PHYSICAL REVIEW D 84, 074019 (2011)

Nonleptonic two-body decays of charmed mesons

Fu-Sheng Yu, Xiao-Xia Wang, and Cai-Dian Lü

Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences,

Beijing 100049, People's Republic of China

(Received 28 January 2011; published 13 October 2011)

Nonleptonic decays of charmed mesons into two pseudoscalar mesons or one pseudoscalar meson and one vector meson are studied on the basis of a generalized factorization method considering the resonance effects in the pole model for the annihilation contributions. Large strong phases between different topological diagrams are considered in this work, simply taking the phase in the coefficients a_i . We find that the annihilation-type contributions calculated in the pole model are large in both of the *PP* and *PV* modes, which make our numerical results agree with the experimental data better than those previous calculations.

DOI: 10.1103/PhysRevD.84.074019

PACS numbers: 13.25.Ft, 11.15.Tk, 11.30.Hv, 12.39.St

I. INTRODUCTION

Nonleptonic decays of charmed mesons are very interesting as they can provide useful information on flavor mixing, *CP* violation, and strong interactions [1]. They may also shed light on any new physics signal through $D^0 - \overline{D}^0$ mixing and rare decays [2–7]. The CLEO-c and two *B* factories experiments have given many new results on this subject and more are expected soon from the BES-III experiment. Besides, theoretical studies have been in progress for decades.

With the heavy quark effective theory, many QCDinspired approaches, such as the QCD factorization approach [8], the perturbative QCD approach [9], and the soft-collinear effective theory [10], successfully describe the hadronic *B* decays. However, this is not the case for the *D* meson decays. These approaches do not work well here, due to the mass of charm quark, of order 1.5 GeV, which is not heavy enough for a sensible heavy quark expansion, neither light enough to apply the chiral perturbation theory.

After decades of studies, the factorization approach is still one of the effective ways to deal with the two-body charmed meson decays [11]. However, it is well known that some difficulties exist in the naive factorization approach: the Wilson coefficients $a_1(\mu)$ and $a_2(\mu)$ of effective operators are renormalization scale and γ_5 -scheme dependent; and the color-suppressed processes are not calculated well due to the smallness of a_2 , etc. In order to solve these problems, the so-called generalized factorization approach was proposed [12]. The Wilson coefficients $a_1(\mu)$ and $a_2(\mu)$ are not from direct calculation any more but are effective coefficients to accommodate the important nonfactorizable corrections [13]. In these naive or generalized factorization approaches, there is almost no strong phase. But large strong phases have been found from D decay experiments. A corresponding large relative strong phase between the factorization coefficients a_1 and a_2 has been discussed in [14,15].

Unsatisfied by the shortcomings of the factorization approach, the model-independent diagrammatic approach, with various topological amplitudes extracted from the data, is recently applied to two-body hadronic D decays [16–19]. They use SU(3) symmetry in their analysis to avoid model calculations. All the parameters are fitted from experiments to give a better agreement with the experimental data but with less predictive power. More precise predictions are limited in this approach due to the uncontrolled SU(3) breaking effect [15]. These analyses also show that large annihilation-type contributions are needed to explain the data, which cannot be calculated in the naive or generalized factorization approaches. The kind of pure annihilation type D meson decays also needs to be systematically analyzed [20].

In another aspect, the hadronic picture description of nonleptonic weak decays has a longer history, because of their nonperturbative feature. Based on the idea of vector dominance, which is discussed on strange particle decays [21], the pole-dominance model of two-body nonleptonic decays is proposed [22]. Beyond the vector dominance pole, this model also involves scalar, pseudoscalar, and axial-vector poles. For simplicity, only the lowest-lying poles are considered. This model has already been applied to charmed meson and bottom meson decays [22,23], where it is approved that this model is more or less equivalent to the factorization approach in the first order approximation.

In this work, we will use the generalized factorization approach but with a relative strong phase between the Wilson coefficients a_1 and a_2 to accommodate the nonfactorizable contributions. For the uncalculable annihilationtype contributions, we use the pole model. In this case, we can really calculate most of the important contributions in the hadronic *D* decays, which are demonstrated in the model-independent diagrammatic analysis [19].

The outline of this paper is as follows. In Sec. II, we give the formulas of this work, showing the generalized factorization approach and the pole-dominance model. In

FU-SHENG YU, XIAO-XIA WANG, AND CAI-DIAN LÜ

Sec. III, our results are given and compared to the experimental data and those of the diagrammatic approach and the calculations considering the final-state interaction of nearby resonances effects. Summary and conclusions are followed.

II. FORMALISM

A. The factorization approach

First we begin with the weak effective Hamiltonian H_{eff} for the $\Delta C = 1$ transition [24]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} (C_1 O_1 + C_2 O_2) + \text{H.c.}, \quad (1)$$

where V_{CKM} is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $C_{1,2}$ are the Wilson coefficients. The current-current operators $O_{1,2}$ are

$$O_{1} = \bar{u}_{\alpha} \gamma_{\mu} (1 - \gamma_{5}) q_{2\beta} \cdot \bar{q}_{3\beta} \gamma^{\mu} (1 - \gamma_{5}) c_{\alpha},$$

$$O_{2} = \bar{u}_{\alpha} \gamma_{\mu} (1 - \gamma_{5}) q_{2\alpha} \cdot \bar{q}_{3\beta} \gamma^{\mu} (1 - \gamma_{5}) c_{\beta},$$
(2)

where α , β are color indices, and $q_{2,3}$ are d or s quarks.

The color-favored emission diagram corresponding to $D \rightarrow PP$ decays, with *P* representing a pseudoscalar meson, is shown in Fig. 1(a). Under the factorization hypothesis, the transition matrix element of hadronic two-body charmed meson decays is factorized into two parts [11]:

$$\langle P_1 P_2 | \mathcal{H}_{\text{eff}} | D \rangle = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} a_1 \langle P_2 | \bar{u} \gamma_\mu (1 - \gamma_5) q_2 | 0 \rangle$$
$$\times \langle P_1 | \bar{q}_3 \gamma^\mu (1 - \gamma_5) c | D \rangle. \tag{3}$$

Similarly, the contribution of the color-suppressed diagram shown in Fig. 1(b) is given as

$$\langle P_1 P_2 | \mathcal{H}_{\text{eff}} | D \rangle = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} a_2 \langle P_1 | \bar{q}_3 \gamma_\mu (1 - \gamma_5) q_2 | 0 \rangle$$
$$\times \langle P_2 | \bar{u} \gamma^\mu (1 - \gamma_5) c | D \rangle, \tag{4}$$

where



FIG. 1. Emission-type diagrams in the factorization approach.

$$a_{1}(\mu) = C_{2}(\mu) + \frac{C_{1}(\mu)}{N_{c}},$$

$$a_{2}(\mu) = C_{1}(\mu) + \frac{C_{2}(\mu)}{N_{c}}$$
(5)

correspond to the color-favored tree diagram (\mathcal{T}) and the color-suppressed diagram (\mathcal{C}) , respectively, in the naive factorization, with the number of colors $N_c = 3$. They are assumed to be universal and process-independent in the native factorization approach. The current matrix elements in Eqs. (3) and (4) are evaluated in terms of transition form factors and decay constants. For $D \rightarrow PP$ decays, the form factor is defined as follows:

$$\langle P(k) | \bar{q}_{3} \gamma_{\mu} (1 - \gamma_{5}) c | D(p) \rangle$$

$$= \left[(p + k)_{\mu} - \frac{m_{D}^{2} - m_{P}^{2}}{q^{2}} q_{\mu} \right] F_{1}^{D \to P}(q^{2})$$

$$+ \frac{m_{D}^{2} - m_{P}^{2}}{q^{2}} q_{\mu} F_{0}^{D \to P}(q^{2}), \qquad (6)$$

where q = p - k, and F_i are the corresponding transition form factors. The decay constants f_P of pseudoscalar mesons are defined as

$$\langle P(q)|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|0\rangle = if_P q_\mu. \tag{7}$$

In terms of decay constant and transition form factors, the decay amplitude of Figs. 1(a) and 1(b) are then

$$\langle P_1 P_2 | \mathcal{H}_{\text{eff}} | D \rangle_T = i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} a_1 f_{P_2} (m_D^2 - m_{P_1}^2) F_0^{D \to P_1} (m_{P_2}^2), \quad (8)$$

$$\langle P_1 P_2 | \mathcal{H}_{eff} | D \rangle_C$$

$$= i \frac{G_F}{\sqrt{2}} V_{CKM} a_2 f_{P_1} (m_D^2 - m_{P_2}^2) F_0^{D \to P_2} (m_{P_1}^2).$$
(9)

The diagrams for $D \rightarrow PV$ decays, with V denoting a vector meson, are shown in Figs. 1(c) and 1(d). If the outemitted particle is a pseudoscalar meson, the matrix element of Fig. 1(c) is

$$\langle PV | \mathcal{H}_{eff} | D \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} a_1 \langle P | \bar{u} \gamma_\mu (1 - \gamma_5) q_2 | 0 \rangle \\ \times \langle V | \bar{q}_3 \gamma^\mu (1 - \gamma_5) c | D \rangle.$$
(10)

