Matrix elements of the electromagnetic operator between kaon and pion states

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We compute the matrix elements of the electromagnetic operator $\bar{s}F_{\mu\nu}\sigma^{\mu\nu}d$ between kaon and pion states, using lattice QCD with maximally twisted-mass fermions and two flavors of dynamical quarks ($N_f = 2$). The operator is renormalized nonperturbatively in the RI'/MOM scheme and our simulations cover pion masses as light as 270 MeV and three values of the lattice spacing from $\simeq 0.07$ up to $\simeq 0.1$ fm. At the physical point our result for the corresponding tensor form factor at zero-momentum transfer is $f_T^{K\pi}(0) = 0.417(14_{\text{stat}})(5_{\text{syst}})$, where the systematic error does not include the effect of quenching the strange and charm quarks. Our result differs significantly from the old quenched result $f_T^{K\pi}(0) = 0.78(6)$ obtained by the SPQ$_{\text{cd}}$R Collaboration with pion masses above 500 MeV. We investigate the source of this difference and conclude that it is mainly related to the chiral extrapolation. We also study the tensor charge of the pion and obtain the value $f_T^{\pi\pi}(0) = 0.195(8_{\text{stat}})(6_{\text{syst}})$ in good agreement with, but more accurate than the result $f_T^{\pi\pi}(0) = 0.216(34)$ obtained by the QCDSF Collaboration using higher pion masses.

I. INTRODUCTION

Accurate measurements of hadron weak decays can constrain the parameters of the standard model and can place bounds on new physics models. In particular, the rare decays of the kaon are ideally suited to search for new, possibly large CP-violating effects in the light-quark sector (see Ref. [1]).

In this work we present a lattice study of the matrix elements of the electromagnetic (EM) operator between kaon and pion states, which may be relevant in the CP-violating part of the $K \to \pi \ell^+ \ell^-$ semileptonic decays. The study has been performed using the gauge configurations generated [2] by the European Twisted Mass Collaboration (ETMC) with $N_f = 2$ maximally twisted-mass fermions [3,4] and preliminary results have been presented already in Ref. [5].

The EM operator involved in the weak $s \to d$ transition is given by $\bar{s}F_{\mu\nu}\sigma^{\mu\nu}d$, where $F_{\mu\nu}$ is the EM field tensor. Therefore its matrix elements between kaon and pion states involve the ones of the weak tensor current, which can be written in terms of a single form factor, $f_T^{K\pi}(q^2)$, as

$$\langle \pi^0 | \bar{s}\sigma^{\mu\nu}d | K^0 \rangle = (p_\pi^\mu p_K^\nu - p_\pi^\nu p_K^\mu)\frac{\sqrt{2}f_T^{K\pi}(q^2)}{M_K + M_\pi}, \quad (1)$$

where $q = (p_K - p_\pi)$ is the 4-momentum transfer and the factor $(M_K + M_\pi)^{-1}$ is conventionally inserted in order to make the tensor form factor dimensionless.

Our simulations cover pion masses as light as 270 MeV and three values of the lattice spacing from $\simeq 0.07$ up to $\simeq 0.1$ fm. At the physical point our result for the $K \to \pi$ tensor form factor at zero-momentum transfer is

$$f_T^{K\pi}(0) = 0.417(14_{\text{stat}})(5_{\text{syst}}) = 0.417(15). \quad (2)$$

where the systematic error does not include any estimate of the effect of quenching the strange and charm quarks. Our finding differs significantly from the old quenched result $f_T^{K\pi}(0) = 0.78(6)$ obtained in Ref. [6] by the SPQ$_{\text{cd}}$R Collaboration with pion masses above ~ 500 MeV. The reason is mainly due to the nonanalytic behavior of the tensor form factor $f_T^{K\pi}(0)$ in terms of the quark masses introduced by the factor $(M_K + M_\pi)^{-1}$ in the parameterization (1). Such a behavior was not taken into account in Ref. [6] (see later on).

