

Visser's massive graviton bimetric theory revisitedAlain de Roany,^{*} Bertrand Chauvineau,[†] and José A. de Freitas Pacheco[‡]*University of Nice-Sophia Antipolis, Observatoire de la Côte d'Azur, Laboratoire Cassiopée, BP 4229, Nice Cedex 4, France*

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A massive gravity theory was proposed by Visser in the late 1990s. This theory, based on a background metric $b_{\alpha\beta}$ and on an usual dynamical metric $g_{\alpha\beta}$ has the advantage of being free of ghosts as well as discontinuities present in other massive theories proposed in the past. In the present investigation, the equations of Visser's theory are revisited with particular care on the related conservation laws. It will be shown that a multiplicative factor is missing in the graviton tensor originally derived by Visser, which has no incidence on the weak field approach but becomes important in the strong field regime when, for instance, cosmological applications are considered. In this case, contrary to some previous claims found in the literature, we conclude that a nonstatic background metric is required in order to obtain a solution able to mimic the Λ CDM cosmology.

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I. INTRODUCTION

Studies of the luminosity distance as a function of the redshift of type Ia supernovae suggest that the expansion of the universe is presently accelerated [1,2]. These data as well as those derived from the space probe WMAP on the cosmic microwave background [3] can be explained by the inclusion of the so-called "cosmological constant" term in Einstein's equations. Arguments against this possibility have been raised in the literature, in particular, the "coincidence" problem and the interpretation of such a term as the vacuum energy density. However, some authors believe that these objections do not represent real difficulties for the theory [4]. They claim that the coincidence problem is ill-defined and that the identification of the cosmological constant with the vacuum energy density is probably a mistake.

Independently of the reality or not of these difficulties for the Λ CDM cosmology, alternative models have been proposed in the literature. In a first class of models, the usual thermodynamic properties of the constituents of the universe are modified and the acceleration of the expansion is driven by a negative pressure term associated either to particle production (see, for instance, [5] and references therein) or to a bulk viscosity term ([6] and references therein). In another class of models, the existence of new fields in nature responsible for the acceleration are postulated (scalar fields in GR) as well as modifications in the Einstein-Hilbert action ($f(R)$ theories) [7–9] or scalar-tensor theories [9–17].

In a particular class of theories (massive gravity), the graviton has a small but nonzero mass. These theories have a long history and present several difficulties. Fierz and Pauli (FP) noticed that the mass term must be quadratic for

a Lorentz invariant massive spin-2 theory, otherwise a "ghost" appears in the spectrum [18]. A major difficulty with the FP theory is the van Dam-Veltman-Zakharov discontinuity [19,20]. In other words, in the linearized FP theory, the extra scalar mode of the graviton does not disappear and remains coupled to matter even in the limit of a vanishing graviton mass and, consequently, in such a limit, the linear equations of general relativity (GR) are not recuperated. Moreover, the prediction of the quadratic FP theory for the light-bending effect differs from GR, practically ruling out the FP theory. However, Vainshtein [21], from the analysis of static spherically symmetric solutions, argued that the linear FP theory is only valid for distances larger than a certain scale dubbed the Vainshtein radius R_V , which goes to infinity as the graviton mass goes to zero. For distances less than R_V , around a static spherically symmetric source of mass M , the full nonlinear strongly coupled theory has to be considered in order to recover GR. In the past years, the problem of the continuous matching between solutions inside and outside the Vainshtein radius have been extensively debated in the literature [22–25].

A different approach was proposed by Visser [26], who introduced a background metric not subjected to any dynamical equation. The mass term in this theory depends both on the dynamical and on the background metric in such a way that in the linear limit, the massive field obeys a Klein-Gordon equation with a source term. GR is recovered when the graviton mass vanishes. The massive field in Visser's theory has six degrees of freedom: five spin-2 and one scalar [26]. This theory was applied to cosmology by different authors [27–30], who claim that the resulting dynamical equations based on such a theory are able to explain the present observed acceleration of the universe and to satisfy other cosmological tests like the distance scale provided by the baryon acoustic peak and the cosmic microwave background shift parameter. In the aforementioned investigations, a flat background (Minkowski space-time) had been adopted despite the fact that according to