The $D \rightarrow V$ transition form factors are usually defined as

$$\langle V(k) | \bar{q}_{3} \gamma_{\mu} (1 - \gamma_{5}) c | D(p) \rangle$$

$$= \frac{2}{m_{D} + m_{V}} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^{\rho} k^{\sigma} V(q^{2})$$

$$- i \left(\epsilon_{\mu}^{*} - \frac{\epsilon^{*} \cdot q}{q^{2}} q_{\mu} \right) (m_{D} + m_{V}) A_{1}^{D \rightarrow V}(q^{2})$$

$$+ i \left((p + k)_{\mu} - \frac{m_{D}^{2} - m_{V}^{2}}{q^{2}} q_{\mu} \right) \frac{\epsilon^{*} \cdot q}{m_{D} + m_{V}} A_{2}^{D \rightarrow V}(q^{2})$$

$$- i \frac{2m_{V}(\epsilon^{*} \cdot q)}{q^{2}} q_{\mu} A_{0}^{D \rightarrow V}(q^{2}), \qquad (11)$$

where ε^* is the polarization vector of the vector meson, and A_i and V are corresponding transition form factors. Utilizing the form factor definitions we get the result for Eq. (10):

$$\langle PV|\mathcal{H}_{\text{eff}}|D\rangle = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} a_1 f_P m_V A_0^{D \to V}(m_P^2) 2(\varepsilon^* \cdot p_D).$$
(12)

If a vector meson is out emitted, the matrix element of Fig. 1(c) is

$$\langle PV | \mathcal{H}_{eff} | D \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} a_1 \langle V | \bar{u} \gamma^{\mu} (1 - \gamma_5) q_2 | 0 \rangle$$
$$\times \langle P | \bar{q}_3 \gamma^{\mu} (1 - \gamma_5) c | D \rangle$$
$$= \frac{G_F}{\sqrt{2}} V_{CKM} a_1 f_V m_V F_1^{D \to P} (m_V^2) 2(\varepsilon^* \cdot p_D),$$
(13)

where the decay constants f_V of vector mesons are defined as

$$\langle V(q)|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|0\rangle = f_V m_V \varepsilon^*_\mu(q).$$
(14)

For the color-suppressed diagram, Fig. 1(d), we have similar formulas as Eqs. (12) and (13), but with the Wilson coefficient changed from a_1 to a_2 .

As mentioned in the Introduction, the Wilson coefficients a_1 and a_2 are renormalization-scale dependent in the naive factorization approach and it fails to describe the color-suppressed processes with too small $a_2 \approx -0.1$. So the generalized factorization method is proposed to include the nonfactorizable contributions [13],

$$a_1^{\text{eff}} = C_2(\mu) + C_1(\mu) \Big(\frac{1}{N_c} + \chi_1(\mu) \Big),$$

$$a_2^{\text{eff}} = C_1(\mu) + C_2(\mu) \Big(\frac{1}{N_c} + \chi_2(\mu) \Big),$$
(15)

where the terms χ_i characterize the nonfactorizable corrections involving vertex corrections, hard spectator interactions, final-state interactions, resonance effects, etc. These $\chi_i(\mu)$ will compensate the scale and scheme dependence of the Wilson coefficients, so that a_i 's are physical now. Without confusion, we will drop the superscript "eff" in the effective Wilson coefficients for convenience in the following discussions. In the large- N_c approach, the $1/N_c$ terms are discarded [25], equally with a universal non-factorizable term $\chi_1 = \chi_2 = -1/N_c$, hence,

$$a_1 \approx C_2(m_c) = 1.274, \quad a_2 \approx C_1(m_c) = -0.529.$$
 (16)

This implies a null relative strong phase between the two kinds of contributions. However, the experimental data tell us that there should be a large strong phase between a_1 and a_2 . On the other hand, the existence of relative phases is reasonable for the importance of inelastic final-state interactions of the *D* meson decays, in which the on-shell intermediate states contribute imaginary parts. Therefore, we consider a relative phase between the coefficients a_1 and a_2 in this work, so that

$$a_1 = |a_1|, \qquad a_2 = |a_2|e^{i\delta},$$
 (17)

where we set a_1 real for convenience.

B. Pole-dominance model

The annihilation-type diagrams are neglected as an approximation in the factorization model. However, considerable contributions come from the weak annihilation diagrams in the D decays, which can be demonstrated by the difference of lifetime between D^0 and D^+ . Hence, we will calculate them in a single pole-dominance model. For simplicity, only the lowest-lying poles are considered in the single-pole model. Taking $D^0 \rightarrow \pi^+ K^{*-}$ as an example, the annihilation-type diagram in the pole model is shown in Fig. 2(a). D^0 goes into \overline{K}^0 via the weak interaction in Eq. (1) shown in terms of quark lines in Fig. 2(b), and then decays into $\pi^+ K^{*-}$ through the strong interaction. Angular momentum should be conserved at the weak vertex and all conservation laws be preserved at the strong vertex. So it is a pseudoscalar meson as a resonant state for $D \rightarrow PV$ decays. The weak matrix element is evaluated in the vacuum insertion approximation [23],

$$\langle \bar{K}^{0} | \mathcal{H}_{\text{eff}} | D^{0} \rangle = \frac{G_{F}}{\sqrt{2}} V_{cs}^{*} V_{ud} a_{E}^{PV} \langle \bar{K}^{0} | \bar{s} \gamma_{\mu} (1 - \gamma_{5}) d | 0 \rangle$$
$$\times \langle 0 | \bar{u} \gamma^{\mu} (1 - \gamma_{5}) c | D^{0} \rangle$$
$$= \frac{G_{F}}{\sqrt{2}} V_{cs}^{*} V_{ud} a_{E}^{PV} f_{K} f_{D} m_{D}^{2}, \qquad (18)$$

where the subscript *E* of the Wilson coefficient a_E^{PV} denotes a W-exchange diagram for $D \rightarrow PV$, otherwise a_A^{PV}



FIG. 2. Annihilation diagrams in the pole model.

FU-SHENG YU, XIAO-XIA WANG, AND CAI-DIAN LÜ

corresponds to W-annihilation contributions. In fact, the effective Wilson coefficients of the W-annihilation diagrams and W-exchange diagrams have the same form as a_1 and a_2 in Eq. (15), that is,

$$a_{E} = C_{1}(\mu) + C_{2}(\mu) \left(\frac{1}{N_{c}} + \chi_{E}(\mu) \right),$$

$$a_{A} = C_{2}(\mu) + C_{1}(\mu) \left(\frac{1}{N_{c}} + \chi_{A}(\mu) \right),$$
(19)

where $\chi_{A(E)}$ represents the nonfactorizable contributions in the annihilation (exchange) process. Since the nonfactorizable contributions in these kinds of diagrams are large and with relatively different strong phases, we use different symbols to avoid confusing in our approach for these collective effective Wilson coefficients as a_E and a_A , respectively. Strong phases relative to the emission diagrams are considered in the Wilson coefficients. The effective strong coupling constant of \bar{K}^0 to $\pi^+ K^{*-}$ is defined through the Lagrangian

$$\mathcal{L}_{VPP} = ig_{VPP} V^{\mu} (P \overleftrightarrow{\partial}_{\mu} P), \qquad (20)$$

where g_{VPP} is dimensionless. Inserting the propagator of the intermediate \bar{K}^0 meson, the decay amplitude is

$$\langle \pi^+ K^{*-} | \mathcal{H}_{\text{eff}} | D^0 \rangle = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_E^{PV} f_K f_D g_{K^* K \pi} \\ \times \frac{m_D^2}{m_D^2 - m_K^2} 2(\varepsilon^* \cdot p_D).$$
(21)

Similarly, in $D \rightarrow PP$ decays, it is a scalar meson as a resonant state. The effective strong coupling constant is described by

$$\mathcal{L}_{SPP} = -g_{SPP}m_SSPP, \qquad (22)$$

where m_S is the mass of the scalar meson. Besides, the scalar meson decay constant of the vector current is defined as

$$\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_S p_\mu. \tag{23}$$

Therefore, the corresponding matrix element is

$$\langle PP|\mathcal{H}_{\text{eff}}|D\rangle = -i\frac{G_F}{\sqrt{2}}V_{\text{CKM}}a_A(a_E)f_Sf_Dg_{SPP}\frac{m_D^2m_S}{m_D^2 - m_S^2}.$$
(24)

As a convenience of reference, the various decay formulas of individual decay modes are collected in the Appendix. Note that some of the intermediate resonances are unstable particles which have large width and therefore contribute large relative phases. These phases are absorbed in the effective Wilson coefficients a_A and a_E for convenience.

III. NUMERICAL ANALYSIS

A. Input parameters

In order to calculate the emission-type diagrams in the factorization approach, we need to know the transition form factors and meson decay constants. The decay constants of π , K, D, and D_s are taken from the particle data group (PDG) [26], others are from [27], all of which are summarized in Table I. There exist many models to parametrize the transition form factors and their q^2 dependence [28–46]. In this work we shall use the dipole model [28]:

$$F(q^2) = \frac{F(0)}{(1 - \alpha_1 \frac{q^2}{m_{\text{pole}}^2} + \alpha_2 \frac{q^4}{m_{\text{pole}}^4})},$$
(25)

where m_{pole} is the mass of the pole. The corresponding poles are D^* for $F_{0,1}^{D\pi,D\eta^{(\prime)},D_sK}$, D_s^* for $F_{0,1}^{DK,D_s\eta^{(\prime)}}$, D for $A_0^{D\rho,D\omega,D_sK^*}$, and D_s for $A_0^{DK^*,D_s\phi}$. The transition form factors and α_i parameters of D to π and K are taken from the recent CLEO-c measurement [29], $D \rightarrow \eta_q$ are from [30], and others from [28], all of which are shown in Table II.