In the case of the degenerate $\pi \to \pi$ transition, using the predictions of the chiral perturbation theory (ChPT) carried out in Ref. [7], we obtain for the tensor form factor $f_T^{\pi\pi}(0)$, known as the tensor charge of the pion, the following value

$$f_T^{\pi\pi}(0) = 0.195(8_{\text{stat}})(6_{\text{syst}}) = 0.195(10), \quad (3)$$

which improves the result $f_T^{\pi\pi}(0) = 0.216(34)$ obtained by the QCDSF Collaboration [8] with simulations at higher pion masses.

II. $K \to \pi$ RESULTS

We have performed the calculations of the relevant 2-point and 3-point correlation functions using the ETMC gauge configurations with $N_f = 2$ dynamical twisted-mass quarks generated [2] at three values of the lattice coupling β, namely, the ensembles $A_2 - A_4$ at $\beta = 3.8$ ($a = 0.098(4)$ fm), $B_1 - B_7$ at $\beta = 3.9$ ($a = 0.085(3)$ fm), and $C_1 - C_3$ at $\beta = 4.05$ ($a = 0.067(2)$ fm). The pion mass M_π ranges between $\simeq 270$ MeV and $\simeq 600$ MeV

and the size L of our lattices guarantees that $M_\pi L$ is larger than ~ 3.3. For each value of the pion mass and of the lattice spacing we have used several values of the (bare) strange quark mass m_s to allow for a smooth, local interpolation of our results to the physical value of m_s (see Ref. [9]). The calculation of the 2- and 3-point correlation functions has been carried out using all-to-all quark propagators evaluated with the *one-end-trick* stochastic procedure and adopting nonperiodic boundary conditions which make arbitrarily small momenta accessible. All the necessary formulae can be easily inferred from Ref. [10], where the degenerate case of the vector pion form factor is illustrated in details. For each pion mass the statistical errors are evaluated with the jackknife procedure.

The tensor current was renormalized nonperturbatively in the RI'/MOM scheme in Ref. [11], including $\mathcal{O}(a^2 g^2)$ corrections evaluated at one-loop in lattice perturbation theory [12]. The numerical values used in our analyses for the tensor renormalization constant are $Z_T(\overline{\text{MS}}, 2 \text{ GeV}) = 0.733(9), 0.743(5), 0.777(6)$ for $\beta = 3.8, 3.9, 4.05$, respectively.

At each pion and kaon masses we determine the tensor form factor $f_T^{K\pi}(q^2)$ for several values of $q^2 < q_{\max}^2 = (M_K - M_\pi)^2$ in order to interpolate at $q^2 = 0$.[1] Note that, because of the vanishing of the Lorentz structure in Eq. (1), it is not possible to determine $f_T^{K\pi}(q^2)$ at $q^2 = q_{\max}^2$. We take advantage of the nonperiodic boundary conditions to reach values of q^2 quite close to $q^2 = 0$. The momentum dependence of $f_T^{K\pi}(q^2)$ can be nicely fitted either by a pole behavior

$$f_T^{K\pi}(q^2) = f_T^{K\pi}(0)/(1 - s_T^{K\pi} q^2) \qquad (4)$$

or by a quadratic fit in q^2

$$f_T^{K\pi}(q^2) = f_T^{K\pi}(0) \cdot (1 + s_T^{K\pi} q^2 + c_T^{K\pi} q^4). \qquad (5)$$

The good quality of both fits is illustrated in Fig. 1, where the results obtained at two different lattice volumes are also compared. It can clearly be seen that (i) finite size effects are well below the statistical precision of our lattice points; (ii) the results for $f_T^{K\pi}(0)$ and (to a less extent) for the slope $s_T^{K\pi}$, obtained using the pole dominance (1), differ only slightly from those corresponding to the quadratic fit (2) in q^2.

