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Thermodynamics of lattice QCD with 2 sextet quarks on $N_t = 8$ lattices

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We continue our lattice simulations of QCD with 2 flavors of color-sextet quarks as a model for conformal or walking technicolor. A 2-loop perturbative calculation of the β function which describes the evolution of this theory's running coupling constant predicts that it has a second zero at a finite coupling. This nontrivial zero would be an infrared stable fixed point, in which case the theory with massless quarks would be a conformal field theory. However, if the interaction between quarks and antiquarks becomes strong enough that a chiral condensate forms before this IR fixed point is reached, the theory is QCD-like with spontaneously broken chiral symmetry and confinement. However, the presence of the nearby IR fixed point means that there is a range of couplings for which the running coupling evolves very slowly, i.e. it "walks." We are simulating the lattice version of this theory with staggered quarks at finite temperature, studying the changes in couplings at the deconfinement and chiral-symmetry restoring transitions as the temporal extent (N_t) of the lattice, measured in lattice units, is increased. Our earlier results on lattices with $N_t = 4$, 6 show both transitions move to weaker couplings as N_t increases consistent with walking behavior. In this paper we extend these calculations to $N_t = 8$. Although both transitions again move to weaker couplings, the change in the coupling at the chiral transition from $N_t = 6$ to $N_t = 8$ is appreciably smaller than that from $N_t = 4$ to $N_t = 6$. This indicates that at $N_t = 4$, 6 we are seeing strong-coupling effects and that we will need results from $N_t > 8$ to determine if the chiral-transition coupling approaches zero as $N_t \to \infty$, as needed for the theory to walk.

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I. INTRODUCTION

We are interested in extensions of the standard model which have a strongly-coupled (composite) Higgs sector. The most promising theories of this type are the so-called technicolor theories [1,2], QCD-like gauge theories with massless (techni-)quarks, where the (techni-)pions play the role of the Higgs field, giving masses to the W and Z. Such theories tend to have phenomenological problems, especially when they are extended to give masses to quarks and leptons. Walking technicolor theories, where gauge group and fermion content are chosen so that the running coupling constant evolves very slowly ("walks"), might be able to avoid such difficulties [3–6]. Deciding whether a candidate gauge theory has the properties needed is a nonperturbative question. Hence lattice gauge theory simulation methods are the only way to answer this reliably.

For a given gauge group with N_f fermions in a specified representation of that group, there is some value of N_f, below which the gauge theory is asymptotically free. Below this value there is a range of N_f for which the second term in the perturbative Callan-Symanzik β function has the opposite sign from the first. Hence, if the 2-loop β function describes the physics the theory with N_f in this range, β has a second nontrivial zero representing an infrared (IR) fixed point. If this is true the theory is a conformal field theory with a continuous spectrum. However, there is a second possibility. If the fermion-antifermion coupling becomes strong enough that a chiral condensate forms before the would-be fixed point is reached, this effectively removes the fermions from consideration for longer distances, the IR zero is avoided, and the coupling approaches infinity at large distances. In this case the theory is QCD-like with confinement as well as chiral-symmetry breaking. However, the presence of the nearby IR fixed point means that the β function becomes small at some value of the coupling, and the coupling constant walks.

If we restrict ourselves to $SU(N_c)$ gauge groups with N_c relatively small, there are a limited number of potential candidates. These have been identified and rough estimates of the value of N_f, which separates conformal from walking behavior, have been made [7–14]. Extensive lattice studies have been made for $N_c = 3$ with fermions in the fundamental representation of the color group [15–36]. There have also been studies with $N_c = 2$ and fermions in the fundamental representation of color [15,37,38] as well as studies with $N_c = 2$ and fermions in the adjoint (symmetric tensor) representation [39–48]. Finally, there have been studies with $N_c = 3$ and fermions in the sextet (symmetric-tensor) representation of the gauge group [49–56].

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We are concentrating our efforts on QCD ($N_c = 3$) with color-sextet quarks. For this choice, asymptotic freedom is lost at $N_f = 3\frac{3}{10}$. This means that only $N_f = 2, 3$ are of interest. Both of these have β functions where the 1 and 2-loop terms are of opposite sign. $N_f = 3$ is close enough to the number of flavors for which asymptotic freedom is lost that the IR fixed point occurs for very weak coupling, for which perturbation theory can probably be trusted. Hence it is believed that this theory is almost certainly a conformal field theory. Estimates of the value of N_f, which separates conformal from walking behavior, suggest that $N_f = 2$ is a good candidate for walking behavior. However, there is enough uncertainty in these methods that a more reliable study of the $N_f = 2$ is warranted. We have thus chosen to study the $N_f = 2$ theory, using lattice gauge simulations with staggered quarks. Lattice studies of QCD with 2 flavors of color-sextet Wilson quarks have been performed by Degrand, Shamir, and Svetitsky. To date these have been unable to tell unambiguously whether this theory is conformal or walking. The Lattice Higgs Collaboration has been studying this theory using improved staggered quarks. Recently they have reported evidence that this theory spontaneously breaks chiral symmetry which would indicate that it has walking behavior (unless there is a bulk chiral transition at even weaker coupling).