For the final states involving η or η' , it is convenient to consider the flavor mixing of η_q and η_s with a mixing angle ϕ ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (26)$$

where η_q and η_s are defined by

$$\eta_q = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \qquad \eta_s = s\bar{s}. \tag{27}$$

A recent experimental measurement from the KLOE collaboration gives the mixing angle $\phi = (40.4 \pm 0.6)^{\circ}$ [47].

The decay constants of η or η' are defined by

$$\langle 0|\bar{u}\gamma_{\mu}\gamma_{5}u|\eta(p)\rangle = if_{\eta}^{u}p_{\mu}, \langle 0|\bar{d}\gamma_{\mu}\gamma_{5}d|\eta(p)\rangle = if_{\eta}^{d}p_{\mu}, \langle 0|\bar{s}\gamma_{\mu}\gamma_{5}s|\eta(p)\rangle = if_{\eta}^{s}p_{\mu}, \langle 0|\bar{u}\gamma_{\mu}\gamma_{5}u|\eta'(p)\rangle = if_{\eta'}^{u}p_{\mu}, \langle 0|\bar{d}\gamma_{\mu}\gamma_{5}d|\eta'(p)\rangle = if_{\eta'}^{u}p_{\mu}, \langle 0|\bar{s}\gamma_{\mu}\gamma_{5}s|\eta'(p)\rangle = if_{\eta'}^{s}p_{\mu},$$

$$\langle 0|\bar{s}\gamma_{\mu}\gamma_{5}s|\eta'(p)\rangle = if_{\eta'}^{s}p_{\mu},$$

$$\langle 0|\bar{s}\gamma_{\mu}\gamma_{5}s|\eta'(p)\rangle = if_{\eta'}^{s}p_{\mu},$$

where

TABLE I. Meson decay constants (MeV). Those of π , *K*, *D*, and *D_s* are from PDG [26], others are from [27].

f_{π}	f_K	f_{ρ}	f_{K^*}	f_{ω}	f_{ϕ}	f_D	f_{D_s}
130	156	216	220	187	215	207	258

TABLE II. The *D* meson transition form factors and dipole model parameters $\alpha_{1,2}$. The parameter $\alpha_1 = \alpha_2 + 1$ if only α_2 is shown in the table. The form factors and α_i parameters of *D* to π and *K* are from [29], $D \rightarrow \eta_q$ from [30], and others from [28].

	$F_0^{D\pi}$	$F_1^{D\pi}$	F_0^{DK}	F_1^{DK}	$F_1^{D_SK}$	$F_1^{D_S\eta_s}$
$\overline{F(0)}$	0.67	0.67	0.74	0.74	0.72	0.78
α_2	0.21	0.24	0.30	0.33	0.20	0.23
	$A_0^{D\rho}$	$A_0^{D\omega}$	$A_0^{DK^*}$	$A_0^{D_S K^*}$	$A_0^{D_S\phi}$	
$\overline{F(0)}$	0.66	0.66	0.76	0.67	0.73	
α_2	0.36	0.36	0.17	0.20	0.10	
	$F_0^{D_s \to K}$	$F_0^{D_S \to \eta_s(M_\eta)}$	$F_0^{D_S \to \eta_s(M_{\eta'})}$	$F_1^{D \to \eta_q}$	$F_0^{D \to \eta_q}$	
F(0)	0.72	0.78	0.78	0.69	0.69	
α_1	0.41	0.33	0.21	1.03	0.39	
α_2	0.70	0.38	0.76	0.29	0.01	

$$f^{u}_{\eta} = f^{d}_{\eta} = \frac{1}{\sqrt{2}} f^{q}_{\eta}, \qquad f^{u}_{\eta'} = f^{d}_{\eta'} = \frac{1}{\sqrt{2}} f^{q}_{\eta'}.$$
(29)

With the ansatz in [48], we have

$$f_{\eta}^{q} = f_{q} \cos\phi, \qquad f_{\eta}^{s} = -f_{s} \sin\phi, \qquad (30)$$
$$f_{\eta'}^{q} = f_{q} \sin\phi, \qquad f_{\eta'}^{s} = f_{s} \cos\phi.$$

It is assumed that $f_{q,s}$ obtained from the $\eta_{q,s}$ components of the wave functions are independent of the meson involved. We use that $f_q = (1.07 \pm 0.02)f_{\pi}$ and $f_s = (1.34 \pm 0.06)f_{\pi}$ from [48].

The form factors of $D \rightarrow \eta_q$ in Table II denote that of $D \rightarrow \eta_{u\bar{u},d\bar{d}}$, not $D \rightarrow \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, hence,

$$F^{D \to \eta} = \frac{1}{\sqrt{2}} F^{D \to \eta_q} \cos\phi, \qquad F^{D_s \to \eta} = -F^{D_s \to \eta_s} \sin\phi,$$
(31)

$$F^{D \to \eta'} = \frac{1}{\sqrt{2}} F^{D \to \eta_q} \sin\phi, \qquad F^{D_s \to \eta'} = F^{D_s \to \eta_s} \cos\phi.$$
(32)

In order to calculate the annihilation-type diagrams in the pole model, we have to know the effective strong coupling constants between the intermediate state and two final states. Some of them are obtained directly from experiments. Some others are related to the known ones using SU(3) symmetry. Although the intermediate states are a little off shell, in the pole model they are used as on-shell resonant states. So these on-shell strong couplings are used to calculate the annihilation diagrams in an approximation.

There are many scalar mesons discovered by the experiments. The existence of the lightest scalar nonet with the mass smaller than or close to 1 GeV has been a problem for many years [26]. It is still controversial that they are primarily the four-quark bound states or two-quark scalar states. In this work, we use $K_0^*(1430)$, $a_0(1450)$, $f_0(1370)$, and $f_0(1500)$ as intermediate mesons in the pole model for $D \rightarrow PP$ decays. The decay constant of K_0^* is calculated in several methods, such as the finite-energy sum rule [49], the generalized Nambu-Jona-Lasinio model [50], and so on. We shall use the results in [49],

$$f_{K_{\alpha}^*} = (42 \pm 2) \text{ MeV.}$$
 (33)

For all other scalar mesons, we take the same value of decay constants as K_0^* in the flavor SU(3) limit for simplification.

The corresponding effective strong coupling constant between K_0^* and the final states of $K\pi$ is evaluated by $g_{K_0^*K\pi} = 2.7$ from the decay of $K_0^*(1430)^0 \rightarrow \pi^- K^+$. Other couplings of g_{SPP} are of the same value in the SU(3) limit. For $D \rightarrow PV$ decays, the intermediate states are pseudoscalar mesons with relatively large decay constants shown in Table I. The corresponding effective strong coupling constants are $g_{\rho\pi\pi} = 4.2$ obtained from $\rho^+ \rightarrow \pi^+ \pi^0$ or $\rho^0 \rightarrow \pi^+ \pi^-$, $g_{K^*K\pi} = 4.6$ from $K^*(892)^0 \rightarrow \pi^+ K^-$, and $g_{\phi KK} = 4.5$ from $\phi \rightarrow K^+ K^-$. For decays involving η or η' , we assume that $g_q = 4.2$, $g_s = 4.6$, and $g_{ss} = 4.5$, where g_q couple to the states with only u or d quarks, g_s to two of the states with squark, and g_{ss} to all the three mesons with s quark, so that some effects of SU(3) breaking are considered.

B. $D \rightarrow PP$

For $D \rightarrow PP$ decays, the decay rate is

$$\Gamma(D \to PP) = \frac{p}{8\pi m_D^2} |\mathcal{A}|^2, \qquad (34)$$

where p is the momentum of either meson in the final state in the center-of-mass frame, $p = \sqrt{(m_D^2 - (m_{P_1} + m_{P_2})^2)(m_D^2 - (m_{P_1} - m_{P_2})^2)/2m_D}$. As is done in the naive factorization model, the Wilson

coefficients a_i 's are universal and process independent, except with a relative strong phase for different topological diagrams. Because of the important nonperturbative effect of QCD in the charm system, a_1 and a_2 should deviate a lot from the naive factorization approach. In order to give the most suitable results, we input the following values by hand, which also used the hint from the fit of diagrammatic approach:

$$a_{1} = 1.25 \pm 0.10,$$

$$a_{2} = (0.85 \pm 0.10)e^{i(153\pm10)^{\circ}},$$

$$a_{A} = (0.90 \pm 0.10)e^{i(160\pm10)^{\circ}},$$

$$a_{E} = (2.4 \pm 0.1)e^{i(55\pm10)^{\circ}},$$
(35)

where a_1 is the coefficient of the color-favored emission tree diagrams, a_2 for the color-suppressed emission

TABLE III. Branching ratios for Cabibbo-favored decays of $D \rightarrow PP(\%)$. The predicted branching ratios with both annihilation- and emission-type contributions (all) and with only emission-type contributions (emission) are given together with the experimental data [51], the recent results from the diagrammatic approach [19], and the calculations considering the final-state interaction (FSI) effects of nearby resonances [52] as comparison.