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Visser, “*in a cosmological setting it is no longer obvious that we should use the Minkowski metric as a background.*” If a flat background is adopted, then the only issue resulting in an expanding dynamic metric is to assume that the divergence of the massive graviton tensor is a source for the divergence of the matter stress-energy tensor [27]. In Visser’s theory [26], the divergence of the graviton tensor is set to zero and, consequently, when the dynamical equations are linearized around the background metric, the Hilbert-Lorentz condition appears naturally and not as a gauge one. It is worth mentioning that other bimetric gravity theories with generally a flat prior geometry have been elaborated in the past, in particular, the approach by Rosen or the vector-bimetric theory by Rastall (see, for instance, [31,32] for reviews). These theories are quite distinct since the Lagrangian density from which the field equations are derived differs drastically from that proposed by Visser.

In the present paper, the Visser’s theory is revisited as well as applications to cosmology. We will show that the graviton tensor derived by Visser [26] must be corrected by a factor equal to the ratio between the background and dynamical metric determinants. The conservation laws are also revisited and the following question is examined: if the divergence of the graviton tensor is equal to zero (as it should be expected from general arguments based on a variational approach of physics), is it possible to find an adequate background metric from which a cosmology compatible with the present data emerges? A positive answer can be given but the solutions able to mimic the Λ CDM cosmology are not completely satisfactory, since they require that the background metric tensor be proportional to the scale factor describing the dynamics of the universe. This paper is organized as follows: in Sec. II, the Visser’s theory is revisited and the correct graviton tensor is derived. An alternative definition of the Lagrangian density describing the graviton field is given as well as the resulting field equations. In Sec. III, an application of the theory to cosmology is discussed. In Sec. IV, the main conclusions are presented.

II. THE FIELD EQUATIONS AND THE CONSERVATION LAWS

In this paper, the notation $m_{\alpha\beta} \equiv \text{diag}(-1, +1, +1, +1)$ (Minkowski metric in Cartesian coordinates) will be adopted as well as units such as $G = c = \hbar = 1$. The action proposed by Visser [26] reads

$$S = \int d^4x \left\{ \frac{1}{16\pi} [\sqrt{-g}R + \sqrt{-b}L_{\text{mass}}(g, b)] + \sqrt{-g}L_{\text{matter}}(g, X) \right\}, \quad (1)$$

where $b_{\alpha\beta}$ represents the background metric tensor, $g_{\alpha\beta}$ the dynamical metric tensor, and X stands for any

nongravitational field. Both metrics are required to have a Lorentzian signature $(-, +, +, +)$. The Lagrangian of the massive graviton is given explicitly by

$$L_{\text{mass}}(g, b) = -\frac{1}{4}m^2 \left\{ b^{\alpha\beta} b^{\mu\nu} (g_{\alpha\mu} - b_{\alpha\mu})(g_{\beta\nu} - b_{\beta\nu}) - \frac{1}{2}[b^{\alpha\beta}(g_{\alpha\beta} - b_{\alpha\beta})]^2 \right\}. \quad (2)$$

The contravariant tensor $b^{\alpha\beta}$ is defined from $b_{\alpha\beta}$ by the inversion relation $b_{\alpha\beta} b^{\beta\gamma} = \delta_{\alpha}^{\gamma}$, while $g^{\alpha\beta}$ is defined from the relation $g_{\alpha\beta} g^{\beta\gamma} = \delta_{\alpha}^{\gamma}$ as usual. The coefficient $-1/4$ ensures that in the usual weak field limit, i.e., when $b_{\alpha\beta} = m_{\alpha\beta}$ and $g_{\alpha\beta} = m_{\alpha\beta} + h_{\alpha\beta}$ with $|h_{\alpha\beta}| \ll 1$, the graviton field in vacuum obeys the Klein-Gordon equation. Since the construction of the action defined in Eq. (1) is mainly motivated by such a requirement, it is worth pointing out that any alternative action leading to the same linearized equations would be acceptable *a priori*. In particular, the graviton Lagrangian density can be defined in terms of the dynamical metric, i.e., $\sqrt{-g}L_{\text{mass}}$, instead of $\sqrt{-b}L_{\text{mass}}$. This corresponds to an alternative to Visser’s proposal that admits the same weak field limit. Such an alternative for the graviton Lagrangian density will be examined in more detail at the end of this section.

