Spectroscopy and decay properties of the D_s meson

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Using hydrogenic and Gaussian wave functions, mass spectra and decay properties of the $D_s(c\bar{s})$ meson are investigated in the framework of phenomenological quark-antiquark potential (Coulomb plus power) model consisting of a relativistic kinetic energy term. The spin-hyperfine, spin-orbit, and tensor interactions are employed to obtain the pseudoscalar and vector meson masses incorporating the effect of mixing. The decay constants ($f_{P/V}$) are computed with QCD correction using the wave function at the origin. The leptonic branching fractions and electromagnetic transition rates are also calculated in this scheme. Our predictions at potential index $\nu = 1$ are in good agreement with experimental results as well as other theoretical models.

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I. INTRODUCTION

For the charm-strange mesons the S-wave states for the quantum numbers $J^P = 0^-(D_s)$ and $1^+(D_s^\star)$, as well as the P-wave states with quantum numbers $0^+(D_{s0}^\star(2317))$, $1^+(D_{s1}(2460)$ and $D_{s1}'(2536))$ and $2^+(D_{s2}^\star(2573))$ are very well established experimentally [1].

Masses of these states along with other properties have been studied in many theoretical schemes [2–11]. There is a fair agreement for ground state masses with experiment, but for the P-wave masses one finds that there is considerable spread from the experimental values. In particular the $D_{s0}^\star(2317)$ and $D_{s1}(2460)$ states are predicted to be heavier than experimental observations.

Recently the *BABAR* Collaboration in 2009, in inclusive e^+e^- collisions, observed two new charmed-strange states $D_{s1}(2710)$ and $D_{sJ}(2860)$ in both DK and $D^\star K$ channels [12]. They also found evidence of $D_{sJ}(3040)$ in the $D^\star K$ channel. Within the $q\bar{Q}$ description, based on the decay modes and the mass spectrum, possible J^P assignments are discussed in various models. In Refs. [12,13], $J^P = 1^-$ was assigned to the states $D_{s1}(2710)$. The state $D_{sJ}(2860)$ is assigned $J^P = 3^-$ [14,15]. While the resonance $D_{sJ}(3040)$ is assumed to be a $2P$ state with $J^P = 1^+$. The possible spin-parity quantum numbers of these open charm states could be as listed in Table I [14]. These states are compared with 2^3S_1, 1^3D_3, and $2P_1$ states, respectively, in Table V.

The availability of experimental data allows one to test the applicability of different models and gives an opportunity to better understand $q\bar{Q}$ dynamics [17].

The presence of the light quark certainly makes the applicability of nonrelativistic potential models questionable. However it is also very well established that for heavy quarkonia potential models have been extremely successful. In this paper we would like to test the applicability of such a potential model, (Coulomb plus power), and would like to study how far we can extend this formulation by including the kinematic relativistic corrections within the Hamiltonian for the system. Such a study will be useful to quantify the regime of applicability of these potentials. This system may be considered to be semirelativistic [2]. Moreover, in the limit that the heavy quark mass becomes infinite, the heavy-light meson behaves analogously to the hydrogen atom, i.e., the heavier quark does not contribute to the orbital degrees of freedom and the properties of the meson are determined by those of the light quark [18,19]. Therefore it will be useful to make a comparative study of the system with a hydrogen-like as well as a Gaussian wave function.

The pseudoscalar decay constants of the heavy-light mesons have also been estimated in the context of many QCD-motivated approximations. The predictions of each of these constants cover a wide range of values from one model to another [20,21]. Phenomenologically, it is important to have reliable estimates of these decay constants as they are useful in many weak processes such as quark mixing, CP violation, etc. The leptonic decay width is important for the branching ratio (BR). We have calculated the BR for D_s meson using the formula of Ref. [22]. The electromagnetic transitions are also calculated in this scheme because these radiative transitions can probe the internal charge structure of mesons and hence are very useful in determining the meson structure.

In this paper we present a comparative study of mass spectra and decay properties of the D_s meson in the potential scheme of Coulomb plus power potential with the

TABLE I. Possible J^P of the open charm states based on the observed decay mode.

State	Observed channel	Possible J^P
$D_{s1}(2710)$	DK, $D^\star K$	1^-
$D_{sJ}(2860)$	DK, $D^\star K$	$1^-, 3^-, \ldots$
$D_{sJ}(3040)$	$D^\star K$	$0^-, 1^+, 2^-, \ldots$

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power index ν varying from 0.5 to 2.0. We have used the hydrogen-like as well as the Gaussian wave function for calculating masses as well as the decay properties. Spin-hyperfine, spin-orbit, and tensor interactions are introduced to get the S-, P-, and D-wave masses of pseudoscalar and vector mesons. We present the details of semirelativistic treatment of heavy-light mesons in Sec. II. The decay constants ($f_{P/V}$) of these mesons are presented in Sec. III. The leptonic branching fractions are computed in Sec. IV, whereas in Sec. V we present the details of electric and magnetic dipole transition rates. Finally we draw our conclusions in Sec. VI.

