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Thermal Abelian monopoles as self-dual dyons

V. G. Bornyakov

High Energy Physics Institute, 142280 Protvino, Russia and Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia

V. V. Braguta

High Energy Physics Institute, 142280 Protvino, Russia

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The properties of the thermal Abelian monopoles are studied in the deconfinement phase of the $SU(2)$ gluodynamics. To remove effects of Gribov copies, the simulated annealing algorithm is applied to fix the maximally Abelian gauge. To study the monopole profile, we complete the first computations of excess of the nonabelian action density as a function of the distance from the center of the thermal Abelian monopole. We have found that, starting from distances of around two lattice spacings, the chromoelectric and chromomagnetic action densities created by the monopole are equal to each other, from which we conclude that the monopole is a dyon. Furthermore, we find that the chromoelectric and chromomagnetic fields decrease exponentially with increasing distance. These findings were confirmed for different temperatures in the range $T/T_c \in (1.5, 4.8)$.

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One of the hypotheses which has been put forward in the recent past is that the quark-gluon plasma properties might be dominated by a magnetic component [1–3]. The monopoles or center vortices might be responsible for unexpected properties of the hadron matter at $T > T_c$: on one hand, it is well known from lattice results that the equation of state is close to that of an ideal gas; on the other hand, the very low viscosity to entropy ratio tells that it is an ideal liquid.

In Ref. [2], such magnetic component has been related to thermal Abelian monopoles evaporating from the magnetic condensate that is believed to induce color confinement at low temperatures. Moreover, it has been proposed to detect such thermal monopoles in finite temperature lattice QCD simulations by identifying them with monopole currents having a nontrivial wrapping in the Euclidean temporal direction [2,4,5].

The way one can study the monopoles' properties on the lattice is via an Abelian projection after fixing the maximally Abelian gauge (MAG) [6–8]. This gauge, as well as the properties of the monopole clusters, has been investigated in numerous papers both at zero and nonzero temperatures (see for extensive list of references, e.g., [9]). The evidence was found that the nonperturbative properties of the gluodynamics—such as confinement, deconfining transition, chiral symmetry breaking, etc.—are closely related to the Abelian monopoles defined in the MAG. This was called a monopole dominance.

First numerical investigations of the wrapping monopole trajectories were performed long ago in Refs. [4,5]. A more systematic study of the thermal monopoles in $SU(2)$ Yang-Mills theory at high temperature has been performed in Refs. [10–12]. In particular, it was found in [10] that the density of monopoles is independent of the lattice spacing, as it should be for a physical quantity.

In paper [11], very interesting properties of the thermal Abelian monopoles were found. The authors measured the excess of chromoelectric and chromomagnetic action density on the surface of the lattice (hyper-)cubes with monopoles inside. The dependence of the excess on the distance from the monopole center was determined through the variation of the lattice spacing a (similar investigation at $T = 0$ was made in [13]). As a result, it was found that with good accuracy the chromomagnetic and chromoelectric action densities created by the monopole have the following behavior: $H^2(r), E^2(r) = a_{H,E}/r^4$. The coefficients a_H and a_E turned out to be equal to each other with a very good accuracy; from what the authors concluded, that monopole is a dyon. It is worth noting that there were other works in the past where dyonic properties of the monopoles were observed [14–16].

The studies of dyons in the deconfinement phase of the Yang-Mills theory have been undertaken both on the lattice and in the continuum. A semiclassical approach to confinement based on dyons developed in a series of papers [17,18] provides an appealing explanation of all main features associated with confinement and confinement-deconfinement transition. In the lattice studies of $SU(2)$ theory, it has been found [16,19,20] that calorons, which appear below transition, are dissociated into dyon pairs above transition. Another observation made for the deconfinement phase was that these pairs consist of light and heavy dyons, and light dyons are much more frequent and, thus, should be responsible for area law of the spatial Wilson loops [21].

The drawback of the study undertaken in [11] is that all results were obtained at the ultraviolet cutoff scale and were thus subjected to both lattice discretization errors and ultraviolet divergences. In view of the importance of the findings of [11], in this paper we are going to study the chromoelectric and chromomagnetic fields created by the monopole and check whether the observed behavior is correct or is just a lattice artifact, i.e., artifact of the ultraviolet cutoff. To accomplish this check, we will measure the chromoelectric and chromomagnetic fields at various distances from the monopole center.