The values obtained for $f_T^{K\pi}(0)$ and $s_T^{K\pi}$ depend on both pion and kaon masses. The dependence on the latter is shown in Fig. 2 for $f_T^{K\pi}(0)$ at $M_\pi \simeq 450$ MeV and it appears to be quite smooth. Thus an interpolation at the physical strange quark mass can be easily performed using quadratic splines. This is obtained by fixing the combination $(2M_K^2 - M_\pi^2)$ at its physical value, obtaining at each pion mass a *reference* kaon mass, M_K^{ref}, given by

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[1]We remind that the tensor form factor $f_T^{K\pi}(q^2)$ computed in this work is $\mathcal{O}(a)$-improved thanks to the use of maximally twisted Wilson quarks [4].
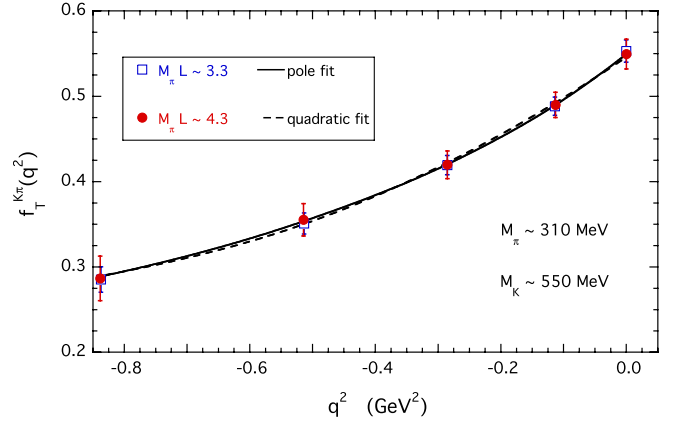

FIG. 1 (color online).   The tensor form factor $f_T^{K\pi}(q^2)$ obtained at M_π 310 MeV and $M_K \simeq 550$ MeV versus q^2 in physical units. The dots and the squares (shifted for better clarity) correspond to the gauge ensembles B_1 and B_7, respectively, which differs only for the lattice size. The solid and dashed lines are the results of the fits based on Eqs. (1) and (2), respectively.
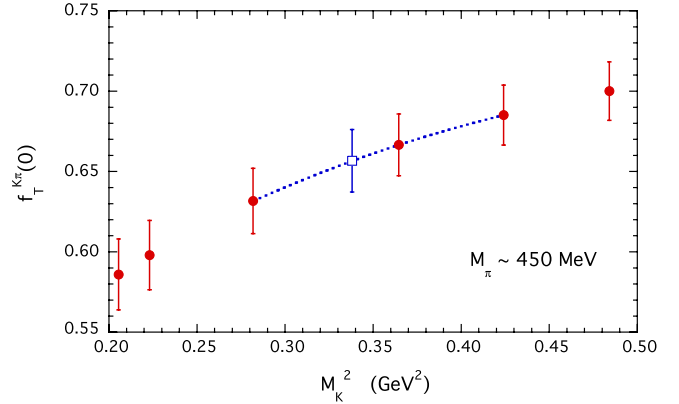

FIG. 2 (color online).   Results for $f_T^{K\pi}(0)$ versus M_K^2 at $M_\pi \simeq$ 450 MeV, obtained assuming the pole dominance (1) for describing the q^2 dependence of our data. The square corresponds to the value of $f_T^{K\pi}(0)$ obtained by local interpolation based on quadratic splines (dotted line) at the reference kaon mass M_K^{ref} 580 MeV from Eq. (3).

$$2[M_K^{\text{ref}}]^2 - M_\pi^2 = 2[M_K^{\text{phys}}]^2 - [M_\pi^{\text{phys}}]^2 \qquad (6)$$

with $M_\pi^{\text{phys}} = 135.0$ MeV and $M_K^{\text{phys}} = 494.4$ MeV.

The results for $f_T^{K\pi}(0)$ and $s_T^{K\pi}$, interpolated at the reference kaon mass M_K^{ref}, are collected in Table I and shown in Figs. 3 and 4, respectively, for the three lattice spacings of our simulations. It can clearly be seen that discretization effects are subdominant.

In Ref. [6] the first lattice calculation of the EM operator matrix element between kaon and pion states was carried out in the quenched approximation and for pion masses above ~ 500 MeV. There the chiral extrapolation was performed adopting a simple linear fit of $f_T^{K\pi}(0)$ in terms

TABLE I. Results for $f_T^{K\pi}(0)$, obtained at the three lattice spacings of our simulations [2], using the pole (1) or quadratic (2) fits and interpolated at the reference kaon mass (3) for each simulated pion mass.

