

Dark degeneracy and interacting cosmic componentsAlejandro Aviles^{1,*} and Jorge L. Cervantes-Cota^{2,†}¹*Instituto de Ciencias Nucleares, UNAM, México*²*Depto. de Física, Instituto Nacional de Investigaciones Nucleares, México*

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We study some properties of the dark degeneracy, which is the fact that what we measure in gravitational experiments is the energy-momentum tensor of the total dark sector, and any split into components (as in dark matter and dark energy) is arbitrary. In fact, just one dark fluid is necessary to obtain exactly the same cosmological and astrophysical phenomenology as the Λ CDM model. We work explicitly the first-order perturbation theory and show that beyond the linear order the dark degeneracy is preserved under some general assumptions. Then we construct the dark fluid from a collection of interacting fluids. Finally, we try to break the degeneracy with a general class of couplings to baryonic matter. Nonetheless, we show that these interactions can also be understood in the context of the Λ CDM model as between dark matter and baryons. For this last investigation we choose two independent parametrizations for the interactions, one inspired by electromagnetism and the other by chameleon theories. Then, we constrain them with a joint analysis of CMB and supernovae observational data.

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I. INTRODUCTION

Current cosmological observations indicate that about 96% of the total energy content of our Universe is made of yet unknown dark components and only 4% is made of particles of the standard model. Among them, there are precision measurements of anisotropies in the cosmic microwave background (CMB) radiation [1–3], baryon acoustic oscillations [4,5], and Type Ia supernovae [6–8]. For a recent review of the nowadays status of cosmology see, e.g., [9].

Usually this *dark fluid* is separated into two components: a clustering *dark matter* piece responsible for forming structure in the Universe and a nonclustering *dark energy* with a negative pressure that is responsible for the current accelerated cosmic expansion.

Moreover, in the concordance model of cosmology—the so-called lambda cold dark matter (Λ CDM) model—the dark energy is of geometric nature and enters as a constant at the Lagrangian level of Einstein’s gravitational theory. For gravity matters, the cosmological constant is indistinguishable from the vacuum contribution of quantum fields, and here appears an intriguing problem, the sum of both contributions is about 1 over 10^{120} times the latter [10]. A second conceptual problem with the Λ CDM model comes out when one considers the ratio of dark energy over dark matter energy densities, which in spite of growing with the third power of the scale factor of the Universe, its value today is of order one. This is the so-called *coincidence problem*. As a consequence, a plethora of alternative proposals has appeared in the literature; as a short sample see [11–20].

In this paper we argue that the separation of the dark fluid into dark matter and dark energy is arbitrary and is only favored for historical reasons and computational simplicity. After all, we define the dark fluid as our lack of knowledge as¹

$$T_{\mu\nu}^{dark} = \frac{1}{8\pi G} G_{\mu\nu} - T_{\mu\nu}^{obs}, \quad (1)$$

where $G_{\mu\nu}$ comes from the observed geometry of the Universe and $T_{\mu\nu}^{obs}$ from its observed energy content. In fact, the dark sector could be composed by a large zoo of particles with complicated interactions between them. Or, it could be even just one exotic unknown dark fluid. This property has been called *dark degeneracy* by M. Kunz [21]; see also [22–28].

In this paper we assume an astrophysical perspective to the dark fluid by defining it as a barotropic fluid with speed of sound equal to zero, as in [29,30] and more recently in [31]; for similar approaches see [32–34].

Making the speed of sound equal to zero ensures that dark fluid energy density perturbations will grow at all scales, but at the same time, it allows the fluid to have a nonzero pressure. This is quite in contrast with canonical scalar fields as quintessence, for which the speed of its perturbations is equal to 1, a feature that makes dark energy perturbations to be quickly damped, so they do not grow inside the horizon.

Interactions within the dark sector has been studied largely in the literature [35–43], mainly as a mechanism to solve the coincidence problem. On the other hand, interactions between dark matter and the standard model of particles are expected, and have been studied also on

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¹We use natural units, $\hbar = 1$ and $c = 1$, otherwise is stated.

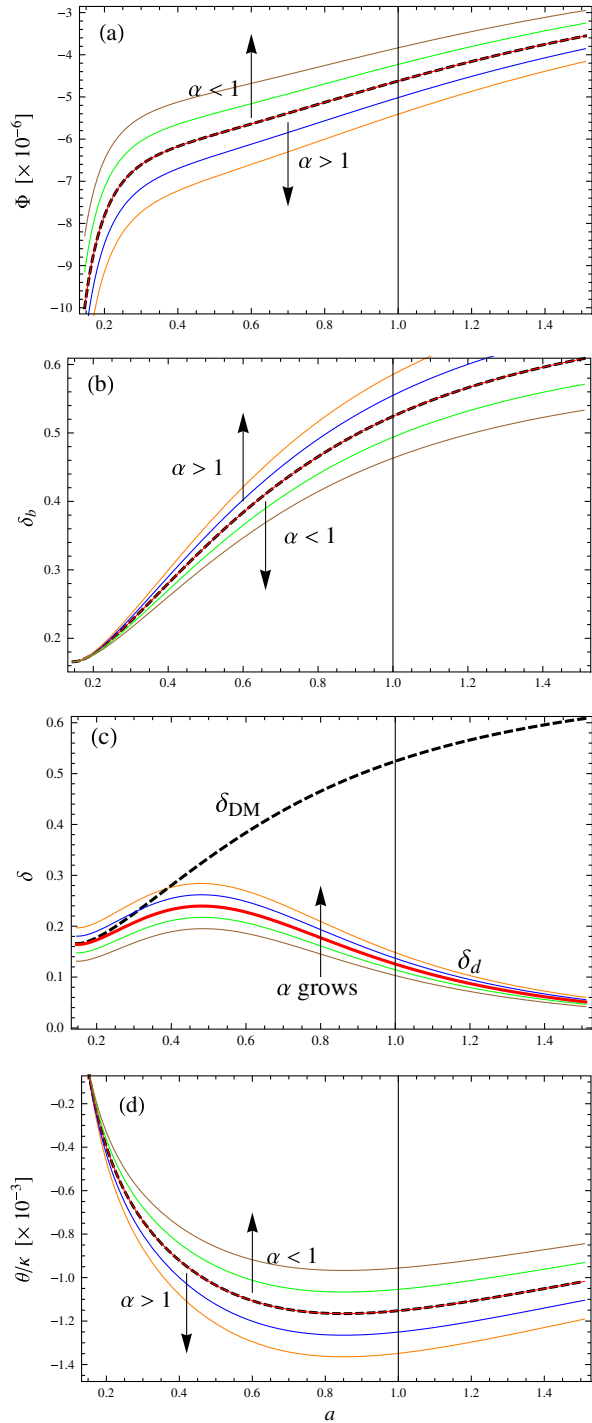


FIG. 1 (color online). Evolution of perturbation variables for a mode $k = 0.05 \text{ Mpc}^{-1}$. Solid curves are obtained from the dark fluid model for different values of the parameter α in the initial conditions $\delta_d(\tau_i) = \alpha \rho_{\text{DM}}(\tau_i) \delta_{\text{DM}}(\tau_i) / \rho_d(\tau_i)$. α takes values from 0.8 to 1.2. Dashed curves are for the Λ CDM model variables. The panels show: (a) Gravitational potential Φ . (b) Baryonic density contrast δ_b . (c) Dark fluid (solid lines) and dark matter (dashed line) density contrasts, δ_d and δ_{DM} . (d) Dark fluid (solid lines) and dark matter (dashed line) velocities, θ_d and θ_{DM} . The solutions for the case $\alpha = 1$ are depicted with the thick (red) lines, which for panels (a), (b), and (d) coincide with the dashed lines.

several occasions. In fact, the weakly interacting massive particles (WIMPs) paradigm has emerged as the predominant scenario to solve the missing mass problem [44–48]. Nevertheless, other alternatives have been proposed. Among them, special attention has been attracted by the strong interacting dark matter scenarios [49–54], in which dark matter interacts with itself and with baryons [50] through the strong force. It has been shown [49,55] that this alternative can alleviate the cuspy halos and overdensity of substructure problems that appear in N-body simulations based on the standard Λ CDM model [56].

In this paper we develop a general class of couplings of the dark fluid to the standard model of particles, and show that they can be also understood as interactions of dark matter to baryons. Then, we impose constraints by using CMB anisotropies and supernovae observations.

This work is organized as follows: In Sec. II we define the dark fluid from the properties of the dark matter itself. In Sec. III we work out the cosmological background solutions for the dark fluid and show that they are identical to the Λ CDM model ones, leading to the dark degeneracy. In Sec. IV we show that the dark degeneracy is preserved under some assumptions when one goes beyond zero order in cosmological perturbation theory. In Sec. V we work a multifluid description of the dark sector, allowing interaction among its components. In Sec. VI we extend these interactions to baryons to try to break the dark degeneracy. Finally, in Sec. VII, we present our conclusions.