Whereas the other groups have concentrated their efforts on determining the nature of QCD with 2 sextet quarks from studies of the zero-temperature behavior of the theory (apart from some early thermodynamics simulations by Degrand, Shamir, and Svetitsky), we are studying the thermodynamics of this theory. Here we are measuring the dependence of the lattice (bare) coupling at the deconfinement and chiral-symmetry restoration transitions on N_t, the temporal extent of the lattice in lattice units. If these are indeed finite-temperature transitions, the couplings at which they occur should tend to zero as $N_t \to \infty$ in a manner controlled by asymptotic freedom. Such behavior would indicate walking. If the theory is conformal, these couplings should approach a nonzero constant as $N_t \to \infty$, indicating a bulk transition. Simulations at $N_t = 4$ and 6, reported in our earlier publication showed that both transition couplings did decrease with increasing N_t. This work reports the results of simulations at $N_t = 8$. While both transitions do tend to weaker couplings as N_t goes from 6 to 8, the change in coupling at the chiral transition, which occurs at a considerably weaker coupling than the deconfinement transition, is much smaller than that between $N_t = 4$ and 6. (Such separation of the deconfinement and chiral-symmetry restoration transitions, which is not observed for fundamental quarks, has been observed with adjoint quarks [57,58].) The most likely interpretation is that between $N_t = 4$ and 6, this transition is in the strong-coupling domain where the quarks have condensed to form a chiral condensate at length scales of order of the lattice spacing and do not participate in the running of the coupling constant, which now runs as in quenched QCD. Between $N_t = 6$ and 8 the chiral-transition coupling finally emerges into the weak-coupling regime where the quarks also participate in the running of the coupling constant. This means that we will need to simulate at even larger N_t's to determine whether this theory is conformal or walking.

As for $N_t = 6$, the $N_t = 8$ lattice shows a clear 3-state signal above the deconfinement transition. These states are characterized by the phase of the Wilson Line (Polyakov Loop) having the values 0, $\pm\frac{2\pi}{3}$, a vestige of the Z_3 color symmetry of the quenched theory. Within the limitations of our simulations, all 3 states appear stable. At even weaker couplings—close to the chiral transition—the 2 states with complex phases disorder to a phase with a negative Wilson Line. This phase structure, which is richer than that for fundamental quarks, where the Wilson Line is always real and positive, was predicted by Machtey and Svetitsky and observed in their simulations with Wilson quarks [59].

In Sec. II we describe our simulation techniques and how one can measure the running of the coupling constant from thermodynamics. Section III describes our simulations and results. Finally in Sec. IV we discuss our results, draw conclusions, and indicate directions for future investigations.

II. METHODOLOGY

For the gauge fields we use the standard Wilson (plaquette) action:

$$S_g = \beta\sum_{\square}\left[1 - \frac{1}{3}\,\mathrm{Re}(\mathrm{Tr}UUUU)\right]. \tag{1}$$

For the fermions we use the unimproved staggered-quark action:

$$S_f = \sum_{\rm sites}\left[\sum_{f=1}^{N_f/4}\psi_f^\dagger[\not{D} + m]\psi_f\right], \tag{2}$$

where $\not{D} = \sum_\mu \eta_\mu D_\mu$ with

$$D_\mu\psi(x) = \frac{1}{2}[U_\mu^{(6)}(x)\psi(x+\hat{\mu}) - U_\mu^{(6)\dagger}(x-\hat{\mu})\psi(x-\hat{\mu})], \tag{3}$$

where $U^{(6)}$ is the sextet representation of U, i.e. the symmetric part of the tensor product $U \otimes U$. When N_f is not a multiple of 4 we use the fermion action:

$$S_f = \sum_{\rm sites}\chi^\dagger\{[\not{D} + m][-\not{D} + m]\}^{N_f/8}\chi. \tag{4}$$

The operator which is raised to a fractional power is positive definite, and we choose the real positive root. This yields a well-defined operator. We assume that this defines a sensible field theory in the zero lattice-spacing limit, ignoring the rooting controversy. (See for example [60] for a review and guide to the literature on rooting.)

We use the RHMC method for our simulations [61], where the required powers of the quadratic Dirac operator are replaced by diagonal rational approximations to the desired precision. By applying a global Metropolis accept/reject step at the end of each trajectory, errors due to the discretization of molecular-dynamics time are removed.

Finite-temperature simulations are performed by using a lattice of finite extent N_t in lattice units in the Euclidean time direction and of infinite extent N_s in the spatial direction. In practice this means we choose $N_s \gg N_t$. The temperature $T = 1/N_t a$, where a is the lattice spacing. (In our earlier equations we set $a = 1$.) Since the deconfinement temperature T_d and the chiral-symmetry restoration temperature T_χ should not depend on a, and since $a = 1/N_t T$, measuring the coupling g at T_d or T_χ as a function of N_t gives $g(a)$ for a series of a values which approach zero as $N_t \to \infty$. If the ultraviolet behavior of the theory is governed by asymptotic freedom, $g(a)$ should approach zero as $a \to 0$, i.e. $N_t \to \infty$. The way g_d and g_χ approach zero should be determined by the perturbative β function. The 2-loop β function

$$\beta(g) = -b_1 g^3 - b_2 g^5. \tag{5}$$

Then expressing our coupling constant evolution in terms of $\beta = 6/g^2$ (we apologize for the fact that we are using β for 2 different purposes)

$$\Delta\beta(\beta) = \beta(a) - \beta(\lambda a) = (12b_1 + 72b_2/\beta)\ln(\lambda) \tag{6}$$

through this order. For N_f flavors of sextet quarks,

$$\begin{aligned} b_1 &= \left(11 - \frac{10}{3}N_f\right)\Big/16\pi^2 \\ b_2 &= \left(102 - \frac{250}{3}N_f\right)\Big/(16\pi^2)^2. \end{aligned} \tag{7}$$