A. Conservation laws

From the variation of the action [Eq. (1)] with respect to the field X , (here, for simplicity, we assume the presence of only one nongravitational field), one obtains the Lagrange equation describing the dynamics of the considered field. Since the matter Lagrangian density L_{matter} depends only on the field X and on the dynamical metric tensor (the field X couples with $g_{\alpha\beta}$ only), then the diffeomorphism invariance of L_{matter} leads immediately to a conservation equation expressed by the null covariant divergence condition

$$\nabla_{\alpha} T^{\alpha\beta} = 0, \quad (3)$$

where $T^{\alpha\beta}$ is the stress-energy tensor of matter [33]. If the condition above is ignored, some inconsistencies may appear in the physical laws describing the dynamics of nongravitational fields. It is worth recalling that the null divergence of the Einstein tensor leads only to the null divergence of the sum of tensors constituting the right-hand side of the field equations derived by varying the complete action of the theory [in our case, Eq. (1)] with respect to the metric tensor $g_{\alpha\beta}$. If different stress-energy tensors are present, the null divergence of the Einstein tensor is not equivalent to Eq. (3). In this case the validity of Eq. (3) implies an additional condition to be fulfilled by the solutions of the field equations.

B. Field equations

Varying the action defined by Eq. (1) with respect to the metric tensor $g_{\alpha\beta}$ leads to

$$G^{\alpha\beta} = 8\pi T^{\alpha\beta} + 8\pi T_{\text{mass}}^{\alpha\beta}, \quad (4)$$

where $G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta}$ is the Einstein tensor, $R_{\alpha\beta}$ is the Ricci tensor, and $R = R^\beta_\beta$ is the Ricci scalar. The matter stress-energy tensor $T^{\alpha\beta}$ is, as usual, defined by

$$T^{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_{\text{mat}})}{\delta g_{\alpha\beta}}, \quad (5)$$

and the so-called graviton tensor by

$$T_{\text{mass}}^{\alpha\beta} = -\frac{m^2}{16\pi} \frac{\sqrt{-b}}{\sqrt{-g}} \left(b^{\mu\alpha} b^{\nu\beta} - \frac{1}{2} b^{\mu\nu} b^{\alpha\beta} \right) (g_{\mu\nu} - b_{\mu\nu}). \quad (6)$$

From the null divergence of the Einstein tensor and Eq. (3), one gets

$$\nabla_\alpha T_{\text{mass}}^{\alpha\beta} = 0. \quad (7)$$

It should be emphasized that both conservation laws expressed by Eqs. (3) and (7) must be satisfied and are necessary conditions to be taken into account when considering solutions of the field equations. From these equations, it is trivial to verify that Visser's theory reduces to GR when the graviton mass m vanishes. As we shall see in detail in Appendix A, a class of solutions in which the dynamical metric tensor is proportional to that of the background metric exists, which can be generated from GR solutions including a cosmological constant.

C. An alternative Lagrangian density

As already mentioned, the requirement that the graviton field obeys the Klein-Gordon equation in the weak field (or linear) limit does not fix the form of the Lagrangian density. An alternative to Visser's proposal is to consider a Lagrangian density defined in terms of the dynamical metric tensor, i.e., $\sqrt{-g}L_{\text{mass}}$ and, in this case, the total action of the theory is

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi} [R + L_{\text{mass}}(g, b)] + L_{\text{matter}}(g, X) \right\}. \quad (8)$$

The resulting field equations are essentially identical to Eq. (4) but now the graviton tensor reads

$$\begin{aligned} T_{\text{mass}}^{\alpha\beta} = & -\frac{m^2}{16\pi} \left[b^{\mu\alpha} b^{\nu\beta} g_{\mu\nu} + b^{\alpha\beta} \left(1 - \frac{1}{2} b^{\mu\nu} g_{\mu\nu} \right) \right. \\ & + g^{\alpha\beta} \left(\frac{1}{4} b^{\mu\sigma} b^{\nu\rho} g_{\mu\nu} g_{\sigma\rho} - \frac{1}{8} (b^{\mu\nu} g_{\mu\nu})^2 \right. \\ & \left. \left. + \frac{1}{2} b^{\mu\nu} g_{\mu\nu} - 1 \right) \right]. \end{aligned} \quad (9)$$

It should be emphasized that as in the case of the original Visser's proposal, the field equations derived from this alternative Lagrangian density admit solutions generated

from GR equations including a cosmological constant (see details in Appendix A).