II. THE SOUND SPEED OF THE DARK FLUID

Considering Newtonian gravity for the moment, the overdensities of matter in an expanding Universe follow the equation in Fourier space

$$\delta'' + 2H\delta' + (c_s^2 k_{\text{phys}}^2 - 4\pi G\rho)\delta = 0, \quad (2)$$

where k_{phys} is a physical wavelength (contrary to the comoving wavelength which we will use in the following sections) related to a physical length scale by $k_{\text{phys}} = 2\pi/l_{\text{phys}}$, c_s^2 is the speed of sound of the fluid under consideration, and prime means derivative with respect to cosmic time. In astrophysical scales it is safe to set $H = 0$ and then the scale factor is equal to a constant that we make equal to one for this discussion matters. It follows that there is a threshold scale, the Jeans length, given by

$$l_J = c_s \sqrt{\frac{\pi}{G\rho}} \quad (3)$$

for which perturbations with physical length above it, $l_{\text{phys}} > l_J$, grow by gravitational collapse, and perturbations with $l_{\text{phys}} < l_J$ develop acoustic oscillations.

We expect to have dark matter structure at a wide range of scales: from the largest cosmological structure to galaxies. In fact, even dwarf galaxies need to have dark matter halos in order to stabilize their disks and to account for the

necessary gravity sources to obtain the observed flat rotation curves which follow the luminous matter in there. The only way to guarantee that dark matter perturbations grow at all scales is demanding that the Jeans length be equal to zero.

For barotropic fluids the adiabatic speed of sound coincides with the speed at which perturbations propagate in the fluid. We define the dark fluid as a barotropic fluid with an adiabatic speed of sound equal to zero,

$$c_s^2 = 0, \quad (4)$$

and that, at a first approximation, does not interact with particles of the standard model.² Without loss of generality we can write its equation of state as

$$P(\rho) = w(\rho)\rho, \quad (5)$$

where w is its equation of state parameter and is a function of the energy density ρ only. Thus we have $c_s^2 = (\partial P/\partial \rho)_s = dP/d\rho$. Using Eq. (4) we obtain that the equation of state parameter is solved by the equation $\rho dw/d\rho + w = 0$, whose solution is

$$w = -\frac{\mathcal{C}}{\rho}, \quad (6)$$

with \mathcal{C} a constant and the minus sign is set for later convenience. This means that the pressure is a constant

$$P = P_0 = -\mathcal{C}. \quad (7)$$

Astrophysical observations constrain this value to be very small, $|P| \ll \rho_A$, where ρ_A is the energy density of typical astrophysical scales where dark matter has been detected. Usually, it is assumed that dark matter is pressureless, but this is by no means necessary; for instance it could be the case that $|P| \sim \rho_{c0}$, where ρ_{c0} is a typical cosmological energy density scale at present, without getting in contradiction with observations. In fact, this is the entrance that leads us to consider the dark fluid to be dark energy as well as dark matter.

III. DARK FLUID AS DARK ENERGY

Now, let us consider a Friedmann-Robertson-Walker description of the Universe at very large scales, filled with standard model particles (b, γ, \dots) and with the above-defined dark fluid, from now on labeled by d . The evolution equations of such a Universe are

$$H^2 = \frac{8\pi G}{3}(\rho_d + \rho_b + \rho_\gamma), \quad (8)$$

²The barotropic condition dismisses, for instance, the possibility of scalar fields for which, although the speed of propagation of its perturbations is equal to the speed of light, this does not coincide with its adiabatic speed of sound, which indeed could be zero.

$$\rho'_b + 3H\rho_b = 0, \quad (9)$$

$$\rho'_\gamma + 4H\rho_\gamma = 0, \quad (10)$$

and

$$\rho'_d + 3H(1 + w_d)\rho_d = 0, \quad (11)$$

where $H \equiv a'/a$ is the Hubble factor. Equations (9) and (10) give $\rho_b = \rho_{b0}a^{-3}$ and $\rho_\gamma = \rho_{\gamma0}a^{-4}$, where a sub-index 0 means that the quantity under consideration is evaluated at present time, and we have normalized the scale factor to be equal to one today, $a_0 = 1$. Integration of Eq. (11) gives, using Eq. (6),

$$\rho_d = \frac{\rho_{d0}}{1 + \mathcal{K}} \left(1 + \frac{\mathcal{K}}{a^3}\right), \quad (12)$$

where we have defined the constant $\mathcal{K} = (\rho_{d0} - \mathcal{C})/\mathcal{C}$. This expression is what we expect for the evolution of a unified fluid: a piece that redshifts with the scale factor as a^{-3} , as a dark matter component does, and a piece that remains constant, as vacuum energy. This result has been found elsewhere in the literature, as in [34], which investigated the Λ CDM limit of a Chaplying gas, or as in the study of barotropic fluids with constant speed of sound [29–32]. In fact, using the language of E. Linder and R. Scherrer [32], in our case the term proportional to a^{-3} is the *aether* piece of a barotropic fluid.

Now the equation of state parameter of the dark fluid (6) becomes

$$w_d = -\frac{1}{1 + \mathcal{K}a^{-3}}. \quad (13)$$

We want to stress that the energy density of the dark fluid is proportional to the inverse of its equation of state parameter

$$\rho_d = -\frac{\rho_{d0}}{(1 + \mathcal{K})} \frac{1}{w_d}, \quad (14)$$

and that its pressure, expressed in terms of the constant \mathcal{K} instead of \mathcal{C} , is

$$P_d = -\frac{\rho_{d0}}{1 + \mathcal{K}}. \quad (15)$$

In order to ensure the positivity of the energy density at all times, \mathcal{K} must be a positive number. This implies that the pressure is negative, a quality that allows the dark fluid to accelerate the Universe, and as we have outlined in the last section it could take values of the order of the critical density ($\sim 3H_0^2/8\pi G$) without affecting the behavior of the dark fluid as dark matter in astrophysical scenarios.

The Friedmann Eq. (8) becomes

$$H^2 = \frac{8\pi G}{3} \left(\frac{\rho_{d0}}{1 + \mathcal{K}} + \frac{\mathcal{K}\rho_{d0}}{1 + \mathcal{K}} a^{-3} + \rho_{b0}a^{-3} + \rho_{\gamma0}a^{-4} \right). \quad (16)$$

This is the same evolution equation for the scale factor as in the Λ CDM model. This is not an accident, if one assumes Λ CDM as the valid model and considers the total energy density of the dark components, $\rho_T = \rho_{\text{DM}} + \rho_\Lambda$, the total equation of state parameter, w_T , defined by

$$w_T \equiv \frac{\sum_a w_a \rho_a}{\sum_a \rho_a}, \quad (17)$$

where the subindex a runs over dark matter (DM) and cosmological constant (Λ), is given by

$$w_T = -\frac{1}{1 + \frac{\Omega_{\text{DM}}}{\Omega_\Lambda} a^{-3}}, \quad (18)$$

where $\Omega_{\text{DM}} = 8\pi G \rho_{\text{DM}0}/3H_0^2$ and $\Omega_{\Lambda 0} = \Lambda/3H_0^2$. (Clearly, the equation $\dot{\rho}_T = -3\mathcal{H}(1 + w_T)\rho_T$ is accomplished for the total energy density.) Then, comparing these results to Eqs. (13) and (16), we note that under the identifications

$$\mathcal{K} = \frac{\Omega_{\text{DM}}}{\Omega_\Lambda}, \quad (19)$$

and

$$\Omega_d = \Omega_{\text{DM}} + \Omega_\Lambda, \quad (20)$$

where $\Omega_d = 8\pi G \rho_{d0}/3H_0^2$, the resulting evolution cosmology in both models are exactly the same, at least at the background level.

This property has been called *dark degeneracy* by M. Kunz in [21]. In fact, it is more general than for the single fluid case worked here: any collection of fluids whose total equation of state parameter is equal to Eq. (18) and that do not interact with baryons and photons will behave exactly as the composed dark matter-cosmological constant fluid, leading to a degeneracy with the Λ CDM model.

IV. BEYOND THE BACKGROUND

In this section we explicitly show that the degeneracy is preserved when one goes beyond the homogeneous and isotropic cosmology, but only under some general assumptions that have been taken for granted in previous works. We divide the discussion into linear and higher orders in cosmological perturbation theory.

A. Linear order

Let us consider cosmological perturbation theory in the conformal Newtonian gauge, the metric is given by (for details see [57])

$$ds^2 = a^2(\tau)[-(1 + 2\Psi)d\tau^2 + (1 - 2\Phi)\delta_{ij}dx^i dx^j], \quad (21)$$

where τ is the conformal time related to the cosmic time by $dt = a d\tau$. As usual, we define the matter perturbation variables through the expressions of the energy-

momentum tensor

$$T^0_0 = -\rho(1 + \delta), \quad (22)$$

$$T^i_0 = -(\rho + P)v^i, \quad (23)$$

$$T^i_j = P((1 + \pi_L)\delta^i_j + \Pi^i_j), \quad (24)$$

where Π^i_j is the anisotropic (traceless) stress tensor. The energy density ρ and the pressure P denote background quantities and are functions of the conformal time only. The vector v^i is called the peculiar velocity and is related to the four velocity u^μ of the fluid by the relation $v^i = u^i/u^0$. In Fourier space we define the velocity $\theta = ik_i v^i$ and the scalar anisotropic stress $\sigma = 2k_i k_j \Pi^{ij} w / 3(1 + w)$. We use the flat space metric (δ_{ij}) to raise and lower indices of intrinsic space geometrical objects like v^i and Π^i_j .

