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Mass and decay constant of the recently observed bound state $h_b(1P)$

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The mass and decay constant of the ground state of spin-singlet $b\bar{b}$ is calculated by using the QCD sum rules method. This meson is an axial vector P wave named $h_b(1P)$ meson with $J^P = 1^+$. It is found that the mass of this meson is $m = (9940 \pm 37)$ MeV, which is consistent with the recent experimental data by the Belle Collaboration.

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I. INTRODUCTION

Heavy quarkonium bound state and their spectroscopy such as $b\bar{b}$ and $c\bar{c}$ have been considered as a good tool for the investigation of the quantum chromodynamics (QCD) interaction. Bottomonium is especially important because of the variety of bound states and decay channels allowing us to study and fix many parameters and aspects of the standard model and QCD. Moreover, measurements of bottomonium masses, total widths, and transition rates serve as important benchmarks for the predictions of QCD-inspired potential models, nonrelativistic QCD, lattice QCD, and QCD sum rules [1,2]. The spin-singlet P-wave bound states of $b\bar{b}$ such as $h_b(1P)$ can be used as tests of the P-wave spin-spin (or hyperfine) interaction. Therefore, theoretical calculations on the physical parameters of this meson and their comparison with experimental data could give essential information about their nature and hyperfine interaction.

The QCD sum rules approach [3–8], which enjoys two peculiar properties, namely, its foundation based on QCD Lagrangian and free of model dependent parameters, as one of the most well-established nonperturbative methods has widely been applied to hadron physics. In the present work, we extend the application of this method to calculate the masses and decay constants of the axial vector P-wave $h_b(1P)$ meson with the quantum numbers $J^P = 1^+$. Since the heavy quark condensates are suppressed by the inverse powers of the heavy quark mass [9,10], as the first nonperturbative contributions, we take into account and calculate the two-gluon condensate diagrams. Note that the Belle Collaboration has recently reported the observation of the $h_b(1P)$ spin-singlet bottomonium state produced in the reaction $e^+e^- \to h_b(1P)\pi^+\pi$ with significances of 5.5σ. They found that $m[h_b(1P)] = (9898.25 \pm 1.06^{+1.03}_{-1.07})$ MeV [1].

The layout of the paper is as follows: In the next section, the sum rules for the mass and decay constant of the ground state axial vector meson is calculated. Section III encompasses our numerical predictions for the mass and leptonic decay constant of the $h_b(1P)$ axial vector meson.

II. QCD SUM RULES FOR THE MASS AND DECAY CONSTANT OF THE $h_b(1P)$ AXIAL VECTOR MESON

The two point correlation function responsible for the mass and decay constant of the axial vector meson can be written as

$$\Pi^A_{\mu\nu} = i \int d^4x e^{ip.x} \langle 0 \mid \mathcal{T}(J^A_\mu(x)\bar{J}^A_\nu(0)) \mid 0\rangle, \qquad (1)$$

where $\mathcal{T}$ is the time ordering product and $J^A_\mu = \bar{b}(x)\gamma_\mu\gamma_5 b(x)$ is responsible for creating the axial vector quarkonia from the vacuum with the same quantum numbers as the interpolating currents.

According to the QCD sum rules, the correlation function has to be calculated in two different ways. Regarding the physical or phenomenological side, it can be evaluated in terms of hadronic parameters such as meson mass and decay constant. On the other hand, in the QCD or theoretical side, it is calculated in terms of QCD degrees of freedom such as quark masses and gluon condensates by means of the operator product expansion in the deep Euclidean region. Matching these two representations of the correlation function via the dispersion relation, the relation among the physical or hadronic parameters i.e., the mass and the decay constant and the fundamental QCD parameters, can be achieved. The Borel transformation with respect to the momentum squared is applied for both sides of the correlation function. This end is necessary to suppress the contributions of higher states and continuum.

