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Nucleon tensor form factors induced by isovector and isoscalar currents in QCD

T. M. Aliev,[1,*,†] K. Azizi,[2,‡] and M. Savcı[1,§]

[1]*Physics Department, Middle East Technical University, 06531 Ankara, Turkey*
[2]*Department of Physics, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey*

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Using the most general form of the nucleon interpolating current, we calculate the tensor form factors of the nucleon within light cone QCD sum rules. A comparison of our results on tensor form factors with those of the chiral-soliton model and lattice QCD is given.

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I. INTRODUCTION

The main problem of QCD is to understand the structure of hadrons and their properties in terms of quarks and gluons. Nucleon charges defined as matrix elements of vector, axial, and tensor currents between nucleon states contain complete information about the quark structure of the nucleon. These charges are connected with the leading twist unpolarized $q(x)$, the helicity $\Delta q(x)$, and transversity $\delta q(x)$ parton distribution functions (PDFs). The first two PDFs have been extensively investigated theoretically and experimentally in many works (for instance, see [1,2] and references therein as well as [3–5]). There is a big experimental problem to measure the transversity of the nucleon because of its chiral odd nature. Only recently, the tensor charge $\delta q(x)$ was extracted [6] using the data from BELLE [7], HERMES [8] and COMPASS [9] collaborations. This extraction is based on analysis of the measured azimuthal asymmetries in semi-inclusive scattering and those in $e^+e^- \to h_1h_2X$ processes. Since $\delta q(x)$ is a spin dependent PDF, it is interesting to investigate whether there is a ''transversity crisis'' similar to the case of ''spin crisis'' in $\Delta q(x)$. Therefore, reliable determination of nucleon tensor charge receives special attention.

Theoretically, tensor charges of hadrons are studied in different frameworks such as, non–relativistic MIT bag model [10], $SU(6)$ quark model [11], quark model with axial vector dominance [12], lattice QCD [13], external field [14], and three-point versions of QCD sum rules [10].

In the present work, using the most general form of the nucleon interpolating field, we study the tensor form factors of nucleons within light cone QCD sum rules (LCQSR). The LCQSR is based on the operator product expansion (OPE) over twist of the operators near the light cone, while in the traditional QCD sum rules, the OPE is performed over dimensions of the operators. This approach has been widely applied to hadron physics (see, for example, [15]). Note that, the tensor form factors of nucleons up to $Q^2 \leq 1$ GeV^2 (where $Q^2 = -q^2$ is the Euclidean momentum transfer square) within the $SU(3)$ chiral soliton model are studied in [16] (see also [17]). The anomalous tensor form factors are studied within the same framework in [18]. These form factors are further studied in lattice QCD (see, for instance, [19]).

The plan of this paper is as follows. In Sec. II, we derive sum rules for the tensor form factors of the nucleon within LQCSR method. In Sec. III, we numerically analyze the sum rules for the tensor form factors. A comparison of our results on form factors with those existing in the literature is also presented in this section.

II. LIGHT CONE SUM RULES FOR THE NUCLEON TENSOR FORM FACTORS

This section is devoted to derivation of LCQSR for the nucleon tensor form factors. The matrix element of the tensor current between initial and final nucleon states is parametrized in terms of four form factors as follows [1,19,20]:

$$\begin{aligned}\langle N(p')|\bar{q}\sigma_{\mu\nu}q|N(p)\rangle &= \bar{u}(p')\Big\{H_T(Q^2)i\sigma_{\mu\nu} - E_T(Q^2)\frac{\gamma_\mu q_\nu - \gamma_\nu q_\mu}{2m_N} \\ &+ E_{1T}(Q^2)\frac{\gamma_\mu \mathcal{P}_\nu - \gamma_\nu \mathcal{P}_\mu}{2m_N} \\ &- \tilde{H}_T(Q^2)\frac{\mathcal{P}_\mu q_\nu - \mathcal{P}_\nu q_\mu}{2m_N^2}\Big\}u(p), \end{aligned} \quad (1)$$

where $q_\mu = (p - p')_\mu$, $\mathcal{P}_\mu = (p + p')_\mu$, and $q^2 = -Q^2$. From T–invariance, it follows that $E_{1T}(Q^2) = 0$.

In order to calculate the remaining three tensor form factors within LCQSR, we consider the correlation function,

$$\Pi_{\mu\nu}(p, q) = i\int d^4x e^{iqx}\langle 0|T\{J^N(0)J_{\mu\nu}(x)\}|N(p)\rangle. \quad (2)$$

This correlation function describes transition of the initial nucleon to the final nucleon with the help of the tensor

*taliev@metu.edu.tr
†Permanent address: Institute of Physics, Baku, Azerbaijan.
‡kazizi@dogus.edu.tr
§savci@metu.edu.tr

current. The most general form of the nucleon interpolating field is given as,

$$J^N(x) = 2\varepsilon^{abc}\sum_{i=1}^{2}[q^{Ta}(x)CA_1^i q'^b(x)]A_2^i q^c(x), \quad (3)$$

where C is the charge conjugation operator, $A_1^1 = I$, $A_1^2 = A_2^1 = \gamma_5$, $A_2^2 = t$ with t being an arbitrary parameter and $t = -1$ corresponds to the Ioffe current and a, b, c are the color indices. The quark flavors are $q = u$, $q' = d$ for the proton and $q = d$, $q' = u$ for the neutron. The tensor current is chosen as,

$$J_{\mu\nu} = \bar{u}\sigma_{\mu\nu}u \pm \bar{d}\sigma_{\mu\nu}d, \quad (4)$$

where the upper and lower signs correspond to the isosinglet and isovector cases, respectively.