II. THEORETICAL FRAMEWORK

For the study of the heavy-light bound state system we consider the relativistic Hamiltonian in which motion of the quarks inside the D_s meson is relativistic [8,23]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}), \tag{1}$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark, m_Q is the heavy quark mass, and $m_{\bar{q}}$ is the light quark mass. The Hamiltonian in Eq. (1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy (KE) part of the Hamiltonian up to $\mathcal{O}(p^6)$, and $V(\mathbf{r})$ is the quark-antiquark potential [24–26],

$$V(\mathbf{r}) = -\frac{\alpha_c}{r} + Ar^{\nu} + V_0, \tag{2}$$

where A is the potential parameter and ν is a general power index, such that the choice of $\nu = 1$ corresponds to the Coulomb plus linear potential with a constant term V_0. $\alpha_c = (4/3)\alpha_S(M^2)$, where $\alpha_S(M^2)$ is the strong running coupling constant. The value of the QCD coupling constant $\alpha_s(M^2)$ is determined through the simplest model with freezing [27,28], namely,

$$\alpha_s(M^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln\frac{M^2+M_B^2}{\Lambda^2}}, \tag{3}$$

where the scale is taken as $M = 2m_Q m_{\bar{q}}/(m_Q + m_{\bar{q}})$, the background mass is $M_B = 2.24\sqrt{A} = 0.95$ GeV [27,28], and $\Lambda = 413$ MeV was fixed from fitting the ρ mass [29].

We have used the hydrogenic radial wave function as well as the Gaussian wave function in the present study. The hydrogenic wave function has the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2}(\mu r)^l e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \tag{4}$$

and the Gaussian wave function has the form

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2(n-1)!}{\Gamma(n+l+1/2)}\right)(\mu r)^l \times e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2). \tag{5}$$

Here, μ is the variational parameter and L is Laguerre polynomial.

For the present study of heavy-light flavor mesons, we employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi. \tag{6}$$

For a chosen value of ν, the variational parameter, μ is determined for each state using the virial theorem

$$\langle \mathrm{KE} \rangle = \frac{1}{2}\left\langle \frac{rdV}{dr} \right\rangle. \tag{7}$$

As the interaction potential assumed here does not contain the spin-dependent part, Eq. (6) gives the spin-averaged masses of the system in terms of the power index ν. The spin-averaged mass for the ground state is computed for the values of ν from 0.5 to 2.0. The spin-averaged (SA) mass is matched with the experimental value for the ground state using the equation [26]

$$M_{\mathrm{SA}} = M_P + \frac{3}{4}(M_V - M_P), \tag{8}$$

where M_V and M_P are the experimentally measured vector and pseudoscalar meson ground state masses. This fixes the parameter V_0, for the chosen value of ν. Using this value of V_0 we calculate S-, P-, and D-wave spin-averaged masses of the D_s mesons which are listed in Tables III and IV. For the comparison for the nJ state, we compute the spin-average or the center of weight (CW) mass from the respective experimental as well as theoretical values as [26]

$$M_{\mathrm{CW},nJ} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)}, \tag{9}$$

where $M_{\mathrm{CW},nJ}$ denotes the spin-averaged mass of the nJ state and M_{nJ} represents the mass of the meson in the nJ state.

The value of the radial wave function $R(0)$ for 0^{-+} and 1^{--} states would be different due to their spin-dependent hyperfine interaction. The spin-hyperfine interactions of the heavy-light flavored mesons are small and this can cause a small shift in the value of the wave function at the origin. Thus, many other models do not consider this contribution to their value of $R(0)$. However, we account this correction to the value of $R(0)$ by considering

$$R_{nJ}(0) = R(0)\left[1 + (\mathrm{SF})_J \frac{\langle \epsilon_{\mathrm{SD}} \rangle_{nJ}}{M_{\mathrm{SA}}}\right], \tag{10}$$

where $(\mathrm{SF})_J$ and $\langle \epsilon_{\mathrm{SD}} \rangle_{nJ}$ is the spin factor and spin-interaction energy of the meson in the nJ state, while $R(0)$ and M_{SA} correspond to the radial wave function at the zero separation and spin-average mass, respectively, of the $Q\bar{q}$ system [26].