First, to calculate the physical part, a complete set of intermediate states with the same quantum numbers as the interpolating current is added. Performing the integral over x and isolating the ground state we get that

$$\Pi^A_{\mu\nu} = \frac{\langle 0 \mid J^A_\mu(0) \mid A\rangle\langle A \mid J^A_\nu(0) \mid 0\rangle}{m_A^2 - p^2} + \cdots, \qquad (2)$$

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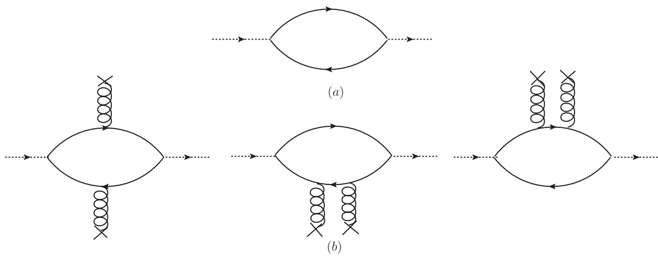

FIG. 1. (a) Bare loop diagram. (b) Diagrams corresponding to gluon condensates.

where $\cdots$ represents contribution of the higher states and continuum, and m_A is the mass of the axial vector meson.

The matrix element, parametrized in terms of the leptonic decay constant, is as follows:

$$\langle 0 \mid J(0) \mid A\rangle = f_A m_A \varepsilon_\mu, \tag{3}$$

where f_A is the leptonic decay constant of the axial vector meson. The summation over the polarization vectors are

$$\epsilon_\mu \epsilon^*_\nu = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_A^2}. \tag{4}$$

Using Eq. (4), the final expressions of the physical side of the correlation function is

$$\Pi^A_{\mu\nu} = \frac{f_A^2 m_A^2}{m_A^2 - p^2}\left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_A^2}\right] + \cdots. \tag{5}$$

To proceed with the calculation of the mass and the decay constant the $g_{\mu\nu}$ structure is used.

In the QCD side, the perturbative (short distance) and nonperturbative (long distance) parts of the correlation function must be separated. In this regard, the operator product expansion method is used and the correlation function is calculated in the deep Euclidean region, $p^2 \ll -\Lambda^2_{QCD}$. So, the correlation function is written as

$$\Pi^{\rm QCD} = \Pi_{\rm pert} + \Pi_{\rm nonpert}. \tag{6}$$

We calculate the short distance contribution (bare loop diagram in Fig. [1(a)]) by means of the perturbation theory, whereas the long distance contributions (diagrams shown in Fig. [1(b)]) are parametrized in terms of gluon condensates. To proceed, we write the perturbative part in terms of a dispersion integral as follows:

$$\Pi^{\rm QCD} = \int \frac{ds\rho(s)}{s - p^2} + \Pi_{\rm nonpert}, \tag{7}$$

where $\rho(s)$ is the spectral density. We evaluate the spectral density having the Feynman amplitude of the bare loop diagram and using the Cutkosky rules. Note that, in the Cutkosky rules the quark propagators are replaced by the Dirac delta function, i.e., $\frac{1}{p^2-m^2} \to (-2\pi i)\delta(p^2 - m^2)$.

Finally, we find the spectral density as

$$\rho(s) = \frac{3}{8\pi^2}\left(1 - \frac{4m_b^2}{s}\right)^{3/2}. \tag{8}$$

The nonperturbative part is calculated via the gluon condensate diagrams represented in Fig. [1(a)]. The Fock-Schwinger gauge, $x^\mu A^a_\mu(x) = 0$, is chosen. Then, the vacuum gluon field is as

$$A^a_\mu(k') = -\frac{i}{2}(2\pi)^4 G^a_{\rho\mu}(0)\frac{\partial}{\partial k'_\rho}\delta^{(4)}(k'), \tag{9}$$

where k' is the gluon momentum and the quark-gluon-quark vertex is used as

$$\Gamma^a_{ij\mu} = ig\gamma_\mu\left(\frac{\lambda^a}{2}\right)_{ij}. \tag{10}$$

Finally, we obtain the nonperturbative part in momentum space is as

$$\Pi^i_{\rm nonpert} = \int_0^1 \langle\alpha_s G^2\rangle \frac{\Theta}{96\pi(m_b^2 - p^2x + p^2x^2)^4} dx. \tag{11}$$