Ensemble $(\beta, L/a)$	M_π (MeV)	M_K^{ref} (MeV)	$f_T^{K\pi}(0)$ (pole)	$f_T^{K\pi}(0)$ (quadratic)
$A_2(3.8, 24)$	422	569	0.631(14)	0.622(14)
$A_3(3.8, 24)$	491	569	0.693(17)	0.674(17)
$A_4(3.8, 24)$	598	644	0.796(18)	0.783(18)
$B_1(3.9, 24)$	319	534	0.544(13)	0.539(12)
$B_2(3.9, 24)$	393	559	0.586(15)	0.578(16)
$B_3(3.9, 24)$	453	581	0.657(19)	0.650(19)
$B_4(3.9, 24)$	490	596	0.681(11)	0.671(11)
$B_5(3.9, 24)$	600	645	0.751(19)	0.736(17)
$B_6(3.9, 32)$	272	522	0.537(16)	0.529(16)
$B_7(3.9, 32)$	311	533	0.542(17)	0.538(16)
$C_1(4.05, 32)$	302	530	0.545(18)	0.539(19)
$C_2(4.05, 32)$	417	568	0.648(17)	0.649(16)
$C_3(4.05, 32)$	486	594	0.702(18)	0.687(17)

of the squared kaon and pion masses, obtaining at the physical point the result $f_T^{K\pi}(0) = 0.78(6)$.

In the degenerate case $M_K = M_\pi$ the chiral expansion of the tensor current has been studied in Ref. [7]. The main finding is that the form factor $f_T^{\pi\pi}(0)$ vanishes like M_π for $M_\pi \to 0$, so that the ratio $f_T^{\pi\pi}(0)/M_\pi$ tends to a nonvanishing value in the chiral limit. The same argument is expected to hold as well in the case of the $K \to \pi$ transition: the form factor $f_T^{K\pi}(0)$ must vanish in the SU(3) chiral limit in such a way that the ratio $f_T^{K\pi}(0)/(M_K + M_\pi)$ remains finite as $(M_K, M_\pi) \to 0$.

Therefore we perform the chiral extrapolation of our lattice data using the Ansatz

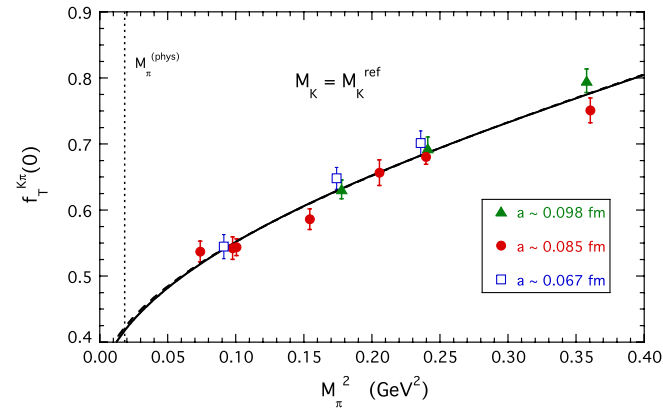FIG. 3 (color online). Results for $f_T^{K\pi}(0)$ versus M_π^2 at $M_K = M_K^{\text{ref}}$ in physical units, assuming the pole dominance (1) for describing the q^2 dependence of our data. The dots, squares and triangles are our results for the three lattice spacings of the ETMC simulations, specified in the inset. The solid and dashed lines correspond to the fit based on Eq. (4) with $B = 0$ and $D = 0$, respectively.
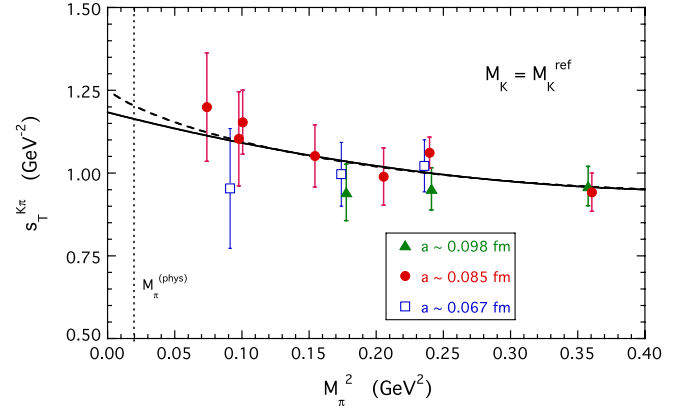