The hydrodynamical equations for a general fluid are [57]

$$\dot{\delta} = -(1 + w)(\theta - 3\dot{\Phi}) - 3\mathcal{H}\left(\frac{\delta P}{\delta \rho} - w\right)\delta, \quad (25)$$

$$\begin{aligned} \dot{\theta} = & -\mathcal{H}(1 - 3w)\theta - \frac{\dot{w}}{1 + w}\theta + \frac{\delta P/\delta \rho}{1 + w}k^2\delta \\ & + k^2\Psi - k^2\sigma, \end{aligned} \quad (26)$$

where $\delta P = P\pi_L$, $\delta \rho = \rho\delta$, $\mathcal{H} = \dot{a}/a$ and a dot means derivative with respect to conformal time. For the dark fluid case, $c_s^2 = \dot{P}_d/\dot{\rho}_d = w_d - \dot{w}_d/3\mathcal{H}(1 + w_d) = 0$ and these equations become

$$\dot{\delta}_d = -(1 + w_d)(\theta_d - 3\dot{\Phi}) + 3\mathcal{H}w_d\delta_d - 3\mathcal{H}\frac{\delta P_d}{\delta \rho_d}\delta_d, \quad (27)$$

$$\dot{\theta}_d = -\mathcal{H}\theta_d + k^2\Psi + \frac{\delta P_d/\delta \rho_d}{1 + w_d}k^2\delta_d - k^2\sigma_d, \quad (28)$$

while for baryons after recombination (when the coupling to photons can be safely neglected)

$$\dot{\delta}_b = -\theta_b + 3\dot{\Phi}, \quad (29)$$

$$\dot{\theta}_b = -\mathcal{H}\theta_b + k^2\Psi. \quad (30)$$

The fluid equations are supplemented with the Einstein's equations

$$k^2\Phi = -4\pi G a^2 \sum_i \rho_i \Delta_i, \quad (31)$$

and

$$k^2(\Phi - \Psi) = 12\pi G a^2 \sum_i (\rho_i + P_i)\sigma_i \quad (32)$$

where the sum runs over all fluid contributions and

$$\Delta_i = \delta_i + 3\mathcal{H}(1 + w_i) \frac{\theta_i}{k^2} \quad (33)$$

is the rest fluid energy density [58].

To solve these equations, we need to add information about the nature of the dark fluid. The barotropic condition implies that $\delta P = c_s^2 \delta \rho$ and thus $\delta P_d = 0$, and because it is a perfect fluid, the anisotropic stress vanishes, $\Pi_{dj}^i = 0$. Accordingly, the space-space components of the perturbed energy-momentum tensor are equal to zero, $\delta T_d^{ij} = 0$. Therefore, in the right-hand side (rhs) of Eq. (27) the last term vanishes, and in the rhs of Eq. (28) only the two first terms survive. If there are only baryons and dark fluid the two gravitational potentials are equal, $\Psi = \Phi$.

Figure 1 shows the evolution of a mode $k = 0.05 \text{ Mpc}^{-1}$ of the perturbations variables. The dashed lines show results for the ΛCDM model, for which we have to use the dark matter perturbations equations,

$$\dot{\delta}_{\text{DM}} = -\theta_{\text{DM}} + 3\dot{\Phi}, \quad (34)$$

$$\dot{\theta}_{\text{DM}} = -\mathcal{H}\theta_{\text{DM}} + k^2\Psi, \quad (35)$$

instead of Eqs. (27) and (28).

In evolving the perturbations for both models, ΛCDM and dark fluid, we have imposed on the initial conditions the relations $\delta_d(\tau_i) = \alpha \rho_{\text{DM}}(\tau_i) \delta_{\text{DM}}(\tau_i) / \rho_d(\tau_i)$ for the density contrasts, and $\theta_d(\tau_i) = \alpha \rho_{\text{DM}}(\tau_i) \theta_{\text{DM}}(\tau_i) / (1 + w_d) \rho_d(\tau_i)$ for the velocities, and we let α to take different values. These initial conditions are given at an initial time well after recombination, so we can use Eqs. (29) and (30) and the relation $\delta_{\text{DM}} \simeq \delta_b$ holds.

We note that the evolution of the density contrast of baryons for the specific mode we have chosen is indistinguishable for both models if we take $\alpha = 1$. Then, although the cosmological observable is the baryonic matter power spectrum which includes a wide range of wavelengths, Fig. 1 suggests that indeed the results are the same for both models for any wavelength, as we show below.

In the cosmological context, imposing these two initial conditions is equivalent to demand that at first order in perturbation theory, the time-time and time-space components of the perturbed energy-momentum tensor of the dark fluid and ΛCDM models are equal at the given initial time τ_i . It is straightforward to show that both conditions, in the case $\alpha = 1$, will be preserved at all times. This means that in obtaining the $\alpha = 1$ solutions in Fig. 1 we have imposed that

$$\rho_d \delta_d = \rho_{\text{DM}} \delta_{\text{DM}}, \quad (36)$$

and

$$\rho_d (1 + w_d) \theta_d = \rho_{\text{DM}} \theta_{\text{DM}}. \quad (37)$$

From now on, we will demand Eqs. (36) and (37) to hold for all considered fluids, staying in degeneracy with the ΛCDM model at first order in cosmological perturbation

theory. Later on, in Sec. VI, we will try to break this degeneracy, but through couplings of the dark fluid to baryonic matter.

From the results of the previous section it is straightforward to see that $\rho_{\text{DM}}/\rho_d = 1 + w_d$, then Eq. (37) implies that $\theta_{\text{DM}} = \theta_d$, which explains the behavior of the curves in Fig. 1(d). Moreover, from Eq. (31) it follows that the gravitational potentials Φ are the same for both models, so the behavior in Fig. 1(a).

Finally, inserting Eqs. (36) and (37) into Eqs. (34) and (35), respectively, and using Eq. (14), we obtain that the hydrodynamical perturbation equations for the dark fluid are

$$\dot{\delta}_d = -(1 + w_d)(\theta_d - 3\dot{\Phi}) + 3\mathcal{H}w_d\delta_d, \quad (38)$$

$$\dot{\theta}_d = -\mathcal{H}\theta_d + k^2\Psi. \quad (39)$$

These equations are equal to (27) and (28) for a barotropic perfect fluid, confirming the dark degeneracy at first-order level.

It is interesting to note the behavior of the density contrast of the dark fluid [Fig. 1(c)], which initially grows as a dark matter component to later decay asymptotically to zero. In fact, this behavior is natural for all wavelengths modes as can be seen from Eqs. (27) and (28): At late times, when w_d tends to -1 , the δ_d source becomes independent of θ_d and is a negative quantity which tends to zero. Meanwhile at early times, for $w_d \rightarrow 0$, $\mathcal{H}w_d \rightarrow 0$, and the equations become equal to the cold dark matter evolution equations, (34) and (35).

B. Beyond linear order

To go beyond the linear order, let us make perturbation expansions to the dark fluid and ΛCDM energy-momentum tensors about the (zero order) background cosmological fluids as

$$T_{\mu\nu} = T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} + \dots \quad (40)$$

If the total energy-momentum tensors of both models are equal ($T_{\mu\nu}^d = T_{\mu\nu}^{\Lambda\text{CDM}}$), clearly each of the terms in the expansion will be equal as well ($T_{\mu\nu}^{d(i)} = T_{\mu\nu}^{\Lambda\text{CDM}(i)}$). This argument is outlined in [21] to argue that the degeneracy is preserved at all orders. Certainly, this is correct.

Nonetheless, we want to stress a different approach: We are affected gravitationally by the total energy-momentum tensor, but usually when comparing observations to models we expand it as in Eq. (40), and after this we assign values to each of the pieces. The fact that both energy-momentum tensors are equal, say at zero order, does not imply that they will be equal at first order. In this situation, equations such as (36) and (37) are conditions of the theory and not consequences of it, and if not imposed, the degeneracy is broken, as seen in Fig. 1 for $\alpha \neq 1$.

This approach, which we will adopt, is very close to that followed in some papers which investigate modified gravity theories that are indistinguishable in the background from the Λ CDM model but could differ in linear order, leaving imprints that are parametrized to compare to current and future observations (see [59] and references therein). In fact, if some of these imprints were detected, they could be a consequence of interactions within the dark sector (or even with baryons, as those we study in Sec. VI) instead of a deviation from general relativity.

Finally, there is a subtle point that is pertinent to assert at this moment. The fact that the Universe is so smooth at very large scales is what allows us to expand the energy-momentum tensor of the cosmic fluids as in Eq. (40). The zero-order term is in fact a spatial average in hypersurfaces of constant cosmic time, $T_{\mu\nu}^{(0)} = \langle T_{\mu\nu} \rangle$, and thereafter we assume that general relativity holds in the form $G_{\mu\nu}(g^{(0)}) = \langle T_{\mu\nu} \rangle$. This is not true in general due to the nonlinear character of gravity; it is well-known that back-reaction effects contributes to this last equation. For recent work in the averaging procedure in cosmology and its caveats see [60].

