Study of the pure annihilation $B_c \rightarrow A_2 A_3$ decays

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In this work, we calculate the *CP*-averaged branching ratios and the polarization fractions of the charmless hadronic $B_c \rightarrow A_2 A_3$ decays within the framework of perturbative QCD (pQCD) approach, where A is either a light $^3P_1$ or $^1P_1$ axial-vector meson. These 32 decay modes can occur through the annihilation topology only. Based on the perturbative calculations and phenomenological analysis, we find the following results: (a) the branching ratios of the considered 32 $B_c \rightarrow A_2 A_3$ decays are in the range of 10^{-5} to 10^{-8}; (b) $B_c \rightarrow a_1 b_1$, $\bar{K}_1^0 K_1^+$ and some other decays have sizable branching ratios and can be measured at the LHC experiments; (c) the branching ratios of $B_c \rightarrow A_2(^1P_1)A_3(^1P_1)$ decays are generally much larger than those of $B_c \rightarrow A_2(^3P_1)A_3(^3P_1)$ decays with a factor around $(10 \sim 100)$; (d) the branching ratios of $B_c \rightarrow \bar{K}_1^0 K_1^+$ decays are sensitive to the value of θ_K, which will be tested by the running LHC and forthcoming SuperB experiments; (e) the large longitudinal polarization contributions govern most considered decays and play the dominant role.

I. INTRODUCTION

From the point of structure, the B_c meson is a ground state of $\bar{b}c$ system: which is likely an intermediate state of the $\bar{c}c$ and $\bar{b}b$-quarkonia, but should be very different from both of them since B_c meson carries flavor $B = -C = 1$. When compared with the heavy-light B_q meson with $q = (u, d, s)$, on the other hand, the decays of the B_c meson must be rather different from those $B_u/B_d/B_s$ mesons since here both b and c can decay while the other serves as a spectator, or annihilating into pairs of leptons or light mesons (such as $K^+ \pi^0$, etc). Physicists therefore believe that the B_c physics must be very rich if the statistics reaches high level [1–3]. In recent years, many theoretical studies on the production and decays of B_c meson have been done [2,3], based on, for example, the Operator Production Expansion [4], NRQCD [5], QCD Sum Rules [6], SU(3) flavor symmetry [7], ISGW II model [8], QCD factorization approach [9], and the perturbative QCD (pQCD) factorization approach [10–13].

On the experimental side, it is impossible to find a pair of $B_c^+ B_c^-$ in the B-factory experiments (*BaBar* and Belle) since its mass is well above 6 GeV. Although the first observation of approximately 20 B_c events in the $B_c \rightarrow J/\Psi l \nu$ decay mode was reported in 1998 by the CDF collaboration [1], it was not until 2008 that two confirming observations in excess of 5σ significance were made by CDF and D0 collaboration [14] at Tevatron via two decay channels: the hadronic $B_c \rightarrow J/\Psi \pi^+$ decay and the semi-leptonic $B_c \rightarrow J/\Psi l^+ \nu_l$ decay.

At the LHC experiment, specifically the LHCb, one could expect around 5×10^{10} B_c events per year [2,3]. And therefore, besides the charmed decays with large branching ratios, many rare B_c decays with a decay rate at the level of 10^{-5} to 10^{-6} can also be measured with a good precision at the LHC experiments [7]. This means that many $B_c \rightarrow h_1 h_2$ decays (h_i are the light scalar (S), pseudoscalar (P), vector (V), axial-vector (A) and tensor (T) mesons, made of light u, d, s quarks) can be observed experimentally. In the SM, such decays can only occur via the annihilation type diagrams. The studies on these pure annihilation B_c decays may open a new window to understand the annihilation mechanism in B physics, an important but very difficult problem to be resolved.

In 2004, by employing the low-energy effective Hamiltonian [15] and the pQCD approach [16–18], we studied the pure annihilation decays $B_s \rightarrow \pi\pi$ and presented the pQCD prediction for its branching ratio [19]: $Br(B_s \rightarrow \pi^+ \pi^-) = (4.2 \pm 0.6) \times 10^{-7}$, which was confirmed by a later theoretical calculation [20] and by a very recent CDF measurement with a significance of 3.7σ [21]: $Br(B_s \rightarrow \pi^+ \pi^-) = (5.7 \pm 1.5 \pm 1.0) \times 10^{-7}$. This good agreement encouraged us to extend our work to the case of B_c decays. Although the charm quark c is massive (relative to the known light quarks u, d, and s), the B_c meson has been treated as a heavy-light structure in this work because of the ratio $m_c/m_{B_c} \sim 0.2$, which means that the large part of the energy is carried by the much heavier b quark in a B_c meson. With this assumption, we also employ the k_T factorization theorem to the b decay in

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B_c meson, in a similar way as for the decays of B_u and B_d mesons.

During past two years, based on the pQCD factorization approach, we have made a systematic study on the two-body charmless hadronic decays of $B_c \rightarrow PP$, PV, VV [10], $B_c \rightarrow SP$, SV [11] and $B_c \rightarrow AP$, AV [12,13]. For all the considered pure annihilation B_c decay channels, we calculated their CP-averaged branching ratios and longitudinal polarization fractions, and found some interesting results to be tested by the LHC experiments.

In this paper, we extend our previous investigation further to the charmless hadronic $B_c \rightarrow AA$ decays. The axial-vector mesons involved are the following:

$$a_1(1260), \quad b_1(1235), \quad K_1(1270), \quad K_1(1400), \\ f_1(1285), \quad f_1(1420), \quad h_1(1170), \quad h_1(1380). \quad (1)$$

All the 32 decay modes are the pure annihilation decay processes in the SM.

The internal structure of the axial-vector mesons has been one of the hot topics in recent years [22–24]. Although many efforts on both theoretical and experimental aspects have been made [25–30], we currently still know little about the nature of the axial-vector mesons. Our study will be helpful to understand the structure of these mesons.

As one of the popular factorization tools based on the QCD dynamics, the pQCD approach can be used to analytically calculate the annihilation type diagrams. Besides the good agreement between the pQCD prediction and the newest CDF measurement for $Br(B_s \rightarrow \pi^+ \pi^-)$, the pQCD prediction of $Br(B^0 \rightarrow D_s^- K^+) \approx (4.6 \pm 1.0) \times 10^{-5}$ for the pure annihilation B^0 decay as presented in Ref. [31] also be consistent well with the data [30]. We therefore believed that the pQCD factorization approach is a powerful and consistent framework to perform the calculation for the annihilation type $B_{u,d,s}$ decays, and extend our work to the cases of B_c decays.

The paper is organized as follows. In Sec. II, we give a brief review about the axial-vector meson spectroscopy, and the theoretical framework of the pQCD factorization approach. We perform the perturbative calculations for considered decay channels in Sec. III. The analytic expressions of the decay amplitudes for all 32 $B_c \rightarrow AA$ decays are also collected in this section. The numerical results and phenomenological analysis are given in Sec. IV. The main conclusions and a short summary are presented in the last section.

II. THEORETICAL FRAMEWORK

A. Axial-vector mesons and mixings

In the quark model, there exist two distinct types of light parity-even p-wave axial-vector mesons, namely, $^3P_1$ ($J_{PC} = 1^{++}$) and $^1P_1$ ($J_{PC} = 1^{+-}$) states:

$$^3P_1 \text{nonet: } a_1(1260), \quad f_1(1285), \quad f_1(1420) \text{ and } K_{1A}; \\ ^1P_1 \text{nonet: } b_1(1235), \quad h_1(1170), \quad h_1(1380) \text{ and } K_{1B}. \quad (2)$$

In the SU(3) flavor limit, the above mesons can not mix with each other. Because the s quark is heavier than u, d quarks, the physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ are not purely $^3P_1$ or $^1P_1$ states, but believed to be mixtures of K_{1A} and K_{1B}.[1] Analogous to η and η' system, the flavor-singlet and flavor-octet axial-vector meson can also mix with each other.

The physical states $K_1(1270)$ and $K_1(1400)$ can be written as the mixtures of the K_{1A} and K_{1B} states:

$$K_1(1270) = \sin\theta_K K_{1A} + \cos\theta_K K_{1B}, \\ K_1(1400) = \cos\theta_K K_{1A} - \sin\theta_K K_{1B}, \quad (3)$$

where θ_K is the mixing angle to be determined by the experiments. But we currently have little knowledge about θ_K due to the absence of the relevant data, although it has been studied for a long time [22–24]. In this paper, for simplicity, we will adopt two reference values as those used in Ref. [24]: $\theta_K = \pm 45°$.