If, on the other hand, the $N_f = 2$ theory is conformal, the continuum, zero coupling ($\beta \to \infty$) limit has an unbroken chiral symmetry (and is unconfined). Hence there will be a bulk chiral transition at a finite coupling, which survives in the $N_t \to \infty$ limit, so the coupling and hence β at the chiral transition will tend to a finite value in this limit. [Since the β value at the deconfinement transition (β_d) is expected to be less than that at the chiral transition (β_χ), it follows that β_d will also approach a finite value as $N_f \to \infty$.]

We determine the position of the deconfinement transition as that value of β where the magnitude of the triplet Wilson Line (Polyakov Loop) increases rapidly from a very small value as β increases. The chiral phase transition is at that value of β beyond which the chiral condensate $\langle\bar\psi\psi\rangle$ vanishes in the chiral limit. Because we are forced to simulate at finite quark mass, this value is difficult to determine directly. We therefore estimate the position of the chiral transition by determining the position of the peak in the chiral susceptibility $\chi_{\bar\psi\psi}$ as a function of quark mass and extrapolating to zero quark mass. The chiral susceptibility is given by

$$\chi_{\bar\psi\psi} = V[\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2], \tag{8}$$

where the $\langle\rangle$ indicates an average over the ensemble of gauge configurations and V is the space-time volume of the lattice. Since the fermion functional integrals have already been performed at this stage, this quantity is actually the disconnected part of the chiral susceptibility. Since we use stochastic estimators for $\bar\psi\psi$, we obtain an unbiased estimator for this quantity by using several independent estimates for each configuration (5, in fact). Our estimate of $(\bar\psi\psi)^2$ is then given by the average of the (10) estimates which are "off diagonal" in the noise.

Our $N_t = 8$ simulations are performed on $16^3 \times 8$ lattices. Near the chiral transition, where finite size effects are a concern, we also perform simulations on a $24^3 \times 8$ lattice for the lowest quark mass. We perform simulations with quark masses $m = 0.005$, $m = 0.01$, and $m = 0.02$ in lattice units, to enable continuation to the chiral ($m = 0$) limit. (Since we do not have any zero-temperature measurements, the more desirable method of choosing lines of constant physics is impossible.) Our trajectory length is chosen to be $\Delta\tau = 1$ where τ is the molecular-dynamics "time" in HEMCGC normalization [62].

A more detailed discussion of our methods of choosing parameters, run lengths, etc. is given in our earlier paper describing our $N_t = 4$, 6 simulations [53].

III. SIMULATIONS AND RESULTS

We simulate QCD with 2-flavors of color-sextet staggered quarks on $16^3 \times 8$ and $24^3 \times 8$ lattices. For the smaller lattice we perform simulations with masses $m = 0.005$, $m = 0.01$, and $m = 0.02$ to allow extrapolation to the chiral limit, for a set of β values covering the range $5.5 \le \beta \le 7.4$. To probe the various phases of the Wilson Line, we use 2 different sets of runs. In the first set of runs we use an ordered start, in which the gauge fields are set to the unit matrix on all links, at the highest β, and use configurations from higher β's to start runs at lower β's. The second set of runs uses a start in which the gauge fields are set to the unit matrix, except for the timelike gauge fields on a single time slice, which are set to the matrix diag$(1, -1, -1)$. This puts the system in a state with a real negative Wilson Loop at large β's.

The length of a typical run at a fixed (β, m) away from the transitions is 10 000 trajectories. Close to the deconfinement transition, this is increased to 50 000 trajectories. Run lengths of 50 000 trajectories are also used close to the transition from a state where the Wilson Line has phase $\pm 2\pi/3$ to one where it has phase π. We have detailed our run lengths in the Appendix.

Since finite (spatial) volume effects are most likely to be present in the weak-coupling domain at small quark masses, where they have the potential to shift the chiral transition, we have also performed a set of simulations on $24^3 \times 8$ lattices at the lowest quark mass. These simulations at $m = 0.005$ cover the range $6.2 \leq \beta \leq 7.4$ with mesh $\delta\beta = 0.1$, and with 10 000 trajectories at each β, from positive Wilson line starts.

A. Results

Starting from large β values, the runs which start from a completely ordered state with Wilson Line $+3$ continue to have positive Wilson Lines down to $\beta = 5.8$ for $m = 0.02$, and down to $\beta = 5.7$ for $m = 0.01$ and $m = 0.005$. Below these β values, which are just above the deconfinement transition, we see a clear 3-state signal, where the system tunnels between states where the Wilson Line has phases 0, $\pm 2\pi/3$. Because of this, we bin our data according to the phase ϕ of the Wilson Line for each configuration. Configurations where $-\pi/3 < \phi < \pi/3$ are considered to be in the $\phi = 0$ bin. Outside of this range the configurations are considered to be in the $\pm 2\pi/3$ bins depending on whether the imaginary part of the Polyakov loop is positive or negative. These last 2 bins are combined by complex conjugating those Wilson Lines which have negative imaginary parts.

