particle modes is not necessarily the same. On the one hand, it may be smaller because either nonrelativistic fields have half as many particle modes as their relativistic counterparts [18] or some field combinations may not control particlelike excitations, with vanishing residuum in the propagator at the "mass shell." On the other hand, it may be more because higher order derivative terms may generate several "bands." Global gauge rotation is applied if necessary to make the scalar condensate, $\bar{\phi}$, real.

B. Fluctuations

According to Sec. III, classical stability analysis is sufficient for the homogeneous components of the fields, and the stability of the fluctuations around the vacuum will be verified by checking the spectrum of the elementary excitations in quantum theory. The energy-momentum tensor of a theory with polynomial, higher order derivative terms can easily be obtained; it is the sum of the usual expression for the energy-momentum tensor plus terms containing higher order derivatives of the fields. Therefore, the energy density of the semiclassical homogeneous vacuum characterized by $\bar{A}_\mu$ and $\bar{\phi}$ is given by the Lagrangian up to a sign,

$$U(e^2\bar{A}^2, \bar{\phi}^2) = -\bar{\phi}^2 L(e^2\bar{A}^2) + V(\bar{\phi}^2). \tag{21}$$

We assume at this point that $L(p^2)$ is bounded from above and that it assumes a maximal value at $p^2 = k^2$, thus the minimization with respect to $\bar{A}^2$,

$$0 = \frac{\partial U(e^2\bar{A}^2, \bar{\phi}^2)}{\partial e^2\bar{A}^2} = -\bar{\phi}^2 L'(e^2\bar{A}^2), \tag{22}$$

sets $e^2\bar{A}^2 = k^2$ and $e\bar{A}_\mu = g_{\mu,0}k$, as mentioned above. The separation of the kinetic and the potential energy terms in the Lagrangian (7) for the scalar field is not unique; the invariance of the action under the transformation $L(p^2) \to L(p^2) + \Delta L$, $V(\phi^2) \to V(\phi^2) - \Delta L \phi^2$ can be used to set $L(k^2) = 0$. We assume the form

$$L(p^2) = -\frac{1}{k^2}(p^2 - k^2)^2, \tag{23}$$

the simplest polynomial satisfying our requirements. The scalar condensate $\bar{\phi}$ is found by minimizing $U(k^2, \phi^2)$, i.e., solving the equation

$$0 = V'(\bar{\phi}^2) - L(k^2), \tag{24}$$

with the auxiliary condition that the first nonvanishing derivative of the potential at the vacuum is positive.

Once the homogeneous field components are found, we turn to the free theory by considering the quadratic part of the action. We use the decomposition $\chi = \chi_1 + i\chi_2$, and $\mathbf{A} = \mathbf{n}A_L + \mathbf{A}_T$, $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$, followed by the separation of the static components $\tilde{\chi}_a$, $\tilde{A}_L$, and $\tilde{\mathbf{A}}_T$ by writing $\chi_a \to \chi_a + \tilde{\chi}_a$, $A_L \to A_L + \tilde{A}_L$, and $\mathbf{A}_T \to \mathbf{A}_T + \tilde{\mathbf{A}}_T$. The quadratic action is written as a sum $S^{(2)} = S^{(2)} + \tilde{S}^{(2)}$, with

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\int d^4x(\chi_1, \chi_2, A_L, \mathbf{A}_T) \\ &\quad\times \begin{pmatrix} K_{11} & K_{12} & K_{1L} & 0 \\ K_{21} & K_{22} & K_{2L} & 0 \\ K_{L1} & K_{L2} & K_{LL} & 0 \\ 0 & 0 & 0 & K_{TT} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ A_L \\ \mathbf{A}_T \end{pmatrix} \\ \tilde{S}^{(2)} &= \frac{t_f - t_i}{2}\int d^3x(\tilde{\chi}_1, \tilde{\chi}_2, \tilde{A}_0, \tilde{A}_L, \tilde{\mathbf{A}}_T) \\ &\quad\times \begin{pmatrix} \tilde{K}_{11} & 0 & \tilde{K}_{10} & \tilde{K}_{1L} & 0 \\ 0 & \tilde{K}_{22} & 0 & \tilde{K}_{2L} & 0 \\ \tilde{K}_{01} & 0 & \tilde{K}_{00} & 0 & 0 \\ \tilde{K}_{L1} & \tilde{K}_{L2} & 0 & \tilde{K}_{LL} & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{TT} \end{pmatrix} \begin{pmatrix} \tilde{\chi}_1 \\ \tilde{\chi}_2 \\ \tilde{A}_0 \\ \tilde{A}_L \\ \tilde{\mathbf{A}}_T \end{pmatrix}. \end{aligned} \tag{25}$$