Analogous to the $\eta - \eta'$ mixing in the pseudoscalar sector, the $h_1(1170)$ and $h_1(1380)$ ($^1P_1$ states) system can be mixed in terms of the pure singlet h_1 and octet h_8,

$$h_1(1170) = \sin\theta_1 h_8 + \cos\theta_1 h_1, \\ h_1(1380) = \cos\theta_1 h_8 - \sin\theta_1 h_1. \quad (4)$$

Likewise, $f_1(1285)$ and $f_1(1420)$ (the $^3P_1$ states) will mix in the same way:

$$f_1(1285) = \sin\theta_3 f_8 + \cos\theta_3 f_1, \\ f_1(1420) = \cos\theta_3 f_8 - \sin\theta_3 f_1. \quad (5)$$

where the flavor contents of $h_{1,8}$ and $f_{1,8}$ can be written as

$$h_1 = f_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s), \\ h_8 = f_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s). \quad (6)$$

The values of the mixing angles $\theta_{1,3}$ can be chosen as [24]:

$$\theta_1 = 10° \quad \text{or} \quad 45°; \qquad \theta_3 = 38° \quad \text{or} \quad 50°. \quad (7)$$

---

[1]For the sake of simplicity, we will adopt the forms a_1, b_1, K', K'', f', f'', h' and h'' to denote the axial-vector mesons $a_1(1260)$, $b_1(1235)$, $K_1(1270)$, $K_1(1400)$, $f_1(1285)$, $f_1(1420)$, $h_1(1170)$ and $h_1(1380)$ correspondingly in the following sections, unless otherwise stated. We will also use K_1, f'_1 and h'_1 to denote $K_1(1270)$ and $K_1(1400)$, $f_1(1285)$ and $f_1(1420)$, and $h_1(1170)$ and $h_1(1380)$ for convenience unless otherwise stated explicitly.

B. Formalism

In the pQCD factorization approach, the four annihilation Feynman diagrams for $B_c \to A_2 A_3$ decays are shown in Fig. 1, where (a) and (b) are factorizable diagrams, while (c) and (d) are the nonfactorizable ones. The initial $\bar{b}$ and c quarks annihilate into u and $\bar{d}/\bar{s}$, and then form a pair of light mesons by hadronizing with another pair of $q\bar{q}$ ($q = (u, d, s)$) produced perturbatively through the one-gluon exchange mechanism. Besides the short-distance contributions based on one-gluon-exchange, the $q\bar{q}$ pair can also be produced through strong interaction in nonperturbative regime (final state interaction(FSI), for example).

For the considered $B_c \to A_2 A_3$ decays, the key point is to calculate the corresponding matrix elements:

$$\mathcal{M} \propto \langle A_2 A_3 | \mathcal{H}_{\text{eff}} | B_c \rangle \tag{8}$$

where the weak effective Hamiltonian $\mathcal{H}_{\text{eff}}$ is given by [15]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{cb}^* V_{uD} (C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu))], \tag{9}$$

with the current-current operators $O_{1,2}$,

$$\begin{aligned} O_1 &= \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha \bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\ O_2 &= \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha, \end{aligned} \tag{10}$$

where V_{cb}, V_{uD} ($D = d, s$) are the CKM matrix elements, $C_i(\mu)$ are Wilson coefficients at the renormalization scale μ.

Although the dominance of the one-gluon exchange diagram seems favored by the data of $B_s^0 \to \pi^+ \pi^-$ and $B^0 \to D_s^- K^+$ decays, according to the good agreement between our calculations based on the pQCD approach [19,31] and the data [21,30], we currently still do not know whether the short-distance or the nonperturbative contribution dominate for B_c annihilation decays. We here first assume that the short-distance contribution is dominant, and then calculate the matrix element in Eq. (8) by employing the pQCD approach, provide the pQCD predictions for the branching ratios and longitudinal polarization fractions, and finally wait for the test by the LHC experiments.

We work in the frame with the B_c meson at rest, i.e., with the B_c meson momentum $P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, \mathbf{0}_T)$ in the light-cone coordinates. We assume that the A_2 (A_3) meson

moves in the plus (minus) z direction carrying the momentum P_2 (P_3) and the polarization vector ϵ_2 (ϵ_3). Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \qquad P_3 = \frac{m_{B_c}}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T), \tag{11}$$

where $r_2 = m_{A_2}/m_B$, and $r_3 = m_{A_3}/m_B$. The longitudinal polarization vectors, ϵ_2^L and ϵ_3^L, can be defined as

$$\begin{aligned} \epsilon_2^L &= \frac{m_{B_c}}{\sqrt{2} m_{A_2}}(1 - r_3^2, -r_2^2, \mathbf{0}_T), \\ \epsilon_3^L &= \frac{m_{B_c}}{\sqrt{2} m_{A_3}}(-r_3^2, 1 - r_2^2, \mathbf{0}_T). \end{aligned} \tag{12}$$

The transverse ones are parameterized as $\epsilon_2^T = (0, 0, 1_T)$, and $\epsilon_3^T = (0, 0, 1_T)$. Putting the (light-) quark momenta in B_c, A_2 and A_3 mesons as k_1, k_2, and k_3, respectively, we can choose

$$\begin{aligned} k_1 &= (x_1 P_1^+, 0, \mathbf{k}_{1T}), \\ k_2 &= (x_2 P_2^+, 0, \mathbf{k}_{2T}), \\ k_3 &= (0, x_3 P_3^-, \mathbf{k}_{3T}). \end{aligned} \tag{13}$$

Then the decay amplitude can be written conceptually as the following form,

$$\begin{aligned} \mathcal{M}(B_c \to A_2 A_3) &= \langle A_2 A_3 | \mathcal{H}_{\text{eff}} | B_c \rangle \\ &\sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \\ &\times \text{Tr}[C(t) \Phi_{B_c}(x_1, b_1) \Phi_{A_2}(x_2, b_2) \\ &\times \Phi_{A_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}] \end{aligned} \tag{14}$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [32], and they lead to $S_t(x_i)$ which smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [33]. Thus it makes the perturbative calculation of the hard part H applicable at intermediate scale, i.e., m_{B_c} scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the
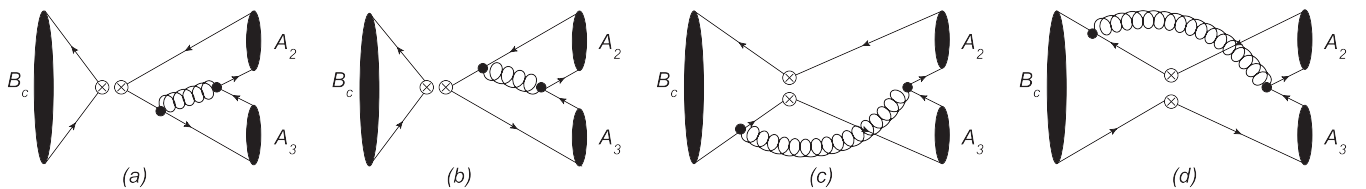

FIG. 1. The annihilation Feynman diagrams for $B_c \to A_2 A_3$ decays. (a) and (b) are factorizable diagrams; while (c) and (d) are the nonfactorizable ones.

considered decays at leading order (LO) in α_s expansion and give the convoluted amplitudes in next section.

III. THE DECAY AMPLITUDES IN THE PQCD APPROACH

For an axial-vector meson, there are three kinds of polarizations, namely, longitudinal (L), normal (N), and transverse (T). The $B_c \rightarrow A_2(\epsilon_2, P_2)A_3(\epsilon_3, P_3)$ decays are characterized by the polarization states of these axial-vector mesons.

A. Decay amplitudes with different polarization

The decay amplitudes $\mathcal{M}_H$ are classified accordingly, with $H = L, N, T$,

$$
\begin{aligned}
\mathcal{M}_H = (m_{B_c}^2 \mathcal{M}_L, m_{B_c}^2 \mathcal{M}_N \epsilon_2^*(T)\epsilon_3^*(T), \\
i\mathcal{M}_T \epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^*(T)\epsilon_{3\beta}^*(T)P_{2\gamma}P_{3\rho}).
\end{aligned} \tag{15}
$$

where $\epsilon(T)$ stands for the transverse polarization vector and we have adopted the notation $\epsilon^{0123} = 1$. Based on the Feynman diagrams shown in Fig. 1, we can combine all contributions to these considered decays and obtain the general expression of total decay amplitude as follows,

$$
\mathcal{M}_H(B_c \rightarrow A_2 A_3) = V_{cb}^* V_{uD}\{f_{B_c} F_{fa;H}^{A_2 A_3} a_1 + M_{na;H}^{A_2 A_3} C_1\}, \tag{16}
$$

where $a_1 = C_1/3 + C_2$,[2] while $F_{fa;H}^{A_2 A_3}$ and $M_{na;H}^{A_2 A_3}$ denote the Feynman amplitudes with three polarizations for factorizable and nonfactorizable annihilation contributions, respectively.