For the specific case that concerns us there is a related issue. Let us consider a fluid with equation of state $P = P(\rho)$ that leads to a unified description of the dark sector. It has been noted that it is not necessarily true that $\langle P \rangle = P(\langle \rho \rangle)$ [61]; in fact higher-order corrections enter into this equation. Therefore, when some scale grows and leaves the linear regime, the naive averaging procedure on the equation of state is no longer valid. This nonlinear effect has been investigated in [29,61,62], and it was shown that instabilities are expected to occur in unified dark models, even on large cosmological scales. Two exceptions are the cases in which the involved equations of state are $P = \text{constant}$ and $P = w\rho$, with w a constant. The former is the case of the dark fluid, and then, it does not suffer for this *averaging problem*. Nonetheless, it is worth emphasizing that in models that depart—although indistinguishable by current observations when calculations are performed using the average procedure—from the dark fluid or from the Λ CDM models, these effects must be considered.

V. MULTIPLE INTERACTING DARK FIELDS

From the last section it is clear that the dark fluid, although not necessarily, could be the sum of a dark matter and a cosmological constant component. Nonetheless, if the interaction to the particles of the standard model is only gravitational (which is the ultimate definition of *dark*), the nature of the dark fluid is fundamentally impossible to elucidate, because of the universality of this force.

In this section, we explore the possibility that the dark fluid is composed of a collection of dark components with possible interactions between them. The equations that we obtain will be generalized in the next section to include interactions to baryons. Let us consider the Einstein field

equations,

$$G^{\mu\nu} = 8\pi G \left(\sum_a T_a^{\mu\nu} + T_{\text{SM}}^{\mu\nu} \right), \quad (41)$$

where the subindex a labels the different dark components. If we forbid dark components to interact with standard model particles, the Bianchi identities imply $\nabla_\mu T_{\text{SM}}^{\mu\nu} = 0$ and

$$\nabla_\mu T_a^{\mu\nu} = Q_a^\nu, \quad (42)$$

where the energy-momentum transfer vectors, Q_a^ν , obey the constraint

$$\sum_a Q_a^\nu = 0. \quad (43)$$

Considering for the moment the homogeneous and isotropic cosmology, the continuity equation for each fluid is

$$\dot{\rho}_a + 3\mathcal{H}(1 + w_a)\rho_a = q_a, \quad (44)$$

where q_a denote the energy transfer between different dark components. This implies that at zero order (ignoring the subindex a 's for the moment),

$$Q^{\nu(0)} = \frac{1}{a^2}(q, \vec{0}), \quad (45)$$

if we define $Q \equiv \sqrt{-g_{\mu\nu}Q^\mu Q^\nu}$, then $Q^{(0)} = q/a$ and $Q^{\nu(0)} = a^{-1}(Q^{(0)}, \vec{0})$. Now, following [63,64], we split the interaction vector as

$$Q_\nu = Qu_\nu + f_\nu, \quad (46)$$

where the momentum transfer, f^ν , is first order and is orthogonal to the four velocity $f^\nu u_\nu = 0$. Thus, under spatial rotations Q transforms as a scalar and f^i as a vector. Accordingly, we decompose f^i as

$$f_i = f_{,i} + \epsilon_i \quad (47)$$

with f a scalar and ϵ_i a transverse vector, $\epsilon^i_{,i} = 0$, and we define the perturbation δQ through

$$Q = Q^{(0)} + \delta Q = \frac{1}{a}(q + \delta q), \quad (48)$$

where in the last equality we defined $\delta q \equiv a\delta Q$. From (46) it follows that up to first order

$$Q^0 = \frac{1}{a^2}(q(1 - \Psi) + \delta q) \quad (49)$$

$$Q^i = \frac{1}{a^2}qv^i + \frac{1}{a^2}f^i + \frac{1}{a^2}\epsilon^i. \quad (50)$$

Finally, from Eq. (43) one obtains (after restoring a 's) the constriction equations

$$\sum_a \delta q_a = 0, \quad \sum_a q_a = 0, \quad (51)$$

and $a^2 \sum_a \mathcal{Q}_a^i = \sum_a (q_a v_a^i + f_a^i + \epsilon_a^i)$, which by taking the divergence and going to Fourier space become

$$\sum_a (q_a \theta_a + k^2 f_a) = 0. \quad (52)$$

Now, to first order in perturbation theory, the divergence of the energy-momentum tensor—for each fluid, omitting the subindex a —is

$$\begin{aligned} \nabla_\mu T^{\mu 0} &= \frac{1}{a^2} (1 + \delta - 2\Psi) (\dot{\rho} + 3\mathcal{H}(1+w)\rho) \\ &+ \frac{\rho}{a^2} \left[\dot{\delta} + (1+w)(\partial_i v^i - 3\dot{\Phi}) + 3\mathcal{H} \left(\frac{\delta P}{\delta \rho} - w \right) \delta \right], \end{aligned} \quad (53)$$

$$\begin{aligned} \nabla_\mu T^{\mu i} &= \frac{P}{a^2} \partial_j \Pi^{ij} + \frac{1}{a^2} (\dot{\rho} + 3\mathcal{H}(1+w)\rho) v^i \\ &+ \frac{\rho + P}{a^2} \left[\dot{v}^i + \mathcal{H} \left(1 + \frac{\dot{\rho} w}{\mathcal{H}(1+w)\rho} \right) v^i \right. \\ &\left. + \frac{\dot{w}}{1+w} v^i + \frac{w \pi_L^i}{1+w} + \Psi^i \right]. \end{aligned} \quad (54)$$

Note that if the interaction vanishes, both equations reduce to the usual ones. We take the divergence of Eq. (54) to isolate the scalar mode perturbations, then Eq. (42) implies

$$\begin{aligned} \dot{\delta}_a + (1+w_a)(\theta_a - 3\dot{\Phi}) + 3\mathcal{H} \left(\frac{\delta P_a}{\delta \rho_a} - w_a \right) \delta_a \\ + \frac{q_a}{\rho_a} (\delta_a - \Psi) - \frac{\delta q_a}{\rho_a} = 0 \end{aligned} \quad (55)$$

and

$$\begin{aligned} \dot{\theta}_a + \mathcal{H} \left(1 - 3w_a + \frac{q_a w_a}{\mathcal{H}(1+w_a)\rho_a} \right) \theta_a + \frac{\dot{w}_a}{1+w_a} \theta_a \\ - \frac{\delta P_a / \delta \rho_a}{1+w_a} k^2 \delta_a - k^2 \Psi + k^2 \sigma_a + \frac{k^2 f_a}{\rho_a(1+w_a)} = 0, \end{aligned} \quad (56)$$

where we have restored the index a 's and back to Fourier space. Also, we have used the decomposition for Π^{ij} in tensor, vector and scalar pieces. Then, when hit with $k_i k_j$ only the scalar anisotropic stress survives and the first term of the rhs of Eq. (56) becomes $2k^2 \Pi^{(s)} P/3 = (\rho + P)k^2 \sigma$.

Defining the total energy density of these dark fluids as $\rho_T = \sum \rho_a$, it follows that the total density contrast, δ_T , is the weighted sum of the individual components

$$\delta_T = \frac{1}{\rho_T} \sum_a \rho_a \delta_a, \quad (57)$$

and the total velocity is given by

$$\theta_T = \frac{1}{\rho_T(1+w_T)} \sum_a \rho_a (1+w_a) \theta_a, \quad (58)$$

where w_T is given by Eq. (18), and the index a runs over all dark components. Using these two last equations it is straightforward to show that the hydrodynamical equations for the total fluid are

$$\dot{\delta}_T + (1+w_T)(\theta_T - 3\dot{\Phi}) + 3\mathcal{H} \left(\frac{\delta P_T}{\delta \rho_T} - w_T \right) \delta_T = 0 \quad (59)$$

and

$$\begin{aligned} \dot{\theta}_T + \mathcal{H}(1-3w_T)\theta_T + \frac{\dot{w}_T}{1+w_T} \theta_T - \frac{\delta P_T / \delta \rho_T}{1+w_T} k^2 \delta_T \\ - k^2 \Psi + k^2 \sigma_T = 0, \end{aligned} \quad (60)$$

where $\delta P_T = \sum \delta P_a$. These are exactly the equations of a single noninteracting fluid, Eqs. (25) and (26). The adiabatic speed of sound of the total fluid is $c_{sT}^2 = \dot{P}_T / \dot{\rho}_T = \sum \rho_a c_{sa}^2 / \rho_T$. Thus, by forcing the condition $c_{sT}^2 = 0$ we obtain that $w_T = w_d$, and if each dark component has positive energy ($\rho_a > 0$), it also implies that $c_{sa}^2 = 0$ and $\delta P_a = 0$. Therefore, we obtain that the hydrodynamical equations for the total fluid perturbations become the same as the dark fluid perturbations (Eqs. (38) and (39)), under the substitution $T \rightarrow d$. Thus, the composed fluid we have constructed is just the dark fluid.