Modes	Br(FSI)	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^+ \rightarrow \pi^+ \bar{K}^0$	2.51	3.08 ± 0.36	3.1 ± 2.0	3.1 ± 2.0	3.074 ± 0.096
$D^0 \rightarrow \pi^+ K^-$	4.03	3.91 ± 0.17	5.8 ± 0.7	3.9 ± 1.0	3.891 ± 0.077
$D^0 \rightarrow \pi^0 \bar{K}^0$	1.35	2.36 ± 0.08	2.1 ± 0.6	2.4 ± 0.7	2.38 ± 0.09
$D^0 \rightarrow \bar{K}^0 \eta$	0.80	0.98 ± 0.05	0.9 ± 0.2	0.8 ± 0.2	0.96 ± 0.06
$D^0 \rightarrow \bar{K}^0 \eta'$	1.51	1.91 ± 0.09	0.3 ± 0.2	1.9 ± 0.3	1.90 ± 0.11
$D_S^+ \rightarrow K^+ \bar{K}^0$	4.79	2.97 ± 0.32	5.1 ± 0.9	3.0 ± 0.9	2.98 ± 0.08
$D_S^+ \to \pi^+ \eta$	1.33	1.82 ± 0.32	3.8 ± 0.4	1.9 ± 0.5	1.84 ± 0.15
$D_S^+ \to \pi^+ \eta'$	5.89	3.82 ± 0.36	2.9 ± 0.6	4.6 ± 0.6	3.95 ± 0.34
$\underline{D_S^+ \to \pi^+ \pi^0}$		0	0	0	< 0.06

diagrams, a_A for the W-annihilation diagrams, and a_E for the W-exchange diagrams. Large relative strong phases are considered in the a_i , due to unneglected inelastic finalstate interactions in the *D* decays. These values of a_1 and a_2 are not far away from the large N_c limit, except that we use quite large strong phases, which are required by the experimental data. As is discussed in the diagrammatic approach [19], the W-annihilation contributions with helicity-suppressed effect are much smaller than those of the W-exchange diagrams. Therefore we use a much larger coefficient of a_E than the W-annihilation coefficient a_A . Besides, we ignore the disconnected hairpin diagrams, *SE* and *SA*, as discussed in [19].

The predicted branching ratios with annihilation-type contributions (all) and without annihilation-type contributions (emission) together with experimental measurements of charmed mesons decay into two pseudoscalar mesons are listed in Tables III, IV, and V, for the Cabibbo-favored decays, the singly Cabibbo-suppressed decays, and the doubly Cabibbo-suppressed decays, respectively. There are many sources of theoretical uncertainties in the calculations. Since the decay constants of pseudoscalar and vector mesons are taken from experiments with very small errors, our numerical results are not very sensitive to the variations of meson decay constants. The branching ratios are truly sensitive to the coefficients of a_1 and a_2 , especially to their relative strong phases. Since the systematic errors from theoretical models are usually difficult to estimate, we show uncertainties at the tables only from the parameters a_i in Eq. (35). For comparison we also show the recent results from the diagrammatic approach [19] and those considering final-state interaction effects of nearby

TABLE IV. Same as Table. III except for singly Cabibbo-suppressed decays of $D \rightarrow PP(\times 10^{-3})$.

Modes	Br(FSI)	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^+ \rightarrow \pi^+ \pi^0$	1.7	0.88 ± 0.10	1.0 ± 0.5	1.0 ± 0.5	1.18 ± 0.07
$D^+ \rightarrow K^+ \bar{K}^0$	8.6	5.46 ± 0.53	11.3 ± 1.6	8.4 ± 1.6	6.12 ± 0.22
$D^+ o \pi^+ \eta$	3.6	1.48 ± 0.26	3.1 ± 1.0	1.6 ± 1.0	3.54 ± 0.21
$D^+ \rightarrow \pi^+ \eta'$	7.9	3.70 ± 0.37	3.7 ± 0.7	5.5 ± 0.8	4.68 ± 0.29
$D^0 ightarrow \pi^+ \pi^-$	1.59	2.24 ± 0.10	3.0 ± 0.4	2.2 ± 0.5	1.45 ± 0.05
$D^0 \rightarrow \pi^0 \pi^0$	1.16	1.35 ± 0.05	0.7 ± 0.2	0.8 ± 0.2	0.81 ± 0.05
$D^0 \rightarrow K^+ K^-$	4.56	1.92 ± 0.08	4.4 ± 0.5	3.0 ± 0.8	4.07 ± 0.10
$D^0 \rightarrow K^0 \bar{K}^0$	0.93	0	0	0.3 ± 0.1	0.64 ± 0.08
$D^0 \rightarrow \pi^0 \eta$	0.58	0.75 ± 0.02	0.7 ± 0.2	1.1 ± 0.3	0.68 ± 0.07
$D^0 ightarrow \pi^0 \eta'$	1.7	0.74 ± 0.02	0.6 ± 0.1	0.6 ± 0.2	0.91 ± 0.13
$D^0 \rightarrow \eta \eta$	1.0	1.44 ± 0.08	1.3 ± 0.4	1.3 ± 0.4	1.67 ± 0.18
$D^0 \rightarrow \eta \eta'$	2.2	1.19 ± 0.07	0.04 ± 0.04	1.1 ± 0.1	1.05 ± 0.26
$D_S^+ \rightarrow \pi^0 K^+$	1.6	0.86 ± 0.09	0.9 ± 0.2	0.5 ± 0.2	0.62 ± 0.23
$D_S^+ \to \pi^+ K^0$	4.3	2.73 ± 0.26	4.1 ± 0.5	2.8 ± 0.6	2.52 ± 0.27
$D_{S}^{+} \rightarrow K^{+} \eta$	2.7	0.78 ± 0.09	0.8 ± 0.5	0.8 ± 0.5	1.76 ± 0.36
$D_S^+ \to K^+ \eta'$	5.2	1.07 ± 0.17	0.7 ± 0.3	1.4 ± 0.4	1.8 ± 0.5

TABLE V. Same as Table. III except for doubly Cabibbo-suppressed decays of $D \rightarrow PP(\times 10^{-4})$.

Modes	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^+ \rightarrow \pi^+ K^0$	1.98 ± 0.22	2.8 ± 0.5	1.7 ± 0.5	
$D^+ \rightarrow \pi^0 K^+$	1.59 ± 0.15	3.0 ± 0.4	2.2 ± 0.4	1.72 ± 0.19
$D^+ \rightarrow K^+ \eta$	0.98 ± 0.04	1.3 ± 0.2	1.2 ± 0.2	1.08 ± 0.17^{a}
$D^+ \rightarrow K^+ \eta'$	0.91 ± 0.17	0.4 ± 0.1	1.0 ± 0.1	1.76 ± 0.22^{b}
$D^0 \rightarrow \pi^0 K^0$	0.67 ± 0.02	0.5 ± 0.2	0.6 ± 0.2	
$D^0 \rightarrow \pi^- K^+$	1.12 ± 0.05	2.3 ± 0.3	1.6 ± 0.4	1.48 ± 0.07
$D^0 \rightarrow K^0 \eta$	0.28 ± 0.02	0.23 ± 0.05	0.22 ± 0.05	
$D^0 \rightarrow K^0 \eta'$	0.55 ± 0.03	0.08 ± 0.06	0.5 ± 0.1	
$D_S^+ \to K^+ K^0$	0.38 ± 0.04	0.7 ± 0.4	0.7 ± 0.4	

^aData from [53].

^bData from [53].

resonances [52]. It is clear that our results with large annihilation-type contributions agree with experiments much better than that of Ref. [52]. For the Cabibbo-favored channels, which are the input data for χ^2 fit in the diagrammatic approach, we have comparable results with the diagrammatic approach [19]. For other channels, we have better agreement with experiments than the diagrammatic approach. The reason is mostly due to the SU(3) breaking effects, which had been fully neglected in the diagrammatic approach.

The branching ratio of the pure annihilation process $D_s^+ \rightarrow \pi^+ \pi^0$ is vanished in our pole model. The resonant state that annihilates to $\pi^+ \pi^0$ is a scalar meson (0^{++}) , whose isospin could be 0, 1, or 2. However, isospin-0 would be ruled out because of charged final states, and isospin-2 is forbidden for the leading order $\Delta C = 1$ weak decay. For the case of isospin-1, its *G* parity would be odd, which conflicts to a system of two pions whose *G* parity is even. Therefore, no resonant states can be produced and then annihilate to $\pi^+ \pi^0$. In another word, no annihilation diagrams contribute to $D_s^+ \rightarrow \pi^+ \pi^0$ and $D^+ \rightarrow \pi^+ \pi^0$. In fact, this kind of contribution is forbidden from the isospin symmetry of π^+ and π^0 as identical particles. Simply, two pions cannot form an *s*-wave isospin 1 state, because of the Bose-Einstein statics.

The pure annihilation process $D^0 \rightarrow K^0 \bar{K}^0$, with nonzero experimental data, also demonstrates the important annihilation-type contributions. There are two kinds of contributions to this mode with $d\bar{d}$ and $s\bar{s}$ produced from weak vertex, respectively. In the flavor SU(3) limit, the rate vanishes due to the cancellation of CKM matrix elements, as predicted in the diagrammatic approach. Therefore, the effect of the SU(3) breaking is the dominant contribution here. In our pole model, we use $f_0(1370)$ and $f_0(1500)$ as two different poles with the $d\bar{d}$ and $s\bar{s}$ components, respectively, to describe the corresponding SU(3) breaking effect. We also refer to the argument of the long distance resonance effect in [19,54], the *t*-channel final-state interaction in [55], the nonfactorizable chiral loop contributions in [56], and the SU(3) breaking effect in the effective Wilson coefficients in [15] for this channel.