Starting from large β's, in those runs which start from the second ordering with Wilson Line -1, the Wilson Line remains negative down to $\beta \approx 6.9$ for $m = 0.02$, $\beta \approx 6.8$,

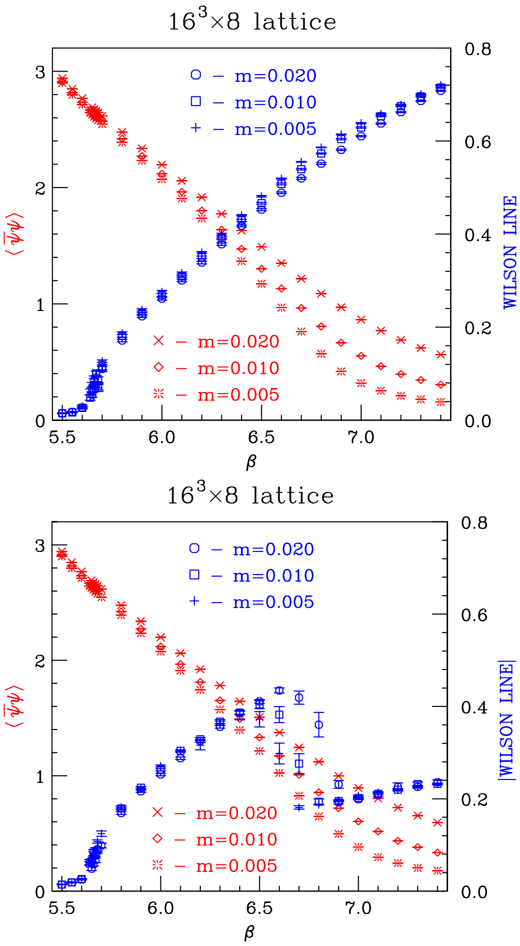

FIG. 1 (color online). a) Wilson Line and chiral condensate for real positive Wilson Line states as functions of β on a $16^3 \times 8$ lattice. b) Magnitude of Wilson Line and chiral condensate for states with complex or real negative Wilson Lines as functions of β on a $16^3 \times 8$ lattice.

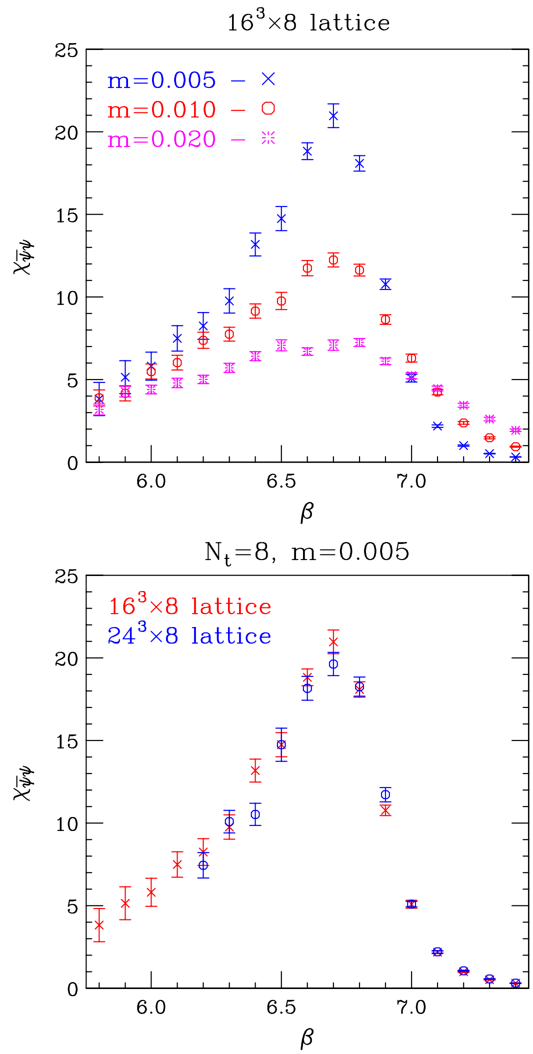

FIG. 2 (color online). a) The chiral susceptibilities as functions of β for each of the 3 masses on a $16^3 \times 8$ lattice. b) The chiral susceptibilities at $m = 0.005$ as functions of β on a $16^3 \times 8$ lattice and on a $24^3 \times 8$ lattice.

$m = 0.01$ and $\beta \approx 6.7$, $m = 0.005$. Below these values the system makes a transition to a state with Wilson Line phase $\pm 2\pi/3$. Below these β values, these runs remain in states with Wilson Line phases $\pm 2\pi/3$ down to $\beta = 5.8$, for each m. For $\beta = 5.7$ and below we see clear 3-state signals where the system tunnels between the 3 states. For this reason we again bin our data according to the Wilson Line phase, ϕ.