If one considers the unphysical case in which not all dark components have positive energy density, the speed of sound of some components can be different from zero, and the total pressure perturbation is equal to

$$\delta P_T = c_{sT}^2 \delta \rho_T - \frac{1}{6\mathcal{H} \dot{\rho}_T} \sum_{a,b} \dot{\rho}_a \dot{\rho}_b (c_{sa}^2 - c_{sb}^2) \mathcal{S}_{ab}, \quad (61)$$

where

$$\mathcal{S}_{ab} = \frac{\delta_a}{1+w_a} - \frac{\delta_b}{1+w_b} \quad (62)$$

is the relative entropy perturbation between fluids a and b [65]. Therefore, to obtain the dark fluid in the case in which some components have negative energy density we have to impose the pressure perturbations given by Eq. (61) to be equal to zero. This condition also implies that the total fluid is barotropic.

VI. INTERACTIONS TO THE STANDARD MODEL OF PARTICLES

In this section we study dark fluid couplings to baryonic matter. We consider models in which the background cosmology is the same as in the Λ CDM model, accordingly we do not allow energy transfer ($q = 0$) between the cosmic components. Nevertheless, we allow a momentum

transfer different from zero. Thus, the interactions affect the fluids only at first order in perturbation theory.

In this work we will not consider interactions to electromagnetism. This is not only for simplicity, many theoretical models present conformal couplings, as the chameleon theories [66,67] (and in general, scalar tensor gravity theories [68]), or even direct couplings to the trace of the energy-momentum tensor [27,69–72]. Also, this is expected in scenarios like the strong interacting dark matter [49,50], where the couplings are through the strong force, for which photons are chargeless.

In the previous section we found the hydrodynamical equations for a system of coupled dark components. It is straightforward to generalize those results to a coupling between the dark fluid and baryonic matter. The conservation equations for the perturbations are, for the dark fluid,

$$\dot{\delta}_d = -(1 + w_d)(\theta_d - 3\dot{\Phi}) + 3\mathcal{H}w_d\delta_d + \frac{\delta q_d}{\rho_d} \quad (63)$$

$$\dot{\theta}_d = -\mathcal{H}\theta_d + k^2\Psi - \frac{k^2 f_d}{\rho_d(1 + w_d)}, \quad (64)$$

and for baryons

$$\dot{\delta}_b = -\theta_b + 3\dot{\Phi} + \frac{\delta q_b}{\rho_b}, \quad (65)$$

$$\dot{\theta}_b = -\mathcal{H}\theta_b + k^2\Psi + c_{sb}^2 k^2 \delta_b - \frac{k^2 f_b}{\rho_b(1 + w_b)}. \quad (66)$$

These equations are supplemented by the constrictions (51) and (52). The interactions to electromagnetism are considered, but not shown in Eq. (66). Note that we have not considered a term $c_{sd}^2 k^2 \delta_d$ in Eq. (64), in agreement with the definition of dark fluid.

In the absence of a fundamental theory we parametrize the coupling. But before do it, let us make a comparison to electromagnetism. Under the formalism we developed in the last section, the hydrodynamical perturbation equations for Thomson scattering (neglecting photon moments beyond the quadrupole) can be obtained if we choose $\delta q_\gamma = 0$ and $k^2 f_\gamma = -\rho_\gamma(1 + w_\gamma)ax_en_e\sigma_T c(\theta_b - \theta_\gamma)$, where $\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$ is the Thomson scattering cross section, n_e is the number density of electrons, x_e is the ionization fraction, and $c = 1$ is the velocity of light.

Using the analogy to electromagnetism we choose the interaction terms to be

$$\delta q_d = 0, \quad (67)$$

and

$$f_d = \rho_d(1 + w_d)an_d\Sigma_I(\theta_b - \theta_d)/k^2, \quad (68)$$

where the parameter Σ_I has units of area times velocity, or thermalized cross section $\langle\sigma v\rangle$, which we identify with some, unknown, fundamental interaction. n_d is the number

density of dark *particles* that we set equal to

$$n_d = \frac{\rho_{d0}}{m_p a^3}, \quad (69)$$

where we use the proton mass, $m_p = 0.938 \text{ GeV}$, as an arbitrary mass scale and ρ_{d0} is the energy density of the dark fluid evaluated today. Here, we have not an analogous to the ionization fraction, in empathy to universal interactions.

The constrictions $f_b = -f_d$ and $\delta q_d = -\delta q_b = 0$, enter into the baryons equations. Because of the relation $\rho_{\text{DM}} = (1 + w_d)\rho_d$ the interaction term goes as $(\theta_d - \theta_b)a^{-2}$. Then, the greatest deviation from the standard model of cosmology will come from the early Universe. Accordingly we expect the interaction to be tightly bounded from CMB anisotropies tests. We have modified the public available code CAMB [73] to account for the interaction.

In Fig. 2 we show the angular power spectrum for the CMB radiation. There several curves are shown for different values of Σ_I , which is expressed in units of 10^{-6} times the Thomson scattering cross section times the speed of light ($c = 1$). We note that the greatest differences with the Λ CDM model come at high multipoles, which is not surprising because the interaction quickly decays and only modes which have entered into the horizon at early times in the Universe were affected by it.

On the other hand, it is well known that interactions between the dark sector and baryons are tightly constrained from equivalence principle and solar system tests [74]. Nonetheless, such interactions could be originated by general predictions of string theory [75,76], leading to the necessity of some screening mechanism [66,67,77,78] to evade these experimental constraints. In chameleon theories, the range of this interaction depends on the energy density of the surrounding medium, leading to long-range

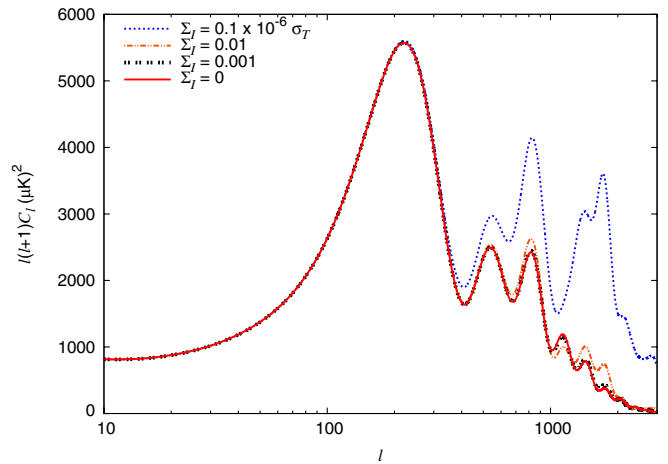


FIG. 2 (color online). The CMB power spectrum considering different values of the interaction parameter Σ_I in units of $10^{-6}\sigma_T$. The other parameters are fixed.

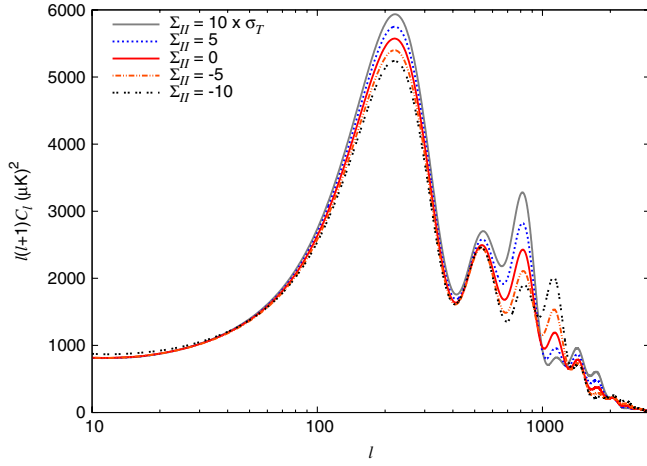


FIG. 3 (color online). The CMB power spectrum considering different values of the interaction parameter Σ_{II} in units of σ_T . The other parameters are fixed.

forces when the density is low, and to short-range force-when it is high. Inspired by these theories, we give a k dependence to the hydrodynamical equations, such that the interactions respond to an effective wave number given by $k_{\text{eff}}^2 = k^2 g(\rho)$, where g is a monotonic growing function of the ambient energy density. By noting that ρ falls with the scale factor, we choose a potential law for the wavelength number, $k_{\text{eff}}^2 = k^2/a^n$. In the following we specialize to the case $n = 2$. We use the substitution $k \rightarrow k_{\text{eff}}$ in Eq. (68) and obtain³

$$f_d = \rho_d(1 + w_d)\Sigma_{II} \frac{\rho_{d0}}{m_p a^2} (\theta_b - \theta_d)/k^2. \quad (70)$$

Clearly, we can also understand this interaction as a dependence of the cross section on the ambient energy density. In Fig. 3, we show the CMB angular power spectrum for different values of the parameter Σ_{II} . We note that, in remarkable contrast to the Σ_I interaction, here all scales are affected in a similar way. This is because the effective interaction remains constant while the scale factor grows and all modes feel it when they enter the horizon.