The explicit expressions of the function $F_{fa;H}^{A_2 A_3}$ and $M_{na;H}^{A_2 A_3}$ in the pQCD approach can be written as the following form:

$$
\begin{aligned}
F_{fa}^L = {}& 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{[x_2 \phi_2(x_2)\phi_3(x_3) + 2r_2 r_3((x_2 + 1)\phi_2^s(x_2) + (x_2 - 1)\phi_2^t(x_2))\phi_3^s(x_3)] \\
& \times E_{fa}(t_a)h_{fa}(1 - x_3, x_2, b_3, b_2) + E_{fa}(t_b)h_{fa}(x_2, 1 - x_3, b_2, b_3) \\
& \times [(x_3 - 1)\phi_2(x_2)\phi_3(x_3) + 2r_2 r_3 \phi_2^s(x_2)((x_3 - 2)\phi_3^s(x_3) - x_3\phi_3^t(x_3))]\},
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
M_{na}^L = {}& \frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{[[(r_c - x_3 + 1)\phi_2(x_2)\phi_3(x_3) + r_2 r_3(\phi_2^s(x_2)((3r_c + x_2 - x_3 + 1) \\
& \times \phi_3^s(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^t(x_3)) + \phi_2^t(x_2)((r_c - x_2 - x_3 + 1)\phi_3^s(x_3) + (r_c - x_2 + x_3 - 1) \\
& \times \phi_3^t(x_3)))]E_{na}(t_c)h_{na}^c(x_2, x_3, b_1, b_2) - [(r_b + r_c + x_2 - 1)\phi_2(x_2)\phi_3(x_3) + r_2 r_3(\phi_2^s(x_2)((4r_b + r_c + x_2 - x_3 - 1) \\
& \times \phi_3^s(x_3) - (r_c + x_2 + x_3 - 1)\phi_3^T(x_3)) + \phi_2^t(x_2)((r_c + x_2 + x_3 - 1)\phi_3^s(x_3) - (r_c + x_2 - x_3 - 1)\phi_3^t(x_3)))] \\
& \times E_{na}(t_d)h_{na}^d(x_2, x_3, b_1, b_2)\},
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
F_{fa}^N = {}& 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2 r_3 \{[(x_2 + 1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) + (x_2 - 1)(\phi_2^v(x_2)\phi_3^a(x_3) \\
& + \phi_2^a(x_2)\phi_3^v(x_3))]E_{fa}(t_a)h_{fa}(1 - x_3, x_2, b_3, b_2) + [(x_3 - 2)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)) - x_3(\phi_2^a(x_2)\phi_3^v(x_3) \\
& + \phi_2^v(x_2)\phi_3^a(x_3))]E_{fa}(t_b)h_{fa}(x_2, 1 - x_3, b_2, b_3)\},
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
M_{na}^N = {}& \frac{32\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2 r_3 \{r_c[\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)]E_{na}(t_c)h_{na}^c(x_2, x_3, b_1, b_2) \\
& - r_b[\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^v(x_2)\phi_3^v(x_3)]E_{na}(t_d)h_{na}^d(x_2, x_3, b_1, b_2)\},
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
F_{fa}^T = {}& 16\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2 r_3 \{[(x_2 + 1)(\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3)) + (x_2 - 1)(\phi_2^a(x_2)\phi_3^a(x_3) \\
& + \phi_2^v(x_2)\phi_3^v(x_3))]E_{fa}(t_a)h_{fa}(1 - x_3, x_2, b_3, b_2) + [(x_3 - 2)(\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3)) - x_3(\phi_2^a(x_2)\phi_3^a(x_3) \\
& + \phi_2^v(x_2)\phi_3^v(x_3))]E_{fa}(t_b)h_{fa}(x_2, 1 - x_3, b_2, b_3)\},
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
M_{na}^T = {}& \frac{64\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2 r_3 \{r_c[\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3)]E_{na}(t_c)h_{na}^c(x_2, x_3, b_1, b_2) \\
& - r_b[\phi_2^a(x_2)\phi_3^v(x_3) + \phi_2^v(x_2)\phi_3^a(x_3)]E_{na}(t_d)h_{na}^d(x_2, x_3, b_1, b_2)\}.
\end{aligned} \tag{22}
$$

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[2]One should note that a_1 here just stands for the combined Wilson coefficient, not the abbreviation for axial-vector meson $a_1(1260)$.

where $r_{2(3)} = m_{A_{2(3)}}/m_{B_c}$, $r_{c(b)} = m_{c(b)}/m_{B_c}$. The explicit expressions for the distribution amplitudes ϕ_A, ϕ_A^t, ϕ_A^s, ϕ_A^T, ϕ_A^v and ϕ_A^a are given in the Appendix. The definitions and expressions of the hard functions (h_{fa}, h_{na}), (E_{fa}, E_{na}) and hard scales (t_a, t_b, t_c, t_d) can be found in Appendix B of Ref. [10] and references therein.

B. Decay amplitudes for the considered decay modes

Now we can write down the total decay amplitudes for all 32 $B_c \to A_2 A_3$ decays. The decay amplitudes of the 16 $\Delta S = 0$ decay modes are the following:

$$\sqrt{2}\mathcal{M}_H(B_c \to a_1^+ a_1^0) = V_{cb}^* V_{ud}\{f_{B_c}(F_{fa;H}^{a_1^+ a_1^0} - F_{fa;H}^{a_1^0 a_1^+})a_1 + (M_{na;H}^{a_1^+ a_1^0} - M_{na;H}^{a_1^0 a_1^+})C_1\}, \tag{23}$$

$$\sqrt{2}\mathcal{M}_H(B_c \to b_1^+ b_1^0) = V_{cb}^* V_{ud}\{f_{B_c}(F_{fa;H}^{b_1^+ b_1^0} - F_{fa;H}^{b_1^0 b_1^+})a_1 + (M_{na;H}^{b_1^+ b_1^0} - M_{na;H}^{b_1^0 b_1^+})C_1\}, \tag{24}$$

$$\sqrt{2}\mathcal{M}_H(B_c \to a_1^+ b_1^0) = V_{cb}^* V_{ud}\{f_{B_c}(F_{fa;H}^{a_1^+ b_1^0} - F_{fa;H}^{b_1^0 a_1^+})a_1 + (M_{na;H}^{a_1^+ b_1^0} - M_{na;H}^{b_1^0 a_1^+})C_1\}, \tag{25}$$

$$\sqrt{2}\mathcal{M}_H(B_c \to b_1^+ a_1^0) = V_{cb}^* V_{ud}\{f_{B_c}(F_{fa;H}^{b_1^+ a_1^0} - F_{fa;H}^{a_1^0 b_1^+})a_1 + (M_{na;H}^{b_1^+ a_1^0} - M_{na;H}^{a_1^0 b_1^+})C_1\}, \tag{26}$$

$$\mathcal{M}_H(B_c \to a_1^+ f') = V_{cb}^* V_{ud}\left\{\frac{\cos\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{a_1^+ f_1^u} + F_{fa;H}^{f_1^d a_1^+})a_1 + (M_{na;H}^{a_1^+ f_1^u} + M_{na;H}^{f_1^d a_1^+})C_1]\right.$$
$$\left. + \frac{\sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{a_1^+ f_8^u} + F_{fa;H}^{f_8^d a_1^+})a_1 + (M_{na;H}^{a_1^+ f_8^u} + M_{na;H}^{f_8^d a_1^+})C_1]\right\}, \tag{27}$$

$$\mathcal{M}_H(B_c \to a_1^+ f'') = V_{cb}^* V_{ud}\left\{\frac{-\sin\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{a_1^+ f_1^u} + F_{fa;H}^{f_1^d a_1^+})a_1 + (M_{na;H}^{a_1^+ f_1^u} + M_{na;H}^{f_1^d a_1^+})C_1]\right.$$
$$\left. + \frac{\cos\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{a_1^+ f_8^u} + F_{fa;H}^{f_8^d a_1^+})a_1 + (M_{na;H}^{a_1^+ f_8^u} + M_{na;H}^{f_8^d a_1^+})C_1]\right\}, \tag{28}$$

$$\mathcal{M}_H(B_c \to b_1^+ f') = \mathcal{M}_H(B_c \to a_1^+ f')(a_1 \to b_1), \qquad \mathcal{M}_H(B_c \to b_1^+ f'') = \mathcal{M}_H(B_c \to a_1^+ f'')(a_1 \to b_1), \tag{29}$$

$$\mathcal{M}_H(B_c \to a_1^+ h') = \mathcal{M}_H(B_c \to a_1^+ f')(f \to h, \theta_3 \to \theta_1), \quad \mathcal{M}_H(B_c \to a_1^+ h'') = \mathcal{M}_H(B_c \to a_1^+ f'')(f \to h, \theta_3 \to \theta_1), \tag{30}$$