In order to obtain sum rules for the form factors, it is necessary to calculate the correlation function in terms of quarks and gluons on one side (QCD side), and in terms of hadrons on the other side (phenomenological side). These two representations of the correlation function are then equated. The final step in this method is to apply the Borel transformation, which is needed to suppress the higher states and the continuum contributions.

Following this strategy, we start to calculate the phenomenological part. Saturating the correlation function with a full set of hadrons carrying the same quantum numbers as nucleon and isolating the contributions of the ground state, we get

$$\Pi_{\mu\nu}(p,q) = \frac{\langle 0|J^N(0)|N(p')\rangle\langle N(p')|J_{\mu\nu}|N(p)\rangle}{m_N^2 - p'^2} + \cdots, \quad (5)$$

where dots stands for contributions of higher states and continuum. The matrix element $\langle 0|J^N(0)|N(p')\rangle$ entering Eq. (5) is defined as

$$\langle 0|J^N(0)|N(p')\rangle = \lambda_N u(p), \quad (6)$$

where λ_N is the residue of the nucleon. Using Eqs. (1), (2), and (6), and performing summation over spins of the nucleon, we get,

$$\Pi_{\mu\nu} = \frac{\lambda_N}{m_N^2 - p'^2}(\not{p}' + m_N)\Big\{H_T(Q^2)i\sigma_{\mu\nu} - E_T(Q^2)\frac{\gamma_\mu q_\nu - \gamma_\nu q_\mu}{2m_N} - \tilde{H}_T(Q^2)\frac{\mathcal{P}_\mu q_\nu - \mathcal{P}_\nu q_\mu}{2m_N^2}\Big\}u(p). \quad (7)$$

From Eq. (7), we see that there are many structures, and all of them play equal role for determination of the tensor form factors of the nucleon. In practical applications, it is more useful to work with $\tilde{E}_T(Q^2) = E_T(Q^2) + 2\tilde{H}_T(Q^2)$ rather than $E_T(Q^2)$. For this reason, we choose the structures $\sigma_{\mu\nu}$, $p_\mu q_\nu$ and $p_\mu q_\nu \not{q}$ for obtaining the sum rules for the form factors H_T, $\tilde{E}_T$ and $\tilde{H}_T$, respectively.

The correlation function $\Pi_{\mu\nu}(p,q)$ is also calculated in terms of quarks and gluons in deep Eucledian domain $p'^2 = (p-q)^2 \ll 0$. After simple calculations, we get the following expression for the correlation function for the proton case:

$$\begin{aligned}(\Pi_{\mu\nu})_\rho = \frac{i}{2}\int d^4x e^{iqx}\sum_{i=1}^{2}\Big\{&(CA_1^i)_{\alpha\tau}[A_2^i S_u(-x)\sigma_{\mu\nu}]_{\rho\sigma} \\ &\times 4\epsilon^{abc}\langle 0|u_\alpha^a(0)u_\sigma^b(x)d_\tau^c(0)|N(p)\rangle \\ &+ (A_2^i)_{\rho\alpha}[(CA_1^i)^T S_u(-x)\sigma_{\mu\nu}]_{\tau\sigma} \\ &\times 4\epsilon^{abc}\langle 0|u_\alpha^a(x)u_\sigma^b(x)d_\tau^c(0)|N(p)\rangle \\ &\pm (A_2^i)_{\rho\sigma}[CA_1^i S_d(-x)\sigma_{\mu\nu}]_{\alpha\tau} \\ &\times 4\epsilon^{abc}\langle 0|u_\alpha^a(0)u_\sigma^b(0)d_\tau^c(x)|N(p)\rangle\Big\}. \end{aligned} \quad (8)$$

Obviously, the correlation function for the neutron case can easily be obtained by making the replacement $u \leftrightarrow d$.

From Eq. (8), it is clear that in order to calculate the correlation function from QCD side, we need to know the matrix element,

$$4\varepsilon^{abc}\langle 0|u_\alpha^a(a_1x)u_\sigma^b(a_2x)d_\tau^a(a_3x)|N(p)\rangle,$$

where a_1, a_2, and a_3 determine the fraction of the nucleon momentum carried by the corresponding quarks. This matrix element is the main nonperturbative ingredient of the sum rules and it is defined in terms of the nucleon distribution amplitudes (DAs). The nucleon DAs are studied in detail in [21–23].

The light cone expanded expression for the light quark propagator $S_q(x)$ is given as,

$$\begin{aligned}S_q(x) = &\frac{i\not{x}}{2\pi^2x^4} - \frac{\langle q\bar{q}\rangle}{12}\left(1 + \frac{m_0^2x^2}{16}\right) \\ &- ig_s\int_0^1 dv\Big[\frac{\not{x}}{16\pi^2x^4}G_{\mu\nu}\sigma^{\mu\nu} \\ &- vx^\mu G_{\mu\nu}\gamma^\nu\frac{i}{4\pi^2x^2}\Big], \end{aligned} \quad (9)$$

where the mass of the light quarks are neglected, $m_0^2 = (0.8 \pm 0.2)$ GeV^2 [24] and $G_{\mu\nu}$ is the gluon field strength tensor. The terms containing $G_{\mu\nu}$ give contributions to four- and five-particle distribution functions. These contributions are negligibly small (for more detail, see [21–23]), and therefore in further analysis, we will neglect these terms. Moreover, Borel transformation kills the terms proportional to the quark condensate, and as a result only the first term is relevant for our discussion.