III. FRW COSMOLOGY

Recently, different authors have considered the Visser's theory to describe the dynamics of the Universe ([27–30]), claiming that the graviton mass term appearing in the Friedman equations is able to drive the observed acceleration of the expansion of the Universe. In these investigations, a Minkowski spacetime was assumed as a background and, in this case, as a consequence of Eq. (7), it results that the scale factor must be constant or, in other words, the Universe must be “static.” In order to maintain a flat and static background (Minkowski), the aforementioned authors assumed that the conservation law expressed by Eq. (7) is violated and, in order to satisfy the condition $\nabla_\alpha G^\alpha_\beta = 0$, they have hypothesized that the divergence of the graviton tensor is a (negative) source term for the divergence of the stress-energy tensor of the matter.

Here, the FRW cosmology is revisited in the context of Visser's theory but with preservation of the conservation laws as discussed in the previous section. Considering the standard FRW form for the dynamical metric, we have

$$ds^2 = -dt^2 + a(t)^2 [d\chi^2 + F_k(\chi)^2 (d\theta^2 + \sin^2\theta d\varphi^2)], \quad (10)$$

where $k = +1, 0, -1$, corresponds, respectively, to spherical, flat, or hyperbolic spatial sections and, accordingly, $F_k(\chi) = \sin\chi, \chi, \sinh\chi$. Following Visser [26], we will search possible nonstatic solutions for the background metric, i.e.,

$$d\sigma^2 = -B(t)^2 dt^2 + A(t)^2 [d\chi^2 + G_k(\chi)^2 (d\theta^2 + \sin^2\theta d\varphi^2)], \quad (11)$$

where A and B are positive functions of the cosmic time. From the spatial components of Eq. (7), we have necessarily $F_k(\chi) = G_k(\chi)$. The function $A(t)$ may be interpreted as a “background scale factor” while a “background cosmic time” may be defined by $\int B(t) dt$. Matter, as usual, is described as a perfect fluid with energy density $\epsilon(t)$ and pressure $P(t)$, linked by an equation of state $P(\epsilon)$.

Besides the time component of matter conservation [Eq. (3)], the other equations describing the dynamics of the Universe are the components (0 0) and (1 1) of the field Eq. (4), since the other components do not provide any additional information. On the other hand, a relation between the coefficients of the background and of the dynamical metric can be directly obtained from Eq. (7). It is useful to define the functions $\Xi(t) = A(t)/a(t)$ and $\Psi(t) = B(t)/\Xi(t)$, which should be positive since A, B and the scale factor a are positive functions. In terms of these new functions, the background metric can be expressed as

$$d\sigma^2 = \Xi(t)^2\{-\Psi(t)^2 dt^2 + a(t)^2[d\chi^2 + F_k(\chi)^2(d\theta^2 + \sin^2\theta d\varphi^2)]\}. \quad (12)$$

The usual procedure in the context of the Visser's theory is to define first a background metric and then, from the field equations and conservation laws, to derive the dynamics of the Universe (supposing that the properties of matter, like the equation of state are given *a priori*). In this case, any cosmological scale parameter $a(t)$ can be obtained by an appropriate choice of the background, but this does not necessarily imply that we have a physically acceptable solution. Another possibility to tackle the problem is to search for metric coefficients of the background spacetime once a dynamical solution is imposed to the field equations or, in other words, to work in the opposite route of the usual procedure. This approach will be adopted here. From the field components (0 0) and (1 1) of Eq. (4), we have