$$\mathcal{M}_H(B_c \to b_1^+ h') = \mathcal{M}_H(B_c \to a_1^+ h')(a_1 \to b_1), \qquad \mathcal{M}_H(B_c \to b_1^+ h'') = \mathcal{M}_H(B_c \to a_1^+ h'')(a_1 \to b_1), \tag{31}$$

$$\mathcal{M}_H(B_c \to \bar{K}'^0 K'^+) = V_{cb}^* V_{ud}\{-\sin^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1A}}C_1) - \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1B}}C_1)$$
$$+ \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1A}}C_1) + \cos^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1B}}C_1)\}, \tag{32}$$

$$\mathcal{M}_H(B_c \to \bar{K}'^0 K''^+) = V_{cb}^* V_{ud}\{-\cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1A}}C_1) + \sin^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1B}}C_1)$$
$$+ \cos^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1A}}C_1) - \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1B}}C_1)\}, \tag{33}$$

$$\mathcal{M}_H(B_c \to \bar{K}''^0 K'^+) = V_{cb}^* V_{ud}\{\cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1A}}C_1) + \cos^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1B}}C_1)$$
$$+ \sin^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1A}}C_1) + \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1B}}C_1)\}, \tag{34}$$

$$\mathcal{M}_H(B_c \to \bar{K}''^0 K''^+) = V_{cb}^* V_{ud}\{\cos^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1A}}C_1) - \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1A}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1A}^0 K_{1B}}C_1)$$
$$+ \cos\theta_K\sin\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1A}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1A}}C_1) - \sin^2\theta_K(f_{B_c}F_{fa;H}^{\bar{K}_{1B}^0 K_{1B}}a_1 + M_{na;H}^{\bar{K}_{1B}^0 K_{1B}}C_1)\}; \tag{35}$$

The decay amplitudes of the 16 $\Delta S = 1$ decay modes are of the form:

$$\mathcal{M}_H(B_c \to K'^0 a_1^+) = \sqrt{2}\mathcal{M}_H(B_c \to K'^+ a_1^0)$$

$$= V_{cb}^* V_{us}\{\sin\theta_K[f_{B_c}F_{fa;H}^{K_{1A}^0 a_1^+}a_1 + M_{na;H}^{K_{1A}^0 a_1^+}C_1] + \cos\theta_K[f_{B_c}F_{fa;H}^{K_{1B}^0 a_1^+}a_1 + M_{na;H}^{K_{1B}^0 a_1^+}C_1]\}, \qquad (36)$$

$$\mathcal{M}_H(B_c \to K''^0 a_1^+) = \sqrt{2}\mathcal{M}_H(B_c \to K''^+ a_1^0)$$

$$= V_{cb}^* V_{us}\{\cos\theta_K[f_{B_c}F_{fa;H}^{K_{1A}^0 a_1^+}a_1 + M_{na;H}^{K_{1A}^0 a_1^+}C_1] - \sin\theta_K[f_{B_c}F_{fa;H}^{K_{1B}^0 a_1^+}a_1 + M_{na;H}^{K_{1B}^0 a_1^+}C_1]\}, \qquad (37)$$

$$\mathcal{M}_H(B_c \to K'^0 b_1^+) = \sqrt{2}\mathcal{M}_H(B_c \to K'^+ b_1^0) = \mathcal{M}_H(B_c \to K'^0 a_1^+)(a_1 \to b_1), \qquad (38)$$

$$\mathcal{M}_H(B_c \to K''^0 b_1^+) = \sqrt{2}\mathcal{M}_H(B_c \to K''^+ b_1^0) = \mathcal{M}_H(B_c \to K''^0 a_1^+)(a_1 \to b_1), \qquad (39)$$

$$\mathcal{M}_H(B_c \to K'^+ f') = V_{cb}^* V_{us}\left\{\frac{\cos\theta_3 \sin\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1A}f_1^u} + F_{fa;H}^{f_1^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_1^u} + M_{na;H}^{f_1^s K_{1A}})C_1]\right.$$

$$+ \frac{\sin\theta_3 \sin\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_8^u} - 2F_{fa;H}^{f_8^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_8^u} - 2M_{na;H}^{f_8^s K_{1A}})C_1]$$

$$+ \frac{\cos\theta_3 \cos\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1B}f_1^u} + F_{fa;H}^{f_1^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_1^u} + M_{na;H}^{f_1^s K_{1B}})C_1]$$

$$\left. + \frac{\cos\theta_K \sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1B}f_8^u} - 2F_{fa;H}^{f_8^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_8^u} - 2M_{na;H}^{f_8^s K_{1B}})C_1]\right\}, \qquad (40)$$

$$\mathcal{M}_H(B_c \to K'^+ f'') = V_{cb}^* V_{us}\left\{\frac{-\sin\theta_3 \sin\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1A}f_1^u} + F_{fa;H}^{f_1^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_1^u} + M_{na;H}^{f_1^s K_{1A}})C_1]\right.$$

$$+ \frac{\cos\theta_3 \sin\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_8^u} - 2F_{fa;H}^{f_8^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_8^u} - 2M_{na;H}^{f_8^s K_{1A}})C_1]$$

$$- \frac{\cos\theta_K \sin\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1B}f_1^u} + F_{fa;H}^{f_1^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_1^u} + M_{na;H}^{f_1^s K_{1B}})C_1]$$

$$\left. + \frac{\cos\theta_K \cos\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1B}f_8^u} - 2F_{fa;H}^{f_8^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_8^u} - 2M_{na;H}^{f_8^s K_{1B}})C_1]\right\}, \qquad (41)$$

$$\mathcal{M}_H(B_c \to K''^+ f') = V_{cb}^* V_{us}\left\{\frac{\cos\theta_3 \cos\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1A}f_1^u} + F_{fa;H}^{f_1^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_1^u}\right.$$

$$+ M_{na;H}^{f_1^s K_{1A}})C_1] + \frac{\cos\theta_K \sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_8^u} - 2F_{fa;H}^{f_8^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_8^u} - 2M_{na;H}^{f_8^s K_{1A}})C_1]$$

$$- \frac{\cos\theta_3 \sin\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1B}f_1^u} + F_{fa;H}^{f_1^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_1^u} + M_{na;H}^{f_1^s K_{1B}})C_1]$$

$$\left. - \frac{\sin\theta_K \sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1B}f_8^u} - 2F_{fa;H}^{f_8^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_8^u} - 2M_{na;H}^{f_8^s K_{1B}})C_1]\right\}, \qquad (42)$$

$$\mathcal{M}_H(B_c \to K''^+ f'') = V_{cb}^* V_{us}\left\{\frac{-\cos\theta_K \sin\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1A}f_1^u} + F_{fa;H}^{f_1^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_1^u} + M_{na;H}^{f_1^s K_{1A}})C_1]\right.$$

$$+ \frac{\cos\theta_3 \cos\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_8^u} - 2F_{fa;H}^{f_8^s K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_8^u} - 2M_{na;H}^{f_8^s K_{1A}})C_1]$$

$$+ \frac{\sin\theta_3 \sin\theta_K}{\sqrt{3}}[f_{B_c}(F_{fa;H}^{K_{1B}f_1^u} + F_{fa;H}^{f_1^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_1^u} + M_{na;H}^{f_1^s K_{1B}})C_1]$$

$$\left. - \frac{\cos\theta_3 \sin\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1B}f_8^u} - 2F_{fa;H}^{f_8^s K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_8^u} - 2M_{na;H}^{f_8^s K_{1B}})C_1]\right\}, \qquad (43)$$

$$\mathcal{M}_H(B_c \to K'^+ h') = \mathcal{M}_H(B_c \to K'^+ f')(f \to h, \theta_3 \to \theta_1),$$
(44)

$$\mathcal{M}_H(B_c \to K'^+ h'') = \mathcal{M}_H(B_c \to K'^+ f'')(f \to h, \theta_3 \to \theta_1),$$
(45)

$$\mathcal{M}_H(B_c \to K''^+ h') = \mathcal{M}_H(B_c \to K''^+ f')(f \to h, \theta_3 \to \theta_1),$$
(46)

$$\mathcal{M}_H(B_c \to K''^+ h'') = \mathcal{M}_H(B_c \to K''^+ f'')(f \to h, \theta_3 \to \theta_1).$$
(47)

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate numerically the BRs and polarization fractions for those considered 32 $B_c \to A_2 A_3$ decay modes. First of all, the central values of the input parameters to be used are the following.