Large branching ratios with η' in the final states are both measured and predicted, which are larger than those with η in most cases, such as D^0 decays into $\bar{K}^0\eta$, $\bar{K}^0\eta'$ and D_s^+ into $\pi^+\eta$, $\pi^+\eta'$, although the phase spaces with η' are smaller than those with η . For η , the contributions from the components of $d\bar{d}$ and $s\bar{s}$ are destructive due to the minus sign in the mixing matrix of Eq. (26) and the positive mixing angle;¹ while they are constructive for η' . Besides, large W-exchange contributions dominate most η' modes, especially for $D^0 \to \eta \eta'$ and $D^0 \to \bar{K}^0 \eta'$, which demonstrates large annihilation-type contributions directly again. We also refer to this issue with $K_0^*(1430)$ as a resonance in the spacelike form factors in the factorization approach in [57], some effects of the inelastic final-state interactions in [58], the final-state phases of the amplitudes in [16], and the two-gluon anomaly effects in [59].

C. $D \rightarrow PV$ decays

The decay rate of $D \rightarrow PV$ decays is

$$\Gamma(D \to PV) = \frac{p}{8\pi m_D^2} \sum_{\text{pol}} |\mathcal{A}|^2, \qquad (36)$$

by summing over all the polarization states of the vector mesons.

We assume that the coefficients a_i are universal and process independent for $D \rightarrow PV$, but they are different from those of $D \rightarrow PP$ as discussed in [14,19]. Their absolute values are larger than those of $D \rightarrow PP$ because the soft final-state interactions make more effects on $D \rightarrow PV$ decays. In our calculations, they are used as

¹The theoretical and phenomenological estimates for mixing angle ϕ is 42.2° and (39.3 ± 1.0)°, respectively [48].

TABLE VI. Branching ratios for Cabibbo-favored decays of $D \rightarrow PV(\%)$. The predicted rates with only emission-type contributions (emission) and with both annihilation- and emission-type contributions (all) are shown in the table, compared with the experimental data [26], the fitted results from the diagrammatic approach [19] in which only the (A, A1) solution is quoted, and the results considering final-state interaction (FSI) effects of nearby resonances [52].

Modes	Br(FSI)	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^0 \rightarrow K^- \rho^+$	11.19	10.8 ± 2.2	12.2 ± 1.8	8.8 ± 2.2	10.8 ± 0.7
$D^0 \rightarrow \bar{K}^0 \rho^0$	0.88	1.54 ± 1.15	0.7 ± 0.5	1.7 ± 0.7	$1.32^{+0.12}_{-0.16}$
$D^0 \rightarrow \pi^0 \bar{K}^{*0}$	3.49	2.82 ± 0.34	2.3 ± 0.7	2.9 ± 1.0	2.82 ± 0.35
$D^0 \rightarrow \pi^+ K^{*-}$	4.69	5.91 ± 0.70	3.8 ± 0.7	3.1 ± 1.0	$5.68^{+0.68}_{-0.53}$
$D^0 \rightarrow \eta \bar{K}^{*0}$	0.51	0.96 ± 0.32	0.7 ± 0.2	0.7 ± 0.2	0.96 ± 0.30
$D^0 ightarrow \eta' ar{K}^{*0}$	0.005	0.012 ± 0.003	0.003 ± 0.001	0.016 ± 0.005	< 0.11
$D^0 \rightarrow \bar{K}^0 \omega$	2.16	2.26 ± 1.38	0.6 ± 0.5	2.5 ± 0.7	2.22 ± 0.12
$D^0 \rightarrow \bar{K}^0 \phi$	0.90	0.868 ± 0.139	0	0.8 ± 0.2	0.868 ± 0.060
$D^+ \rightarrow \pi^+ \bar{K}^{*0}$	0.64	1.83 ± 0.49	1.4 ± 1.3	1.4 ± 1.3	1.56 ± 0.18
$D^+ \rightarrow \bar{K}^0 \rho^+$	11.77	9.2 ± 6.7	15.1 ± 3.8	15.1 ± 3.8	9.4 ± 2.0
$D_S^+ \to K^+ \bar{K}^{*0}$	3.86		5.6 ± 1.9	4.2 ± 1.7	3.90 ± 0.23
$D_S^+ \rightarrow \bar{K}^0 K^{*+}$	3.37		1.7 ± 0.7	1.0 ± 0.6	5.4 ± 1.2
$D_S^+ \rightarrow \eta \rho^+$	9.49		8.3 ± 1.3	8.3 ± 1.3	8.9 ± 0.8 [60]
$D_S^+ \rightarrow \eta' \rho^+$	2.61		3.0 ± 0.5	3.0 ± 0.5	12.2 ± 2.0
$D_S^+ \rightarrow \pi^+ \phi$	2.89	4.38 ± 0.35	4.3 ± 0.6	4.3 ± 0.6	4.5 ± 0.4
$D_S^+ \rightarrow \pi^+ \rho^0$	0.080		0	0.4 ± 0.4	0.02 ± 0.012
$D^+_S ightarrow \pi^0 ho^+$	0.080		0	0.4 ± 0.4	
$D_S^+ \to \pi^+ \omega$	0.0		0	0	0.23 ± 0.06

$$a_1^{PV} = 1.32 \pm 0.10,$$

$$a_2^{PV} = (0.75 \pm 0.10)e^{i(160\pm10)^\circ},$$

$$a_A^{PV} = (0.12 \pm 0.10)e^{i(345\pm10)^\circ},$$

$$a_E^{PV} = (0.62 \pm 0.10)e^{i(238\pm10)^\circ}.$$

(37)

Similarly to the *PP* modes, large relative strong phases due to inelastic final-state interactions are considered in the a_i . Again, the contributions from W-annihilation diagrams are smaller than the W-exchange ones. Besides, the relative strong phase between a_1^{PV} and a_2^{PV} is in accordance with the results from the diagrammatic approach [14,19].

Our prediction of branching ratios of the Cabibbofavored, the singly Cabibbo-suppressed, and the doubly Cabibbo-suppressed $D \rightarrow PV$ decays are shown in Tables VI, VII, and VIII, respectively. The results in the third column (emission) in each of these tables are the predictions of rates with only the emission-type processes; while the results in the fourth column (all) also include the annihilation-type contributions. It is obvious that the annihilation-type contributions are of the same order as the emission-type diagrams, since the intermediate states here in the pole model are pseudoscalar mesons with relatively larger decay constants than those scalar mesons of the $D \rightarrow PP$ case. Again, for theoretical uncertainty estimation, we use only those from the parameters a_i shown in Eq. (37), as illustration. For comparison, we also list the results of the diagrammatic approach [19] and the experimental date in

these tables. It is easy to see that our results with the annihilation-type contributions agree with the experimental data. This means that the single-pole contribution dominates the annihilation-type contribution in most $D \rightarrow PV$ decay channels. For example, although the $D^0 \rightarrow \overline{K}^0 \phi$ channel has no emission-type contribution, with vanishing branching ratio in the factorization approach, our pole model gives the right branching ratios agreeing with the experiment. This also confirms the calculation done in the perturbative QCD approach [20]. Besides, some of the SU(3) flavor symmetry breaking effects are considered in this work since the decay constants, transition form factors, and effective strong coupling constants are involved.

There is no resonant state contributing to the W-exchange diagram of $D^0 \rightarrow \pi^0 \rho^0$ in the pole model, because a π^0 would violate the C parity, similarly to the case of $D^0 \rightarrow \eta(\eta') \omega$, $\eta \phi$. Besides, the single-pole annihilation diagrams cannot contribute to the $D \rightarrow \rho \eta, \pi \omega$ decays because of G parity violation. The isospin of resonant state for $\rho \eta$ or $\pi \omega$ is one, so the G parity of the intermediate state is odd since it is a pseudoscalar meson. However, the G parity of ρ and η are both even, and that of π and ω are both odd, so the total G parity of the final states is even. Therefore, no resonant states are available for the decays of D mesons into $\rho \eta$ and $\pi \omega$. It is even worse for the pure annihilation process $D_s^+ \rightarrow \pi^+ \omega$, since its decay rate is predicted to be zero in the single-pole model, but it is not small in the experiment. This has already been discussed that this channel may be dominated

TABLE VII. Same as Table. VI except for singly Cabibbo-suppressed decays of $D \rightarrow PV(\times 10^{-3})$.