The momentum space representation of the quadratic form,

$$K(p) = \int dx e^{ip(x-y)} K(x, y), \tag{26}$$

is calculated in Appendix , with the result

$$
\begin{aligned}
K_{11} &= L_d^+(p) - 4V''\bar{\phi}^2 = K_{22},\\
K_{12} &= iL_d^-(p) = -K_{21},\\
K_{1L} &= -|\mathbf{p}|[z(p)L_d(p)]^- = K_{L1},\\
K_{2L} &= i|\mathbf{p}|[z(p)L_d(p)]^+ = -K_{L2},\\
K_{LL} &= \mathbf{p}^2[z^2(p)L_d(p)]^+ + \omega^2,\\
K_{TT} &= \omega^2 - \mathbf{p}^2,
\end{aligned}
\tag{27}
$$

where the notation $f^{\pm}(p) = f(p) \pm f(-p)$ has been introduced with

$$L_d(p) = L((p + e\bar{A})^2) - L(k^2), \tag{28}$$

$z(p) = e\bar{\phi}/(p^2 + 2\omega k)$, and $p = (\omega, \mathbf{p})$ for the four-dimensional fields. The three-dimensional, static sector has the quadratic forms $\tilde{K}(\mathbf{p}) = K(p)_{|\omega=0}$, obtained from the equations in (27) and

$$
\begin{aligned}
\tilde{K}_{10} &= -\frac{4e\bar{\phi}k}{\mathbf{p}^2} L_d(p)_{|\omega=0} = K_{01},\\
\tilde{K}_{00} &= \frac{8e^2\bar{\phi}^2k^2}{(\mathbf{p}^2)^2} L_d(p)_{|\omega=0} + \mathbf{p}^2.
\end{aligned}
\tag{29}
$$

C. Unitarity

We turn now to the question of unitarity of the time evolution within the positive norm subspace of the Fock space. Two circumstances require us to go beyond the argument based on the reconstruction theorem for Euclidean theories [19]. One is that the manifest $O(4)$/Lorentz invariance of the Euclidean/Minkowski Green functions, one of the numerous conditions of the theorem, is lost in our case. The other point is that states belonging to excitations generated by the time component of the gauge field have negative norm in Minkowski spacetime and are thus nonphysical. Rather than attempting to generalize the reconstruction theorem, we choose a simpler argument, valid in any finite order of the perturbation expansion.

The partial fraction decomposition of the propagator is now made in terms of ω^2 rather than p^2, and the realness of the one-particle energies guarantees the unitarity of the perturbative model within the Fock space with indefinite norm. Perturbation expansion, based on the vacuum with homogeneous fields $\bar{\phi}$ and $\bar{A}_\mu$, leads to a stable and unitary theory if all solutions of the equation $\det K(p) = 0$ of the quadratic form K of Eq. (25),

$$
\begin{aligned}
\det K(p) = \frac{4}{k^4}(\omega^2 - \mathbf{p}^2)^2\{&\omega^{10} - 4\omega^8(\mathbf{p}^2 + 2k^2) + \omega^6[16k^4 + 16k^2\mathbf{p}^2 + 6(\mathbf{p}^2)^2 + 4V''\bar{\phi}^2k^2]\\
&- \omega^4[4V''\bar{\phi}^2(e^2\mathbf{p}^2\bar{\phi}^2 + 2k^2\mathbf{p}^2 - 4k^4) + 4(\mathbf{p}^2)^3 + 8k^2(\mathbf{p}^2)^2] + \omega^2[64V''^2\bar{\phi}^4 + 4V''\bar{\phi}^2k^2(\mathbf{p}^2)^2\\
&+ 2e^2\bar{\phi}^2(\mathbf{p}^2)^2 - 4e^2\bar{\phi}^2k^2\mathbf{p}^2] - 2e^2k^2\mathbf{p}^2V''\bar{\phi}^3 - 4e^2(\mathbf{p}^2)^3V''\bar{\phi}^3\},
\end{aligned}
\tag{30}
$$

obtained for the kinetic term (23) have real frequency components, $\omega^2 > 0$. It is easy to see that this expression has negative or complex ω^2 as roots; there are instable modes in the scalar particle, longitudinal gauge-field sector. These instabilities can be excluded by imposing the condition

$$V''(\bar{\phi}^2) = 0 \tag{31}$$

on the local potential, which is not a natural relation; it requires fine-tuning to cancel the scalar particle-scattering amplitude at vanishing momentum.