Masses (GeV):

$$
\begin{aligned}
&m_W = 80.41; &&m_{B_c} = 6.286; &&m_b = 4.8; \\
&m_c = 1.5; &&m_{a_1} = 1.23; &&m_{K_{1A}} = 1.32; \\
&m_{f_1} = 1.28; &&m_{f_8} = 1.28; &&m_{b_1} = 1.21; \\
&m_{K_{1B}} = 1.34; &&m_{h_1} = 1.23; &&m_{h_8} = 1.37;
\end{aligned}
$$
(48)

Decay constants (GeV):

$$
\begin{aligned}
&f_{a_1} = 0.238; &&f_{K_{1A}} = 0.250; &&f_{f_1} = 0.245; \\
&f_{f_8} = 0.239; &&f_{b_1} = 0.180; &&f_{K_{1B}} = 0.190; \\
&f_{h_1} = 0.180; &&f_{h_8} = 0.190; &&f_{B_c} = 0.489;
\end{aligned}
$$
(49)

QCD scale and B_c meson lifetime:

$$\Lambda_{\overline{\text{MS}}}^{(f=4)} = 0.250 \text{ GeV}, \qquad \tau_{B_c} = 0.46 \text{ ps}.$$
(50)

For the CKM matrix elements we use $A = 0.814$ and $\lambda = 0.2257$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [34]. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

For these considered $B_c \to A_2 A_3$ decays, the decay rate can be written explicitly as,

$$\Gamma = \frac{G_F^2 |\mathbf{P_c}|}{16\pi m_{B_c}^2} \sum_{\sigma=L,T} \mathcal{M}^{(\sigma)\dagger} \mathcal{M}^{(\sigma)}$$
(51)

where $|\mathbf{P_c}| \equiv |\mathbf{P_{2z}}| = |\mathbf{P_{3z}}|$ is the momentum of either of the outgoing axial-vector mesons.

The polarization fractions $f_{L(\|,\perp)}$ can be defined as [35],

$$f_{L(\|,\perp)} = \frac{|\mathcal{A}_{L(\|,\perp)}|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\|\|^2 + |\mathcal{A}_\perp|^2},$$
(52)

where the amplitudes $\mathcal{A}_i (i = L, \|, \perp)$ are defined as,

$$
\begin{aligned}
\mathcal{A}_L &= -\xi m_{B_c}^2 \mathcal{M}_L, \qquad \mathcal{A}_\| = \xi\sqrt{2} m_{B_c}^2 \mathcal{M}_N, \\
\mathcal{A}_\perp &= \xi m_{A_2} m_{A_3} \sqrt{2(r^2 - 1)} \mathcal{M}_T,
\end{aligned}
$$
(53)

for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor $\xi = \sqrt{G_F^2 \mathbf{P_c}/(16\pi m_{B_c}^2 \Gamma)}$ and the ratio $r = P_2 \cdot P_3/(m_{A_2} m_{A_3})$. These amplitudes satisfy the relation,

$$|\mathcal{A}_L|^2 + |\mathcal{A}_\|\|^2 + |\mathcal{A}_\perp|^2 = 1.$$
(54)

following the summation in Eq. (51).

By using the analytic expressions for the complete decay amplitudes and the input parameters as given explicitly in Eqs. (23)–(50), we calculate and then present the pQCD predictions for the CP-averaged BRs and longitudinal polarization fractions (LPFs) of the considered decays with errors in Tables I, II, III, IV, and V. The dominant errors arise from the uncertainties of charm quark mass $m_c = 1.5 \pm 0.15$ GeV and the combined Gegenbauer moments a_i of the axial-vector meson distribution amplitudes, respectively.

A. The pQCD predictions for $\Delta S = 0$ decays

In Table I and II, we show the pQCD predictions for the branching ratios and the longitudinal polarization fractions of the 16 $\Delta S = 0$ decays.

For both the $B_c \to a_1^+ a_1^0$ and $b_1^+ b_1^0$ decays, since the quark structure of a_1^0 and b_1^0 are the same one, $(u\bar{u} - d\bar{d})/\sqrt{2}$, the contributions from $u\bar{u}$ and $d\bar{d}$ components to the corresponding decay amplitude as shown in Eqs. (23) and (24) will interfere destructively, and therefore will cancel each other exactly at leading order and result in the zero BRs for these two channels, as illustrated in the Table I. For the possible high order contributions, they will also cancel each other due to the isospin symmetry between u and d quarks. As for the nonperturbative part, we currently do not know how to calculate it reliably. But we generally believe that it is small in magnitude for B meson

TABLE I. The pQCD predictions of BRs and LPFs for $B_c \to (a_1, b_1)(a_1, b_1)$ decays. The source of the dominant errors is explained in the text.

$\Delta S = 0$ Decay modes	BRs (10^{-5})	LPFs (%)	$\Delta S = 0$ Decay modes	BRs (10^{-5})	LPFs (%)
$B_c \to a_1^+ a_1^0$	0.0	-	$B_c \to b_1^+ b_1^0$	0.0	-
$B_c \to a_1^+ b_1^0$	$2.2^{+0.6}_{-0.5}(m_c)^{+1.1}_{-0.9}(a_i)$	$92.4^{+1.9}_{-2.8}$	$B_c \to b_1^+ a_1^0$	$2.2^{+0.6}_{-0.5}(m_c)^{+1.1}_{-0.8}(a_i)$	$91.8^{+2.0}_{-2.6}$

TABLE II. Same as Table I but for $B_c \to (a_1^+, b_1^+)(f_1', h_1')$ decays.

$\Delta S = 0$ Decay modes	$\theta_3 = 38°$ BRs (10^{-6})	LPFs (%)	$\theta_3 = 50°$ BRs (10^{-6})	LPFs (%)
$B_c \to a_1(1260)^+ f_1(1285)$	$6.5^{+1.0}_{-0.9}(m_c)^{+0.5}_{-1.0}(a_i)$	$83.6^{+2.4}_{-4.1}$	$6.1^{+1.0}_{-0.9}(m_c)^{+0.4}_{-0.9}(a_i)$	$84.0^{+2.3}_{-4.0}$
$B_c \to a_1(1260)^+ f_1(1420) \times 10$	$0.3^{+0.1}_{-0.1}(m_c)^{+0.7}_{-0.3}(a_i)$	$56.8^{+43.2}_{-56.8}$	$3.9^{+0.7}_{-0.0}(m_c)^{+1.3}_{-1.6}(a_i)$	$78.5^{+7.4}_{-13.9}$

$\Delta S = 0$ Decay modes	$\theta_3 = 38°$ BRs (10^{-7})	LPFs (%)	$\theta_3 = 50°$ BRs (10^{-7})	LPFs (%)
$B_c \to b_1(1235)^+ f_1(1285)$	$2.8^{+4.1}_{-0.5}(m_c)^{+1.8}_{-0.9}(a_i)$	$65.2^{+28.3}_{-16.4}$	$3.0^{+4.4}_{-0.8}(m_c)^{+1.5}_{-0.9}(a_i)$	$68.7^{+21.7}_{-14.6}$
$B_c \to b_1(1235)^+ f_1(1420)$	$1.4^{+0.2}_{-0.1}(m_c)^{+0.7}_{-0.9}(a_i)$	100.0 ± 0.0	$1.2^{+0.4}_{-0.4}(m_c)^{+1.1}_{-0.8}(a_i)$	$100.0^{+0.0}_{-0.8}$

$\Delta S = 0$ Decay modes	$\theta_1 = 10°$ BRs (10^{-6})	LPFs (%)	$\theta_1 = 45°$ BRs (10^{-6})	LPFs (%)
$B_c \to a_1(1260)^+ h_1(1170)$	$1.3^{+0.2}_{-0.5}(m_c)^{+0.5}_{-0.4}(a_i)$	$86.3^{+2.0}_{-9.8}$	$0.7^{+0.2}_{-0.4}(m_c)^{+0.3}_{-0.2}(a_i)$	$73.1^{+7.4}_{-28.8}$
$B_c \to a_1(1260)^+ h_1(1380) \times 10$	$1.1^{+0.7}_{-0.0}(m_c)^{+1.3}_{-0.5}(a_i)$	$68.8^{+23.2}_{-11.5}$	$6.8^{+0.2}_{-1.1}(m_c)^{+2.3}_{-2.4}(a_i)$	$100.0^{+0.0}_{-1.2}$

$\Delta S = 0$ Decay modes	$\theta_1 = 10°$ BRs (10^{-5})	LPFs (%)	$\theta_1 = 45°$ BRs (10^{-5})	LPFs (%)
$B_c \to b_1(1235)^+ h_1(1170)$	$8.1^{+3.6}_{-2.8}(m_c)^{+3.9}_{-3.4}(a_i)$	$96.4^{+1.0}_{-1.6}$	$10.3^{+4.0}_{-3.3}(m_c)^{+4.7}_{-3.8}(a_i)$	$96.4^{+0.9}_{-1.4}$
$B_c \to b_1(1235)^+ h_1(1380)$	$2.5^{+0.5}_{-0.7}(m_c)^{+1.4}_{-1.2}(a_i)$	$100.0^{+0.0}_{-0.8}$	$0.3^{+0.2}_{-0.1}(m_c)^{+0.5}_{-0.2}(a_i)$	100.0 ± 0.0