Modes	Br(FSI)	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^0 \rightarrow \pi^- \rho^+$	8.2	8.34 ± 1.69	7.4 ± 1.3	10.2 ± 1.5	9.8 ± 0.4
$D^0 \rightarrow \pi^+ \rho^-$	6.5	3.92 ± 0.46	1.8 ± 0.5	3.5 ± 0.6	4.97 ± 0.23
$D^0 \rightarrow \pi^0 \rho^0$	1.7	2.96 ± 0.98	1.4 ± 0.6	1.4 ± 0.6	3.73 ± 0.22
$D^0 \longrightarrow K^- K^{*+}$	4.5	4.25 ± 0.86	5.5 ± 0.8	4.7 ± 0.8	4.38 ± 0.21
$D^0 \longrightarrow K^+ K^{*-}$	2.8	1.99 ± 0.24	2.0 ± 0.3	1.6 ± 0.3	1.56 ± 0.12
$D^0 \rightarrow \bar{K}^0 K^{*0}$	0.99	0.29 ± 0.22	0	0.16 ± 0.05	< 0.9
$D^0 \rightarrow K^0 \bar{K}^{*0}$	0.99	0.29 ± 0.22	0	0.16 ± 0.05	<1.8
$D^0 \rightarrow \pi^0 \omega$	0.08	0.10 ± 0.18	0.08 ± 0.02	0.08 ± 0.02	< 0.26
$D^0 \rightarrow \pi^0 \phi$	1.1	1.22 ± 0.08	1.0 ± 0.3	1.0 ± 0.3	0.76 ± 0.05
$D^0 \rightarrow \eta \phi$	0.57	0.31 ± 0.10	0.23 ± 0.06	0.23 ± 0.06	0.14 ± 0.05
$D^0 \rightarrow \eta \rho^0$	0.24	1.11 ± 0.86	0.05 ± 0.01	0.05 ± 0.01	
$D^0 \rightarrow \eta' \rho^0$	0.10	0.14 ± 0.02	0.08 ± 0.02	0.08 ± 0.02	
$D^0 \rightarrow \eta \omega$	1.9	3.08 ± 1.42	1.2 ± 0.3	1.2 ± 0.3	2.21 ± 0.23
$D^0 \rightarrow \eta' \omega$	0.001	0.07 ± 0.02	0.0001 ± 0.0001	0.0001 ± 0.0001	
$D^+ \rightarrow \pi^+ \rho^0$	1.7		0.4 ± 0.4	0.8 ± 0.7	0.83 ± 0.15
$D^+ ightarrow \pi^0 ho^+$	3.7		5.3 ± 1.7	3.5 ± 1.6	
$D^+ \rightarrow K^+ \bar{K}^{*0}$	2.5		5.1 ± 1.1	4.1 ± 1.0	$3.76^{+0.20}_{-0.26}$
$D^+ \rightarrow \bar{K}^0 K^{*+}$	1.70		14.0 ± 2.5	12.4 ± 2.4	32 ± 14
$D^+ \rightarrow \eta \rho^+$	0.002		0.4 ± 0.4	0.4 ± 0.4	<7
$D^+ ightarrow \eta' ho^+$	1.3		0.8 ± 0.1	0.8 ± 0.1	<5
$D^+ \rightarrow \pi^+ \phi$	5.9	6.21 ± 0.43	5.1 ± 1.4	5.1 ± 1.4	5.44 ± 0.26
$D^+ \rightarrow \pi^+ \omega$	0.35		0.3 ± 0.3	0.3 ± 0.3	< 0.34
$D_S^+ \rightarrow \pi^+ K^{*0}$	3.3		2.3 ± 0.8	1.5 ± 0.7	2.25 ± 0.39
$D_S^+ \rightarrow \pi^0 K^{*+}$	0.29		0.4 ± 0.2	0.1 ± 0.1	
$D_S^+ \to K^+ \rho^0$	2.4		1.6 ± 0.6	1.0 ± 0.6	2.7 ± 0.5
$D_S^+ \rightarrow K^0 \rho^+$	19.5		9.7 ± 2.2	7.5 ± 2.1	
$D_S^+ \rightarrow \eta K^{*+}$	0.24		1.0 ± 0.4	1.0 ± 0.4	
$D_S^+ \rightarrow \eta' K^{*+}$	0.24		0.4 ± 0.2	0.6 ± 0.2	
$D_S^+ \to K^+ \omega$	0.72		1.1 ± 0.7	1.8 ± 0.7	<2.4
$D_S^+ \to K^+ \phi$	0.15		0.3 ± 0.3	0.3 ± 0.3	< 0.6

by the final-state rescattering via quark exchange in [19,54], and by hidden strangeness final-state interactions in [61]. Besides, the pure annihilation mode $D_s^+ \rightarrow \pi^+ \rho^0$ is predicted much larger in the pole model than the experiment data. The contributions from the two diagrams in this channel are constructive since the minus sign in the normalization of ρ^0 is compensated by the asymmetric space wave function of the two final states which are in the *P*-wave state. These two channels make such trouble that we fail to find a reasonable solution of A_P and A_V and predict the *PV* modes with the W-annihilation contributions in the diagrammatic approach [19]. Hence, further discussions are still needed for these two channels.

 $D_S^+ \rightarrow \rho \eta'$ is predicted much smaller than the mode of $D_S^+ \rightarrow \rho \eta$, but the experimental branching ratios of the former is larger. This is a puzzle that the phase space of the former mode is much smaller than the latter, so its branching ratio should be smaller. In fact, the experimental measurement of $D_S^+ \rightarrow \rho \eta'$ [62] is already too old. It is already questioned by the PDG [26], since this branching fraction

 $(12.5 \pm 2.2)\%$ considerably exceeds the recent inclusive η' fraction of $(11.7 \pm 1.8)\%$.

IV. SUMMARY

We have calculated the branching ratios for the twobody hadronic decays of charmed mesons into *PP* and *PV* using the generalized factorization approach for the emission-type diagrams and the pole-dominance model for the annihilation-type diagrams. Relative strong phases between different topological diagrams, which are important in the charmed decays, are considered in this work. Most of our predicted branching fractions are in accordance with the experimental data. Besides, compared to the naive and generalized factorization models ever before, the results in this work are much better since we have considered the annihilation-type diagrams and the relative strong phases between diagrams.

We find that the annihilation-type contributions in the pole model are large for both *PP* and *PV* modes, which is

TABLE VIII. Same as Table VI except for doubly Cabibbo-suppressed decays of $D \rightarrow PV(\times 10^{-4})$.

Modes	Br(diagrammatic)	Br(emission)	Br(all)	Br(exp)
$D^0 \rightarrow \pi^- K^{*+}$	3.59 ± 0.72	3.7 ± 0.6	2.7 ± 0.6	$3.54^{+1.80}_{-1.05}$
$D^0 \rightarrow \pi^0 K^{*0}$	0.54 ± 0.18	0.6 ± 0.2	0.8 ± 0.3	1.05
$D^0 \rightarrow K^+ \rho^-$	1.45 ± 0.17	1.1 ± 0.2	0.9 ± 0.3	
$D^0 \rightarrow K^0 \rho^0$	0.91 ± 0.51	0.2 ± 0.1	0.5 ± 0.2	
$D^0 \rightarrow K^0 \omega$	0.58 ± 0.40	0.2 ± 0.1	0.7 ± 0.2	
$D^0 \rightarrow K^0 \phi$	0.06 ± 0.05	0	0.20 ± 0.06	
$D^0 \rightarrow \eta K^{*0}$	0.33 ± 0.08	0.18 ± 0.05	0.17 ± 0.05	
$D^0 \rightarrow \eta' K^{*0}$	0.0040 ± 0.0006	0.001 ± 0.001	0.004 ± 0.001	
$D^+ \rightarrow \pi^+ K^{*0}$		3.0 ± 1.0	2.2 ± 0.9	3.75 ± 0.75
$D^+ \rightarrow \pi^0 K^{*+}$		4.7 ± 0.9	4.0 ± 0.9	
$D^+ \rightarrow K^+ \rho^0$		1.4 ± 0.4	1.0 ± 0.4	2.1 ± 0.5
$D^+ \rightarrow K^0 \rho^+$		0.9 ± 0.4	0.5 ± 0.4	
$D^+ \rightarrow K^+ \omega$		1.4 ± 0.5	1.8 ± 0.5	
$D^+ \rightarrow K^+ \phi$		0	0.2 ± 0.2	
$D^+ \rightarrow \eta K^{*+}$		1.5 ± 0.2	1.4 ± 0.2	
$D^+ \rightarrow \eta' K^{*+}$		0.013 ± 0.006	0.020 ± 0.07	
$D_S^+ \rightarrow K^+ K^{*0}$	0.20 ± 0.05	0.2 ± 0.2	0.2 ± 0.2	
$D_S^+ \to K^0 K^{*+}$	1.17 ± 0.86	2.3 ± 0.6	2.3 ± 0.6	

also indicated by the difference between the lifetime of D^+ and D^0 . Comparing with the model-independent diagrammatic approach, we reproduce their results with our specific model considering some SU(3) breaking effects. Furthermore, we get more predictions in many $D \rightarrow PV$ decay channels, which are absent in the diagrammatic approach [19]. Most of the results have a better agreement with experimental data than previous calculations.

ACKNOWLEDGMENTS

We are grateful to Hai-Yang Cheng, Wei Wang, Yu-Ming Wang, Dong-Sheng Du, Ping Wang, Qiang Zhao, Run-Hui Li, and Cheng Li for useful discussions. This work is partially supported by National Natural Science Foundation of China under the Grants No. 10735080 and No. 11075168; National Basic Research Program of China (973) No. 2010CB833000; Natural Science Foundation of Zhejiang Province of China, Grant No. Y606252, and Scientific Research Fund of Zhejiang Provincial Education Department of China, Grant No. 20051357.