According to the Goldstone theorem, the minimization of the vacuum energy with respect to the strength of condensate cancels the gap for certain modes. The Goldstone mode arising from the breakdown of global gauge invariance is made by Eq. (24). As far as the three soft-field combinations, which correspond to the breakdown of Lorentz symmetry, are concerned, let us introduce a mass term for the gauge field by the extension $\mathcal{L} \to \mathcal{L} + m^2B^2/2$ of the Lagrangian (7) as in Proca theory, which leads to the modified potential $U(e^2\bar{A}^2, \bar{\phi}^2) \to U(e^2\bar{A}^2, \bar{\phi}^2) - m^2\bar{A}^2/2$ in Eq. (21). The minimization with respect to the gauge-field condensate, Eq. (22), generates three soft-field combinations. Two of them are the nonvanishing helicity components of the transverse gauge field even for $m^2 \neq 0$, and the third is a combination of $\partial_\mu A^\mu$, $\bar{A}_\mu A^\mu$, χ_1, and χ_2. To simplify matters, we return in our discussion to scalar electrodynamics, $m^2 = 0$, where the determinant of Eq. (30), whose vanishing identifies the normal mode dispersion relation, reads

$$
\begin{aligned}
\det K(p) = \frac{4}{k^4}(\omega^2 - \mathbf{p}^2)^2\omega^2(\omega^2 - 2k\omega - \mathbf{p}^2)^2(\omega^2\\
+ 2k\omega - \mathbf{p}^2)^2.
\end{aligned}
\tag{32}
$$

The energy spectrum is real; transverse gauge fields make up two Goldstone modes with $\omega = \pm|\mathbf{p}|$. The scalar field, together with the longitudinal components of the gauge field, produce the dispersion relations

$$\omega = \sigma_1 k + \sigma_2\sqrt{k^2 + \mathbf{p}^2}, \tag{33}$$

where σ_1, $\sigma_2 = \pm 1$. The choice $\sigma_1\sigma_2 = -1$ in Eq. (33) belongs to two other Goldstone modes. The determinant (30) corresponds to nonstatic fluctuations; therefore, the factor $\omega = 0$ in Eq. (32) is never vanishing.

Once the unitarity has been established in the whole Fock space, let us turn to the physical subspace. The argument in Ref. [14] was presented for the Yang-Mills-Higgs model, given by the Lagrangian (7), although some additional care is required in this case to draw conclusions for Minkowski space-time theories. The Wick rotation is more involved for gauge than for scalar fields because the norm of state created by A_0 changes sign during Wick rotation between Euclidean and Minkowski space-time. This leads to the following two problems. One has already been mentioned in Sec. IV: the usual path integration formulas require the orthogonality of the field eigenstates, and we should use skew-adjoint representation for A_0 in Minkowski space-time. This amounts to integration over an imaginary A_0 field, which is in obvious conflict with the usual interpretation of A_0 as the temporal component of a Hermitian quantum field. The solution of this apparent contradiction is well known: the treatment of A_0 as a nondynamical, auxiliary variable. This is what happens in the static temporal gauge where A_0 is the (real) integral variable of the projection operator to restrict the dynamics into the subspace with Gauss's law. Once the real, static A_0 configurations are accepted in the path integral of Eq. (14), then we may return to the gauge-free case, Eq. (16) in the calculation of gauge-invariant quantities. In other words, in the usual path integral formalism for real time, available for gauge theories with higher order derivatives as well, it is better to interpret the temporal component of the gauge field as an auxiliary variable to handle Gauss's law rather than as a quantum field handling physical excitations. The situation is reminiscent of conventional QED where elementary excitations, stability, renormalizability, etc. are trivial in relativistic gauges but one has to go into another, physical gauge, usually chosen to be the Coulomb gauge, to recover unitarity in the physical subspace in an obvious manner.