The parameters used to calculate the low-lying masses of the D_s meson are $A = 0.14$ GeV^2, $m_s = 0.52$ GeV,

TABLE II. Value of V_0 (in GeV).

ν	Hydrogenic	Gaussian
0.5	−0.113	−0.282
1.0	−0.268	−0.282
1.5	−0.478	−0.440
2.0	−0.808	−0.639

$m_c = 1.55$ GeV and the value of the constant V_0 was found to be as given in Table II.

We have calculated the spin-averaged masses of the D_s mesons in Tables III and IV using the hydrogenic and Gaussian wave functions for $\nu = 0.5$ to 2.0. Looking at the values of the wave functions at the origin for 1S to 4S states, the value of hydrogen-like wave function obtained is approximately double the Gaussian-like wave function. The spin-averaged mass at potential index $\nu = 1$ is close to the experimental as well as other theoretical predictions. We have thus calculated the mass spectra of the D_s meson at $\nu = 1$, which is consistent with lattice predictions.

A. Excited states

We add separately [in Eq. (6)] the spin-dependent part of the usual one gluon exchange potential between the quark and antiquark for computing the hyperfine and spin-orbit shifting of the low-lying S-, P-, and D-states. Thus to take into account the spin-dependent and spin-orbit interaction, causing the splitting of the nL levels one introduces an additional term in the Hamiltonian [30–32]

$$V_{\rm SD}(\mathbf{r}) = \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2} + \frac{\mathbf{L}\cdot\mathbf{S_{\bar q}}}{2m_{\bar q}^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right) + \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar q}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar q}}\mathbf{S_Q}\cdot\mathbf{S_{\bar q}}4\pi\delta(\mathbf{r}) + \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar q}}\{3(\mathbf{S_Q}\cdot\mathbf{n})(\mathbf{S_{\bar q}}\cdot\mathbf{n}) - (\mathbf{S_Q}\cdot\mathbf{S_{\bar q}})\}\frac{1}{r^3},$$
$$\mathbf{n} = \frac{\mathbf{r}}{r}, \tag{11}$$

where $V(r)$ is the phenomenological potential, the first term takes into account the relativistic corrections to the potential $V(r)$, the second term accounts spin orbital interaction, the third term is the usual spin-spin interaction part which is responsible for pseudoscalar and vector meson splitting [Eqs. (15) and (16)], and the fourth term stands for tensor interaction.

The angular momentum of the heavy quark is described by its spin $\mathbf{S_Q}$, and that of the light degrees of freedom are described by $\mathbf{j_{\bar q}} = \mathbf{s_{\bar q}} + \mathbf{L}$, where $\mathbf{s_{\bar q}}$ is the light quark spin and $\mathbf{L}$ is the orbital angular momentum of the light quark. The quantum numbers $\mathbf{S_Q}$ and $\mathbf{j_{\bar q}}$ are individually conserved. The quantum numbers of the excited $\mathbf{L = 1}$ states are formed by combining $\mathbf{S_Q}$ and $\mathbf{j_{\bar q}}$. For $\mathbf{L} = 1$ we have $\mathbf{j_{\bar q}} = 1/2$ ($\mathbf{J} = 0, 1$) and $\mathbf{j_{\bar q}} = 3/2$ ($\mathbf{J} = 1, 2$) states. These states will be denoted as $D^\star_{s0}$, D'_{s1} ($\mathbf{j_{\bar q}} = 1/2$), D_{s1} ($\mathbf{j_{\bar q}} = 3/2$), and $D^\star_{s2}$ in the case of the D_s meson.

For unequal-mass quarks, mass eigenstates are constructed by jj coupling, first coupling $\mathbf{L} + \mathbf{s_{\bar q}} = \mathbf{j_{\bar q}}$ and

TABLE III. S-wave spin-averaged mass.