In this paper, we study the $SU(2)$ lattice gauge theory with the standard Wilson action

$$S = \beta\sum_x \sum_{\mu>\nu}\left[1 - \frac{1}{2}\,\mathrm{Tr}(U_{x\mu}U_{x+\mu;\nu}U^\dagger_{x+\nu;\mu}U^\dagger_{x\nu})\right],$$

where $\beta = 4/g_0^2$ and g_0 is a bare coupling constant. The link variables $U_{x\mu} \in SU(2)$ transform under gauge transformations g_x as follows:

$$U_{x\mu} \mapsto^g U^g_{x\mu} = g^\dagger_x U_{x\mu} g_{x+\mu}; \qquad g_x \in SU(2). \tag{1}$$

Our calculations were performed on the asymmetric lattices with lattice volume $V = L_t L_s^3$, where $L_{t,s}$ is the number of sites in the time (space) direction. The temperature T is given by

$$T = \frac{1}{aL_t}, \tag{2}$$

where a is the lattice spacing.

The MAG is fixed by finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V}\sum_{x\mu}\frac{1}{2}\,\mathrm{Tr}(U^g_{x\mu}\sigma_3 U^{g\dagger}_{x\mu}\sigma_3), \tag{3}$$

with respect to gauge transformations g_x. We apply the simulated annealing algorithm, which proved to be very efficient for this gauge [22], as well as for other gauges such as center gauges [23] and Landau gauge [24]. To further decrease the Gribov copy effects, we generated 10 Gribov copies for every configuration. For each copy gauge fixing procedure started from a randomly selected gauge copy of the original Monte Carlo configuration.

In Table I, we provide the information about the gauge field ensembles used in our study.

The chromomagnetic action density at a site x is defined as

$$S_M(x) = \frac{1}{12}\sum_{P_s \ni x}\left(1 - \frac{1}{2}\,\mathrm{Tr}U_{P_s}\right). \tag{4}$$

The sum is taken over all spatial plaquettes P_s that contain the lattice site x. In the continuum limit, this expression is proportional to $\sim\mathrm{Tr}(G_{23}^2 + G_{13}^2 + G_{12}^2) = \mathrm{Tr}(H_1^2 + H_2^2 + H_3^2)$. So, this expression can be taken as a measure of the chromomagnetic action $\mathrm{Tr}(\mathbf{H}^2)$ at the site x.

Analogously, for the chromoelectric action density at a site x, we take

$$S_E(x) = \frac{1}{12}\sum_{P_t \ni x}\left(1 - \frac{1}{2}\,\mathrm{Tr}U_{P_t}\right). \tag{5}$$

Here, the sum is taken over all timelike plaquettes P_t that contain the site x. In the continuum limit, this expression is proportional to $\sim\mathrm{Tr}(G_{01}^2 + G_{02}^2 + G_{03}^2) = \mathrm{Tr}(E_1^2 + E_2^2 + E_3^2)$ and, thus, it can be taken as a measure of the chromoelectric action $\mathrm{Tr}(\mathbf{E}^2)$. Note that our definitions for $S_{M,E}(x)$ differ from those used in Ref. [11]. Although definitions of [11] were natural for the surface of a cube with a monopole, our definitions are more suitable for measurements at some distance from such cube.

Since we are studying the fields created by a monopole, we should subtract the vacuum fluctuations of the chromomagnetic and chromoelectric actions from the Eqs. (4) and (5). We define the excess of the action density as

$$\langle\delta S_{M,E}(d)\rangle = \langle\overline{S_{M,E}(x)}\rangle - \langle S_{M,E}\rangle, \tag{6}$$

where $\langle\ldots\rangle$ means ensemble average, d is the distance from a monopole, and bar means averaging over all wrapped monopoles and all sites x at the distance d from monopole centers.

The monopole currents and their wrapping numbers are defined in a standard way (see, e.g., [10]). Moving along wrapped monopole clusters on a dual lattice, we detect all three-dimensional cubes in all time slices on the original lattice that contain monopoles corresponding to the j_4 component of the magnetic current. Having detected all such three-dimensional cubes, we do the measurements of the chromomagnetic and chromoelectric action densities $\langle\delta S_{M,E}(d)\rangle$ at various distances from a given monopole, and then we take the average over all thermal monopoles found on the lattice.