The other problem, caused by an exceptional feature of $A_0(x)$ during Wick rotation, is that Eq. (6), used to identify the signature of the norm, is no longer valid for this component of the gauge field in Minkowski space-time. A generalization valid for the gauge field is

$$\sigma[\psi] = \tau[\psi]\pi[\psi], \tag{34}$$

where ψ is any local combination of the elementary bosonic fields $\partial_0^a\phi$, $\partial_0^a\phi^*$, and $\partial_0^a\mathbf{A}$, and space inversion acts as $P\psi(t,\mathbf{x}) = \psi(t,-\mathbf{x})$ with $\pi[\phi] = -\pi[\mathbf{A}] = 1$. Combined space- and time-reversal invariance yields the conservation of σ and assures unitarity within the positive norm, physical subspace [20].

VI. QUASIPHOTONS

It has been established so far that our model has unitary time evolution within the positive norm subspace and is therefore physically interpretable. The next question is about its physical content, which will be assessed by comparing it with standard electrodynamics. The usual Higgs mechanism renders photons massive. The Goldstone modes arising from the spontaneous breakdown of the Lorentz invariance make three combinations of the fields soft. Two of them are the transverse, nonvanishing helicity components of the gauge field, and they keep the radiation field massless, just as in standard electrodynamics. Two further soft field combinations are made from the longitudinal gauge and the scalar-field components.

The double pole of (23) may render the normal modes of the scalar field nonparticlelike because scattering amplitude wave packets, constructed by this kind of excitation may be vanishing according to the reduction formulas. Thus, we take the point of view that the scalar field corresponds to so far nonobserved excitations and seek only the dynamics of the gauge field. To simplify matters further, we ignore radiative corrections due to the charged scalar field and restrict ourselves to the $\mathcal{O}(A^2)$ part of the action where the normal modes are quasiphotons. We consider below two aspects of the model: the number of propagating, dynamical degrees of freedom and their dispersion relation. It is worthwhile to separate two different kinds of dynamics for the gauge field, the first arising through the field strength tensor in the Maxwell action [the first term in the Lagrangian (7)] and the second coming directly from the connection term of the covariant derivative in the minimal coupling. The former, field-strength tensor dynamics, represents conventional electrodynamics and the latter, connection term dynamics, is the source of genuine quantum and topological effects.

Let us first have a look into the Proca theory, the simplest model with the massive vector field, and use the standard three-dimensional notation $A^\mu = (\varphi, \mathbf{A})$, $j^\mu = (\rho, \mathbf{j})$, $\mathbf{E} = -\nabla\varphi - \partial_0\mathbf{A}$, $\mathbf{H} = \nabla\times\mathbf{A}$. We separate the transverse and longitudinal components, $\mathbf{A} = \mathbf{A}_T + \nabla\Phi$, $\mathbf{j} = \mathbf{j}_T + \nabla\kappa$, where current conservation implies $\partial_0\rho + \Delta\kappa = 0$. The Lagrangian

$$\mathcal{L} = \frac{1}{2}\mathbf{E}^2 - \frac{1}{2}\mathbf{B}^2 - \rho\varphi + \frac{m^2}{2}(\varphi^2 - \mathbf{A}^2) + \mathbf{j}\mathbf{A} \tag{35}$$

can be written as $L = L_T + L_{L0}$, where the first and second terms contain the transverse and longitudinal and temporal components,

$$\begin{aligned}\mathcal{L}_T &= \frac{1}{2}(\partial_0\mathbf{A}_T)^2 - \frac{1}{2}\mathbf{B}^2 - \frac{m^2}{2}\mathbf{A}_T^2 + \mathbf{j}_T\mathbf{A}_T,\\ \mathcal{L}_{L0} &= \frac{1}{2}(\nabla\varphi + \partial_0\nabla\Phi)^2 + \frac{m^2}{2}[\varphi^2 - (\nabla\Phi)^2]\\ &\quad - \rho\varphi + \nabla\Phi\nabla\kappa.\end{aligned} \tag{36}$$