nL	ν	Hydrogenic μ (GeV)	$\lvert R(0)\rvert$ GeV$^{3/2}$	$E(\mu)$ (GeV)	Gaussian μ (GeV)	$\lvert R(0)\rvert$ GeV$^{3/2}$	$E(\mu)$ (GeV)	Experiment [1] (GeV)	Theory (GeV)
1S	0.5	0.902	0.606	2.076	0.349	0.309	2.076		2.076 [3]
	1.0	1.203	0.933	2.076	0.467	0.480	2.076	2.076 [1]	2.082 [6]
	1.5	1.480	1.273	2.076	0.576	0.658	2.076		2.074 [9]
	2.0	1.697	1.564	2.076	0.663	0.811	2.076		2.072 [14]
									2.075 [16]
2S	0.5	0.745	0.227	2.364	0.199	0.109	2.346		2.779 [3]
	1.0	1.248	0.493	2.682	0.336	0.239	2.713		2.700 [6]
	1.5	1.688	0.775	3.163	0.480	0.408	3.207		2.706 [9]
	2.0	1.991	0.993	4.022	0.601	0.572	3.905		2.695 [14]
									2.720 [16]
3S	0.5	0.735	0.149	2.505	0.159	0.070	2.489		3.323 [3]
	1.0	1.359	0.373	3.099	0.298	0.179	3.175		3.165 [6]
	1.5	1.892	0.613	4.140	0.452	0.334	4.180		3.165 [6]
	1.5	1.892	0.613	4.140	0.450	0.334	4.180		3.076 [9]
	2.0	2.245	0.793	6.219	0.583	0.488	5.765		3.236 [16]
4S	0.5	0.746	0.114	2.605	0.139	0.053	2.594		
	1.0	1.461	0.312	3.451	0.278	0.149	3.567		3.356 [9]
	1.5	2.058	0.522	5.108	0.437	0.294	5.086		3.665 [16]
	2.0	2.450	0.678	8.694	0.571	0.438	7.687		

then adding the spin of the heavier quark, $\mathbf{S_Q} + \mathbf{j_{\bar{q}}} = \mathbf{J}$. Independently of the total spin J projection one has

$$|^{2L+1}L_{L+1}\rangle = |J = L+1, S = 1\rangle, \tag{12}$$

$$|^{2L+1}L_L\rangle = \sqrt{\frac{L}{L+1}}|J=L, S=1\rangle + \sqrt{\frac{L+1}{2L+1}}|J=L, S=0\rangle, \tag{13}$$

$$|^{2L-1}L_L\rangle = \sqrt{\frac{L+1}{2L+1}}|J=L, S=1\rangle - \sqrt{\frac{L}{2L+1}}|J=L, S=0\rangle, \tag{14}$$

where $|J, S\rangle$ are the state vectors with the given values of the total quark spin $\mathbf{S} = \mathbf{s_{\bar{q}}} + \mathbf{S_Q}$, so that the potential terms of the order of $1/m_{\bar{q}}m_Q$, $1/m_Q^2$, lead to the mixing of the levels with the different $j_{\bar{q}}$ values at the given J values. The tensor forces [the last term in Eq. (11)] are equal to zero at $L = 0$ or $S = 0$.

Using these relations for the level shifts, calculated in the perturbation theory at $S = 1$, one gets the following formulas [32]:

$$\Delta E_{n^1S_0} = -\alpha_S \frac{2}{3m_{\bar{q}}m_Q}|R_{nS}(0)|^2, \tag{15}$$

$$\Delta E_{n^3S_1} = \alpha_S \frac{2}{9m_{\bar{q}}m_Q}|R_{nS}(0)|^2, \tag{16}$$

$$\Delta E_{n^3P_2} = \alpha_S \frac{6}{5m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{4}\left(\frac{1}{m_{\bar{q}}^2} + \frac{1}{m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{17}$$

$$\Delta E_{n^3P_0} = -\alpha_S \frac{4}{m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle - \frac{1}{2}\left(\frac{1}{m_{\bar{q}}^2} + \frac{1}{m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{18}$$

$$\Delta E_{n^3D_3} = \alpha_S \frac{52}{21m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{2}\left(\frac{1}{m_{\bar{q}}^2} + \frac{1}{m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{19}$$

$$\Delta E_{n^3D_1} = -\alpha_S \frac{92}{21m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle - \frac{3}{4}\left(\frac{1}{m_{\bar{q}}^2} + \frac{1}{m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{20}$$

where $R_{nS}(0)$ denotes the radial wave functions at $L = 0$and $\langle\cdots\rangle$ denotes the average values, calculated under the wave functions $R_{nL}(r)$. The mixing matrix elements have the forms [32]

$$\langle^3P_1|\Delta E|^3P_1\rangle = \frac{-2\alpha_S}{9m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \left(\frac{1}{4m_{\bar{q}}^2} - \frac{1}{12m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{21}$$

$$\langle^1P_1|\Delta E|^1P_1\rangle = \frac{-4\alpha_S}{9m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \left(-\frac{1}{2m_{\bar{q}}^2} + \frac{1}{6m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{22}$$

$$\langle^3P_1|\Delta E|^1P_1\rangle = -\alpha_S\frac{2\sqrt{2}}{9m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle - \frac{\sqrt{2}}{6m_q^2}\left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{23}$$