$\Delta S = 0$ Decay modes	$\theta_K = 45°$ BRs (10^{-5})	LPFs (%)	$\theta_K = -45°$ BRs (10^{-5})	LPFs (%)
$B_c \to \bar{K}_1(1270)^0 K_1(1270)^+$	$1.2^{+0.2}_{-0.1}(m_c)^{+1.8}_{-0.9}(a_i)$	$99.7^{+0.1}_{-1.0}$	$2.9^{+1.2}_{-1.0}(m_c)^{+4.4}_{-2.3}(a_i)$	$71.9^{+16.2}_{-24.6}$
$B_c \to \bar{K}_1(1400)^0 K_1(1400)^+$	$2.8^{+1.2}_{-1.0}(m_c)^{+4.3}_{-2.3}(a_i)$	$72.7^{+15.8}_{-24.3}$	$1.1^{+0.2}_{-0.0}(m_c)^{+1.9}_{-0.9}(a_i)$	$99.7^{+0.0}_{-1.0}$
$B_c \to \bar{K}_1(1270)^0 K_1(1400)^+$	$3.7^{+1.3}_{-1.1}(m_c)^{+3.1}_{-2.2}(a_i)$	$96.2^{+3.5}_{-8.4}$	$1.9^{+0.5}_{-0.5}(m_c)^{+2.2}_{-1.4}(a_i)$	$94.8^{+3.4}_{-10.3}$
$B_c \to \bar{K}_1(1400)^0 K_1(1270)^+$	$1.9^{+0.5}_{-0.5}(m_c)^{+2.2}_{-1.4}(a_i)$	$94.6^{+3.6}_{-10.7}$	$3.7^{+1.3}_{-1.1}(m_c)^{+3.2}_{-2.1}(a_i)$	$96.1^{+3.6}_{-8.6}$

[a]Here, the factor 10 is specifically used for the BRs. The following one has the same meaning.

decays. Consequently, we think that a nonzero measurement for the branching ratios of these two decays may be a signal of the effects of new physics beyond the SM.

For $B_c \to a_1^+ b_1^0$ and $B_c \to b_1^+ a_1^0$ decays, however, the pQCD predictions for their BRs are rather large, as given in Table I

$$Br(B_c \to a_1^+ b_1^0) = Br(B_c \to b_1^+ a_1^0) \approx 2.2 \times 10^{-5}. \quad (55)$$

Besides $B_c \to a_1^+ b_1^0$ and $b_1^+ a_1^0$ decays, other six $\Delta S = 0$ decays, such as the $B_c \to b_1 h_1$ and $B_c \to \bar{K}_1^0 K_1^+$ decays, also have a large branching ratios at the 10^{-5} level, as listed in Table II. According to the studies in Ref. [7], these

TABLE III. Same as Table I but for $B_c \to K_1 a_1$, $K_1 b_1$ decays.

$\Delta S = 1$ Decay modes	$\theta_K = 45°$ BRs (10^{-7})	LPFs (%)	$\theta_K = -45°$ BRs (10^{-7})	LPFs (%)
$B_c \to K_1(1270)^0 a_1(1260)^+$	$4.6^{+1.3}_{-1.0}(m_c)^{+4.7}_{-2.4}(a_i)$	$79.2^{+12.4}_{-16.3}$	$8.3^{+1.3}_{-1.8}(m_c)^{+3.6}_{-3.9}(a_i)$	$99.3^{+0.8}_{-5.5}$
$B_c \to K_1(1400)^0 a_1(1260)^+$	$8.0^{+1.3}_{-1.7}(m_c)^{+3.5}_{-3.7}(a_i)$	$100.0^{+0.0}_{-3.8}$	$4.5^{+1.2}_{-1.1}(m_c)^{+4.4}_{-2.5}(a_i)$	$81.3^{+12.5}_{-16.6}$
$B_c \to K_1(1270)^+ a_1(1260)^0$	$2.3^{+0.6}_{-0.5}(m_c)^{+2.4}_{-1.3}(a_i)$	$79.2^{+12.4}_{-16.3}$	$4.2^{+0.6}_{-1.0}(m_c)^{+1.8}_{-2.0}(a_i)$	$99.3^{+0.8}_{-5.5}$
$B_c \to K_1(1400)^+ a_1(1260)^0$	$4.0^{+0.7}_{-0.9}(m_c)^{+1.8}_{-1.9}(a_i)$	$100.0^{+0.0}_{-3.8}$	$2.2^{+0.6}_{-0.5}(m_c)^{+2.3}_{-1.1}(a_i)$	$81.3^{+12.5}_{-16.6}$

$\Delta S = 1$ Decay modes	$\theta_K = 45°$ BRs (10^{-6})	LPFs (%)	$\theta_K = -45°$ BRs (10^{-6})	LPFs (%)
$B_c \to K_1(1270)^0 b_1(1235)^+$	$1.6^{+0.8}_{-0.5}(m_c)^{+1.3}_{-0.9}(a_i)$	$91.3^{+5.0}_{-5.1}$	$1.4^{+0.4}_{-0.2}(m_c)^{+0.8}_{-0.7}(a_i)$	$100.0^{+0.0}_{-0.3}$
$B_c \to K_1(1400)^0 b_1(1235)^+$	$1.3^{+0.4}_{-0.2}(m_c)^{+0.9}_{-0.5}(a_i)$	100.0 ± 0.0	$1.5^{+0.8}_{-0.5}(m_c)^{+1.3}_{-0.9}(a_i)$	$93.6^{+5.0}_{-5.1}$
$B_c \to K_1(1270)^+ b_1(1235)^0$	$0.8^{+0.4}_{-0.3}(m_c)^{+0.6}_{-0.5}(a_i)$	$91.4^{+4.9}_{-5.1}$	$0.7^{+0.2}_{-0.1}(m_c)^{+0.4}_{-0.4}(a_i)$	$100.0^{+0.0}_{-0.3}$
$B_c \to K_1(1400)^+ b_1(1235)^0$	$0.7^{+0.2}_{-0.2}(m_c)^{+0.4}_{-0.4}(a_i)$	100.0 ± 0.0	$0.8^{+0.3}_{-0.3}(m_c)^{+0.6}_{-0.5}(a_i)$	$93.6^{+5.1}_{-4.9}$

TABLE IV.   Same as Table I but for $B_c \to K_1^+ f_1'$ decays with $\theta_3 = 38°$ (1st entry) and $\theta_3 = 50°$ (2nd entry), respectively.

$\Delta S = 1$ Decay modes	$\theta_K = 45°$ BRs (10^{-7})	LPFs (%)	$\theta_K = -45°$ BRs (10^{-7})	LPFs (%)
$B_c \to K_1(1270)^+ f_1(1285)$	$1.4^{+0.9}_{-0.4}(m_c)^{+2.0}_{-0.7}(a_i)$	$65.1^{+27.4}_{-19.4}$	$1.6^{+0.1}_{-0.5}(m_c)^{+1.1}_{-1.0}(a_i)$	$96.7^{+2.7}_{-11.6}$
	$1.7^{+1.1}_{-0.4}(m_c)^{+2.3}_{-1.0}(a_i)$	$69.1^{+22.1}_{-19.6}$	$1.5^{+0.3}_{-0.6}(m_c)^{+1.6}_{-1.2}(a_i)$	$92.1^{+2.8}_{-13.0}$
$B_c \to K_1(1400)^+ f_1(1285)$	$1.5^{+0.2}_{-0.4}(m_c)^{+1.2}_{-0.8}(a_i)$	$96.7^{+2.7}_{-11.5}$	$1.4^{+0.8}_{-0.4}(m_c)^{+1.8}_{-0.8}(a_i)$	$65.5^{+27.2}_{-19.4}$
	$1.5^{+0.3}_{-0.6}(m_c)^{+1.6}_{-1.2}(a_i)$	$92.1^{+4.0}_{-12.8}$	$1.7^{+1.1}_{-0.5}(m_c)^{+2.2}_{-1.0}(a_i)$	$69.5^{+21.9}_{-19.6}$
$B_c \to K_1(1270)^+ f_1(1420)$	$0.9^{+0.4}_{-0.3}(m_c)^{+0.8}_{-0.9}(a_i)$	$81.6^{+13.5}_{-34.6}$	$4.4^{+0.6}_{-0.4}(m_c)^{+1.5}_{-1.7}(a_i)$	$71.5^{+4.8}_{-8.9}$
	$0.6^{+0.1}_{-0.2}(m_c)^{+0.4}_{-0.6}(a_i)$	$78.5^{+16.9}_{-48.1}$	$4.4^{+0.5}_{-0.3}(m_c)^{+1.2}_{-1.5}(a_i)$	$73.2^{+4.8}_{-9.3}$
$B_c \to K_1(1400)^+ f_1(1420)$	$4.3^{+0.6}_{-0.4}(m_c)^{+1.6}_{-1.7}(a_i)$	$71.9^{+4.8}_{-9.3}$	$0.9^{+0.4}_{-0.3}(m_c)^{+0.8}_{-0.9}(a_i)$	$81.9^{+13.2}_{-34.4}$
	$4.4^{+0.5}_{-0.3}(m_c)^{+1.1}_{-1.6}(a_i)$	$73.6^{+4.8}_{-8.8}$	$0.6^{+0.1}_{-0.2}(m_c)^{+0.4}_{-0.7}(a_i)$	$78.7^{+16.8}_{-47.3}$

B_c decay modes with a branching ratio at 10^{-5} level could be measured at the LHC experiments [7].