In Fig. 1(a) we present the Wilson Line and chiral condensate for the states with a real positive Wilson Line plotted against β for each of the 3 masses. In Fig. 1(b) we plot the magnitude of the Wilson Line and the chiral condensate for those states with complex or real negative Wilson Lines.

In both sets of graphs in Fig. 1 we observe a rapid increase in the (magnitude of the) Wilson Line at $\beta \approx 5.65$, corresponding to the deconfinement transition. The sudden drop in the magnitudes of the Wilson Lines of Fig. 1(b) for $6.6 \lesssim \beta \lesssim 7.0$ marks the transition where states with complex Wilson Lines ($\phi = \pm 2\pi/3$) disorder to a state with a real negative Wilson Line $\phi = \pi$.

It is clear that the chiral condensate becomes small for large β and decreases with decreasing quark mass, which suggests that it will vanish in the chiral limit, for β large enough. However, extrapolating the chiral condensate to zero quark mass to determine the chiral transition from these quark masses where the β dependence is so smooth would be exceedingly difficult. We therefore estimate the position of the chiral transition from determinations of the positions of the peaks in the chiral susceptibilities for each mass. These are plotted in Fig. 2(a). For the lower two masses, the peaks in the chiral susceptibilities are well defined. (This is the best evidence we have that our quark masses are small enough to perform the chiral extrapolation.) In addition, for the limited set of β values of our simulations, both the $m = 0.01$ and the $m = 0.005$ "data" peak at the same β, namely, $\beta = 6.7$. We therefore estimate that the position of the peak and thus the chiral phase transition at $m = 0$ are at $\beta_\chi = 6.7(1)$. This means that β_d and β_χ are well separated as was observed for $N_t = 4$ and 6. At $m = 0.005$, close to the chiral transition, we have also performed simulations on larger ($24^3 \times 8$) lattices. The

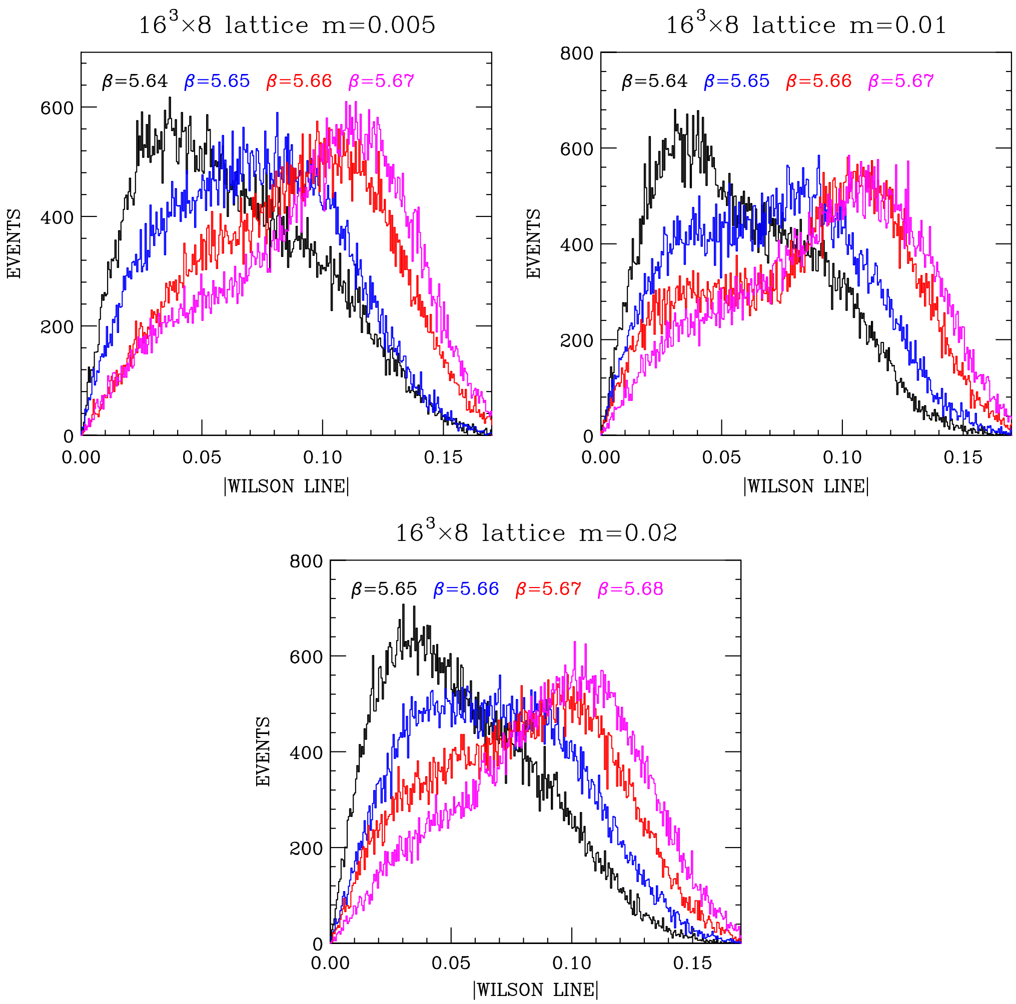

FIG. 3 (color online). Histograms of magnitudes of Wilson Lines: a) for $m = 0.005$, b) for $m = 0.01$, and c) for $m = 0.02$.