The explicit expressions for Θ is as follows:

$$\begin{aligned}\Theta = {}& -\tfrac{1}{2}x^2\{2m_b^4x^3(m_b^2(4-3x) + p^2(x^2+x-2)) + m_b^4x(m_b^2(x(17-2x(18(x-3)x+47)) + 8)\\ &+ p^2x(x(27x-25)-7)(x-1)^2) + m_b^2(-m_b^2p^2(x-1)x(x(x(2x(27x-82)+149)-32)-11)\\ &+ m_b^4(3x(x(x(3x(4x-17)+76)-46)+8)+5) + p^4(x-1)^3x^2(24x^2-22x-5))\\ &+ m_b^2(x-1)x^2(m_b^2p^2(7-x(2x+3)) + m_b^4(3x-5) + p^4(x-1)(2(x-1)x+3))\\ &+ (x-1)(-m_b^2p^4(x-1)x(2x(x(2x(6x-19)+37)-10)-3)\\ &+ m_b^4p^2(x(x(x(x(27x-112)+162)-97)+14)+5) + m_b^6(x(57-2x(3x(2x-9)+43))-15)\\ &+ p^6(x-1)^3x^2(6(x-1)x-1)) + 3m_b^6x^4 + 3m_b^6(x-1)^2x^2(4x+1)\}.\end{aligned} \tag{12}$$

The next step is to apply Borel transformation over p^2 for suppressing the contribution of the higher states and continuum. Then, we get the sum rules as

$$m_A^2 f_A^2 e^{((-m_A^2)/M^2)} = \int_{4m_b^2}^{s_0} ds\rho(s)e^{-(s/M^2)} + \hat{B}\Pi_{\rm nonpert}, \tag{13}$$

where M^2 and s_0 are the Borel mass parameter and the continuum threshold, respectively.

We extract the meson mass (m_A) with simple algebra (applying derivative with respect to $-\frac{1}{M^2}$ to both sides of the above sum rule and dividing by itself). Then, the meson mass is as follows:

$$m_A^2 = \frac{-\frac{d}{d\left(\frac{1}{M^2}\right)}\left[\int_{4m_b^2}^{s_0} ds\rho^A(s)e^{-(s/M^2)} + \hat{B}\Pi^A_{\rm nonpert}\right]}{\int_{4m_b^2}^{s_0} ds\rho^A(s)e^{-(s/M^2)} + \hat{B}\Pi^A_{\rm nonpert}}, \tag{14}$$

where

$$\hat{B}\Pi^i_{\rm nonpert} = \int_0^1 e^{m_b^2/M^2x(x-1)} \frac{\Delta}{\pi 96 M^6(x-1)^4x^3}\langle\alpha_s G^2\rangle dx, \tag{15}$$

and we get

$$\begin{aligned}\Delta = &-m_b^4(x-1)x^2(m_b^2(2x^2-5x+2)+2M^2x(x^2-4x+3)) - m_b^4(x-1)x^3(m_b^2(3x^2-6x+2)+M^2x(3x^2-4x+1))\\ &+ m_b^2(x-1)x(m_b^2M^2x(2x^3-11x^2+17x-6)+m_b^4(x^3-4x^2+4x-1)+2M^4x^2(2x^3-4x^2+5x-3))\\ &+ m_b^2x(m_b^2M^2x(6x^5-26x^4+43x^3-31x^2+9x-1)+m_b^4(3x^5-15x^4+27x^3-20x^2+7x-1)\\ &+ 4M^4x^2(3x+1)(x-1)^4) + (x-1)(-2m_b^2M^4x^3(6x^4-28x^3+45x^2-31x+8)\\ &+ m_b^4M^2x(-3x^5+16x^4-33x^3+28x^2-11x+2) - m_b^6(x^5-6x^4+13x^3-11x^2+5x-1)\\ &+ 2M^6(x-1)^3x^3(6x^2-6x-1)) + m_b^6(x-1)x^5 + m_b^6(x-1)^2x^3. \end{aligned} \tag{16}$$