Now, let us consider a three-dimensional cube with the monopole belonging to a wrapped cluster. Below, it will be assumed that the monopole is located in the center of this cube, and we take the center as a coordinate origin. Actually, one cannot assert that the monopole is exactly located at the center of the cube. However, since we take an average over all monopoles, this approximation can be considered a good one. We have measured the action densities $\langle\delta S_{M,E}(d)\rangle$ at various distances d from the centers of the cubes with a monopole. The distance was defined as

TABLE I. Values of β, lattice sizes, temperatures, number of measurements, and number of gauge copies used throughout this paper. To fix the scale, we take $\sqrt{\sigma} = 440$ MeV.

β	a [fm]	L_t	L_s	T/T_c	N_{meas}
2.43	0.108	4	32	1.5	1000
2.5115	0.081	4	28	2.0	400
2.635	0.054	4	36	3.0	500
2.80	0.034	4	48	4.8	400

a length of the vector $\vec{d} = \{n_1 \pm 1/2, n_2 \pm 1/2, n_3 \pm 1/2\}$ from the coordinate origin to lattice site x under consideration. We present results of measurements for $\vec{d} = \frac{1}{2}\{m, m, m\}$, $m = 1$, 3, and $\vec{d} = \frac{1}{2}\{1, 1, 3\}$, $\frac{1}{2}\{1, 3, 3\}$, $\frac{1}{2}\{1, 1, 5\}$, $\frac{1}{2}\{1, 3, 5\}$. For $T/T_c = 1.5$, additional results for $\vec{d} = \frac{1}{2}\{1, 5, 5\}$, $\frac{1}{2}\{1, 1, 7\}$ are presented. For longer distances, the statistical errors were too large.

In Figs. 1 and 2, results are shown for the $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ as functions of the dimensionless distance $rT = d/4$. From these figures, one clearly sees that at least at large distances both $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ decrease with distance in agreement with exponential falloff $\sim \exp(-2M_{m,e} r)$. The dependence $1/r^4$ found in [11] is ruled out. We do not have enough data points to determine the preexponential function by fitting. Respectively, it is rather difficult to find the parameters $M_{m,e}$ with a good accuracy. In this paper, we just make rough estimation of these parameters fitting the last 3 data points for $\langle \delta S_M \rangle$ to the exponential falloff with constant prefactor. We get the following results: $M_m/T = 3.5(2), 4.0(4), 4.3(2), 3.7(8)$ for the temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$, respectively.

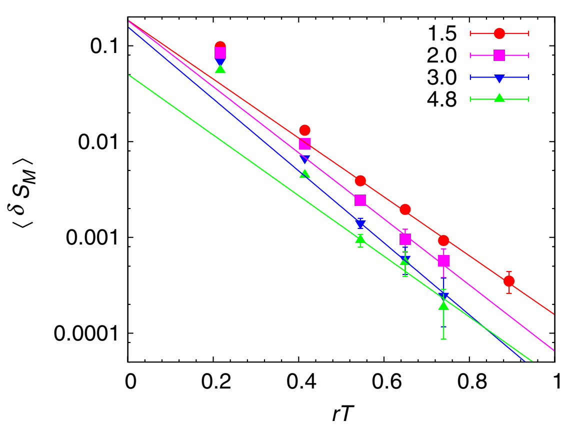

FIG. 1 (color online). $\langle \delta S_M \rangle$ defined in Eq. (6) as function of the dimensionless distance rT from the monopole center at the temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$.

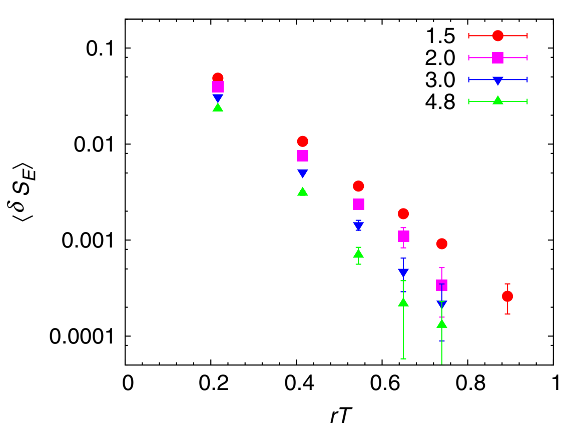

FIG. 2 (color online). Same as in Fig. 1 for $\langle \delta S_E \rangle$.