Using the explicit expressions of DAs for the proton and light quark propagators, performing Fourier transformation and then applying Borel transformation with respect to the variable $p'^2 = (p-q)^2$, which suppresses the contributions of continuum and higher states, and choosing the coefficients of the structures $\sigma_{\mu\nu}$, $p_\mu q_\nu$, and $p_\mu q_\nu \not{q}$, we get the following sum rules for the tensor form factors of nucleon:

$$
\begin{aligned}
H_T(Q^2) = {} & \frac{1}{2m_N\lambda_N} e^{m_N^2/M^2}\Bigg\{\int_{x_0}^1 \frac{dt_2}{t_2} e^{-s(t_2)/M^2}[(1-t)F_{H_T}^1(t_2) + (1+t)F_{H_T}^2(t_2)] \\
& \pm \int_{x_0}^1 \frac{dt_3}{t_3} e^{-s(t_3)/M^2}[(1-t)F_{H_T}^3(t_3) + (1+t)F_{H_T}^4(t_3)] + \int_{x_0}^1 \frac{dt_2}{t_2} e^{-s(t_2)/M^2}[(1-t)F_{H_T}^5(t_2) + (1+t)F_{H_T}^6(t_2)] \\
& \pm \int_{x_0}^1 \frac{dt_3}{t_3} e^{-s(t_3)/M^2}[(1-t)F_{H_T}^7(t_3) + (1+t)F_{H_T}^8(t_3)] + \int_{x_0}^1 \frac{dt_2}{t_2} e^{-s(t_2)/M^2}(1-t)F_{H_T}^9(t_2) \\
& \pm \int_{x_0}^1 \frac{dt_3}{t_3} e^{-s(t_3)/M^2}(1+t)F_{H_T}^{10}(t_3) + \frac{1}{M^2}\int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}[(1-t)F_{H_T}^{11}(t_2) + (1+t)F_{H_T}^{12}(t_2)] \\
& + \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}[(1-t)F_{H_T}^{11}(x_0) + (1+t)F_{H_T}^{12}(x_0)] \pm \frac{1}{M^2}\int_{x_0}^1 \frac{dt_3}{t_3^2} e^{-s(t_3)/M^2}[(1-t)F_{H_T}^{13}(t_3) + (1+t)F_{H_T}^{14}(t_3)] \\
& \pm \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}[(1-t)F_{H_T}^{13}(x_0) + (1+t)F_{H_T}^{14}(x_0)] + \frac{1}{M^2}\int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}[(1-t)F_{H_T}^{15}(t_2) + (1+t)F_{H_T}^{16}(t_2)] \\
& + \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}[(1-t)F_{H_T}^{15}(x_0) + (1+t)F_{H_T}^{16}(x_0)] \pm \frac{1}{M^2}\int_{x_0}^1 \frac{dt_3}{t_3^2} e^{-s(t_3)/M^2}(1+t)F_{H_T}^{17}(t_3) \\
& \pm \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1+t)F_{H_T}^{17}(x_0)\Bigg\},
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
F_{H_T}^1(t_2) &= \int_0^{1-t_2} dt_1 \Bigg\{\frac{2m_N^2}{t_2}[\tilde{\mathcal{T}}_1^M + t_2^2(\tilde{\mathcal{P}}_1 - 3\tilde{\mathcal{T}}_3 - \tilde{\mathcal{T}}_4)](t_1, t_2, 1-t_1-t_2) + \frac{2(Q^2 + m_N^2 t_2^2)}{t_2}\tilde{\mathcal{T}}_1(t_1, t_2, 1-t_1-t_2)\Bigg\}, \\
F_{H_T}^2(t_2) &= \int_0^{1-t_2} dt_1 \Bigg\{\frac{m_N^2}{t_2}[\tilde{\mathcal{V}}_1^M - \tilde{\mathcal{A}}_1^M - t_2^2(\tilde{\mathcal{A}}_2 + 3\tilde{\mathcal{A}}_3 + \tilde{\mathcal{V}}_2 + 3\tilde{\mathcal{V}}_3)](t_1, t_2, 1-t_1-t_2) \\
&\quad - \frac{Q^2 + m_N^2 t_2^2}{t_2}[\tilde{\mathcal{A}}_1 - \tilde{\mathcal{V}}_1](t_1, t_2, 1-t_1-t_2)\Bigg\}, \\
F_{H_T}^3(t_3) &= \int_0^{1-t_3} dt_1 \frac{1}{t_3}[m_N^2(\tilde{\mathcal{A}}_1^M + \tilde{\mathcal{V}}_1^M) + m_N^2 t_3^2(\tilde{\mathcal{A}}_3 - \tilde{\mathcal{V}}_3) + Q^2(\tilde{\mathcal{A}}_1 + \tilde{\mathcal{V}}_1)](t_1, 1-t_1-t_3, t_3), \\
F_{H_T}^4(t_3) &= \int_0^{1-t_3} dt_1 [\frac{2}{t_3}(m_N^2\tilde{\mathcal{T}}_1^M + Q^2\tilde{\mathcal{T}}_1) - \frac{m_N^2 t_3}{2}(2\tilde{\mathcal{P}}_1 - 2\tilde{\mathcal{S}}_1 + 2\tilde{\mathcal{T}}_1 - \tilde{\mathcal{T}}_2 - \tilde{\mathcal{T}}_4)](t_1, 1-t_1-t_3, t_3), \\
F_{H_T}^5(t_2) &= \frac{m_N^2}{2}\int_1^{t_2} d\rho \int_0^{1-\rho} dt_1 [4\tilde{\mathcal{T}}_4 + 4\tilde{\mathcal{T}}_5 - 3\tilde{\mathcal{T}}_6 + 12\tilde{\mathcal{T}}_7 - 4\tilde{\mathcal{S}}_2](t_1, \rho, 1-t_1-\rho), \\
F_{H_T}^6(t_2) &= m_N^2\int_1^{t_2} d\rho \int_0^{1-\rho} dt_1 [2\tilde{\mathcal{A}}_2 - \tilde{\mathcal{A}}_4 + 2\tilde{\mathcal{A}}_5 + 2\tilde{\mathcal{V}}_2 + \tilde{\mathcal{V}}_4 - 2\tilde{\mathcal{N}}_5](t_1, \rho, 1-t_1-\rho), \\
F_{H_T}^7(t_3) &= \frac{m_N^2}{2}\int_1^{t_3} d\rho \int_0^{1-\rho} dt_1 [2\tilde{\mathcal{A}}_2 - \tilde{\mathcal{A}}_4 + \tilde{\mathcal{A}}_5 - 2\tilde{\mathcal{V}}_2 - \tilde{\mathcal{V}}_4 + \tilde{\mathcal{V}}_5](t_1, 1-t_1-\rho, \rho), \\
F_{H_T}^8(t_3) &= \frac{m_N^2}{2}\int_1^{t_3} d\rho \int_0^{1-\rho} dt_1 [2\tilde{\mathcal{T}}_2 + 2\tilde{\mathcal{T}}_5 - \tilde{\mathcal{T}}_6 - 2\tilde{\mathcal{P}}_2 - 2\tilde{\mathcal{S}}_4](t_1, 1-t_1-\rho, \rho), \\
F_{H_T}^9(t_2) &= \frac{m_N^2}{2}\int_1^{t_2} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \frac{1}{\rho}\tilde{\mathcal{T}}_6(t_1, \rho, 1-t_1-\rho),
\end{aligned}
$$