III. NUMERICAL RESULTS

The sum rules for the decay constant and the mass of the $h_b(1P)$ depend on the bottom quark mass, QCD parameters and two auxiliary parameters, namely, Borel mass parameter M^2 and continuum threshold s_0. The values $m_b = 4.8$ GeV and $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0\rangle = 0.012$ GeV^4 [11] are chosen. Even though the physical observables are supposed to be independent of auxiliary parameters, the weak dependency of physical observables on auxiliary parameters can be tolerated. We have analyzed the dependence of the mass and the decay constant on the Borel mass parameter M^2 and the continuum threshold s_0 as shown by Figs. 2 and 3. With these figures, the lower limit of Borel mass parameter can be fixed in principle when the sensitivity to the variation of these parameters are weak, where we take the lower limit as 15 GeV$^2 \leq M^2$. Furthermore, we postulate that the contributions of the higher states, continuum, and the highest order operators should be less than the 30% of the result of the correlation

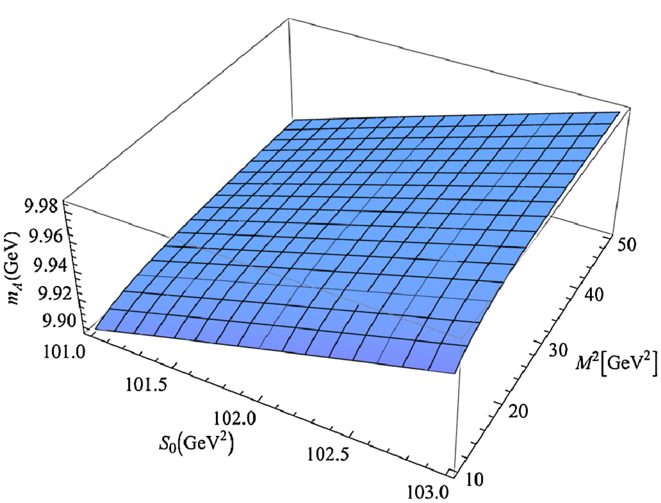

FIG. 2 (color online). Dependence of the mass of the axial vector $\bar{b}\gamma_\mu\gamma_5 b$ on the Borel parameter, M^2, and the continuum threshold s_0.

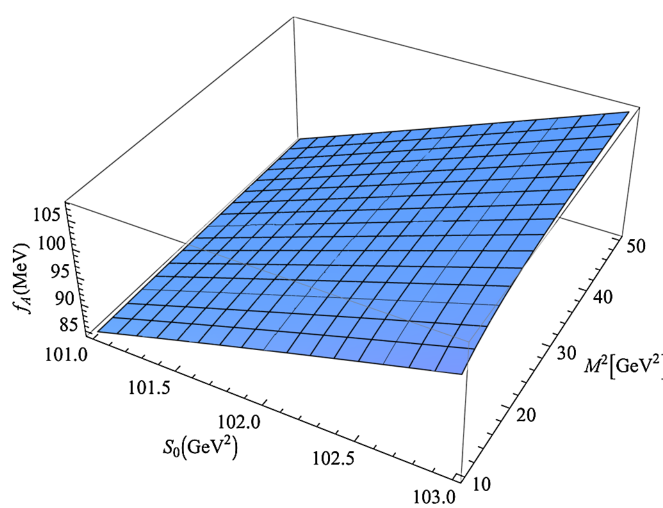

FIG. 3 (color online). Dependence of decay constant of the axial vector $\bar{b}\gamma_\mu\gamma_5 b$ on the Borel parameter, M^2, and the continuum threshold s_0.

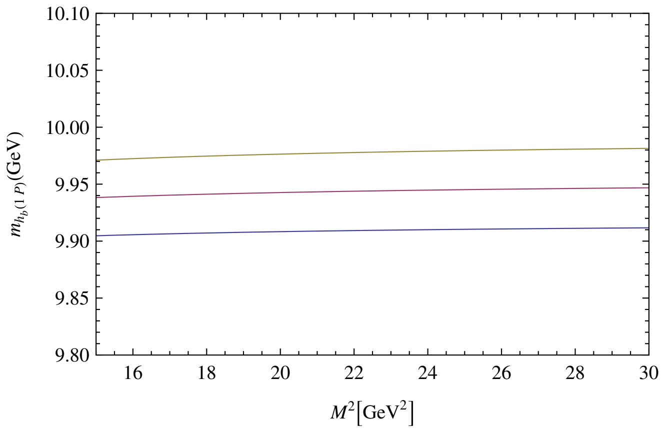

FIG. 4 (color online). Dependence of the mass of the axial vector $\bar{b}\gamma_\mu\gamma_5 b$ on the Borel parameter, M^2, at three fixed values of the continuum threshold. The upper, middle, and lower lines belong to the values $s_0 = 103$ GeV^2, $s_0 = 102$ GeV^2, and $s_0 = 101$ GeV^2, respectively.