As is well known, the temporal component φ is not a dynamical degree of freedom and can be eliminated by solving its algebraic equation of motion in time,

$$\varphi = \frac{1}{m^2 - \Delta}(\rho + \partial_0\Delta\Phi), \tag{37}$$

without generating nonlocal effects in time, and the resulting effective Lagrangian for Φ is

$$\mathcal{L}_{L0} = -\frac{1}{2}\rho\frac{1}{m^2-\Delta}\rho + \frac{m^2}{2}\Phi\frac{\Delta(\Box + m^2)}{m^2-\Delta}\Phi + \Phi\left[\frac{\Delta}{m^2-\Delta}\partial_0\rho - \Delta\kappa\right]. \tag{38}$$

For a massless photon, $m^2 = 0$, the equation of motion for Φ is the current conservation and longitudinal photons drop out from the field strength dynamics. But the mass term, arising from the connection term dynamics, may bring the longitudinal component back as a genuine dynamical variable. Gauge transformations may make the separation of auxiliary and truly dynamical variables difficult. For instance, there are gauges, such as the static temporal gauge, where the longitudinal component appears to be dynamical but it drops out from gauge-invariant observables. When higher order derivative terms appear in the connection term dynamics, then either the temporal or the transverse component of the gauge field may acquire nontrivial dynamics. The formal gauge invariance always makes the theory redundant; therefore, one expects three dynamical, propagating degrees of freedom for the theory (7) from the photon field, just as in the usual Higgs mechanism. But their dispersion relations differ from those of the Higgs mechanism, betraying the different underlying symmetry-breaking patterns.

Let us look into the dispersion relation of the model (7) in the Coulomb gauge which offers a particularly clear view in our model with spontaneously broken Lorentz symmetry. The Lagrangian $L = L_T + L_{0m}$ is written as the sum of the transverse part, given by the first equation in Eqs. (36), and the rest, whose quadratic part is

$$\mathcal{L}^{(2)}_{0m} = \frac{1}{2}(\chi_1, \chi_2, \varphi)K_C\begin{pmatrix}\chi_1\\ \chi_2\\ \varphi\end{pmatrix}, \tag{39}$$

where $\chi = \chi_1 + i\chi_2$ and

$$K_C = \begin{pmatrix} L_d^+(p) & iL_d^-(p) & [(p^0+2k)z(p)L_d(p)]^+ \\ -iL_d^-(p) & L_d^+(p) & -i[(p^0+2k)z(p)L_d(p)]^- \\ [(p^0+2k)z(p)L_d(p)]^+ & i[(p^0+2k)z(p)L_d(p)]^- & [(2k+p^0)^2z^2(p)L_d(p)]^+ + \mathbf{p}^2 \end{pmatrix}. \tag{40}$$

The dispersion relation is defined by the roots of the determinant of the quadratic form:

$$\det K_C(p) = \frac{4}{k^4}\mathbf{p}^2(\omega^2 - 2k\omega - \mathbf{p}^2)^2(\omega^2 + 2k\omega - \mathbf{p}^2)^2. \tag{41}$$

Comparing this expression with Eq. (32), the determinant of the small fluctuations in the static temporal gauge, apart from the obvious absence of two massless modes, corresponding to nonvanishing helicity transverse modes of the gauge field, one notices the appearance of a new root, $\mathbf{p}^2$, suggesting the emergence of a conventional Coulomb propagator. One can obtain a more detailed view of the normal modes by the inspection of the propagators. The inverse of K_C is a full matrix with rather involved matrix elements. Matrix elements of K_C^{-1} between the matter field contain the factor $(\omega^2 - 2k\omega - \mathbf{p}^2)^2(\omega^2 + 2k\omega - \mathbf{p}^2)^2$, indicating the nonparticlelike behavior. The matrix elements between the matter field and φ have the factors $(\omega^2 - 2k\omega - \mathbf{p}^2)(\omega^2 + 2k\omega - \mathbf{p}^2)$ and $\mathbf{p}^2$ in the denominators. Finally, the simplest inverse matrix element is the diagonal one for φ,

$$(K_C^{-1})_{00} = \frac{1}{\mathbf{p}^2}, \tag{42}$$

confirming that the factor $\mathbf{p}^2$ in Eq. (41) corresponds to the unchanged Coulomb law.