$$
\begin{aligned}
F_{H_T}^{10}(t_3) &= m_N^2 \int_1^{t_3} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \frac{1}{\rho} \tilde{\mathcal{T}}_6(t_1, 1-t_1-\rho, \rho),\\
F_{H_T}^{11}(t_2) &= 2m_N^2 \int_0^{1-t_2} dt_1 \frac{Q^2 + m_N^2 t_2^2}{t_2} \tilde{\mathcal{T}}_1^M(t_1, t_2, 1-t_1-t_2),\\
F_{H_T}^{12}(t_2) &= m_N^2 \int_0^{1-t_2} dt_1 \frac{Q^2 + m_N^2 t_2^2}{t_2} [\tilde{\mathcal{V}}_1^M - \tilde{\mathcal{A}}_1^M](t_1, t_2, 1-t_1-t_2),\\
F_{H_T}^{13}(t_3) &= m_N^2 \int_0^{1-t_3} dt_1 \frac{Q^2}{t_3} [\tilde{\mathcal{A}}_1^M + \tilde{\mathcal{V}}_1^M](t_1, 1-t_1-t_3, t_3),\\
F_{H_T}^{14}(t_3) &= m_N^2 \int_0^{1-t_3} dt_1 \frac{2Q^2 - m_N^2 t_3^2}{t_3} \tilde{\mathcal{T}}_1^M(t_1, 1-t_1-t_3, t_3),\\
F_{H_T}^{15}(t_2) &= \frac{m_N^2}{2} \int_1^{t_2} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \frac{1}{\rho} [(Q^2 + m_N^2\rho^2)\tilde{\mathcal{T}}_6 + 8m_N^2\rho^2\tilde{\mathcal{T}}_8](t_1, \rho, 1-t_1-\rho),\\
F_{H_T}^{16}(t_2) &= 2m_N^4(1+t) \int_1^{t_2} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \rho [\tilde{\mathcal{A}}_6 + \tilde{\mathcal{V}}_6](t_1, \rho, 1-t_1-\rho),\\
F_{H_T}^{17}(t_3) &= m_N^2 \int_1^{t_3} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \frac{1}{\rho} [m_N^2\rho^2\tilde{\mathcal{T}}_8 - Q^2\tilde{\mathcal{T}}_6](t_1, 1-t_1-\rho, \rho).
\end{aligned} \tag{11}
$$

For the form factor $\tilde{E}_T(Q^2)$, we obtain the following sum rule:

$$
\begin{aligned}
\tilde{E}_T(Q^2) = \frac{1}{m_N\lambda_N} e^{m_N^2/M^2} \Bigg\{ & \frac{1}{M^2} \int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}(1-t) F_{\tilde{E}_T}^1(t_2) + \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1-t) F_{\tilde{E}_T}^1(x_0)\\
& + \frac{1}{M^2} \int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}(1-t) F_{\tilde{E}_T}^3(t_2) + \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1-t) F_{\tilde{E}_T}^3(x_0)\\
& \pm \frac{1}{M^2} \int_{x_0}^1 \frac{dt_3}{t_3^2} e^{-s(t_3)/M^2}(1-t) F_{\tilde{E}_T}^4(t_3) \pm \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1-t) F_{\tilde{E}_T}^4(x_0)\\
& + \frac{1}{M^2} \int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}(1-t) F_{\tilde{E}_T}^5(t_2) + \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1-t) F_{\tilde{E}_T}^5(x_0)\\
& \pm \frac{1}{M^2} \int_{x_0}^1 \frac{dt_3}{t_3^2} e^{-s(t_3)/M^2}(1-t) F_{\tilde{E}_T}^6(t_3) \pm \frac{1}{Q^2 + x_0^2 m_N^2} e^{-s_0/M^2}(1-t) F_{\tilde{E}_T}^6(x_0)\\
& + \int_{x_0}^1 \frac{dt_2}{t_2} e^{-s(t_2)/M^2}(1-t) F_{\tilde{E}_T}^7(t_2) \pm \int_{x_0}^1 \frac{dt_3}{t_3} e^{-s(t_3)/M^2}(1-t) F_{\tilde{E}_T}^8(t_3) \Bigg\},
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
F_{\tilde{E}_T}^1(t_2) &= -4m_N^2 \int_1^{t_2} d\lambda \int_1^{\lambda} d\rho \int_0^{1-\rho} dt_1 \tilde{\mathcal{T}}_6(t_1, \rho, 1-t_1-\rho),\\
F_{\tilde{E}_T}^3(t_2) &= -4m_N^2 \int_1^{t_2} d\rho \int_0^{1-\rho} dt_1 \rho [\tilde{\mathcal{T}}_2 + \tilde{\mathcal{T}}_4](t_1, \rho, 1-t_1-\rho),\\
F_{\tilde{E}_T}^4(t_3) &= 4m_N^2 \int_1^{t_2} d\rho \int_0^{1-\rho} dt_1 \rho [\tilde{\mathcal{A}}_2 - \tilde{\mathcal{V}}_2](t_1, 1-t_1-\rho, \rho),\\
F_{\tilde{E}_T}^5(t_2) &= 8m_N^2 \int_0^{1-t_2} dt_1 \tilde{\mathcal{T}}_1^M(t_1, t_2, 1-t_1-t_2),\\
F_{\tilde{E}_T}^6(t_3) &= 4m_N^2 \int_0^{1-t_3} dt_1 [\tilde{\mathcal{A}}_1^M + \tilde{\mathcal{V}}_1^M](t_1, 1-t_1-t_3, t_3),\\
F_{\tilde{E}_T}^7(t_2) &= 8 \int_0^{1-t_2} dt_1 \tilde{\mathcal{T}}_1(t_1, t_2, 1-t_1-t_2)),\\
F_{\tilde{E}_T}^8(t_3) &= 4 \int_0^{1-t_3} dt_1 [\tilde{\mathcal{A}}_1 + \tilde{\mathcal{V}}_1](t_1, 1-t_1-t_3, t_3).
\end{aligned}
$$

Finally, for the form factor $\tilde{H}_T(Q^2)$, we get the following sum rule:

$$\tilde{H}_T(Q^2) = \frac{1}{m_N^2\lambda_N} e^{m_N^2/M^2}\Big\{\frac{1}{M^2}\int_{x_0}^1 \frac{dt_2}{t_2^2} e^{-s(t_2)/M^2}(1-t)F^1_{\tilde{H}_T}(t_2) + \frac{1}{Q^2+x_0^2m_N^2} e^{-s_0/M^2}(1-t)F^1_{\tilde{H}_T}(x_0) \pm \frac{1}{M^2}\int_{x_0}^1 \frac{dt_3}{t_3^2} e^{-s(t_3)/M^2}(1-t)F^2_{\tilde{H}_T}(t_3) \pm \frac{1}{Q^2+x_0^2m_N^2} e^{-s_0/M^2}(1-t)F^2_{\tilde{H}_T}(x_0)\Big\}, \quad (13)$$

where

$$F^1_{\tilde{H}_T}(t_2) = 4m_N\int_1^{t_2} d\rho \int_0^{1-\rho} dt_1[\tilde{\mathcal{T}}_2 + \tilde{\mathcal{T}}_4] \times (t_1, \rho, 1-t_1-\rho),$$
$$F^2_{\tilde{H}_T}(t_3) = 4m_N\int_1^{\rho} d\rho \int_0^{1-\rho} dt_1[-\tilde{\mathcal{A}}_2 + \tilde{\mathcal{V}}_2] \times (t_1, 1-t_1-\rho, \rho),$$

and we use

$$\tilde{\mathcal{V}}_2(t_i) = V_1(t_i) - V_2(t_i) - V_3(t_i),$$
$$\tilde{\mathcal{A}}_2(t_i) = -A_1(t_i) + A_2(t_i) - A_3(t_i),$$
$$\tilde{\mathcal{A}}_4(t_i) = -2A_1(t_i) - A_3(t_i) - A_4(t_i) + 2A_5(t_i),$$
$$\tilde{\mathcal{A}}_5(t_i) = A_3(t_i) - A_4(t_i),$$
$$\tilde{\mathcal{A}}_6(t_i) = A_1(t_i) - A_2(t_i) + A_3(t_i) + A_4(t_i) - A_5(t_i) + A_6(t_i),$$
$$\tilde{\mathcal{T}}_2(t_i) = T_1(t_i) + T_2(t_i) - 2T_3(t_i),$$
$$\tilde{\mathcal{T}}_4(t_i) = T_1(t_i) - T_2(t_i) - 2T_7(t_i),$$
$$\tilde{\mathcal{T}}_5(t_i) = -T_1(t_i) + T_5(t_i) + 2T_8(t_i),$$
$$\tilde{\mathcal{T}}_6(t_i) = 2[T_2(t_i) - T_3(t_i) - T_4(t_i) + T_5(t_i) + T_7(t_i) + T_8(t_i)],$$
$$\tilde{\mathcal{T}}_7(t_i) = T_7(t_i) - T_8(t_i),$$
$$\tilde{\mathcal{S}}_2(t_i) = S_1(t_i) - S_2(t_i),$$
$$\tilde{\mathcal{P}}_2(t_i) = P_2(t_i) - P_1(t_i),$$

In these expressions, we also use

$$\mathcal{F}(x_i) = \mathcal{F}(x_1, x_2, 1-x_1-x_2),$$
$$\mathcal{F}(x_i') = \mathcal{F}(x_1, 1-x_1-x_3, x_3),$$
$$s(x, Q^2) = (1-x)m_N^2 + \frac{(1-x)}{x}Q^2,$$

where $x_0(s_0, Q^2)$ is the solution to the equation $s(x_0, Q^2) = s_0$.