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi}{3}\epsilon + \frac{m^2}{12}\Sigma \quad (13)$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi P + \frac{m^2}{4} \frac{(2\Psi^2\Xi^2 - \Psi^2 - 1)}{\Psi}, \quad (14)$$

where the notation $\dot{X} \equiv dX/dt$ was adopted (It is worth mentioning that in above equations Ξ and Ψ are functions of the cosmic time t). The new function Σ in Eq. (13) is defined as

$$\Sigma = \frac{1}{\Psi^3}(1 + 2\Psi^2\Xi^2 - 3\Psi^2). \quad (15)$$

The space components of Eq. (7) are trivially satisfied while the time component provides a relation between the coefficients of the background metric and the scale factor of the dynamical metric, namely,

$$\frac{d\Sigma}{dt} + 3\frac{\dot{a}}{a}\left(\Sigma + \frac{1 + \Psi^2 - 2\Psi^2\Xi^2}{\Psi}\right) = 0, \quad (16)$$

or, equivalently,

$$\frac{d\Sigma}{dt} = 3\frac{(\Psi^2 - 1)}{\Psi^3}(1 - \Psi^2 + 2\Psi^2\Xi^2)\frac{d\ln a}{dt}. \quad (17)$$

For the sake of completeness, we mention that the time component of Eq. (3) can be written explicitly as

$$\frac{d(\epsilon a^3)}{dt} + 3Pa^2\dot{a} = 0. \quad (18)$$

Combining Eqs. (13)–(15), one obtains

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\epsilon + 3P) + \frac{m^2}{24\Psi^3}[2\Psi^2\Xi^2(3\Psi^2 - 1) - (3\Psi^4 + 1)]. \quad (19)$$

The equation above shows that the last term on the right-hand side (depending on the graviton mass) can give a positive contribution to the acceleration of the expansion of the Universe. According to our adopted approach, a solution for the background metric functions $\Xi(t)$ and $\Psi(t)$ can be obtained by adopting the following procedure: first, from the matter equation of state and Eq. (18), the variation of the energy density and of the pressure can be derived as a function of the scale parameter a or, equivalently, of the redshift z . Then, if the acceleration parameter $q(z) = -\ddot{a}a/\dot{a}^2$ and the Hubble parameter $H(z)$ are known from observations, the metric functions $\Xi(z)$ and $\Psi(z)$ can be determined from Eqs. (13) and (19). Acceptable solutions require that the background metric functions be positive and at the present time, in order to have a positive acceleration, the following condition must be satisfied (including all the physical constants)

$$\frac{1}{\Psi^3}[2\Psi^2\Xi^2(3\Psi^2 - 1) - (3\Psi^4 + 1)] > \frac{12\Omega_m H_0^2 \hbar^2}{m^2 c^4}, \quad (20)$$

where Ω_m is the present matter-density parameter and the metric functions Ξ and Ψ are taken at the present time. The other symbols have their usual meaning.

Although static solutions have only an academic interest, it is worth mentioning that these solutions necessarily imply also a static background geometry and vice versa. Hence, the Λ CDM cosmology cannot be reproduced if a Minkowski background is adopted. This point will be considered in some more detail in Appendix B.

A. Back to the Λ CDM cosmology

The present observations [luminosity distance of type Ia supernovae, the baryon acoustic peak (BAO) and the cosmic microwave background (CMB) shift parameter] are quite well fitted by the so-called Λ CDM model. Is it possible to mimic such a cosmological model within the framework of the Visser's theory? A positive answer can be obtained if one identifies the last term on the right-hand side of Eq. (13) as

$$\frac{m^2}{12}\Sigma = \frac{1}{3}\Lambda. \quad (21)$$

In this case, the function Σ must be a constant and two solutions are possible. The first corresponds to $\Psi^2 = 1$ and $\Xi^2 = 1 + 2m^{-2}\Lambda$. Since $\Lambda > 0$, we have $\Xi^2 > 1$. This particular solution corresponds to a class of solutions discussed in Appendix A, in which applications to cosmology are considered. The second possibility corresponds to $\Psi^2 = (1 - 2\Xi^2)^{-1}$ and $\Sigma = -2\sqrt{1 - 2\Xi^2}$. However, this solution implies a negative cosmological constant, which is not supported by observations and will be not discussed further.