FIG. 4 (color online). The same as in Fig. 3, but for the slope $s_T^{K\pi}$ of the tensor form factor at $q^2 = 0$, obtained assuming the pole dominance (1) for describing the q^2 dependence of our data.

$$f_T^{K\pi}(0) = (M_K^{\text{ref}} + M_\pi)A[1 + BM_\pi^2 \log(M_\pi^2) + CM_\pi^2 + DM_\pi^4],$$
(7)

where A, B, C and D are low-energy constants, depending on the strange quark mass, which are kept as free parameters in our fits. The results of the fit (4) assuming either $B = 0$ (no chiral log) or $D = 0$ are shown in Fig. 3 by the solid and dashed lines, respectively. It can be seen that the effects of the chiral log cannot be appreciated with our data.

At the physical point, after averaging the results obtained with and without the chiral log in the fitting function (4) and assuming for the momentum dependence either the pole (1) or the quadratic (2) functional forms, we get

$$f_T^{K\pi}(0) = 0.417(14_{\text{stat}}) \qquad \text{(ETMC)},$$
(8)

where the error is statistical only.

Had we neglected the factor $(M_K^{\text{ref}} + M_\pi)$ in Eq. (4) the result at the physical point would change only marginally: $f_T^{K\pi}(0) = 0.446(19_{\text{stat}})$. On the contrary, in the case of the quenched data of Ref. [6], which were determined at pion masses above ~ 500 MeV, the inclusion of the factor $(M_K + M_\pi)$ in the chiral extrapolation changes significantly the result at the physical point by many standard deviations, namely, from $f_T^{K\pi}(0) = 0.78(6)$ to

$$f_T^{K\pi}(0) = 0.48(4) \qquad \text{(SPQ}_{\text{cd}}\text{R)}.$$
(9)

These findings indicate that the effect of the quenched approximation does not exceed 15%, provided the correct mass factor is included in the chiral extrapolation.

In the case of the slope $s_T^{K\pi}$ no mass factor should be considered in the chiral extrapolation of the lattice data shown in Fig. 4. After averaging the results obtained assuming for the momentum dependence either the pole (1)

or the quadratic (2) functional forms, we get at the physical point the value

$$s_T^{K\pi} = 1.10(8_{\text{stat}}) \text{ GeV}^{-2} \qquad \text{(ETMC)}, \qquad (10)$$

which is consistent with the quenched result $s_T^{K\pi} = 1.11(5) \text{ GeV}^{-2}$ from Ref. [6].

We now present our estimates of the systematic effects.

Momentum dependence. We have fitted the q^2 dependence of our data using either the pole (1) or the quadratic (2) fits (see Fig. 1). We have found a systematic error due to these different choices equal to 0.002 for $f_T^{K\pi}(0)$ and 0.09 GeV^{-2} for $s_T^{K\pi}$.

Finite size: The comparison between the simulations corresponding to the gauge ensembles B_1 and B_7 (see Table I and Fig. 1), which differs only for the lattice size, indicates a very small volume effect of the order of 0.4% on $f_T^{K\pi}(0)$ and of 4% on $s_T^{K\pi}$ at $M_\pi \sim 310$ MeV. We quote therefore a systematic error due to finite size effects equal to 0.002 for $f_T^{K\pi}(0)$ and 0.05 GeV^{-2} for $s_T^{K\pi}$.

Discretization: The results obtained using only the data at a single lattice spacing $a \sim 0.085$ fm, where the number of simulated pion masses ensures the most reliable chiral extrapolation, are $f_T^{K\pi}(0) = 0.420(10_{\text{stat}})$ and $s_T^{K\pi} = 1.12(6_{\text{stat}})$. We estimate therefore a systematic error due to discretization effects equal to 0.003 for $f_T^{K\pi}(0)$ and 0.02 GeV^{-2} for $s_T^{K\pi}$.