The residue λ_N is determined from two-point sum rule. For the general form of the interpolating current, it is calculated in [25], whose expression is given as

$$\lambda_N^2 = e^{m_N^2/M^2}\Big\{\frac{M^6}{256\pi^4}E_2(x)(5+2t+t^2) - \frac{\langle\bar{u}u\rangle}{6}[6(1-t^2)\langle\bar{d}d\rangle - (1-t)^2\langle\bar{u}u\rangle] + \frac{m_0^2}{24M^2}\langle\bar{u}u\rangle[12(1-t^2)\langle\bar{d}d\rangle - (1-t)^2\langle\bar{u}u\rangle]\Big\},$$

where

$$E_2(s_0/M^2) = 1 - e^{s_0/M^2}\sum_{i=0}^{2}\frac{(s_0/M^2)^i}{i!}.$$

The Borel transformations are implemented by the following subtraction rules [21–23],

$$\int dx \frac{\rho(x)}{(q-xp)^2} \to -\int \frac{dx}{x}\rho(x)e^{-s(x)/M^2},$$
$$\int dx \frac{\rho(x)}{(q-xp)^4} \to \frac{1}{M^2}\int \frac{dx}{x^2}\rho(x)e^{-s(x)/M^2} + \frac{\rho(x_0)}{Q^2+x_0^2m_N^2}e^{-s_0/M^2},$$
$$\int dx \frac{\rho(x)}{(q-xp)^6} \to -\frac{1}{2M^2}\int \frac{dx}{x^3}\rho(x)e^{-s(x)/M^2} - \frac{1}{2}\frac{\rho(x_0)}{x_0(Q^2+x_0^2m_N^2)M^2}e^{-s_0/M^2} + \frac{1}{2}\frac{x_0^2}{Q^2+x_0^2m_N^2} \times \Big\{\frac{d}{dx_0}\Big[\frac{1}{x_0}\frac{\rho(x_0)}{Q^2+x_0^2m_N^2}\Big]\Big\}e^{-s_0/M^2}. \quad (14)$$

III. NUMERICAL ANALYSIS OF THE SUM RULES FOR THE TENSOR FORM FACTORS OF NUCLEON

In this section, numerical results of the tensor form factors of nucleon are presented. It follows from sum rules for the form factors that the main input parameters are the DAs of nucleon, whose explicit expressions and the values of the parameters f_N, λ_1, λ_2, f_1^u, f_1^d, A_1^u, and V_1^d in the DAs are all given in [21–23].

In the numerical analysis, we use two different sets of parameters:

(a) All eight nonperturbative parameters f_N, λ_1, λ_2, f_1^u, f_1^d, f_2^d, A_1^u, and V_1^d are estimated from QCD sum rules (set 1).

(b) Requiring that all higher conformal spin contributions vanish, fixes five A_1^u, V_1^d, f_1^u, f_1^d, and A_2^d, and the values of the parameters f_N, λ_1, λ_2 are taken from QCD sum rules. This set is called asymptotic set or set 2.

The values of all eight nonperturbative parameters (see, for example, [26]) are presented in Table I.

TABLE I. The values of eight input parameters entering the DAs of nucleon.

	Set 1	Asymptotic set (set 2)
f_N	$(5.0 \pm 0.5) \times 10^{-3}$ GeV^2	$(5.0 \pm 0.5) \times 10^{-3}$ GeV^2
λ_1	$(-2.7 \pm 0.9) \times 10^{-2}$ GeV^2	$(-2.7 \pm 0.9) \times 10^{-2}$ GeV^2
λ_2	$(5.4 \pm 1.9) \times 10^{-2}$ GeV^2	$(5.4 \pm 1.9) \times 10^{-2}$ GeV^2
A_1^u	0.38 ± 0.15	0
V_1^d	0.23 ± 0.03	1/3
f_1^d	0.40 ± 0.05	1/3
f_2^d	0.22 ± 0.05	4/15
f_1^u	0.07 ± 0.05	1/10

The next input parameter of the LCQSR for the tensor form factors is the continuum threshold s_0. This parameter is determined from the two-point sum rules whose value is in the domain $s_0 = (2.25\text{–}2.50)$ GeV^2. The sum rules also contain two extra auxiliary parameters, namely, Borel parameter M^2 and the parameter t entering the expression of the interpolating current for nucleon. Obviously, any physical quantity should be independent of these artificial parameters. Therefore, we try to find such regions of M^2 and t, where the tensor form factors are insensitive to the variation of these parameters.

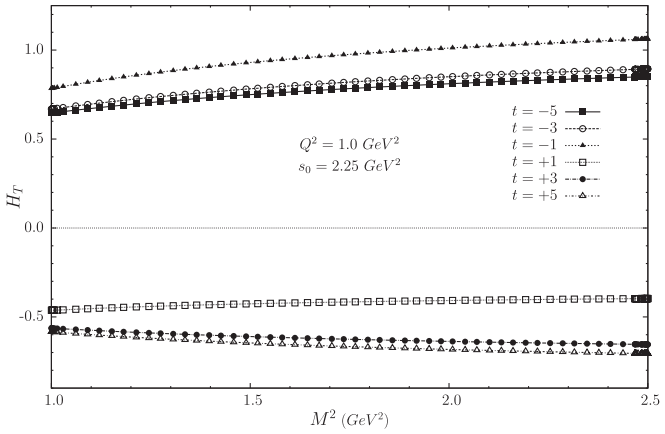

FIG. 1. The dependence of the form factor H_T of nucleon on M^2 at $Q^2 = 1$ GeV^2 and $s_0 = 2.25$ GeV^2, at six different values of t: $t = -5; -3; -1; 1; 3; 5$, using the first set of DAs for the isoscalar current.