TABLE I. $N_f = 2$ deconfinement and chiral transitions for $N_t = 4, 6, 8$.

N_t	β_d	β_χ
4	5.40(1)	6.3(1)
6	5.54(1)	6.6(1)
8	5.65(1)	6.7(1)

Wilson Lines and chiral condensates show little difference between the two lattice sizes. The chiral susceptibilities plotted in Fig. 2 indicate that finite size effects are indeed small. This is more significant, since such fluctuation quantities are most sensitive to finite volume effects.

We have also looked at the chiral susceptibilities for the states with real negative or complex Wilson lines and find peaks at $\beta = 6.8(1)$ for $m = 0.02$, $\beta = 6.7(1)$ for $m = 0.01$, and $\beta = 6.6(1)$ for $m = 0.005$. Since these values are close to the transitions from the state with a negative Wilson Line to states with complex Wilson Lines, there is a possibility of interference between these two transitions. For this reason we concentrate our studies on the chiral transition measured in the positive Wilson Line state.

We now turn our attention to more precise estimates of β_d. For this purpose, we histogram the magnitudes of the Wilson Lines in the neighborhood of the deconfinement transition. Such histograms are shown in Fig. 3 for each quark mass. We estimate that the transition occurs at $\beta = \beta_d = 5.66(1)$ for $m = 0.02$, at $\beta_d = 5.65(1)$ for $m = 0.01$, and at $\beta_d = 5.65(1)$ for $m = 0.005$.

The positions of the deconfinement and chiral transitions, extrapolated to the chiral limit, are given in Table I for each of the 3 N_t values ($N_t = 4$, 6 from [53], $N_t = 8$ this work).

B. Interpretation

We now compare the changes in β_d and β_χ with what would be expected if the running of the lattice coupling constant is given by the 2-loop (perturbative) β function—Eq. (6).

For the deconfinement transition

$$\Delta\beta_d(6,4) = \beta_d(N_t = 6) - \beta_d(N_t = 4) \approx 0.14 \tag{9}$$

compared with the prediction of Eq. (6) which predicts $\Delta\beta_d(6,4) \approx 0.12$, whereas

$$\Delta\beta_d(8,6) = \beta_d(N_t = 8) - \beta_d(N_t = 6) \approx 0.11. \tag{10}$$

compared with the 2-loop prediction $\Delta\beta_d(8,6) \approx 0.09$. If the quarks were actively screening color at these couplings ($5.40 \lesssim \beta \lesssim 5.65$), it would not be unreasonable to assume that these deconfinement couplings were weak enough to be governed by the perturbative β function. The fact that the measured $\Delta\beta$'s are within $\approx 20\%$ of those predicted by 2-loop perturbation theory would tend to support this interpretation. However, examining the running of the coupling constant at the chiral transition, will lead us to a different conclusion.

For the chiral transition, we find

$$\Delta\beta_\chi(6,4) = \beta_\chi(N_t = 6) - \beta_\chi(N_t = 4) \approx 0.3 \tag{11}$$

while

$$\Delta\beta_\chi(8,6) = \beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) \approx 0.1. \tag{12}$$

Using Eq. (6) to estimate $\Delta\beta_\chi$ yields

$$\Delta\beta_\chi(6,4) \approx 0.122 \tag{13}$$

and

$$\Delta\beta_\chi(8,6) \approx 0.087. \tag{14}$$

While $\Delta\beta_\chi(8,6)$ appears consistent with our measurements, $\Delta\beta_\chi(6,4)$ does not. What this suggests is that for N_t in the range 6–8, β_χ is in the weakly-coupled domain where scaling is controlled by asymptotic freedom, while N_t in the range 4–6 is in the strongly-coupled domain.

In the strongly-coupled domain, the fermions have formed a chiral condensate, which effectively stops them from contributing significantly to the running of the coupling constant. Hence we expect that the running of the coupling in this region will be that of the quenched theory, i.e. that for Eq. (6) with $N_f = 0$. This yields

$$\Delta\beta_\chi(6,4) \approx 0.357, \tag{15}$$

which is consistent with what we observe. (It also gives

$$\Delta\beta_\chi(8,6) \approx 0.253, \tag{16}$$

which is larger than what we observe.) Thus we conclude that the chiral transition emerges from the strongly-coupled domain, where the quarks play little part in the coupling constant evolution, into the weak-coupling regime, where the running of the coupling is determined by asymptotic freedom, around $\beta_\chi(N_t = 6)$.

One might argue that both $\Delta\beta_\chi$'s are consistent with either $N_f = 0$ or $N_f = 2$ scaling because of the relatively large error-bars in Table I. However, comparing the graphs of the chiral condensates for fixed masses for each N_t—see Fig. 4—we see that $\Delta\beta_\chi(6,4)$ really does appear to be much larger than $\Delta\beta_\chi(8,6)$ and that the estimates of Eqs. (11) and (12) are more accurate than the errors in the individual β_χ's would suggest.