It is possible to treat both parametrizations together if we choose

$$f_d = \rho_d(1 + w_d) \frac{(\Sigma_I + \Sigma_{II} a^2)\rho_{d0}}{m_p a^2} (\theta_b - \theta_d)/k^2. \quad (71)$$

The hydrodynamical equations become

$$\dot{\delta}_d = -(1 + w_d)(\theta_d - 3\dot{\Phi}) + 3\mathcal{H}w_d\delta_d, \quad (72)$$

³Chameleon theories are modifications of gravity and their perturbation equations are different from those used here; it is out of the scope of this work to treat the precise equations of the chameleons. For such a treatment see [79,80].

$$\dot{\theta}_d = -\mathcal{H}\theta_d + k^2\Psi - \frac{(\Sigma_I + \Sigma_{II}a^2)\rho_{d0}}{m_p a^2} (\theta_b - \theta_d), \quad (73)$$

and for baryons

$$\dot{\delta}_b = -\theta_b + 3\dot{\Phi}, \quad (74)$$

$$\begin{aligned} \dot{\theta}_b = & -\mathcal{H}\theta_b + k^2\Psi + c_{sb}^2 k^2 \delta_b - \frac{\rho_d}{\rho_b} (1 + w_d) \\ & \times \frac{(\Sigma_I + \Sigma_{II}a^2)\rho_{d0}}{m_p a^2} (\theta_d - \theta_b). \end{aligned} \quad (75)$$

If one assumes the Λ CDM decomposition of the dark fluid in dark matter and cosmological constant, it is easy to see that instead of Eqs. (63) and (64), the following equations govern the evolution of the dark matter perturbation

$$\dot{\delta}_{\text{DM}} = -\theta_{\text{DM}} + 3\dot{\Phi} + \frac{\delta q_{\text{DM}}}{\rho_{\text{DM}}}, \quad (76)$$

$$\dot{\theta}_{\text{DM}} = -\mathcal{H}\theta_{\text{DM}} + k^2\Psi + \frac{k^2 f_{\text{DM}}}{\rho_{\text{DM}}}, \quad (77)$$

where we have used the conditions (36) and (37). The transfer energy and momentum terms are related by

$$\delta q_{\text{DM}} = \delta q_d \quad \text{and} \quad f_{\text{DM}} = f_d. \quad (78)$$

Thus, although the degeneracy with the Λ CDM has been broken at first order in perturbation theory, there exist degeneracies to other models, as in this case to Λ CDM with the same interactions, which means that the general class of interactions given by Eqs. (63) and (64) does not help us to elucidate the actual decomposition of the dark fluid (if it exists). Accordingly, we can treat the above described couplings as interactions between dark matter and baryonic matter, without loss of generality, as we will do for numerical purposes.⁴ Thereafter, if desired, we can go back and forth between both models' results using Eqs. (19) and (20).

To constrain the interactions, we perform a Monte Carlo Markov Chain (MCMC) analysis over the nine-parameter space (Model A) $\{\Omega_b h^2, \Omega_{\text{DM}} h^2, \theta, \tau, n_s, \log A_s, A_{sz}, \Sigma_I, \Sigma_{II}\}$ using the code COSMOMC [81]. θ is the ratio of the sound horizon to the angular diameter distance at recombination, τ is the reionization optical depth, n_s is the spectral index of the primordial scalar perturbations and A_s is its amplitude at a pivot scale of $k_0 = 0.05 \text{ Mpc}^{-1}$.

⁴Special care must be taken because numerical codes, as CAMB, use synchronous gauge and fix the residual gauge freedom by taking $\theta_{\text{DM}} = 0$. In cases in which there are sources in the θ evolution equation, like ours, this is not possible to do.

We have imposed flat priors on the two interaction parameters: $0 < \Sigma_I < 10^{-7} \times \sigma_T$ and $-11 \times \sigma_T < \Sigma_{II} < 10 \times \sigma_T$. For the CMB anisotropies and polarization data we used the Wilkinson Microwave Anisotropy Probe (WMAP) seven-year observations results [3]. For the joint analysis we use also Hubble Space Telescope measurements (HST) [82] to impose a Gaussian prior on the Hubble constant today of $H_0 = 74 \pm 3.6$ km/s/Mpc, and the supernovae type Ia Union 2 data set compilation by the Supernovae Cosmology Project [8].

We also study two other models: Model B, only considering the interaction Σ_I , it has an eight-parameter space $\{\Omega_b h^2, \Omega_{\text{DM}} h^2, \theta, \tau, n_s, \log A_s, A_{sz}, \Sigma_I\}$; and Model C, which does not consider any interaction, a seven-parameter space $\{\Omega_b h^2, \Omega_{\text{DM}} h^2, \theta, \tau, n_s, \log A_s, A_{sz}\}$, corresponding to the standard Λ CDM model.

The summary of the posterior one-dimensional marginalized probabilities is outlined in Fig. 4 and Table I. In

Fig. 5 we show the contour confidence intervals for the marginalized $\Sigma_I - \Sigma_{II}$ space at 0.68 and 0.95 confidence levels (CLs). There the high degeneracy between both parameters is shown: while Σ_{II} takes values closer to zero, Σ_I also does. It is interesting that nonzero values of the interactions (when introduced) are consistent and preferred by the considered data at 0.95 CL.

We note that the addition of the Σ_I and Σ_{II} interactions to the theory produce remarkable differences between the parameters estimations of Model A and Model C. This is more evident for the baryonic matter energy density $\Omega_b h^2$, as can be observed from Figs. 4, 6, and 7, or read directly from Table I. For this reason, we study Model B, which only considers the Σ_I interaction. In this case, the tensions between the parameters estimations are alleviated. The discrepancies are also evident in the primordial power spectrum parameters A_s and n_s , which also can be seen from Figs. 4

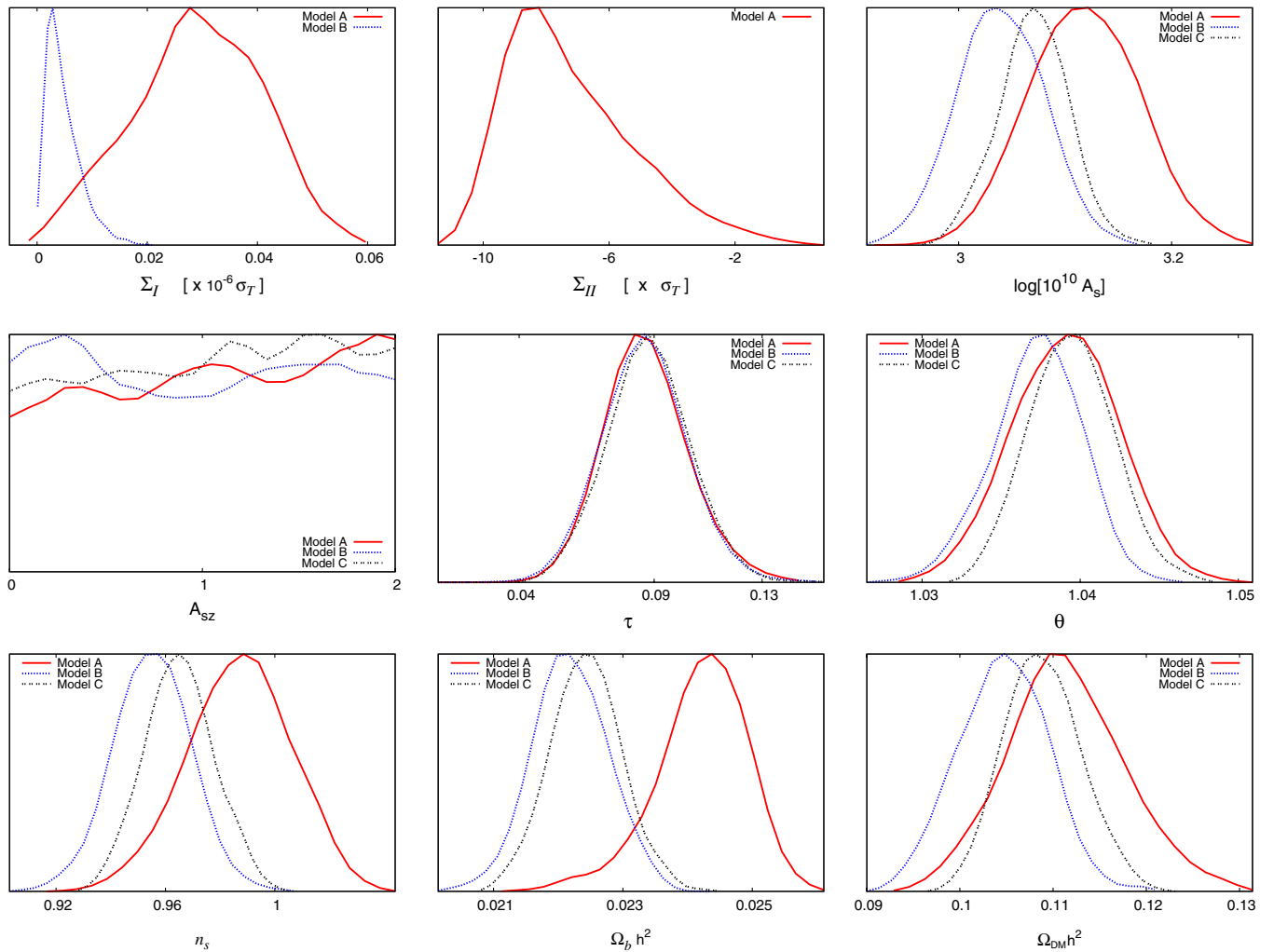


FIG. 4 (color online). (color online) Marginalized probability for the complete set of parameters. Solid lines (red) are for Model A, dotted lines (blue) for Model B, and double-dotted (black) lines for Model C. The data used are the WMAP seven-year results, Union 2 data set supernovae compilation and a prior of HST on the Hubble constant.