The expectation of three dynamical, propagating components for the gauge field, mentioned after Eq. (38) above, turned out to be wrong, and the nontrivial dynamics for the longitudinal component, expected by analogy with the Proca case, Eq. (38), was too naïve. The higher order derivative terms render the nontrivial dispersion relation for the longitudinal component a gauge artifact, and the usual dispersion relation is recovered for the electromagnetic field.

The surprising simplicity of Eq. (42) is the result of nontrivial cancellations. This can be seen most easily by calculating the A_0 propagator directly. To this end, we eliminate the charged field by its equation of motion which is simplest to carry out in the complex χ basis, where

$$\mathcal{L}^{(2)} = \frac{1}{2}(\chi^*, \chi, \varphi)\begin{pmatrix} K_- & 0 & K_0\\ 0 & 0 & 0\\ K_0 & 0 & K_{00}\end{pmatrix}\begin{pmatrix}\chi\\ \chi^*\\ \varphi\end{pmatrix}, \tag{43}$$

with $K_- = -2(\Box - 2ik\partial_0)^2/k^2$, $K_0 = 2e\bar{\phi}(2k + i\partial_0)(\Box - 2ik\partial_0)/k^2$, and $L_{00} = 2e^2\bar{\phi}^2(\partial_0^2 - 4k^2)/k^2 - \Delta$. The equations of motion for χ^* and χ, $0 = K_-\chi + K_0 A_0$, and $0 = \chi^\dagger K_- + A_0 K_0$, used to eliminate the scalar field, yield

$$\mathcal{L}^{(2)} = \frac{1}{2}\varphi D_{00}^{-1}\varphi, \tag{44}$$

where

$$D_{00}^{-1} = K_{00} - \frac{1}{2}(K_0 K_-^{-1} K_0 + K_0^{tr} K_-^{-1tr} K_0^{tr}) \tag{45}$$

give $D_{00}^{-1} = \mathbf{p}^2$ after some cancellations. Therefore, the deviation from usual electrodynamics and the impact of the higher order derivative terms are seen by the charged scalar field in our approximation.

VII. CONCLUSION

A novel spontaneous symmetry breaking is discussed in the framework of a scalar QED which involves higher order covariant derivatives. One finds a nonvanishing expectation value for the gauge field and unitary, physically acceptable interactions in properly fine-tuned models.

The unitarity is proven in the physical, positive norm subspace in two steps. First, it is assured in the whole Fock space by fine-tuning the self-interactions for the charged scalar field. Second, it is shown that combined space- and time-reversal invariance makes the physical subspace closed under time evolution.

The particle content of our model is radically different than the one found in the conventional Higgs mechanism. The Goldstone theorem renders the radiation field massless. Furthermore, a particular model is proposed where all components of the gauge field are massless and Maxwell equations are recovered in the linearized equations of motion.

We sought in this work a vacuum which supports Galilean invariance; therefore, the temporal component of the gauge field was allowed to develop vacuum expectation value. It acts as some dynamically generated chemical potential for the charged scalar particle. The scalar particle condensate remains electrically neutral due to the equal number of particles and antiparticles it contains as a result of the higher order derivative terms in their dispersion relation.

The status of Lorentz symmetry, broken by the vacuum expectation values to the Galilean group, is rather peculiar in the Abelian model. Despite the breakdown of Lorentz invariance, Goldstone modes display relativistic dispersion relations. Furthermore, three components of the gauge field become Goldstone modes corresponding to the spontaneous breakdown of relativistic symmetries and hence remain massless even if one starts with massive Proca action for photons. The quadratic part of the Lagrangian in the fluctuations of the gauge field is identical to that of QED, leaving the Lorentz noninvariant part of the photon dynamics to be generated by radiative corrections. The deviation of this model from standard electrodynamics is due to radiative corrections only.

There are numerous extensions one may consider. Similar models with non-Abelian gauge symmetry should lead to some massive gauge-field components because the Goldstone theorem can no longer protect all components of the gauge field against mass generation. Using a basis in internal space where the massless gauge bosons are diagonal, the other, noncommuting components of the gauge field are charged and allow us to construct models with an unbroken $U(1)$ subgroup, as in the standard model. It remains to be seen if natural models, requiring no fine-tuning, can be constructed by the eventual inclusion of charged fermions. Another issue, the scale-dependence of the breakdown of Lorentz invariance, is interesting, too. Being a spontaneous symmetry breaking, it should be strong at low energy. But some interesting results about non-Lorentz-invariant quadratic terms in gauge theories [21] suggest that certain Lorentz symmetry-breaking parameters of the dynamics tend to be suppressed in the low energy limit. A systematic renormalization-group study of the model would be needed to reveal the true scale dependence of this symmetry breaking. Finally, an extension for gravity opens new questions since the spontaneous breakdown of Lorentz symmetry may generate massive gravitons by a gravitational Higgs effect.