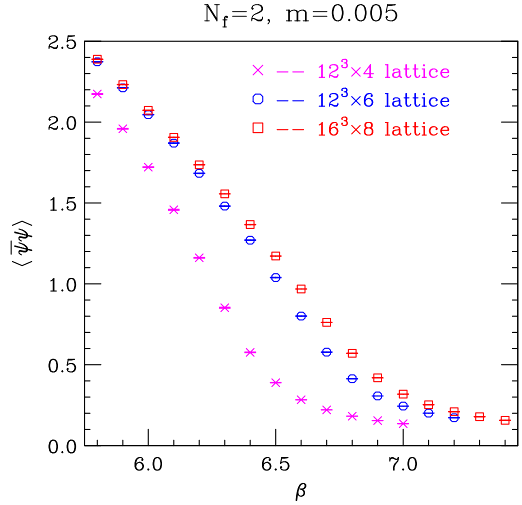

FIG. 4 (color online). Chiral condensates for $m = 0.005$ for $N_t = 4$, 6, and 8, in the high β (weak coupling) regime.

This interpretation of the running of the lattice coupling constant at the chiral transition indicates that the region $\beta \lesssim 6.6$ is one of strong coupling, governed by quenched dynamics. In particular, the change in β_d as N_t is varied from 4 to 6 to 8 should be governed by quenched dynamics. However, it has been determined that the evolution of the deconfinement coupling in quenched QCD is only well described by 2-loop perturbation theory for $\beta \gtrsim 6.1$ [63,64]. Hence the changes in β_d that we observe are not expected to be described by quenched perturbation theory. We note, however, that $\beta_d(N_t = 6) = 5.54(1)$ is close to $\beta_d(N_t = 3) \approx 5.55$ found for the quenched theory [65], while $\beta_d(N_t = 8) = 5.65(1)$, compared to $\beta_d(N_t = 4) \approx 5.69$ for the quenched theory [65–67]. Since the ratios of lattice spacings in these 2 cases are identical, such comparison is justified. Taking into account the fact that small N_t effects are expected to make the $N_t = 3$ quenched β_d anomalously small, the comparison is remarkably good. Unfortunately we cannot expect similar comparisons between $\beta_d(N_t = 4)$ and $\beta_d(N_t = 2)$ for the quenched theory to work, since it is well known that $\beta_d(N_t = 2) \approx 5.1$ for quenched lattice QCD is anomalously low [65]. Still it is reasonable that a strong-coupling quenched interpretation of the running of β_d is correct for $N_t = 4$, 6, 8. The fact that the changes in β_d between $N_t = 4$, 6, and 8 are appreciably less than predicted by quenched perturbation theory is a well-known feature of the strong-coupling domain of quenched lattice QCD [63,64].

IV. DISCUSSION AND CONCLUSIONS

We are studying thermodynamics of QCD with 2 massless color-sextet "quarks" in an attempt to distinguish whether it is a conformal field theory or if it walks, i.e. is a confined, chiral-symmetry broken theory, with a slowly evolving coupling. Simulations are performed on lattices with spatial extent $N_s a$ and temporal extent $N_t a$ (a is the lattice spacing) with $N_s \gg N_t$. We use staggered quarks with several small masses to extrapolate to the massless quark limit. The temperature $T = 1/N_t a$. Hence for fixed T, chosen to be either the deconfinement temperature T_d or the chiral-symmetry restoration temperature T_χ, we can vary a by varying N_t, thus studying the running of the coupling $g(a)$ as $a \to 0$ by simulating at a series of N_t's increasing towards infinity. Our earlier simulations were performed using $N_t = 4$, 6. Those we report here use $N_t = 8$.

At $N_t = 8$, as at the smaller N_t's, the chiral transition occurs at a much weaker coupling than the deconfinement transition (see Table I). (This contrasts with the case with quarks in the fundamental representation of color, where these transitions appear coincident.) Between $N_t = 4$ and 6 both transitions move to appreciably smaller couplings. While this trend continues between $N_t = 6$ and 8, the change in couplings at the chiral transition is much smaller than that between $N_t = 4$ and 6. A possible explanation is that for couplings between those at the $N_t = 4$ and the $N_t = 6$ chiral transitions, the system is in the strong-coupling regime, where the quarks are bound in a chiral condensate at distances $\lesssim a$ and thus do not contribute significantly to the evolution of the coupling constant, which thus evolves as in quenched ($N_f = 0$) QCD. Between the couplings for the $N_t = 6$ and $N_t = 8$ transitions, the system emerges into the weak-coupling domain, where the fermions contribute and the coupling evolves according to $N_f = 2$ asymptotic freedom. Although we have given semiquantitative evidence for this interpretation, we cannot rule out the possibility that the coupling at the chiral transition is approaching a fixed nonzero value. This would be evidence for a bulk chiral transition, implying that the continuum theory has unbroken chiral symmetry and is thus conformal.

In order to distinguish conformal from walking behavior, we will need to perform simulations at larger N_t values. We have already begun simulations on $N_t = 12$ lattices. In addition, since the changes in β_χ between $N_t = 6$ and $N_t = 8$ and those expected between $N_t = 8$ and $N_t = 12$ are of order 0.1, we will need more β values in the neighborhood of β_χ to determine this value more accurately. We are currently performing such simulations and additional simulations with a smaller quark mass ($m = 0.0025$) at $N_t = 8$, to aid with the chiral extrapolation. With these new simulations, we are concentrating on the chiral transition, since it would require N_t values much greater than what is currently feasible to have β_d in the weak-coupling ($\beta \gtrsim 6.6$) regime.