Looking at Figs. 1 and 2, one can see that the data lie on the smooth curves. This means that the data obey rotational invariance, since the data at different distances were measured in different directions. Moreover, vectors $\vec{d} = \frac{1}{2}\{3, 3, 3\}$ and $\vec{d} = \frac{1}{2}\{1, 1, 5\}$ have equal length, and one can check the rotational invariance directly. Indeed, we find for these two vectors consistent results with deviations within the 2σ interval. In all figures, we show averaged data for these vectors $\vec{d}$.

At large enough distances (beginning from the distance $d = 2.18$), the monopole chromomagnetic and chromoelectric action density seem to be equal to each other. To demonstrate this important property, we plot the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle = H^2/E^2$ in Fig. 3. From this figure, we see that within the error bars at distances $rT \gtrsim 0.5$ the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle$ is compatible with 1. From this observation, one can draw a conclusion: at least at large enough distances, $H^2(r) = E^2(r)$. This implies that *monopoles carry both chromoelectric and chromomagnetic charges* and that *they are equal*. So, *monopoles are self-dual dyons.* These statements are the main results of this paper.

To understand the reason for lack of self-duality at small distances, let us look at Fig. 3. For all temperatures, we observe similar behavior. At the distance $d = 0.87$ in lattice units, the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle \sim 2$ for all temperatures. At the distance $d = 1.66$, the ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle \sim 1.3$. At larger distances, the ratio is compatible with unity. We believe that deviation of the ratio from one at small distances can be explained by the discretization effects. Notice that our definition of the chromomagnetic and chromoelectric action densities at site x is nonlocal involving all plaquettes in respective planes that own site x. This nonlocality is different for two action densities: it is purely spatial for the chromomagnetic action density and is both spatial and temporal for the chromoelectric one. Thus, at

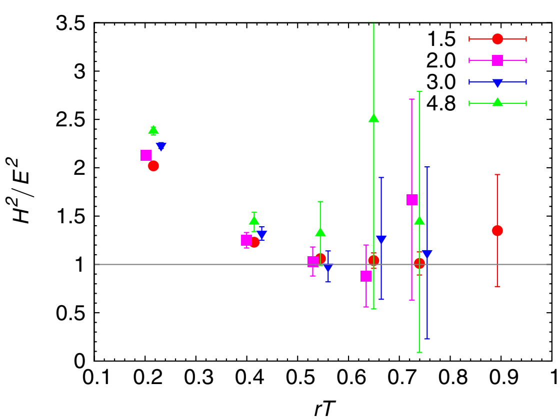

FIG. 3 (color online). The ratio $\langle \delta S_M \rangle / \langle \delta S_E \rangle = H^2/E^2$ as function of the distance rT from the monopole center at temperatures $T/T_c = 1.5, 2.0, 3.0, 4.8$. The data sets at $T/T_c = 2.0$ and 3.0 are shifted along horizontal axis to improve readability of the figure.

distances of order of one lattice spacing, we evidently measure the fields taken at different points and different distances. These arguments should be checked by computations with smaller lattice spacing, i.e., with $L_t > 4$. This will be done in a forthcoming paper. In that paper, we will also present our data on the density and interactions of the thermal Abelian monopoles.

It is clear that results and conclusions of Ref. [11], where $\langle \delta S_M \rangle$ and $\langle \delta S_E \rangle$ were measured at the nearest possible distance to the monopole center, are subjected to the same discretization effects as discussed above for our data at small distances. Then, the dependence $1/r^4$ found in Ref. [11] seems to be an ultraviolet divergence effect.

Thus, we established that the Abelian thermal monopoles carry both chromoelectric and chromomagnetic charges and that they are equal. So, monopoles are self-dual dyons. Furthermore, respective action densities are screened. We believe that these results are important for understanding QCD in the quark-gluon plasma phase since many recent theoretical models of this phase include monopoles as an important ingredient. One example is the model of Refs. [1,3] based on competition between magnetic and electric quasiparticles. The model based on dyons [17] has been already mentioned above. The lattice study of the caloron model has been undertaken recently in [25]. We expect that the thermal dyons, the properties of which have been studied in this paper, are related to dyons studied on the lattice by different methods [16,19,20].

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