APPENDIX A: QUADRATIC ACTION IN MOMENTUM SPACE

To find the momentum dependence of the quadratic form $K(p)$, we evaluate the quadratic action (25) for the test functions

$$\chi(x) = \chi' e^{-ipx}, \qquad A_\mu(x) = A'_\mu e^{-ipx}, \tag{A1}$$

before gauge fixing for the sake of simplicity. The quadratic form of the $\mathcal{O}(\chi^* \chi)$ part can easily be written as

$$K_{\chi^*\chi} = 2L_d(p) - 4V'' \bar{\phi}^2 \tag{A2}$$

by means of Eq. (24) with $L_d(p)$ introduced in Eq. (28).

To find the other terms, it is advantageous to represent the higher derivative kinetic term of the scalar field as a polynomial,

$$L(p^2) = \sum_{n=0}^{n_d} c_n p^{2n}. \tag{A3}$$

The block that mixes the scalar and the gauge field originates from the $\mathcal{O}(B)$ piece in

$$L(-D^2)\bar{\phi} = \sum_{n=0}^{n_d} c_n[-(\partial - ie\bar{A} - ieA)^2]^n \bar{\phi}, \tag{A4}$$

and we find for the $\mathcal{O}(A\chi)$ contributions

$$\begin{aligned} \frac{1}{2}\int dxdy \chi(x) K_{\chi A}(x,y) A(y) &= ie\int dx \chi(x) \sum_{n=0}^{n_d} c_n \sum_{\ell=1}^{n} (-\bar{\Box}) \cdots \\ &\quad \times (2A(x)\bar{\partial} + \partial A(x)) \cdots (-\bar{\Box})\bar{\phi}, \end{aligned} \tag{A5}$$

where $\bar{\partial}_\mu = \partial_0 - ie\bar{A}_\mu$, $\bar{\Box} = \bar{\partial}_\mu \bar{\partial}^\mu$, and the ℓ-th factor of the term $\mathcal{O}((-D^2)^n)$ is replaced by the $\mathcal{O}(A)$ part of $-D^2$ in the right-hand side. The choice (A1) leads to

$$K_{\chi A}(p)A' = 2ie\sum_{n=0}^{n_d} c_n \sum_{\ell=1}^{n}(p + e\bar{A})^2\cdots \times (-2iA'_0k - ipA')k^2\cdots\bar{\phi}, \quad \text{(A6)}$$

written as

$$K_{\chi A}(p)A' = 2e(2A'_0k + pA')k^{-2}\sum_{n=0}^{n_d} c_n k^{2n} \times \sum_{\ell=0}^{n-1}\left(1 + \frac{p^2 + 2p^0k}{k^2}\right)^{\ell}\bar{\phi}. \quad \text{(A7)}$$

The geometric series can be summed,

$$K_{\chi A}(p)A' = 2e\frac{2A'_0k + pA'}{p^2 + 2p^0k} \times \sum_{n=0}^{n_d} c_n k^{2n}\left[\left(1 + \frac{p^2 + 2p^0k}{k^2}\right)^n - 1\right]\bar{\phi}, \quad \text{(A8)}$$

and we have

$$K_{\chi A_\mu}(p) = 2e\bar{\phi}\left[L_d(p)\frac{2g^{\mu 0}k + p^\mu}{p^2 + 2p^0k}\right]. \quad \text{(A9)}$$