Our runs at $N_t = 8$ show a phase structure similar to what was observed at $N_t = 6$. Above the deconfinement transition, the Wilson Line exhibits a definite 3-state structure. In addition to the state with a positive Wilson Line, which is all that is observed for fundamental quarks, there are states characterized by Wilson Lines oriented (at least approximately) in the directions of the 2 complex cube roots of unity. The existence of this 3-state signal is probably because chiral symmetry is still broken in this regime, effectively removing the fermions from the dynamics so that it behaves as a quenched theory. This suggests that the 3-state signal is the vestige of the spontaneously broken Z_3 symmetry of the deconfined pure gauge theory. The fact that this Z_3 symmetry is explicitly broken manifests itself in the fact that the magnitude of the Wilson Line in the complex Wilson Line states is smaller than that in the positive Wilson Line state. Within the limitations of our simulations, all 3 states appear to be stable. At some β value close to the chiral transition, the complex Wilson Line states disorder to a state with a negative Wilson Line. Above this transition the magnitude of the Wilson Line in the negative Wilson Line state is around $1/3$ of that for the positive Wilson line state. This leads us to speculate that the transition indicates a breaking of color $SU(3)$ to color $SU(2) \times U(1)_Y$. Arguments for the existence of states with Wilson Lines having phases $\pm 2\pi/3$ and π in addition to that with phase 0 have been given by Machtey and Svetitsky, who showed evidence for them in their simulations with Wilson quarks.

We also plan simulations to measure the zero-temperature properties of this theory. In the weak-coupling regime $\beta > \beta_\chi$, we will check whether the theory is a conformal field theory or if it is in the quark-gluon plasma phase of a QCD-like gauge theory. If the theory is a conformal field theory (for massless fermions) all "hadron" masses will vanish with the same anomalous dimension, and the chiral condensate will also vanish with an anomalous dimension in the chiral limit. Because such anomalous dimensions are determined by the infrared attractive fixed point, they should be independent of β.

If we do not find evidence of conformal behavior, we will check for QCD-like behavior in the chirally-broken phase and for evidence that this phase has a continuum limit controlled by asymptotic freedom. This will require very large lattices, since we need to choose β values large enough for asymptotic freedom to control the renormalization group scaling of observables, while keeping $\beta < \beta_d (< \beta_\chi)$. Here we will need to measure the masses of the "hadrons" to determine if our quark masses are small enough and our lattices large enough to observe that the "pion" masses vanish in the chiral limit proportional to $\sqrt{m}$, while the other hadrons remain massive. We will also need to check for evidence that the chiral condensate remains finite in the chiral limit. In addition we will measure f_π and study propagators of vector and axial vector mesons which contribute to the S parameter, as is being done by the Lattice Strong Dynamics Collaboration, for fundamental quarks [35]. Simulations will be performed at several β values to determine the running of the bare coupling and of some appropriately-defined renormalized coupling. This is necessary to check that the theory has the correct ultraviolet completion.

Zero-temperature simulations with sextet quarks are already being performed using improved staggered quarks by the Lattice Higgs Collaboration, who presented preliminary results at Lattice 2010 [56].

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TABLE II. Numbers of trajectories for each β and m for runs on $16^3 \times 8$ lattices. The first number is for simulations starting with positive Wilson Lines; the second is for simulations starting with negative or complex Wilson Lines.

β	$m = 0.005$	$m = 0.01$	$m = 0.02$
5.5	10 000	10 000	10 000
5.55	10 000	10 000	10 000
5.6	50 000	50 000	50 000
5.64	50 000 + 50 000	50 000 + 50 000	...
5.65	50 000 + 50 000	50 000 + 50 000	50 000 + 50 000
5.66	50 000 + 50 000	50 000 + 50 000	50 000 + 50 000
5.67	50 000 + 50 000	50 000 + 50 000	50 000 + 50 000
5.68	50 000 + 50 000	50 000 + 50 000	50 000 + 50 000
5.7	50 000 + 50 000	50 000 + 50 000	50 000 + 50 000
5.8	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
5.9	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.0	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.1	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.2	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.3	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.4	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
6.5	10 000 + 50 000	10 000 + 10 000	10 000 + 10 000
6.6	20 000 + 50 000	10 000 + 50 000	10 000 + 10 000
6.7	20 000 + 50 000	10 000 + 50 000	10 000 + 10 000
6.8	20 000 + 50 000	10 000 + 50 000	10 000 + 50 000
6.9	10 000 + 20 000	10 000 + 50 000	10 000 + 50 000
7.0	10 000 + 10 000	10 000 + 50 000	10 000 + 50 000
7.1	10 000 + 10 000	10 000 + 10 000	10 000 + 30 000
7.2	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
7.3	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000
7.4	10 000 + 10 000	10 000 + 10 000	10 000 + 10 000

APPENDIX: RUN DETAILS

Table II gives the length of our $16^3 \times 8$ runs in length-1 trajectories, for each β and mass. Where 2 numbers are given, the first is for a series of runs which started from an ordered configuration at large β, while the second is from a start which gives negative Wilson Loops at large β.

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