Chiral extrapolation: As shown in Figs. 3 and 4, the difference in the extrapolations to the physical point including or excluding a chiral log in the fitting function (4) is quite small, being equal to 0.002 for $f_T^{K\pi}(0)$ and 0.02 GeV^{-2} for $s_T^{K\pi}$. We take these numbers as our estimates of the uncertainty due to the chiral extrapolation.

Adding all the systematic errors in quadrature, our final results are

$$f_T^{K\pi}(0) = 0.417(14_{\text{stat}})(5_{\text{syst}}) = 0.417(15), \qquad (11)$$

$$s_T^{K\pi} = 1.10(8_{\text{stat}})(11_{\text{syst}}) \text{ GeV}^{-2} = 1.10(14) \text{ GeV}^{-2}, \qquad (12)$$

where the systematic errors do not include any estimate of the effect of quenching the strange and charm quarks.

Bound on the supersymmetric coupling δ_+

We want now to use our new determination (8) for the tensor form factor $f_T^{K\pi}(0)$ as well as the recent experimental bound [13] on the branching ratio $\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ to update the bound on the supersymmetric coupling δ_+, related to the splitting of the off-diagonal entries in the down-type squark mass matrix, already calculated in Ref. [6] using the quenched estimate $f_T^{K\pi}(0) = 0.78(6)$.

The master formula, expressing the *CP* violating part of the contribution of the EM operator to the rare decay $K_L \to \pi^0 e^+ e^-$, can be written as (see Ref. [6])

$$\mathcal{B}(K_L \to \pi^0 e^+ e^-) = 5.3 \cdot 10^{-4} B_T^2 (\text{Im}\delta_+)^2, \qquad (13)$$

where the numerical coefficient $(5.3 \cdot 10^{-4})$ is the appropriate one for the EM operator renormalized in the $\overline{\text{MS}}$ scheme at a renormalization scale of 2 GeV and for a gluino and average squark masses of 500 GeV. In Eq. (10) the quantity B_T is the main hadronic parameter related to the tensor form factor at zero-momentum transfer $f_T^{K\pi}(0)$ by

$$B_T = \frac{2M_K}{M_K + M_\pi} \frac{f_T^{K\pi}(0)}{f_+^{K\pi}(0)} \cdot T, \qquad (14)$$

where $f_+^{K\pi}(0)$ is the vector form factor at zero-momentum transfer, appearing in the $K_{\ell 3}$ semileptonic decay $K^+ \to \pi^0 e^+ \nu_e$, while T is a correction which takes into account the different q^2-dependence of the vector and the tensor form factors (see Ref. [6] for its definition). The quantity T can be related to the slopes $s_+^{K\pi}$ and $s_T^{K\pi}$ of the vector and tensor form factors at $q^2 = 0$ by the following relation

$$T = 1 - 2M_\pi^2(s_+^{K\pi} - s_T^{K\pi})R, \qquad (15)$$

where R is a simple kinematical factor depending on the kaon and pion masses. At the physical point one has $R = 1.855$.

In Ref. [14] we investigated the vector and scalar form factors entering the $K_{\ell 3}$ semileptonic decay, obtaining the results $f_+^{K\pi}(0) = 0.9560(57_{\text{stat}})(62_{\text{syst}})$ and $s_+^{K\pi} = 1.30(13_{\text{stat}})(12_{\text{syst}}) \text{ GeV}^{-2}$. Using the latter values and the results (8) and (9), the correction factor T turns out to be quite close to unity, namely $T = 0.986(10_{\text{stat}})(11_{\text{syst}})$, and for the quantity B_T we get

$$B_T = 0.676(24_{\text{stat}})(12_{\text{syst}}) = 0.676(27). \qquad (16)$$

Therefore, using in Eq. (10) the experimental upper bound $\mathcal{B}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$ (95% C.L.) [13] and our result (13), we obtain the bound

$$|\text{Im}\delta_+| < 1.1 \cdot 10^{-3} \qquad (90\%\text{C.L.}). \qquad (17)$$

III. PION TENSOR CHARGE

Following Ref. [7] the chiral expansion of the pion tensor charge $f_T^{\pi\pi}(0)$ has the form

$$f_T^{\pi\pi}(0) = M_\pi A' \left[1 + \frac{M_\pi^2}{(4\pi f)^2} \log(M_\pi^2) + C'M_\pi^2 + D'M_\pi^4 \right], \qquad (18)$$

where $f \simeq 122$ MeV [9] is the pion decay constant in the SU(2) chiral limit and the presence of the factor M_π is expected to have an important, bending effect on the value extrapolated to the physical point.