$$\langle^3D_2|\Delta E|^3D_2\rangle = \frac{-4\alpha_S}{15m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \left(\frac{2}{5m_{\bar{q}}^2} - \frac{1}{5m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{24}$$

$$\langle^1D_2|\Delta E|^1D_2\rangle = \frac{-8\alpha_S}{15m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle + \left(-\frac{3}{4m_{\bar{q}}^2} + \frac{9}{20m_Q^2}\right) \times \left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle, \tag{25}$$

$$\langle^3D_2|\Delta E|^1D_2\rangle = -\alpha_S\frac{2\sqrt{6}}{15m_{\bar{q}}m_Q}\left\langle\frac{1}{r^3}\right\rangle - \frac{\sqrt{6}}{10m_Q^2}\left\langle -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right\rangle. \tag{26}$$

The heavy-light flavored meson states with $J = L$ are a mixture of spin-triplet $|^3L_L\rangle$ and spin-singlet $|^1L_L\rangle$ states, $J = L = 1, 2, 3, \ldots$

$$|\psi_J\rangle = |^1L_L\rangle\cos\phi + |^3L_L\rangle\sin\phi, \tag{27}$$

$$|\psi'_J\rangle = -|^1L_L\rangle\sin\phi + |^3L_L\rangle\cos\phi, \tag{28}$$

where ϕ is the mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in Eq. (9). The masses of the physical states were obtained by diagonalizing the mixing matrix obtained using Eqs. (21)–(26). The calculated values of the mass spectra of the D_s meson are listed in Table V. We are following spectroscopic notation $n^{2S+1}L_J$ in Tables V.

The results from the Gaussian wave function are very close to experimental as well as to other theoretical results (see Table V) but in case of the hydrogenic wave function the ground state masses are far from experimental results. This is primarily because of the large value of the wave function at the origin (see Table III). The difference of masses between pseudoscalar and vector masses for the $1S$ state is 447 MeV compared to 147 MeV in the case of Gaussian, 144 MeV and 142 MeV in the cases of the experimental [1] and Ref. [16], respectively. Similarly for the $2S$ state it is 156 MeV in the case of hydrogenic compared to 39 MeV and 43 MeV in the cases of the Gaussian and Ref. [16], respectively. Similar trend is seen in case of $3S$ and $4S$ states as well.

III. DECAY CONSTANTS ($f_{P/V}$)

The decay constants of mesons are important parameters in the study of leptonic or nonleptonic weak decay processes. The decay constants of pseudoscalar (f_P) and vector (f_V) mesons are obtained by parametrizing the matrix elements of weak current between the corresponding mesons and the vacuum as

$$\langle 0|\bar{Q}\gamma^{\mu}\gamma_5 Q|P_{\mu}(k)\rangle = i f_P k^{\mu}, \tag{29}$$

$$\langle 0|\bar{Q}\gamma^{\mu} Q|V(k,\epsilon)\rangle = f_V M_V \epsilon^{\mu}, \tag{30}$$

where k is the meson momentum, ϵ^{μ} and M_V are the polarization vector and mass of the vector meson. In the relativistic quark model, the decay constant can be expressed through the meson wave function $\Phi_{P,V}(p)$ in the momentum space as [4,5]

$$f_{P/V} = \left(\frac{12}{M_{P,V}}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\left(\frac{E_Q(p)+m_Q}{2E_Q(p)}\right)^{1/2}\left(\frac{E_{\bar{q}}(p)+m_{\bar{q}}}{2E_{\bar{q}}(p)}\right) \times\left\{1+\frac{\lambda p^2}{[E_Q(p)+m_Q][E_{\bar{q}}(p)+m_{\bar{q}}]}\right\}\Phi_{P,V}(p), \tag{31}$$

with $\lambda_P = -1$ and $\lambda_V = -1/3$. In the nonrelativistic limit $\frac{p^2}{m^2}\rightarrow 0$, this expression reduces to the well-known relation between $f_{P,V}$ and the ground state wave function at the origin $\psi_{P/V}(0)$, the Van-Royen-Weisskopf formula [33]. Incorporating a first-order QCD correction factor, we compute the decay constants using the relation

$$f^2_{P/V} = \frac{12|\psi_{P/V}(0)|^2}{M_{P/V}}\bar{C}^2(\alpha_S), \tag{32}$$

where $\bar{C}^2(\alpha_S)$ is the QCD correction factor given by [34]

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right]. \tag{33}$$

The computed f_P and f_V for the D_s meson using Eq. (32) are tabulated in Tables VI and VII. Equation (31) also gives the inequality [41]

$$\sqrt{m_v}f_v \geq \sqrt{m_p}f_p. \tag{34}$$

Our results are in accordance with Eq. (34). We have listed the results with and without corrections in Tables VI and VII. The bracketed values represent the corrected decay constant.