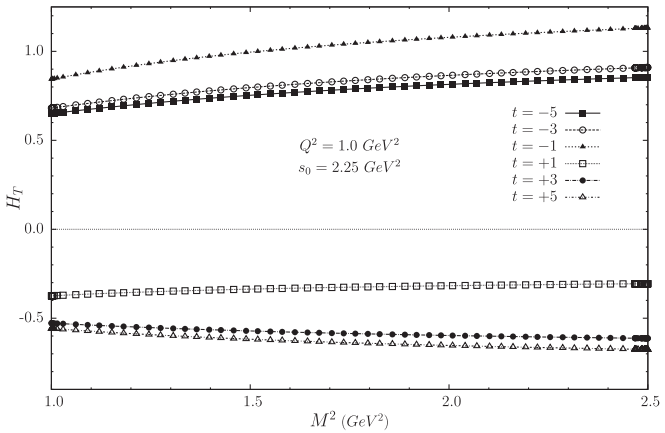

FIG. 2. The same as in Fig. 1, but at $s_0 = 2.5$ GeV^2 and using the second set of DAs.

First, we try to obtain the working region of M^2, where the tensor form factors are independent of it, at fixed values of s_0 and t. As an example, in Figs. 1 and 2, we present the dependence of the tensor form factor $H_T(Q^2)$ induced by the isoscalar current on M^2 at different fixed values of Q^2 and t, and at $s_0 = 2.25$ GeV^2 and $s_0 = 2.50$ GeV^2 for sets 1 and 2, respectively. From these figures, we see that $H_T(Q^2)$ is practically independent of M^2 at fixed values of the parameters Q^2, s_0 and t for both sets 1 and 2. Our calculations also show that the results are approximately the same for two sets, therefore in further discussion, we present the results only for set 1. We perform similar analysis also at $s_0 = 2.40$ GeV^2 and observe that the results change maximally about 5%. The upper limit of M^2 is determined by requiring that the series of light cone expansion with increasing twist converges, i.e., higher twist contributions should be small. Our analysis indeed confirms that the twist-4 contributions to the sum rules constitute maximally about 8% of the total result when $M^2 \leq 2.5$ GeV^2. The lower bound of M^2 is determined by requiring that the contribution of the highest power of M^2 is less than, say, 30% of the higher powers of M^2. Our numerical analysis shows that this condition is satisfied when $M^2 \geq 1.0$ GeV^2. Hence, the working region of M^2 is decided to be in the interval 1.0 GeV$^2 \leq M^2 \leq 2.5$ GeV^2. The working region of the parameter t is determined in such a way that the tensor form factors are also independent of it. Our numerical analysis shows that the form factors are insensitive to $\cos\theta$ (with $t = \tan\theta$) when it varies in the region $-0.5 \leq \cos\theta \leq 0.3$.

In Figs. 3–5, we present the dependence of the form factors $H_T(Q^2)$, $\tilde{E}_T(Q^2)$ and $\tilde{H}_T(Q^2)$ on Q^2 at

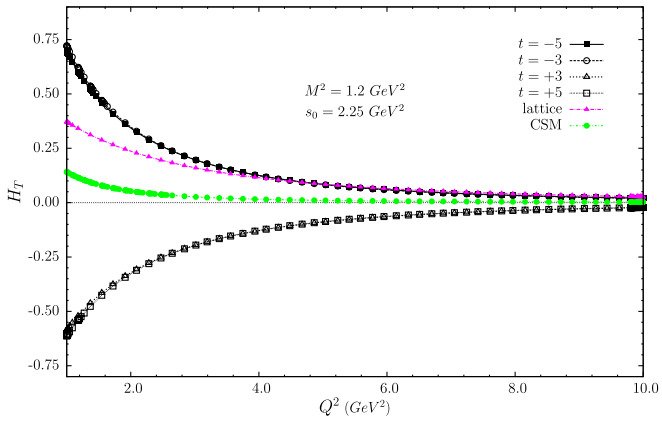

FIG. 3 (color online). The dependence of H_T on Q^2 at $M^2 = 1.2$ GeV^2 and $s_0 = 2.25$ GeV^2 and four fixed values of t: $t = -5; -3; 3; 5$, for the isoscalar current.

$s_0 = 2.25$ GeV^2, $M^2 = 1.2$ GeV^2 and fixed values of t, respectively, using the central values of all input parameters in set 1 for the isoscalar current. For a comparison, we also present the predictions of self consistent chiral soliton model [16] and lattice QCD calculations [19,20] in these figures (note that, chiral soliton model result exists only for $H_T(Q^2)$).

We see from Fig. 3 that our results on $H_T(Q^2)$ are close to the lattice QCD results for $Q^2 \geq 2.0$ GeV^2, while the results of two models differ from each other in the region 1.0 GeV$^2 \leq Q^2 \leq 2.0$ GeV^2. Our and lattice QCD results differ considerably from the predictions of the chiral soliton model. It also follows from these figures that the form factors get positive (negative) at negative (positive) values of the parameter t.

In Figs. 6–8, we present the dependence of the form factors $H_T(Q^2)$, $\tilde{E}_T(Q^2)$ and $\tilde{H}_T(Q^2)$ for the isovector current, i.e., for the $\bar{u}\sigma_{\mu\nu}u - \bar{d}\sigma_{\mu\nu}d$ current. Our observations for set 1 can be summarized as follows:

(i) The Q^2 dependence of $H_T(Q^2)$ is similar to the isoscalar current case, but the values are slightly larger compared to the previous case.

(ii) Similar to the isoscalar case, the form factors $H_T(Q^2)$ and $\tilde{E}_T(Q^2)$ get positive (negative) at negative (positive) values of the parameter t.