B. Λ CDM and the alternative Visser's theory

In the framework of the alternative definition of the graviton Lagrangian density, the (00) component of the field equations is formally identical to Eq. (13) but now the function Σ is defined as

$$\Sigma = \frac{5 - (18 - 12\Xi^2)\Psi^2 - (3 - 12\Xi^2 + 8\Xi^4)\Psi^4}{4\Psi^4\Xi^4}. \quad (22)$$

Another equation relating the metric coefficients can be obtained from the conservation law expressed by Eq. (7) whose time component is

$$\frac{d\Sigma}{dt} = 3 \frac{\Psi^2 - 1}{\Psi^4\Xi^4} (1 - \Psi^2 + 2\Psi^2\Xi^2) \frac{d \ln a}{dt}. \quad (23)$$

Again, to mimic the Λ CDM cosmology, we require that $\Sigma = \text{constant}$ and, in this case, two solutions are possible: $\Psi^2 = 1$ and $\Psi^2 = (1 - 2\Xi^2)^{-1}$ (Notice that the solution $\Psi^2 = 1$ corresponds to a case discussed in Appendix A). Contrary to the solutions derived from equations based on the original Visser proposal, here the two solutions are compatible with a positive cosmological constant and a lower limit for the graviton mass can be derived from the observed value of Λ . In order to have at least one solution with a positive cosmological constant, the mass of the graviton must be higher than a critical value given by

$$m > 2\sqrt{3\Omega_\Lambda} \frac{\hbar H_0}{c^2} \sim 7.7 \times 10^{-66} \text{ g}, \quad (24)$$

where we have adopted $\Omega_\Lambda = 0.7$ and $H_0 = 70 \text{ kms}^{-1} \text{ Mpc}^{-1}$.

IV. CONCLUSIONS

In the present investigation, the Visser's theory was revisited and, in particular, we have found that the original graviton tensor must be corrected by a factor equal to the ratio between the square root of the determinant of the background metric and that of the dynamical metric. This correction is not relevant when the usual weak-field approximation is considered but is of fundamental importance in cosmological applications. We have also considered an alternative to the graviton Lagrangian density proposed by Visser and the consequent modifications in the graviton tensor but both approaches lead to the same linear field equations.

We have also shown that the field equations of the theory when combined to the conservation laws are able to mimic a Λ CDM cosmology only if the background metric functions are proportional to the scale factor defining the dynamical metric. This is in contradiction with claims in the literature based on investigations considering a Minkowski background and abandoning the conservation laws expressed either by Eq. (3) or Eq. (7). However, the

results are somewhat disappointing since only a particular form of the background metric, whose choice was based on *a priori* cosmological considerations leads to satisfactory results.

Since the background required for compatibility between the theory and cosmological observations turns out to be time dependent, a natural question arises: how this affects locally physical processes occurring in the linear regime? As argued by Visser, in the weak-field approximation, the background metric should be Minkowskian and, in fact, an expanding background metric can be put in the form $m_{\alpha\beta}$ by a suitable choice of coordinates if the relevant time scales are orders of magnitude smaller than the cosmological time scale. This is the case when planetary motions or the propagation of light during time intervals up to some years are considered. However, if this is true at a given instant, this may not be the case (say) some billions years later. In this situation, by performing an expansion of the dynamical metric, a Klein-Gordon equation is not obtained. These considerations may lead to specific cosmological signatures on the local dynamics, opening eventual new constraints on the graviton mass by local observations. These aspects will be analyzed in a forthcoming paper.

APPENDIX A: Λ GR SOLUTIONS VS VISSER'S THEORY

For a given background metric $b_{\mu\nu}$, let us search for solutions in which the dynamical metric $g_{\mu\nu}$ is proportional to the background metric tensor, i.e.,

$$g_{\mu\nu} = \lambda^{-2} b_{\mu\nu}, \quad (A1)$$

where λ^2 is a positive constant. In this case, from Eq. (6) (see text), one obtains

$$T_{\text{mass}}^{\alpha\beta} = \frac{1 - \lambda^2}{16\pi} m^2 g^{\alpha\beta}. \quad (A2)$$

Consequently, the field equation formally takes the following structure,

$$G^{\alpha\beta} + \frac{\lambda^2 - 1}{2} m^2 g^{\alpha\beta} = 8\pi T^{\alpha\beta}. \quad (A3)$$

Notice that the above equation satisfies consistently Eq. (7) and indicates that all GR solutions including a cosmological constant Λ are also solutions of Visser's massive gravity with a background metric tensor

$$b_{\alpha\beta} = \left(1 + \frac{2\Lambda}{m^2}\right) g_{\alpha\beta}. \quad (A4)$$

This result indicates that solutions based on Eq. (A1) are possible only if the background metric tensor is proportional to a solution of Einstein equations including a cosmological constant and with the same matter stress-energy tensor.