Decay constants calculated using the hydrogenic wave function are overestimated largely due to the value of the wave function at the origin. Results of the decay constants calculated using the Gaussian wave function are compared with experimental as well as other theoretical predictions and it is found that for the $1S$ state the decay constant with

TABLE IV. P and D-wave spin-averaged mass.

nL	ν	Hydrogenic μ (GeV)	Hydrogenic $E(\mu)$ (GeV)	Gaussian μ (GeV)	Gaussian $E(\mu)$ (GeV)	Experiment (GeV)	Theory (GeV)
$1P$	0.5	0.721	2.332	0.225	2.309		2.568 [3]
	1.0	1.174	2.574	0.365	2.540		2.531 [6]
	1.5	1.628	2.838	0.499	2.806	2.514 [1]	2.538 [9]
	2.0	2.011	3.131	0.608	3.137		2.511 [14]
							2.537 [16]
$2P$	0.5	0.723	2.487	0.169	2.464		3.142 [3]
	1.0	1.326	3.030	0.310	3.026		3.008 [6]
	1.5	1.925	3.798	0.460	3.800		2.954 [9]
	2.0	2.358	5.129	0.583	4.975		2.991 [14]
							3.119 [16]
$1D$	0.5	0.703	2.451	0.186	2.425		2.917 [3]
	1.0	1.250	2.892	0.331	2.852		2.873 [6]
	1.5	1.814	3.421	0.474	3.382		2.850 [9]
	2.0	2.291	4.091	0.587	4.129		2.814 [14]
							2.950 [16]
$2D$	0.5	0.726	2.570	0.154	2.544		
	1.0	1.401	3.303	0.296	3.277		3.288 [6]
	1.5	2.105	4.365	0.449	4.322		3.161 [9]
	2.0	2.628	6.235	0.571	6.028		3.236 [14]
							3.436 [16]

TABLE V. Mass spectrum of D_s meson (in GeV).

States	This work		Experiment [1]	Ref. [16]	Ref. [9]	Ref. [6]	Ref. [3]	Ref. [14]	Ref. [8]	Ref. [10]
	Hydrogenic	Gaussian								
1^1S_0	1.801	1.970	1.968	1.969	1.975	1.940	1.965	1.969	1.968	1.969
1^3S_1	2.248	2.117	2.112	2.111	2.108	2.130	2.113	2.107	2.110	2.109
D_{s0} (1^3P_0)	2.335	2.444	2.318	2.509	2.455	2.380	2.487	2.344	2.387	2.369
D'_{s1}	2.569	2.540	2.535	2.574	2.522	2.520	2.605	2.510	2.536	2.534
D_{s1}	2.529	2.530	2.460	2.536	2.502	2.510	2.535	2.488	2.521	2.507
D_{s2} (1^3P_2)	2.652	2.566	2.573	2.571	2.586	2.580	2.581	2.559	2.573	2.584
2^1S_0	2.569	2.684		2.688	2.659	2.610	2.700	2.640	2.656	2.823
2^3S_1	2.725	2.723	2.710^{+12}_{-7}	2.731	2.722	2.730	2.806	2.714	2.757	2.879
1^3D_1	2.874	2.873		2.913	2.838	2.820	2.900	2.804		
$1D'_2$	2.914	2.896		2.931	2.845	2.860	2.913	2.849		
$1D_2$	2.877	2.816		2.961	2.856	2.880	2.953	2.788		
1^3D_3	2.891	2.834	2.862^{+6}_{-3}	2.971	2.857	2.900	2.925	2.811		
2^3P_0	2.628	2.947		3.054	2.901	2.900	3.067	2.830		
$2P_1$	3.046	3.019	3.044^{+30}_{-9}	3.067	2.928	3.000	3.114	2.958		
$2P'_1$	2.913	3.023		3.154	2.942	3.010	3.165	2.995		
2^3P_2	3.171	3.048		3.142	2.98	3.060	3.157	3.040		
3^1S_0	3.030	3.158		3.219	3.044	3.090	3.259			
3^3S_1	3.123	3.180		3.242	3.087	3.190	3.345			
2^3D_1	3.243	3.292		3.383	3.144	3.250		3.217		
$2D'_2$	3.303	3.312		3.403	3.172	3.280		3.260		
$2D_2$	3.296	3.248		3.456	3.167	3.290		3.217		
2^3D_3	3.318	3.263		3.469	3.157	3.310		3.240		
4^1S_0	3.402	3.556		3.652	3.331					
4^3S_1	3.467	3.571		3.669	3.364					

QCD correction is around 25 MeV less than that of the Heavy Flavor Averaging Group [35] and lattice results [36].