APPENDIX: INDIVIDUAL FORMULAS FOR VARIOUS DECAY CHANNELS OF D MESONS

The different contribution formulas for Cabibbo-favored decays of $D \rightarrow PP$ are listed as

$$\begin{aligned} \mathcal{A}(D^{+} \to \pi^{+} \bar{K}^{0}) &= i \frac{G_{F}}{\sqrt{2}} V_{cs}^{*} V_{ud} (a_{2} f_{K}(m_{D}^{2} - m_{\pi}^{2}) F_{0}^{D\pi}(m_{K}^{2}) + a_{1} f_{\pi}(m_{D}^{2} - m_{K}^{2}) F_{0}^{DK}(m_{\pi}^{2})), \\ \mathcal{A}(D^{0} \to \pi^{+} K^{-}) &= i \frac{G_{F}}{\sqrt{2}} V_{cs}^{*} V_{ud} \left(a_{1} f_{\pi}(m_{D}^{2} - m_{K}^{2}) F_{0}^{DK}(m_{\pi}^{2}) - a_{E} g_{1} f_{S} f_{D} \frac{m_{D}^{2} m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} \right), \\ \mathcal{A}(D^{0} \to \pi^{0} \bar{K}^{0}) &= i \frac{G_{F}}{2} V_{cs}^{*} V_{ud} \left(a_{E} g_{1} f_{S} f_{D} \frac{m_{D}^{2} m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} + a_{2} f_{K}(m_{D}^{2} - m_{\pi}^{2}) F_{0}^{D\pi}(m_{K}^{2}) \right), \\ \mathcal{A}(D^{0} \to \bar{K}^{0} \eta) &= i \frac{G_{F}}{2} V_{cs}^{*} V_{ud} \left(a_{2} f_{K}(m_{D}^{2} - m_{\eta}^{2}) F_{0}^{D\eta_{q}}(m_{K}^{2}) \cos\phi - a_{E} g_{1} f_{S} f_{D} \frac{m_{D}^{2} m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} \left(\cos\phi - \sqrt{2} \sin\phi \right) \right), \\ \mathcal{A}(D^{0} \to \bar{K}^{0} \eta') &= i \frac{G_{F}}{2} V_{cs}^{*} V_{ud} \left(a_{2} f_{K}(m_{D}^{2} - m_{\eta}^{2}) F_{0}^{D\eta_{q}}(m_{K}^{2}) \sin\phi - a_{E} g_{1} f_{S} f_{D} \frac{m_{D}^{2} m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} \left(\sin\phi + \sqrt{2} \cos\phi \right) \right), \end{aligned}$$

NONLEPTONIC TWO-BODY DECAYS OF CHARMED MESONS

$$\mathcal{A}(D_{S}^{+} \to K^{+}\bar{K}^{0}) = i\frac{G_{F}}{\sqrt{2}}V_{cs}^{*}V_{ud}\left(a_{2}f_{K}(m_{D_{S}}^{2} - m_{K}^{2})F_{0}^{D_{S}K}(m_{K}^{2}) - a_{A}g_{1}f_{S}f_{D_{S}}\frac{m_{D_{S}}^{2}m_{a_{0}}}{m_{D_{S}}^{2} - m_{a_{0}}^{2}}\right),$$

$$\mathcal{A}(D_{S}^{+} \to \pi^{+}\eta) = -iG_{F}V_{cs}^{*}V_{ud}\left(\frac{1}{\sqrt{2}}a_{1}f_{\pi}(m_{D_{S}}^{2} - m_{\eta}^{2})F_{0}^{D_{S}\eta_{s}}(m_{\pi}^{2})\sin\phi + a_{A}g_{1}f_{S}f_{D_{S}}\frac{m_{D_{S}}^{2}m_{a_{0}}}{m_{D_{S}}^{2} - m_{a_{0}}^{2}}\cos\phi\right),$$

$$\mathcal{A}(D_{S}^{+} \to \pi^{+}\eta') = iG_{F}V_{cs}^{*}V_{ud}\left(\frac{1}{\sqrt{2}}a_{1}f_{\pi}(m_{D_{S}}^{2} - m_{\eta'}^{2})F_{0}^{D_{S}\eta_{s}}(m_{\pi}^{2})\cos\phi - a_{A}g_{1}f_{S}f_{D_{S}}\frac{m_{D_{S}}^{2}m_{a_{0}}}{m_{D_{S}}^{2} - m_{a_{0}}^{2}}\sin\phi\right),$$
(A1)

where f_s and $g_1 = 2.7$ are, respectively, denoted as the decay constant of scalar mesons and effective strong coupling constant between the intermediate state and final states in the limit of SU(3) symmetry. Some phases from the propagators of the intermediate resonances are absorbed in the effective Wilson coefficients a_A and a_E .

The formulas for singly Cabibbo-suppressed decays of $D \rightarrow PP$ are shown as

$$\begin{split} \mathcal{A}(D^+ \to \pi^+ \pi^0) &= -i \frac{G_F}{2} V_{cd}^* V_{ud} f_\pi(m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) (a_1 + a_2), \\ \mathcal{A}(D^+ \to K^+ \bar{K}^0) &= i \frac{G_F}{\sqrt{2}} \left(a_1 V_{cs}^* V_{us} f_K(m_D^2 - m_K^2) F_0^{DK}(m_K^2) - a_A V_{cd}^* V_{ud} g_1 f_S f_D \frac{m_D^2 m_{a_0}}{m_D^2 - m_{a_0}^2} \right), \\ \mathcal{A}(D^+ \to \pi^+ \eta) &= i \frac{G_F}{2} \left(\cos \phi V_{cd}^* V_{ud} \left[a_1 f_\pi(m_D^2 - m_\eta^2) F_0^{D\eta_v}(m_\pi^2) - 2a_A g_1 f_S f_D \frac{m_D^2 m_{a_0}}{m_D^2 - m_{a_0}^2} \right] \right. \\ &\quad + \sqrt{2} a_2 [V_{cd}^* V_{ud} f_\eta^d + V_{cs}^* V_{us} f_\eta^s] (m_D^2 - m_\eta^2) F_0^{D\eta_v}(m_\eta^2) - 2a_A g_1 f_S f_D \frac{m_D^2 m_{a_0}}{m_D^2 - m_{a_0}^2} \right] \\ &\quad + \sqrt{2} a_2 [V_{cd}^* V_{ud} f_\eta^d + V_{cs}^* V_{us} f_\eta^s] (m_D^2 - m_\eta^2) F_0^{D\eta_v}(m_\eta^2) - 2a_A g_1 f_S f_D \frac{m_D^2 m_{a_0}}{m_D^2 - m_{a_0}^2} \right] \\ &\quad + \sqrt{2} a_2 [V_{cd}^* V_{ud} f_\eta^d + V_{cs}^* V_{us} f_\eta^s] (m_D^2 - m_\eta^2) F_0^{D\pi}(m_\eta^2) \right), \\ \mathcal{A}(D^0 \to \pi^+ \pi^-) &= i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} \left(a_1 f_\pi(m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) - a_E g_1 f_S f_D \frac{m_D^2 m_{f_0}(1370)}{m_D^2 - m_{f_0}^2(1370)} \right), \\ \mathcal{A}(D^0 \to \pi^0 \pi^0) &= -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} \left(a_2 f_\pi(m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) + a_E g_1 f_S f_D \frac{m_D^2 m_{f_0}(1370)}{m_D^2 - m_{f_0}^2(1370)} \right), \\ \mathcal{A}(D^0 \to \pi^0 \eta^0) &= -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{us} \left(a_1 f_K (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) - a_E g_1 f_S f_D \frac{m_D^2 m_{f_0}(1370)}{m_D^2 - m_{f_0}^2(1370)} \right), \\ \mathcal{A}(D^0 \to \pi^0 \eta^0) &= -i \frac{G_F}{\sqrt{2}} a_E g_1 f_S f_D \left(V_{cs}^* V_{us} \frac{m_D^2 m_{f_0}(1500)}{m_D^2 - m_{f_0}^2(1500)} + V_{cd}^* V_{ud} \frac{m_D^2 m_{f_0}(1500)}{m_D^2 - m_{f_0}^2(1500)} \right), \\ \mathcal{A}(D^0 \to \pi^0 \eta) &= i \frac{G_F}{\sqrt{2}} \left(a_2 (m_D^2 - m_\pi^2) F_D^{Dm}(m_\eta^2) \right) \left[V_{cd}^* V_{ud} f_\eta^* + V_{cs}^* V_{us} f_\eta^* \right] - \frac{1}{\sqrt{2}} V_{cd}^* V_{ud} a_2 f_\pi(m_D^2 - m_\eta^2) F_0^{D\eta_e}(m_\pi^2) \cos\phi \right. \\ &\quad + \sqrt{2} a_E g_1 f_S f_D \frac{m_D^2 m_{a_0}}{m_D^2 - m_{a_0}^2}} \cos\phi \right). \\ \mathcal{A}(D^0 \to \pi^0 \eta') &= i \frac{G_F}{2} \left(a_2 (m_D^2 - m_\pi^2) F_D^{Dm}(m_\eta^2) \left[V_{cd}^* V_{ud} f_\eta^* + V_{cs}^* V_{us} f_\eta^* \right] - \frac{1}{\sqrt{2}} V_{cd}^* V_{ud} a_2 f_\pi(m_D^2 - m_\eta^2) F_0^{D\eta_e}(m_\pi^2) \sin\phi \right. \\ \left. + \sqrt$$