The $\mathcal{O}(A^2)$ quadratic form for the real field requires more care. Since it acts on the real field, it must be symmetrical. We shall consider a complex plane wave component of the gauge field in the actual calculation of this term and carry out the symmetrization only at the end. This term is the sum of two contributions. One of them is the standard Maxwell piece,

$$K^{(1)}_{A_\mu A_\nu}(p) = -T^{\mu\nu}p^2, \quad \text{(A10)}$$

where $T^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu/p^2$ is the projection into the transverse polarization subspace. The other part is the $\mathcal{O}(A^2)$ contribution in

$$\bar{\phi}L(-D^2)\bar{\phi} = \sum_n c_n\bar{\phi}[-(\partial - ie\bar{A} - ieA)^2]^n\bar{\phi}, \quad \text{(A11)}$$

which will be written as the sum $K^{(2)}_{AA}(p) + K^{(3)}_{AA}(p)$. The first term stands for the $\mathcal{O}(A^2)$ contributions of the $-D^2$ factor,

$$A'K^{(2)}_{AA}A' = 2e^2\sum_{n=0}^{n_d} c_n\sum_{\ell=1}^{n}\bar{\phi}(-\Box')\cdots A^2(x)\cdots(-\Box')\bar{\phi}, \quad \text{(A12)}$$

which is vanishing,

$$K^{(2)}_{A_\mu A_\nu}(p) = 2g^{\mu\nu}\bar{\phi}^2e^2L'(k^2) = 0. \quad \text{(A13)}$$

The other contribution is for the product of two $\mathcal{O}(A)$ terms,

$$\begin{aligned} A'K^{(3)}_{AA}(p)A' &= -2e^2\sum_{n=0}^{n_d} c_n\sum_{\ell=1}^{n-1}\sum_{\ell'=\ell+1}^{n}\bar{\phi}(-\Box')\cdots \\ &\quad\times(2A\partial' + \partial A)\cdots(2A\partial' + \partial A)\cdots(-\Box')\bar{\phi} \\ &= 2e^2\sum_{n=0}^{n_d} c_n\sum_{\ell=1}^{n-1}\sum_{\ell'=\ell+1}^{n}\bar{\phi}k^2\cdots(2A_0k + pA)\cdots \\ &\quad\times(k^2 + p^2 + 2p^0k)\cdots(2A_0k + pA)\cdots k^2\bar{\phi}, \end{aligned} \quad \text{(A14)}$$

which is written as

$$A'K^{(3)}_{AA}(p)A' = 2e^2\bar{\phi}^2(2A_0k + pA)^2k^{-4} \times \sum_{n=0}^{n_d} c_n k^{2n}\sum_{\ell=1}^{n-1}\sum_{\ell'=\ell+1}^{n}\left(1 + \frac{p^2 + 2p^0k}{k^2}\right)^{\ell'-\ell-1}. \quad \text{(A15)}$$

The summation of this geometric series gives

$$A'K^{(3)}_{AA}(p)A' = 2e^2\bar{\phi}^2\frac{(2A_0k + pA)^2}{p^2 + 2p^0k}k^{-2}\sum_{n=0}^{n_d} c_n k^{2n} \times\left[\sum_{\ell=1}^{n-1}\left(1 + \frac{p^2 + 2p^0k}{k^2}\right)^{n-\ell} - n + 1\right]. \quad \text{(A16)}$$

The resulting geometrical series in the square bracket can again be summed with the result

$$A'K^{(3)}_{AA}(p)A' = 2e^2\bar{\phi}^2\frac{(2A_0k + pA)^2}{(p^2 + 2p^0k)^2}\sum_{n=0}^{n_d} c_n k^{2n} \times\left[\left(1 + \frac{p^2 + 2p^0k}{k^2}\right)^n - 1 - n\frac{p^2 + 2p^0k}{k^2}\right], \quad \text{(A17)}$$

yielding finally

$$A'K^{(3)}_{AA}(p)A' = 2e^2\bar{\phi}^2L_d(p)\frac{(2A_0k + pA)^2}{(p^2 + 2p^0k)^2} \quad \text{(A18)}$$

and

$$\begin{aligned} K_{A_\mu A_\nu}(p) &= -T^{\mu\nu}p^2 + e^2\bar{\phi}^2L_d(p) \times\frac{(2g^{\mu 0}k + p^\mu)(2g^{\nu 0}k + p^\nu)}{(p^2 + 2p^0k)^2} \\ &\quad + e^2\bar{\phi}^2L_d(-p)\frac{(2g^{\mu 0}k - p^\mu)(2g^{\nu 0}k - p^\nu)}{(p^2 - 2p^0k)^2} \end{aligned} \quad \text{(A19)}$$

after symmetrization.

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