IV. LEPTONIC BRANCHING FRACTIONS

The leptonic branching fractions for the (1^1S_0) D_s mesons are obtained using the formula

$$\mathrm{BR} = \Gamma \times \tau, \tag{35}$$

where Γ (leptonic decay width) for D_s^+ is given by [22]

$$\Gamma(D_s^+ \to l^+\nu_l) = \frac{G_F^2}{8\pi} f_{D_s}^2 |V_{cs}|^2 m_l^2 \left(1 - \frac{m_l^2}{M_{D_s}^2}\right)^2 M_{D_s} \tag{36}$$

and $\tau_{D_s} = 0.5$ ps [1]. For the calculation of the branching fractions using Eq. (36) we employ the calculated values of the pseudoscalar decay constants with QCD corrections from Table VI and the masses obtained from Table V. Results are tabulated in Table VIII.

V. ELECTROMAGNETIC TRANSITIONS

A. Electric dipole transitions

The radiative widths are calculated in the dipole approximation. The E1 matrix elements are determined by using the variational radial wave functions of initial and final state and explicitly performing the angular integration given by [2]

TABLE VI. Pseudoscalar decay constants of D_s mesons (in GeV).

1S		2S		3S		4S	
Hydrogenic	Gaussian	Hydrogenic	Gaussian	Hydrogenic	Gaussian	Hydrogenic	Gaussian
0.533 (0.379)	0.315 (0.224)	0.286 (0.204)	0.141 (0.100)	0.205 (0.146)	0.098 (0.070)	0.163 (0.116)	0.077 (0.055)
0.254 ± 0.006 [35]							
0.248 ± 0.002 [36]							
0.235 ± 0.024 [37]							

TABLE VII. Vector decay constants of D_s mesons (in GeV).

1S		1S		3S		4S	
Hydrogenic	Gaussian	Hydrogenic	Gaussian	Hydrogenic	Gaussian	Hydrogenic	Gaussian
0.652 (0.464)	0.329 (0.234)	0.296 (0.211)	0.142 (0.101)	0.208 (0.148)	0.098 (0.070)	0.165 (0.117)	0.077 (0.055)
0.335 [4,5]							
$0.326^{+0.021}_{-0.017}$ [38]							
0.254 [39]							
0.242 [40]							
0.298 ± 0.011 [41]							

$$\Gamma_{fi} = \frac{4\alpha}{9}\left(\frac{e_Q m_{\bar q} - e_{\bar q} m_Q}{m_{\bar q} + m_Q}\right)^2 k^3 |\langle f|r|i\rangle|^2 \frac{E_f}{M_i} \times \begin{cases} 1 & \text{for } {}^3P_J \to {}^3S_1 \\ 1 & \text{for } {}^1P_1 \to {}^1S_0 \\ (2J+1)/3 & \text{for } {}^3S_1 \to {}^3P_J \\ 3 & \text{for } {}^1S_0 \to {}^1P_1 \end{cases}. \tag{37}$$

Here, α is the fine structure constant, k is the photon energy, $e_{\bar q}$ and e_Q are the quark charges in units of the proton charge, E_f is the energy of the final meson state, M_i is the mass of the initial meson state, and $m_{\bar q}$ and m_Q are the quark masses.

The E1 radiative transition widths are listed in Table IX. In the absence of any precise experimental measurements we have compared our calculated results with Refs. [14,15,42] which are not in mutual agreement.

B. Magnetic dipole transitions

The M1 rate for transitions between s-wave levels is given by [14,43,44]

$$\Gamma_{\rm M1}(i \to f + \gamma) = \frac{16\alpha}{3}\mu^2 k^3 (2J_f + 1)|\langle f|j_0(kr/2)|i\rangle|^2,$$

where the magnetic dipole moment is

$$\mu = \frac{m_{\bar q} e_Q - m_Q e_{\bar q}}{4 m_{\bar q} m_Q}$$

and k is the photon energy. Rates for the allowed transitions between spin-triplet and spin-singlet states are given in Table X

We have noted that the electromagnetic transitions in general are found to be very sensitive to the choice of the constituent quark masses employed in various models. It is to be noted also that we have not imposed the orthogonality condition on the radial wave functions of the initial and

TABLE VIII. Leptonic branching fractions.