Our results for $f_T^{\pi\pi}(0)$, obtained at three values of the lattice spacing and for 270 MeV $\lesssim M_\pi \lesssim$ 600 MeV, are

TABLE II. Results for $f_T^{\pi\pi}(0)$, obtained at the three lattice spacings of our simulations [2], using the pole or quadratic fits for describing the q^2 dependence of our data.

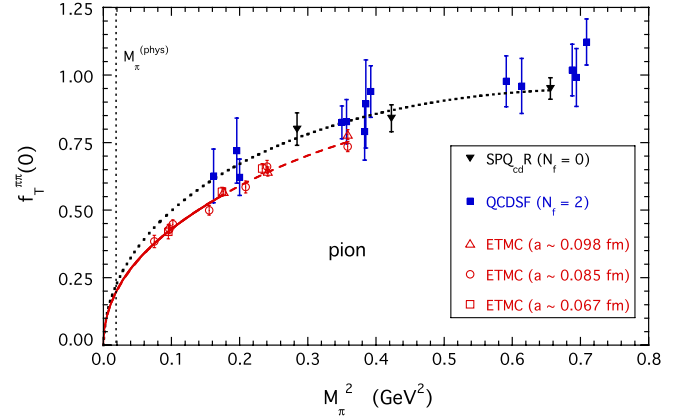| Ensemble $(\beta, L/a)$	M_π (MeV)	$f_T^{\pi\pi}(0)$ (pole)	$f_T^{\pi\pi}(0)$ (quadratic)
$A_2(3.8, 24)$	422	0.568(15)	0.555(13)
$A_3(3.8, 24)$	491	0.643(17)	0.623(16)
$A_4(3.8, 24)$	598	0.780(18)	0.767(17)
$B_1(3.9, 24)$	319	0.448(16)	0.432(14)
$B_2(3.9, 24)$	393	0.500(18)	0.487(18)
$B_3(3.9, 24)$	453	0.586(22)	0.580(20)
$B_4(3.9, 24)$	490	0.661(24)	0.641(22)
$B_5(3.9, 24)$	600	0.736(19)	0.720(18)
$B_6(3.9, 32)$	272	0.384(23)	0.371(20)
$B_7(3.9, 32)$	311	0.431(18)	0.426(15)
$C_1(4.05, 32)$	302	0.420(26)	0.404(23)
$C_2(4.05, 32)$	417	0.569(16)	0.562(15)
$C_3(4.05, 32)$	486	0.654(19)	0.640(17)

FIG. 5 (color online). Results for the pion tensor charge $f_T^{\pi\pi}(0)$ versus M_π^2 in physical units from our simulations at the three different values of the lattice spacing (open markers), obtained assuming the pole dominance (1) for describing the q^2 dependence of our data, and from Refs. [6] (full triangles) and [8] (full squares). The dashed line is the result of the fit (1) with $D' \neq 0$ applied to all ETMC points, while the solid line corresponds to the use of the fitting function (1) with $D' = 0$ applied to the ETMC data with $M_\pi \lesssim 420$ MeV. The dotted line represents the fit described in the text and applied to the quenched data of Ref. [6].

shown in Fig. 5 and compared with the $N_f = 2$ results of Ref. [8], having $M_\pi \gtrsim 440$ MeV, and with the quenched calculations of Ref. [6], ranging from $M_\pi \sim 530$ MeV up to $M_\pi \sim 800$ MeV. It can be seen that our results have a much better statistical precision and cover much lighter pion masses, where the bending effect due to the overall factor M_π is clearly visible.