Besides the large branching ratio at 10^{-5} level, the $B_c \to \bar{K}_1^0 K_1^+$ decay modes also have a strong dependence on the value of the mixing angle θ_K, as shown by the numbers in Table II. If these channels are measured at LHC experiments with enough precision, one can determine the θ_K by compare the pQCD predictions with the data. In order to reduce the effects of the choice of input parameters, we define the ratio of the branching ratios between relevant decay modes:

$$\frac{Br(B_c \to \bar{K}_1(1270)^0 K_1(1400)^+)_{pQCD}}{Br(B_c \to \bar{K}_1(1270)^0 K_1(1270)^+)_{pQCD}}$$
$$\approx \begin{cases} 3.0, & \text{for } \theta_K = 45°, \\ 0.7, & \text{for } \theta_K = -45°; \end{cases} \tag{56}$$

$$\frac{Br(B_c \to \bar{K}_1(1270)^0 K_1(1400)^+)_{pQCD}}{Br(B_c \to \bar{K}_1(1400)^0 K_1(1270)^+)_{pQCD}}$$
$$\approx \begin{cases} 2.0, & \text{for } \theta_K = 45°, \\ 0.5, & \text{for } \theta_K = -45°; \end{cases} \tag{57}$$

$$\frac{Br(B_c \to \bar{K}_1(1400)^0 K_1(1400)^+)_{pQCD}}{Br(B_c \to \bar{K}_1(1270)^0 K_1(1270)^+)_{pQCD}}$$
$$\approx \begin{cases} 2.3, & \text{for } \theta_K = 45°, \\ 0.4, & \text{for } \theta_K = -45°; \end{cases} \tag{58}$$

The LHC experiments can measure these ratios with a better precision than that for a direct measurement of branching ratios for individual decays. We suggest such measurements as a way to determine the mixing angle θ_K at LHC.

B. The pQCD predictions for $\Delta S = 1$ decays

In Tables III, IV, and V, we show the pQCD predictions for the branching ratios and the longitudinal polarization fractions of the 16 $\Delta S = 1$ decays.

First of all, when compared with those $\Delta S = 0$ decays, these $\Delta S = 1$ decays are CKM suppressed due to the factor $|V_{us}/V_{ud}|^2 \sim 0.04$, as can be seen easily from the expressions for the decay amplitudes as given in Eqs. (23)–(47). The pQCD predictions for the branching ratios of these B_c decays are at the level of 10^{-6} to 10^{-8}, much smaller than that for those $\Delta S = 0$ decays. Most of them, for example

TABLE V.   Same as Table I but for $B_c \to K_1^+ h_1'$ decays $\theta_1 = 10°$ (1st entry) and $\theta_1 = 45°$ (2nd entry), respectively.

$\Delta S = 1$ Decay modes	$\theta_K = 45°$ BRs (10^{-6})	LPFs (%)	$\theta_K = -45°$ BRs (10^{-6})	LPFs (%)
$B_c \to K_1(1270)^+ h_1(1170)$	$1.4^{+0.6}_{-0.6}(m_c)^{+1.3}_{-0.8}(a_i)$	$94.5^{+2.3}_{-3.9}$	$1.6^{+0.7}_{-0.5}(m_c)^{+1.0}_{-1.0}(a_i)$	$98.5^{+0.6}_{-0.9}$
	$0.6^{+0.3}_{-0.3}(m_c)^{+0.3}_{-0.4}(a_i)$	$87.9^{+6.5}_{-14.6}$	$0.2^{+0.2}_{-0.0}(m_c)^{+0.3}_{-0.0}(a_i)$	$92.9^{+7.5}_{-15.1}$
$B_c \to K_1(1400)^+ h_1(1170)$	$1.6^{+0.7}_{-0.5}(m_c)^{+1.0}_{-1.1}(a_i)$	$98.5^{+0.6}_{-0.9}$	$1.4^{+0.6}_{-0.6}(m_c)^{+1.2}_{-0.9}(a_i)$	$94.6^{+2.3}_{-3.9}$
	$0.2^{+0.2}_{-0.0}(m_c)^{+0.3}_{-0.0}(a_i)$	$93.0^{+7.3}_{-14.8}$	$0.5^{+0.4}_{-0.3}(m_c)^{+0.6}_{-0.2}(a_i)$	$88.1^{+6.4}_{-14.4}$
$B_c \to K_1(1270)^+ h_1(1380)$	$0.9^{+0.3}_{-0.0}(m_c)^{+0.8}_{-0.3}(a_i)$	$98.5^{+0.8}_{-1.4}$	$1.5^{+0.5}_{-0.4}(m_c)^{+0.9}_{-0.7}(a_i)$	$89.6^{+2.9}_{-4.0}$
	$1.8^{+0.5}_{-0.4}(m_c)^{+1.1}_{-0.9}(a_i)$	$98.6^{+0.8}_{-0.7}$	$2.8^{+1.1}_{-0.8}(m_c)^{+1.7}_{-1.3}(a_i)$	$94.3^{+1.7}_{-2.9}$
$B_c \to K_1(1400)^+ h_1(1380)$	$1.5^{+0.4}_{-0.4}(m_c)^{+0.8}_{-0.7}(a_i)$	$89.8^{+2.8}_{-3.9}$	$0.9^{+0.3}_{-0.1}(m_c)^{+0.8}_{-0.4}(a_i)$	$98.5^{+0.9}_{-1.3}$
	$2.8^{+1.1}_{-0.8}(m_c)^{+1.6}_{-1.3}(a_i)$	$94.4^{+1.7}_{-2.7}$	$1.7^{+0.6}_{-0.3}(m_c)^{+1.4}_{-0.7}(a_i)$	$98.6^{+0.9}_{-0.5}$

$B_c \rightarrow K_1 a_1$ and $K_1 f_1$ decays with BRs around 10^{-7} or less, are hardly to be detected even at the LHC experiments.

For the $B_c \rightarrow K_1 b_1$ decays, the pQCD predictions for the BRs are in the order of 10^{-6}, much larger than the BRs of the $B_c \rightarrow K_1 a_1$ decays, since the $^1P_1$ meson behaves very different from the $^3P_1$ state. From the numerical values in Table III, we can also define the following ratio

$$\frac{Br(B_c \rightarrow K_1(1270)^0 b_1^+)_{pQCD}}{Br(B_c \rightarrow K_1(1270)^+ b_1^0)_{pQCD}}$$
$$\approx \frac{Br(B_c \rightarrow K_1(1400)^0 b_1^+)_{pQCD}}{Br(B_c \rightarrow K_1(1400)^+ b_1^0)_{pQCD}} \approx 2 \qquad (59)$$

for both $\theta_K = \pm 45°$. Such decays have a weak dependence on the variation of θ_K.

In Table IV, we show the pQCD predictions for the BRs and LPFs for $B_c \rightarrow K_1^+ f_1'$ decays with $\theta_3 = 38°$ (1st entry) and $\theta_3 = 50°$ (2nd entry), respectively. In Table V, similarly, we show the pQCD predictions for the BRs and LPFs for $B_c \rightarrow K_1^+ h_1'$ decays with $\theta_1 = 10°$ (1st entry) and $\theta_3 = 45°$ (2nd entry), respectively.

One can see from the numerical results in these two tables that all $B_c \rightarrow K_1^+ (f_1', h_1')$ decays have a weak or moderate dependence on the mixing angles θ_1 and θ_3. It is difficult to measure θ_1 and θ_3 through the considered B_c decays.

For $B_c \rightarrow K_1^+ h_1(1380)$ decays, the pQCD predictions for their BRs show a relatively strong dependence on the mixing angle θ_K. The LHC measurement of these decays may also help to constrain the size and sign of θ_K.