(iii) In contrast to the isoscalar current case, the values of $\tilde{H}_T(Q^2)$ are positive (negative) for negative (positive) values of t.

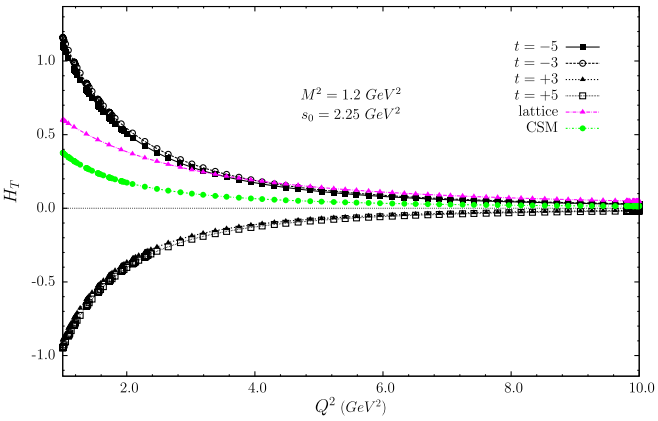

FIG. 6 (color online). The same as in Fig. 3, but for the isovector current.

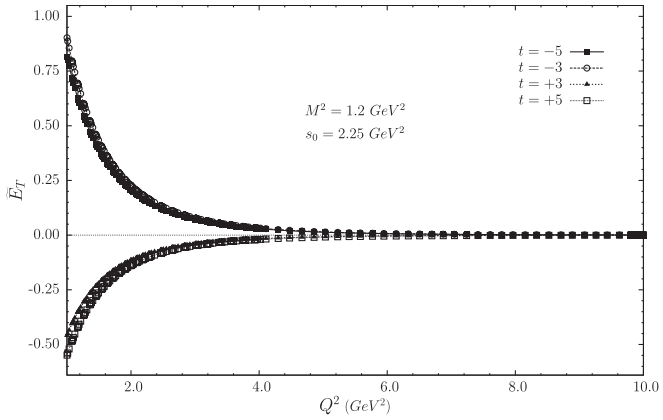

FIG. 4. The same as in Fig. 3, but for the form factor $\tilde{E}_T(Q^2)$.

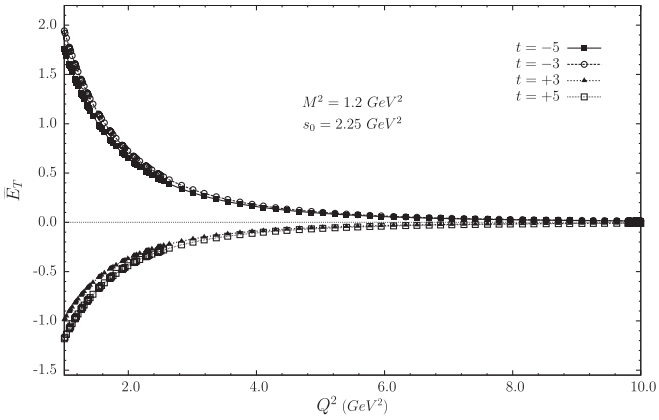

FIG. 7. The same as in Fig. 4, but for the isovector current.

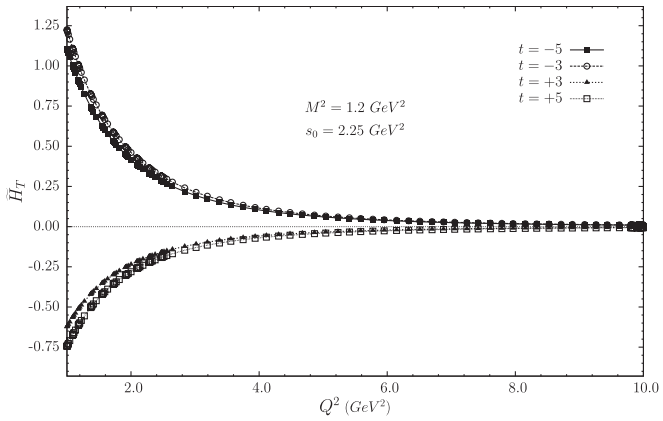

FIG. 5. The same as in Fig. 3, but for the form factor $\tilde{H}_T(Q^2)$.

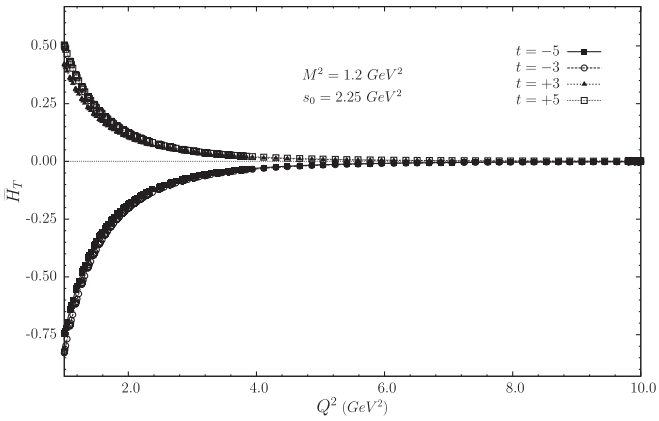

FIG. 8. The same as in Fig. 5, but for the isovector current.

(iv) Our final remark is that the LCQSR results on the form factors can be improved by taking into account the α_s corrections.

In conclusion, using the most general form of the nucleon interpolating current, we calculate the tensor form factors of nucleon within the LCQSR. Our results on these form factors are compared with the lattice QCD and chiral soliton model predictions.

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Note added.—After completing this work, we become aware of a very recent paper [27] in which part of this work is studied.

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