TABLE I. Summary of constraints. The upper panel contains the parameter spaces explored with MCMC for each one of the three models. The bottom panel contains derived parameters. The data used are the WMAP seven-years data, Union 2 compilation and HST.

| Parameter | Model A ^a | Model B ^a | Model C ^a |
|------------------------------|---------------------------------|-------------------------------|------------------------------|
| $10^2 \Omega_b h^2$ | $2.420^{+0.066}_{-0.064}$ | $2.219^{+0.056}_{-0.056}$ | $2.243^{+0.053}_{-0.053}$ |
| $\Omega_c h^2$ | $0.1114^{+0.0061}_{-0.0060}$ | $0.1046^{+0.0047}_{-0.0049}$ | $0.1089^{+0.0041}_{-0.0041}$ |
| θ | $1.039^{+0.003}_{-0.003}$ | $1.037^{+0.003}_{-0.003}$ | $1.039^{+0.003}_{-0.003}$ |
| τ | $0.08712^{+0.00565}_{-0.00721}$ | $0.08646^{+0.0061}_{-0.0067}$ | $0.08797^{+0.003}_{-0.003}$ |
| $10^8 \Sigma_I$ ^b | $2.910^{+1.169}_{-1.959}$ | $0.4845^{+0.2980}_{-0.3824}$ | - |
| Σ_{II} ^b | $-7.169^{+2.218}_{-1.959}$ | - | - |
| n_s | $0.9869^{+0.0192}_{-0.0184}$ | $0.9551^{+0.0135}_{-0.0137}$ | $0.9651^{+0.0123}_{-0.0124}$ |
| $\log[10^{10} A_s]$ | $3.118^{+0.051}_{-0.051}$ | $3.039^{+0.040}_{-0.040}$ | $3.070^{+0.031}_{-0.033}$ |
| A_{sz} ^c | 1.054 ± 0.578 | 0.9544 ± 0.5911 | 1.040 ± 0.574 |
| Ω_d | $0.952^{+0.033}_{-0.033}$ | $0.956^{+0.031}_{-0.030}$ | $0.955^{+0.027}_{-0.027}$ |
| \mathcal{K} | $0.296^{+0.033}_{-0.033}$ | $0.270^{+0.034}_{-0.036}$ | $0.291^{+0.034}_{-0.032}$ |
| t_0 | $13.64^{+0.12}_{-0.13}$ Gyr | $13.85^{+0.11}_{-0.12}$ Gyr | $13.79^{+0.12}_{-0.11}$ Gyr |
| Ω_Λ | $0.734^{+0.024}_{-0.024}$ | $0.754^{+0.022}_{-0.021}$ | $0.740^{+0.019}_{-0.020}$ |
| H_0 ^d | $71.55^{+1.86}_{-1.91}$ | $71.95^{+2.09}_{-1.96}$ | $71.14^{+1.71}_{-1.85}$ |

^aThe mean values of the posterior distribution for each parameter. The quoted errors show the 68% confidence levels.

^b Σ_I and Σ_{II} are given in units of the Thomson scattering cross section times the speed of light, $\sigma_T = 6.65 \times 10^{-25}$ cm² and $c = 1$.

^cThe quoted errors in A_{sz} are the standard deviations of the distributions.

^d H_0 is given in Km/s/Mpc.

and 8. Nonetheless, these are not alleviated when one considers Model B.

Instead of using the proton mass as the scale in the interactions, we can use an arbitrary associated mass to the dark matter or dark fluid particles, m_d . We obtain the following constraints at 0.68 CL on the ratio Σ/m_d (we use $c = 3 \times 10^{10}$ cm/s):

For the case in which we consider both interactions (Model A)

$$3.58 \times 10^{-22} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2} < \frac{\Sigma_I}{m_d} < 8.68 \times 10^{-22} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2}, \quad (79)$$

and

$$-1.94 \times 10^{-13} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2} < \frac{\Sigma_{II}}{m_d} < -1.05 \times 10^{-13} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2}. \quad (80)$$

While for Model B

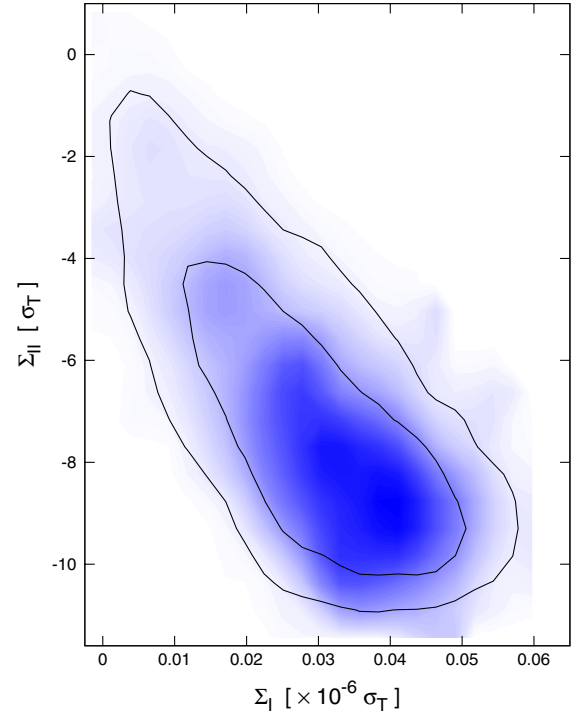


FIG. 5 (color online). Contour confidence intervals for Σ_I vs Σ_{II} at 68% and 95% CL. The shading shows the mean likelihood of the samples.

$$0.22 \times 10^{-22} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2} < \frac{\Sigma_I}{m_d} < 1.66 \times 10^{-22} \frac{\text{cm}^3/\text{s}}{\text{GeV}/c^2}. \quad (81)$$

We only obtained bounds of m_d and Σ in the combination Σ/m_d , although it is possible to find constrictions of them separately, even to the cross section and the particles' thermal velocity, leading to constraints in the $\sigma - m_d$ plane. Nevertheless, to do this we have to allow $c_s^2 \neq 0$, and moreover, make further assumptions on the thermodynamics of the dark matter (or the dark fluid) [52].

Note that the *effective* thermalized cross section for interaction *II* is $a^2 \Sigma_{II}$, and this is equal to Σ_I at about a redshift $z \sim 10^5$. Before this epoch interaction *I* dominates, and after that, interaction *II* starts to do it. Just before recombination, at $z \simeq 1100$, interactions *I* and *II* are smaller in strength than the Thomson interaction by about 9 and 5 orders of magnitude, respectively. After this time, the ionization fraction x_e falls exponentially and Thomson scattering becomes subdominant very quickly.

In Fig. 9 we plot the CMB power spectrum for the mean values estimated by the MCMC analysis. The reported difference is $\Delta = l(l+1)(C_l^{\text{Model C}} - C_l)$. We note that the three models are well inside the error bars of the binned measurements from WMAP seven-year observations results. Nonetheless, at higher multipoles ($l > 1000$) the three models start to have notable discrepancies. The

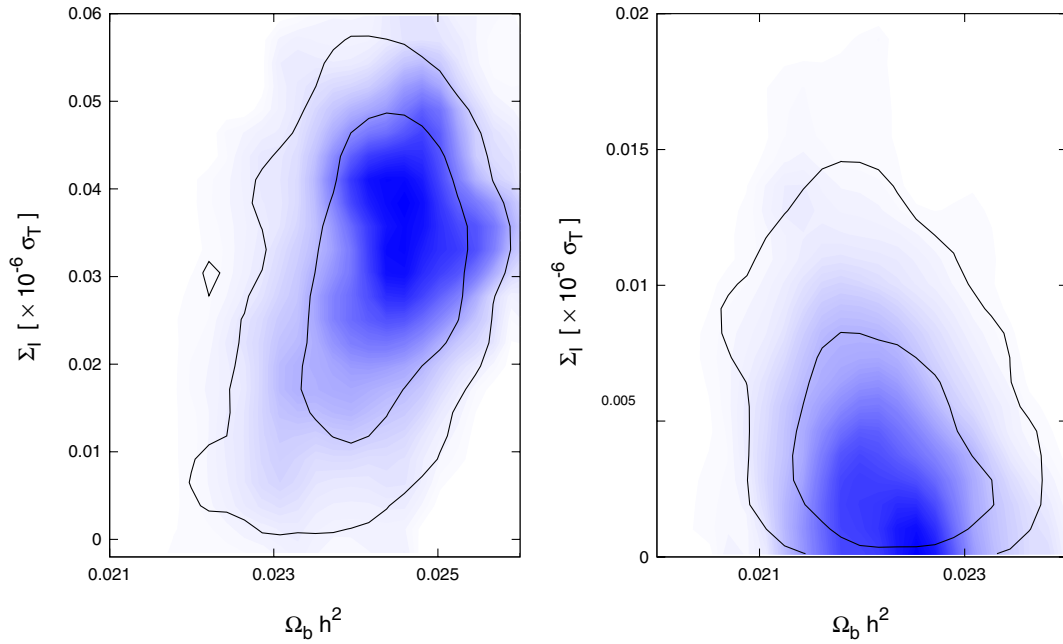


FIG. 6 (color online). 2D Posterior confidence intervals for $\Omega_b h^2$ vs Σ_I at 68% and 95% CL. Left panel: Considering both interactions (model A). Right panel: considering only interaction Σ_I (model B). The shading shows the mean likelihood of the samples.