Similarly, if the alternative Lagrangian density for the graviton field is considered, one obtains for the graviton tensor

$$T_{\text{mass}}^{\alpha\beta} = \frac{m^2}{16\pi} \left(\frac{1}{\lambda^2} - 1 \right) \left(\frac{2}{\lambda^2} - 1 \right) g^{\alpha\beta} \quad (\text{A5})$$

that, replaced in the field equation, leads to

$$G^{\alpha\beta} + m^2 \left(1 - \frac{1}{\lambda^2} \right) \left(\frac{1}{\lambda^2} - \frac{1}{2} \right) g^{\alpha\beta} = 8\pi T^{\alpha\beta}. \quad (\text{A6})$$

Thus, as in Visser's original theory, all GR solutions including a cosmological constant term are also solutions of the massive gravity field equations but now the proportional constant is given by

$$(1 - \lambda^{-2})(2\lambda^{-2} - 1) = \frac{2\Lambda}{m^2}. \quad (\text{A7})$$

APPENDIX B: STATIC CASES IN VISSER'S THEORY

Static solutions are those with a constant scale factor a (or $\dot{a} = 0$). The equations to be considered in this case are Eqs. (13), (14), and (16) (see text). From these equations, one obtains

$$\frac{k}{a^2} = \frac{8\pi}{3} \epsilon + \frac{1}{12} m^2 \Sigma, \quad (\text{B1})$$

where

$$\Sigma = \frac{1}{\Psi^3} [1 + 2\Psi^2 \Xi^2 - 3\Psi^2], \quad (\text{B2})$$

$$\frac{k}{a^2} = -8\pi P + \frac{1}{4} m^2 \frac{1}{\Psi} [2\Psi^2 \Xi^2 - \Psi^2 - 1], \quad (\text{B3})$$

with $\dot{\Sigma} = 0$ (static case), which implies that $\Sigma = \text{constant}$. The constancy of the energy density and of the pressure can be derived from the first equation and the equation of state. Thus, both Σ and the quantity $\Psi^{-1}[1 + \Psi^2 - 2\Psi^2 \Xi^2]$ are constants, indicating that the background metric is necessarily static.

For a given ϵ , P , k and a (the case $P = 0$ corresponds to a dust-filled static universe in the context of Visser's theory), the metric coefficient Ψ is obtained by solving the equation

$$2 \left(\frac{3k}{a^2} - 8\pi\epsilon \right) \Psi^3 - m^2 \Psi^2 - 2 \left(\frac{k}{a^2} + 8\pi P \right) \Psi + m^2 = 0. \quad (\text{B4})$$

And, consequently, Ξ satisfies

$$\Psi^2 \Xi^2 = \frac{\Psi^2 + 1}{2} - \frac{2}{m^2} \left(\frac{k}{a^2} + 8\pi P \right) \Psi. \quad (\text{B5})$$

The equations above permit one to obtain the metric coefficients Ψ and Ξ .

Reciprocally, if we have a static background, we have necessarily A and B or $\Psi\Xi$ and Ξa constants (Notice that these imply also that a/Ψ is a constant). Inserting these relations into Eq. (17) and after a straightforward calculation, one obtains

$$(2\Psi^2 \Xi^2 - \Psi^2 + 1) \frac{d\Psi}{dt} = 0. \quad (\text{B6})$$

Therefore, one has either $2\Psi^2 \Xi^2 - \Psi^2 + 1 = 0$ or $d\Psi/dt = 0$. Both possibilities imply $\Psi = \text{constant}$ and $a = \text{constant}$, i.e., a static universe.

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