Frankly speaking, the theoretical predictions in the pQCD factorization approach still have large theoretical errors induced by the large uncertainties of many input parameters and the meson distribution amplitudes. Any progress in reducing the error of input parameters will help us to improve the precision of the pQCD predictions.

It is worth of stressing that we here calculated only the short-distance contributions in the considered decay modes and do not consider the possible long-distance contributions, such as the rescattering effects, although they may be large and affect the theoretical predictions. Strictly speaking, it is the task after the first measurements of the B_c meson decays and thus beyond the scope of this work.

V. SUMMARY

In this paper, we studied the 32 charmless hadronic $B_c \rightarrow A_2 A_3$ decays by employing the pQCD factorization approach. These considered decay channels can only occur via the annihilation type diagrams in the SM. The pQCD predictions for the CP-averaged branching ratios and longitudinal polarization fractions are analyzed phenomenologically.

From our perturbative evaluations and phenomenological analysis, we found the following results:

(1) The branching ratios of the considered 32 $B_c \rightarrow AA$ decays are in the range of 10^{-5} to 10^{-8}; $B_c \rightarrow a_1 b_1$, $\bar{K}_1^0 K_1^+$ and some other decays have sizable branching ratios ($\sim 10^{-5}$) and can be measured at the LHC experiments;

(2) The branching ratios of $B_c \rightarrow A_2(^1P_1) A_3(^1P_1)$ decays are generally much larger than those of $B_c \rightarrow A_2(^3P_1) A_3(^3P_1)$ decays with a factor around ($10 \sim 100$) because of the rather different QCD behavior between $^1P_1$ and $^3P_1$ states;

(3) For $B_c \rightarrow AA$ decays, the branching ratios of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.

(4) The branching ratios of $B_c \rightarrow \bar{K}_1^0 K_1^+$ decays are sensitive to the value of θ_K, which will be tested by the running LHC and forthcoming SuperB experiments;

(5) The LPFs is larger than 80% for almost all decay modes. That means that these pure annihilation decays of B_c meson are dominated by the longitudinal polarization fraction.

These charmless hadronic B_c meson decays will provide an important platform for studying the mechanism of annihilation contributions, understanding the helicity structure of these considered channels and the content of the axial-vector mesons.

APPENDIX: WAVE FUNCTIONS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy B_c meson, we adopt the form (see Ref. [10], and references therein) as follows,

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{6}}[(\not{P} + m_{B_c})\gamma_5 \phi_{B_c}(x)]_{\alpha\beta}. \qquad (A1)$$

where the distribution amplitude ϕ_{B_c} is of the form [36] in the nonrelativistic limit,

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{6}}\delta(x - m_c/m_{B_c}). \qquad (A2)$$

In fact, we know little about ϕ_{B_c} for heavy B_c meson. Because of embracing b and c quarks simultaneously, B_c meson can be approximated as a nonrelativistic bound

state. At the nonrelativistic limit, the leading 2-particle distribution amplitude ϕ_{B_c} can be approximated by delta function [36], fixing the light-cone momenta of the quarks according to their masses. According to Ref. [36], this form will become a smooth function after considering the evolution effect from relativistic gluon exchange.

For the wave function of axial-vector meson, the longitudinal (L) and transverse (T) polarizations are involved, and can be written as,

$$\Phi_A^L(x) = \frac{1}{\sqrt{6}} \gamma_5 \{m_A \epsilon_A^{*L} \phi_A(x) + \epsilon_A^{*L} \slashed{P} \phi_A^t(x)$$

$$+ m_A \phi_A^s(x)\}_{\alpha\beta}, \qquad (A3)$$

$$\Phi_A^T(x) = \frac{1}{\sqrt{6}} \gamma_5 \{m_A \epsilon_A^{*T} \phi_A^v(x) + \epsilon_A^{*T} \slashed{P} \phi_A^T(x)$$

$$+ m_A i\epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{*\nu} n^\rho v^\sigma \phi_A^a(x)\}_{\alpha\beta}, \qquad (A4)$$

where $\epsilon_A^{L,T}$ denotes the longitudinal and transverse polarization vectors of axial-vector meson, satisfying $P \cdot \epsilon = 0$ in each polarization, x denotes the momentum fraction carried by quark in the meson, and $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are dimensionless lightlike unit vectors. We here adopt the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$.

The twist-2 distribution amplitudes $\phi_A(x)$ and $\phi_A^T(x)$ in Eqs. (A3) and (A4) can be parameterized as [24,29]:

$$\phi_A(x) = \frac{3f_A}{\sqrt{6}} x(1-x) \left[a_{0A}^{\|} + 3a_{1A}^{\|}(2x-1) \right.$$

$$\left. + a_{2A}^{\|} \frac{3}{2}(5(2x-1)^2 - 1) \right], \qquad (A5)$$

$$\phi_A^T(x) = \frac{3f_A}{\sqrt{6}} x(1-x) \left[a_{0A}^{\perp} + 3a_{1A}^{\perp}(2x-1) \right.$$

$$\left. + a_{2A}^{\perp} \frac{3}{2}(5(2x-1)^2 - 1) \right], \qquad (A6)$$

Here, the definition of these distribution amplitudes $\phi_A(x)$ and $\phi_A^T(x)$ satisfy the following normalization relations:

$$\int_0^1 \phi_{3P_1}(x) = \frac{f_{3P_1}}{2\sqrt{6}}, \qquad \int_0^1 \phi_{3P_1}^T(x) = a_{0^3P_1}^{\perp} \frac{f_{3P_1}}{2\sqrt{6}};$$

$$\int_0^1 \phi_{1P_1}(x) = a_{0^1P_1}^{\|} \frac{f_{1P_1}}{2\sqrt{6}}, \qquad \int_0^1 \phi_{1P_1}^T(x) = \frac{f_{1P_1}}{2\sqrt{6}}. \qquad (A7)$$

where $a_{0^3P_1}^{\|} = 1$ and $a_{0^1P_1}^{\perp} = 1$ have been used.

As for the twist-3 distribution amplitudes in Eqs. (A3) and (A4), we use the following form [29]:

$$\phi_A^t(x) = \frac{3f_A}{2\sqrt{6}} \left\{ a_{0A}^{\perp}(2x-1)^2 + \frac{1}{2}a_{1A}^{\perp}(2x-1)(3(2x-1)^2 - 1) \right\},$$

$$(A8)$$

$$\phi_A^s(x) = \frac{3f_A}{2\sqrt{6}} \frac{d}{dx}\{x(1-x)(a_{0A}^{\perp} + a_{1A}^{\perp}(2x-1))\}. \quad (A9)$$

$$\phi_A^v(x) = \frac{3f_A}{4\sqrt{6}} \left\{ \frac{1}{2}a_{0A}^{\|}(1 + (2x-1)^2) + a_{1A}^{\|}(2x-1)^3 \right\},$$

$$(A10)$$

$$\phi_A^a(x) = \frac{3f_A}{4\sqrt{6}} \frac{d}{dx}\{x(1-x)(a_{0A}^{\|} + a_{1A}^{\|}(2x-1))\}. \quad (A11)$$

where f_A is the decay constant of the relevant axial-vector meson. When the axial-vector mesons are K_{1A} and K_{1B}, x in the distribution amplitudes stands for the momentum fraction carrying by the s quark.

The Gegenbauer moments have been studied extensively in the literatures (see Ref. [24] and references therein). Here we adopt the following values:

$$a_{2a_1}^{\|} = -0.02 \pm 0.02; \qquad a_{1a_1}^{\perp} = -1.04 \pm 0.34;$$

$$a_{1b_1}^{\|} = -1.95 \pm 0.35; \qquad a_{2f_1}^{\|} = -0.04 \pm 0.03;$$

$$a_{1f_1}^{\perp} = -1.06 \pm 0.36; \qquad a_{1h_1}^{\|} = -2.00 \pm 0.35;$$

$$a_{2f_8}^{\|} = -0.07 \pm 0.04; \qquad a_{1f_8}^{\perp} = -1.11 \pm 0.31;$$

$$a_{1h_8}^{\|} = -1.95 \pm 0.35; \qquad a_{1K_{1A}}^{\|} = 0.00 \pm 0.26; \qquad (A12)$$

$$a_{2K_{1A}}^{\|} = -0.05 \pm 0.03; \qquad a_{0K_{1A}}^{\perp} = 0.08 \pm 0.09;$$

$$a_{1K_{1A}}^{\perp} = -1.08 \pm 0.48; \qquad a_{0K_{1B}}^{\|} = 0.14 \pm 0.15;$$

$$a_{1K_{1B}}^{\|} = -1.95 \pm 0.45; \qquad a_{2K_{1B}}^{\|} = 0.02 \pm 0.10;$$

$$a_{1K_{1B}}^{\perp} = 0.17 \pm 0.22.$$

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