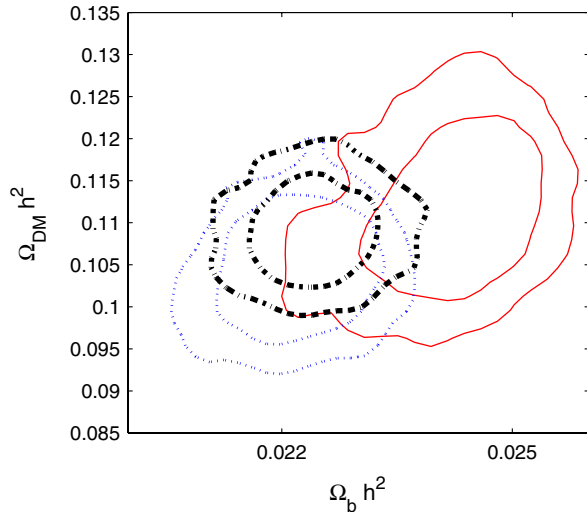


FIG. 7 (color online). Contour confidence intervals for $\Omega_b h^2$ vs $\Omega_{DM} h^2$ at 68% and 95% CL. Solid (red) lines are for Model A, dot-dashed (blue) for Model B, and thick dashed (black) for Model C.

higher differences appear in the region $1000 < l < 2000$, just inside the window in which the primordial power spectrum parameters are expected to be estimated with higher precision ($1000 < l < 3000$). For $l > 3000$ secondary anisotropies, mainly the Sunyaev-Zeldovich effect, are expected to dominate the CMB spectrum, while for multipoles $l < 1000$ the spectrum is more sensible to the other parameters. These high- l power spectrum multi-

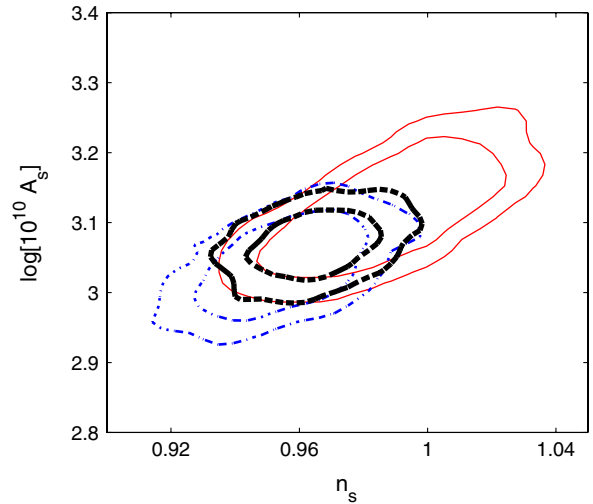


FIG. 8 (color online). Contour confidence intervals for n_s vs A_s at 68% and 95% CL. Solid (red) lines are for Model A, dot-dashed (blue) for Model B, and thick dashed (black) for Model C.

poles will be probed by the PLANCK mission [83], so we expect to obtain tighter constraints in the near future.

In this work we have not considered the effect that interactions I and II could have on big bang nucleosynthesis. This is because our phenomenological model only includes the thermalized cross sections Σ_I and Σ_{II} of elastic collisions, whose strengths are at least 9 orders of magnitude weaker than the Thomson scattering

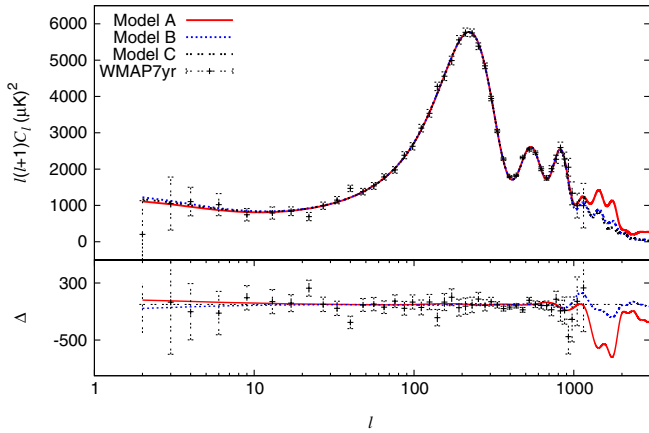


FIG. 9 (color online). CMB power spectrum for the mean values estimated with the MCMC analysis by CosmoMC for the three models. The reported difference is $\Delta = l(l+1) \times (C_l^{\text{ModelC}} - C_l)$.

at this epoch and, more important, do not annihilate baryons and maintain the baryon-to-photon ratio unaltered. Accordingly, we expect the effect over this process to be quite weak.

VII. CONCLUSIONS

In this paper we worked out some properties of the so-called dark degeneracy [21]: the fact that we can only measure the total energy-momentum tensor of the dark sector and any split into different pieces (as in dark matter and dark energy) is artificial, although it could be mathematically convenient.

We start by defining the *dark fluid* as a barotropic fluid with speed of sound equal to zero, as in [29,30] and, more recently, in [31] (for similar approaches see [32,33]). This is motivated by astrophysical constraints in the dark matter besides its cosmological properties. Making the speed of sound equal to zero allows dark fluid energy density perturbations to grow at all length scales, as a cold dark matter component does, while it leaves room for a constant pressure different from zero. Astrophysical scenarios forbid this pressure to be very high, but it well could take values of the cosmological critical density today. From the positivity of the energy density of the dark fluid it follows that this pressure has to be negative. Thus, we conclude that the dark fluid could act as dark energy. Then we study the background cosmology for the dark fluid and show that, not surprisingly, the same evolution equations as in the Λ CDM model are obtained, leading to a degeneracy between the two models. We conclude that any collection of fluids for which its total equation of state parameter is equal to Eq. (18) will lead us to the same result at the background cosmological level.

This is not necessarily the case when we consider linear perturbations. In order to preserve the dark degeneracy, we

demand that the dark fluid be a perfect fluid. For a barotropic fluid, this last condition implies that the first-order space-space part of the energy-momentum tensor is equal to zero. Furthermore, one has to add that the energy-momentum flux density is equal to that of the Λ CDM. Thus, we have demanded that the energy-momentum tensor of the dark fluid be equal to the Λ CDM one at first order in perturbation theory, otherwise the degeneracy is broken.

When considering a collection of multiple interacting fluids and demanding that the adiabatic speed of sound of the total composed fluid be equal to zero, we obtain exactly the perturbation equations of the dark fluid. This shows that the degeneracy is present also for more complicated dark sector schemes.

From the first five sections, we conclude that there exist an infinity of cosmological models that give exactly the same observational signatures. Accordingly, it is fundamentally impossible to elucidate the actual structure of the dark sector. Thus, it is a matter of taste to prefer the Λ CDM model over any one of the exposed in this paper. In fact, to economize it is better to take the one fluid choice (the dark fluid) as the correct model.

The only hope we have to understand the actual nature of the dark sector is that it interacts in some way with the particles of the standard model. Of course interactions would in general break the degeneracy, but this is not necessarily true. This subject is studied in Sec. VI, where we allow the degeneracy of the dark fluid and the Λ CDM model in the homogeneous and isotropic cosmology, and then we try to break it by adding interactions to baryons at first order in perturbations theory. We show that a general class of interactions defined by Eqs. (63) and (64) also can be understood as between dark matter and baryons in the Λ CDM model context, leading to a new degeneracy, although not with the concordance model, but with a Λ CDM-plus-interactions model. As a consequence, these interactions does not help us to favor any decomposition of the dark fluid.

For the latter investigation we have chosen two independent interactions: One that resembles the Thomson scattering between photons and baryons, regulated by a parameter Σ_I , and a second, density-dependent interaction, inspired by chameleon theories and parametrized by Σ_{II} . For the analysis we have used a combination of the seven-year results of WMAP, the Union 2 data set compilation of supernovae measurements, and a prior in the Hubble constant of the Hubble Space Telescope.

When we consider both interactions together, we obtain that other parameters of the theory differ too much from its standard values, up to almost 10% in the case of the baryonic density parameter $\Omega_b h^2$. Then, we study the case in which Σ_{II} is equal to zero, allowing Σ_I to vary (Model B). Here, we show that the tension between the parameters estimation is reduced.

The introduction of the two parameterized interactions is allowed by current data, and if done, it is remarkable that nonzero values of them are preferred at 0.95 confidence level.

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