Applying Eq. (1) with $D' = 0$ to our lattice points with $M_\pi \lesssim 420$ MeV (see the solid line in Fig. 5), we get at the physical point $f_T^{\pi\pi}(0) = 0.195(8_{\text{stat}})$.

As for the estimates of the systematic effects, we follow the same procedures described in the previous Section for the $K \to \pi$ transition, obtaining an effect equal to 0.004 from fitting the momentum dependence with either a pole or a quadratic forms, 0.003 from the finite size,[2] 0.001 from discretization and 0.003 from the chiral extrapolation. Adding all the systematic errors in quadrature, our final result is

$$f_T^{\pi\pi}(0) = 0.195(8_{\text{stat}})(6_{\text{syst}}) = 0.195(10) \qquad \text{(ETMC)} \tag{19}$$

which improves the QCDSF result [8]

$$f_T^{\pi\pi}(0) = 0.216(34) \qquad \text{(QCDSF).} \tag{20}$$

Finally, we also apply a simple fit of the form $f_T^{\pi\pi}(0) = M_\pi A'[1 + C'M_\pi^2]$ to the three quenched data of Ref. [6]

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[2]The comparison between the simulations corresponding o the gauge ensembles B_1 and B_7, which differs only for the lattice size, indicates a volume effect of the order of $2 \div 4\%$ on $f_T^{\pi\pi}(0)$ [see Table II], which is close to the statistical error. Such an effect is however mostly due to the volume effect on the pion mass. Indeed, the finite size effect on the ratio $f_T^{\pi\pi}(0)/M_\pi$ reduces to $\simeq 1.5\%$, which we therefore take as our estimate of the corresponding systematic error on our final result.

(see the dotted line in Fig. 5), obtaining at the physical point the result

$$f_T^{\pi\pi}(0) = 0.221(21_{\text{stat}}) \qquad \text{(SPQ}_{\text{cd}}\text{R),} \tag{21}$$

which clearly shows that quenching effects on the pion tensor charge are subdominant with respect to the present precision.

Before closing we want to comment briefly on the $SU(3)$ breaking effect on the tensor form factor at zero-momentum transfer. Our final values obtained for $f_T^{K\pi}(0)$ and $f_T^{\pi\pi}(0)$ [i.e., Eqs. (8) and (2)] differ by a factor of ≈ 2. This huge difference is however mostly due to the factor $(M_K + M_\pi)$, conventionally inserted in Eq. (1). By introducing the quantities

$$F_T^{K\pi}(0) \equiv f_T^{K\pi}(0)/(M_K + M_\pi) \tag{22}$$

$$F_T^{\pi\pi}(0) \equiv f_T^{\pi\pi}(0)/2M_\pi \tag{23}$$

one gets from Eqs. (8) and (2) the values

$$F_T^{K\pi}(0) = 0.663(22_{\text{stat}})(8_{\text{syst}}) \text{ GeV}^{-1} = 0.663(24) \text{ GeV}^{-1} \tag{24}$$

$$\begin{aligned} F_T^{\pi\pi}(0) &\equiv 0.722(30_{\text{stat}})(22_{\text{syst}}) \text{ GeV}^{-1} \\ &= 0.722(37) \text{ GeV}^{-1} \end{aligned} \tag{25}$$

which yield a ratio $F_T^{K\pi}(0)/F_T^{\pi\pi}(0)$ equal to 0.92(6). This indicates that the $SU(3)$ breaking effect on the matrix element of the EM operator $\bar{s}F_{\mu\nu}\sigma^{\mu\nu}d$ at zero-momentum transfer is equal to $-8(6)\%$. This value is of the same sign and order of magnitude of the $SU(3)$ breaking effect on the corresponding matrix element of the vector current, namely, on the vector form factor at zero-momentum transfer $f_+^{K\pi}(0)$. Indeed, taking into account that $f_+^{\pi\pi}(0) = 1$ because of charge conservation and $f_+^{K\pi}(0) = 0.9560(84)$ from Ref. [14], the $SU(3)$ breaking effect on the matrix element of the vector current at zero-momentum transfer is equal to $-4(1)\%$.

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