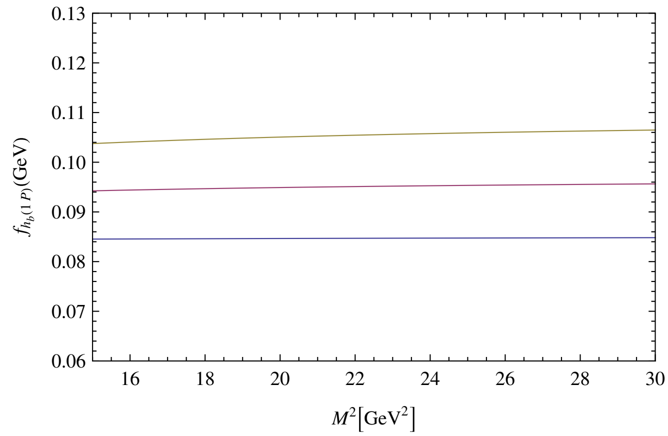

FIG. 5 (color online). Dependence of the decay constant of the axial vector $\bar{b}b$ on the Borel parameter, M^2, at three fixed values of the continuum threshold. The upper, middle, and lower lines belong to the values 103 GeV^2, $s_0 = 102$ GeV^2, and $s_0 = 101$ GeV^2, respectively.

function. As a result the upper limit of the Borel mass parameter is determined as $M^2 \leq 30$ GeV^2. Then, the so-called ''working region'' of M^2 lies in the region 15 GeV$^2 \leq M^2 \leq 30$ GeV^2. Note that the similar region has been found for analyzing the mass and the decay constant for χ_{b0} [12]. The continuum threshold s_0 is not completely arbitrary but it is related to the energy of the first exited states $[h_b(2P)]$ with the same quantum numbers as the interpolating currents. In other words, the interval of s_0 must be in between the mass square of $[h_b(1P)]$ and $[h_b(2P)]$, i.e., $m^2[h_b(1P)] < s_0 < m^2[h_b(2P)]$, where $m[h_b(1P)] = (9898.25 \pm 1.06^{+1.03}_{-1.07})$ MeV and $m[h_b(2P)] = (10259.76 \pm 1.064^{+1.43}_{-1.03})$ MeV. Then, the interval 101 GeV$^2 \leq s_0 \leq 103$ GeV^2 for the continuum threshold s_0 is chosen. Here, it is worth mentioning that in the above regions for Borel mass parameter M^2 and continuum threshold s_0 the dependence of the results on auxiliary parameters is the weakest one.

Regarding the intervals of the Borel mass parameter M^2, continuum threshold s_0, and input parameters mentioned above, the following values for the mass and the decay constant of $h_b(1P)$ are obtained:

$$m[h_b(1P)] = (9940 \pm 37) \text{ MeV}, \tag{17}$$

$$f[h_b(1P)] = (94 \pm 10) \text{ MeV}. \tag{18}$$

Note that, we use the experimental value of the mass $m[h_b(1P)] = 9898.25$ MeV rather than the theoretical value of that in order to determine the decay constant. It is also worth mentioning that the result for the value of the mass is in good agreement with the recent data on the mass of $h_b(1P)$ by the Belle Collaboration [1] where they found

$$m[h_b(1P)] = (9898.25 \pm 1.06^{+1.03}_{-1.07}) \text{ MeV}. \tag{19}$$

The mass and the decay constant of axial vector $h_b(1P)$ are depicted in Figs. 4 and 5 at three fixed values of the continuum threshold in terms of Borel mass parameter M^2. These figures indicate a good stability of the mass and decay constant with respect to the Borel mass parameter M^2.

In summary, this study presents the first theoretical calculation on the mass and decay constant of the newly found axial vector meson $h_b(1P)$ in the framework of the two point QCD sum rules. While the result for the mass is consistent with the measured value by the Belle Collaboration, the decay constant has not yet been measured.

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