$BR_\tau \times 10^{-2}$		$BR_\mu \times 10^{-3}$		$BR_e \times 10^{-8}$	
Hydrogenic	Gaussian	Hydrogenic	Gaussian	Hydrogenic	Gaussian
0.22	4.22	11.09	4.25	26.1	10.0
5.6 ± 0.4 [1]		5.8 ± 0.4 [1]		$<1.2 \times 10^{-4}$ [1]	

TABLE IX. E1 transitions in the D_s meson.

	k (MeV)		Γ (keV)					
Transition	Hydrogenic	Gaussian	Hydrogenic	Gaussian	Ref. [15]	Ref. [2]	Ref. [42]	Ref. [14]
$D_{s2} \to D_s^\star \gamma$	0.374	0.410	9.3	8.7	8.8	44.1	19	
$D'_{s1} \to D_s^\star \gamma$	0.301	0.388	4.5	6.2	4.76	8.90	5.6	
$D'_{s1} \to D_s \gamma$	0.654	0.506	43.8	13.5	3.49	54.5	15	
$D_{s1} \to D_s \gamma$	0.624	0.498	4.0	2.6	4.90	12.8	6.2	
$D_{s1} \to D_s^\star \gamma$	0.266	0.380	0.3	1.2	0.13	15.5	5.5	
$D_{s0} \to D_s^\star \gamma$	0.086	0.305	0.1	3.6	1.0	4.92	1.9	
$2^3S_1[D_s(2710)] \to D_{s2}\gamma$	0.072	0.153	0.3	0.7				0.1
$2^3S_1[D_s(2710)] \to D_{s0}\gamma$	0.362	0.264	6.5	0.7				6.9

TABLE X. M1 transitions in the D_s meson.

Transition	k (GeV) Hydrogenic	Gaussian	Γ (keV) Hydrogenic	Gaussian	Ref. [2]	Ref. [45]
$1^3S_1 \to 1^1S_0\gamma$	0.403	0.063	5.980	0.3017	1.91	0.2
$2^3S_1 \to 2^1S_0\gamma$	0.152	0.008	0.352	0.0061		
$3^3S_1 \to 3^1S_0\gamma$	0.091	0.003	0.078	0.0011		
$4^3S_1 \to 4^1S_0\gamma$	0.065	0.002	0.029	0.0004		

final states. This may be important for some decays which are sensitive to the presence of nodes in the initial and final state amplitudes [46].

VI. CONCLUSION

In this paper, we have done a comparative study (hydrogenic and Gaussian) of the mass spectra and decay properties of the D_s meson in the framework of phenomenological quark-antiquark potential (Coulomb plus power) model. The spin-averaged masses of the ground state as well as excited states are calculated and listed in Tables III and IV. The spin-hyperfine, spin-orbit, and tensor interaction are employed to Eq. (6) to get the pseudoscalar and vector masses listed in Table V. The decay constants which are very important for weak decays are calculated with QCD correction given in Table VI. The leptonic branching fractions and electromagnetic transitions are also evaluated in this scheme and are listed in Tables VIII, IX, and X.

In the case of the D_s meson, while the problem for $L = 1$ states remains with most models including the present work, it is certainly true that the newly observed states $D_{s1}(2710)$, $D_{sJ}(2860)$, and $D_{sJ}(3040)$ will provide further help toward finding a consistent approach once their J^P is confirmed. We have compared these states with 2^3S_1, 1^3D_3, and $2P_1$ states in our predictions. The calculated results are reasonably close to experimental results as well as to Refs. [9,14].

The spin-averaged masses of this meson are fairly close to the experimental results and other theoretical predictions at the same potential index $\nu = 1$ in both the cases, but the spin-spin interaction contribution in hydrogenic wave function is very large because of which the difference between the pseudoscalar and vector mesons is also large. This also explains why the decay constants and branching ratios are overestimated. The mass spectra for excited states of the D_s meson calculated by several authors are far from each other with a difference of 100 to 150 MeV, but the ground state pseudoscalar and vector meson masses of all the models are fairly close to the experimental results [1].

In conclusion we have successfully studied the mass spectra, decay constants, leptonic branching fractions, E1 and M1 radiative transitions of the D_s meson using the hydrogen-like wave function and the Gaussian wave function. Overall results suggest that the Gaussian wave function adequately describes the heavy-light system compared to the hydrogenic wave function with the same set of chosen parameters. We have also noted that, in the case of the hydrogenic wave function, if the value of A is taken to be lower than what we have used along with a different set of mass parameters then the predictions could be improved. Now we would like to extend this model to study similar properties of B, B_s, and B_c mesons.

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