FU-SHENG YU, XIAO-XIA WANG, AND CAI-DIAN LÜ

$$\begin{aligned} \mathcal{A}(D^{0} \to \eta \eta) &= i \frac{G_{F}}{\sqrt{2}} \Big(a_{2}(m_{D}^{2} - m_{\eta}^{2}) F_{0}^{D\eta_{q}}(m_{\eta}^{2}) [V_{cs}^{*}V_{ud}f_{\eta}^{d} + V_{cs}^{*}V_{us}f_{\eta}^{*}] \cos\phi \\ &\quad - a_{E}g_{1}f_{S}f_{D} \Big(\sqrt{2}V_{cs}^{*}V_{us} \frac{m_{D}^{2}m_{f_{0}}(1500)}{m_{D}^{2} - m_{f_{0}}^{2}(1500)} \sin^{2}\phi + \frac{1}{\sqrt{2}} V_{cd}^{*}V_{ud} \frac{m_{D}^{2}m_{f_{0}}(1370)}{m_{D}^{2} - m_{f_{0}}^{2}(1370)} \cos^{2}\phi \Big) \Big), \\ \mathcal{A}(D^{0} \to \eta \eta') &= i \frac{G_{F}}{2} \Big(a_{2}(m_{D}^{2} - m_{\eta}^{2}) F_{0}^{D\eta_{q}}(m_{\eta}^{2}) [V_{cd}^{*}V_{ud}f_{\eta}^{d} + V_{cs}^{*}V_{us}f_{\eta}^{*}] \cos\phi \\ &\quad + a_{2}(m_{D}^{2} - m_{\eta'}^{2}) F_{0}^{D\eta_{q}}(m_{\eta'}^{2}) [V_{cd}^{*}V_{ud}f_{\eta}^{d} + V_{cs}^{*}V_{us}f_{\eta}^{*}] \sin\phi \\ &\quad + a_{E}g_{1}f_{S}f_{D} \Big(\sqrt{2}V_{cs}^{*}V_{us} \frac{m_{D}^{2}m_{f_{0}}(1500)}{m_{D}^{2} - m_{f_{0}}^{2}(1500)} \sin2\phi - \frac{1}{\sqrt{2}} V_{cd}^{*}V_{ud} \frac{m_{D}^{2}m_{f_{0}}(1370)}{m_{D}^{2} - m_{f_{0}}^{2}(1370)} \sin2\phi \Big) \Big), \\ \mathcal{A}(D_{S}^{+} \to K^{+}\pi^{0}) &= -i \frac{G_{F}}{2} \Big(V_{cs}^{*}V_{us}a_{A}g_{1}f_{S}f_{D_{S}} \frac{m_{D}^{2}m_{h_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} + V_{cd}^{*}V_{ud}a_{2}f_{\pi}(m_{D_{S}^{*}}^{2} - m_{K}^{2})F_{0}^{D_{S}K}(m_{\pi}^{2}) \Big), \\ \mathcal{A}(D_{S}^{+} \to \pi^{+}K^{0}) &= -i \frac{G_{F}}{\sqrt{2}} \Big(V_{cs}^{*}V_{us}a_{A}g_{1}f_{S}f_{D_{S}} \frac{m_{D_{S}}^{2}m_{h_{0}^{*}}}{m_{D_{S}^{*}}^{2} - m_{K_{0}^{*}}^{2}} - V_{cd}^{*}V_{ud}a_{1}f_{\pi}(m_{D_{S}^{*}}^{2} - m_{K}^{2})F_{0}^{D_{S}K}(m_{\pi}^{2}) \Big), \\ \mathcal{A}(D_{S}^{+} \to \pi^{+}K^{0}) &= -i \frac{G_{F}}{\sqrt{2}} \Big(V_{cs}^{*}V_{us}a_{A}g_{1}f_{S}f_{D_{S}} \frac{m_{D_{S}}^{2}m_{h_{0}^{*}}}{m_{D_{S}^{*}}^{2} - m_{K_{0}^{*}}^{2} \Big) \sin\phi - a_{2}(m_{D}^{2} - m_{K}^{2})F_{0}^{D_{S}K}(m_{\pi}^{2}) \Big), \\ \mathcal{A}(D_{S}^{+} \to K^{+}\eta) &= -i \frac{G_{F}}{\sqrt{2}} \Big(V_{cs}^{*}V_{us}a_{A}g_{1}f_{S}f_{D_{S}} \frac{m_{D_{S}}^{2}m_{h_{0}^{*}}}{m_{D_{S}^{*}}^{2} - m_{K_{0}^{*}}^{2} \Big) \sin\phi - a_{2}(m_{D}^{2} - m_{K}^{2})F_{0}^{D_{K}}(m_{\eta}^{2}) [V_{cd}^{*}V_{ud}f_{\eta}^{d} + V_{cs}^{*}V_{us}f_{\eta}^{*}] \Big), \\ \mathcal{A}(D_{S}^{+} \to K^{+}\eta') &= -i \frac{G_{F}}{\sqrt{2}} \Big(V_{cs}^{*}V_{us}a_{A}g_{1}f_{S}f_{D_{S}} \frac{m_{D_{S}}^{2}m_{h_{0}^{*}}}{m_{D_{S}^{*}}^{2} - m_{K_{0}^{*}}^{2}$$

The formulas for doubly Cabibbo-suppressed decays of $D \rightarrow PP$ are listed as

$$\begin{aligned} \mathcal{A}(D^+ \to K^+ \pi^0) &= -i \frac{G_F}{2} V_{cd}^* V_{us} \Big(a_A g_1 f_S f_D \frac{m_D^2 m_{K_0^*}}{m_D^2 - m_{K_0^*}^2} + a_1 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2) \Big), \\ \mathcal{A}(D^+ \to \pi^+ K^0) &= -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{us} \Big(a_A g_1 f_S f_D \frac{m_D^2 m_{K_0^*}}{m_D^2 - m_{K_0^*}^2} - a_2 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2) \Big), \\ \mathcal{A}(D^+ \to K^+ \eta) &= i \frac{G_F}{2} V_{cd}^* V_{us} \Big(a_A g_1 f_S f_D \frac{m_D^2 m_{K_0^*}}{m_D^2 - m_{K_0^*}^2} (\sqrt{2} \sin\phi - \cos\phi) + a_1 f_K (m_D^2 - m_\eta^2) F_0^{D\eta_q}(m_K^2) \cos\phi \Big), \\ \mathcal{A}(D^+ \to K^+ \eta') &= i \frac{G_F}{2} V_{cd}^* V_{us} \Big(-a_A g_1 f_S f_D \frac{m_D^2 m_{K_0^*}}{m_D^2 - m_{K_0^*}^2} (\sin\phi + \sqrt{2} \cos\phi) + a_1 f_K (m_D^2 - m_\eta^2) F_0^{D\eta_q}(m_K^2) \sin\phi \Big), \\ \mathcal{A}(D^0 \to K^+ \pi^-) &= -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{us} \Big(a_E g_1 f_S f_D \frac{m_D^2 m_{K_0^*}}{m_D^2 - m_{K_0^*}^2} - a_1 f_K (m_D^2 - m_\eta^2) F_0^{D\pi}(m_K^2) \Big), \end{aligned}$$

NONLEPTONIC TWO-BODY DECAYS OF CHARMED MESONS

$$\mathcal{A}(D^{0} \to K^{0}\pi^{0}) = i\frac{G_{F}}{2}V_{cd}^{*}V_{us}\left(a_{E}g_{1}f_{S}f_{D}\frac{m_{D}^{2}m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}} + a_{2}f_{K}(m_{D}^{2} - m_{\pi}^{2})F_{0}^{D\pi}(m_{K}^{2})\right),$$

$$\mathcal{A}(D^{0} \to K^{0}\eta) = i\frac{G_{F}}{2}V_{cd}^{*}V_{us}\left(a_{E}g_{1}f_{S}f_{D}\frac{m_{D}^{2}m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}}(\sqrt{2}\sin\phi - \cos\phi) + a_{2}f_{K}(m_{D}^{2} - m_{\eta}^{2})F_{0}^{D\eta_{q}}(m_{K}^{2})\cos\phi\right),$$

$$\mathcal{A}(D^{0} \to K^{0}\eta') = i\frac{G_{F}}{2}V_{cd}^{*}V_{us}\left(-a_{E}g_{1}f_{S}f_{D}\frac{m_{D}^{2}m_{K_{0}^{*}}}{m_{D}^{2} - m_{K_{0}^{*}}^{2}}(\sqrt{2}\cos\phi + \sin\phi) + a_{2}f_{K}(m_{D}^{2} - m_{\eta'}^{2})F_{0}^{D\eta_{q}}(m_{K}^{2})\sin\phi\right),$$

$$\mathcal{A}(D_{s}^{+} \to K^{0}K^{+}) = i\frac{G_{F}}{\sqrt{2}}V_{cd}^{*}V_{us}f_{K}(m_{D_{s}}^{2} - m_{K}^{2})F_{0}^{D_{s}K}(m_{K}^{2})(a_{1} + a_{2}).$$
(A4)

The formulas for Cabibbo-favored decays of $D \rightarrow PV$ are shown as

$$\begin{split} \mathcal{A}(D^{0} \to \pi^{+}K^{*-}) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}g_{sf}K_{fD}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} + a_{F}^{PV}m_{K}\cdot f_{\pi}A_{0}^{DK^{*}}(m_{\pi}^{2})\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{-}\rho^{+}) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}g_{sf}K_{fD}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} + a_{F}^{PV}m_{\rho}f_{\rho}F_{0}^{PK}(m_{\rho}^{2})\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{0}\bar{\kappa}^{*0}) &= G_{F}V_{cs}^{*}V_{ud} \left(-a_{E}^{FV}g_{sf}K_{fD}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} + a_{2}^{PV}m_{\kappa}f_{\kappa}F_{1}^{Dm}(m_{\kappa}^{2})\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{\kappa}^{0}\rho^{0}) &= G_{F}V_{cs}^{*}V_{ud} \left(-a_{E}^{PV}g_{sf}f_{\kappa}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} + a_{2}^{PV}m_{\rho}f_{\rho}A_{0}^{Dn}(m_{K}^{2})\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{\kappa}^{0}\rho^{0}) &= G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}f_{\kappa}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} (g_{s}\cos\phi - \sqrt{2}g_{ss}\sin\phi) + a_{2}^{PV}m_{\kappa}f_{\kappa}F_{1}^{D\eta_{v}}(m_{K}^{2})\cos\phi\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \eta^{K^{*0}}) &= G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}f_{\kappa}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} (g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{2}^{PV}m_{\kappa}f_{\kappa}F_{1}^{D\eta_{v}}(m_{K}^{2})\sin\phi\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{\kappa}^{0}\phi) &= G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}g_{sf}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} (g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{2}^{PV}m_{\kappa}f_{\kappa}F_{1}^{-D\eta_{v}}(m_{K}^{2})\sin\phi\right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{\kappa}^{0}\phi) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}g_{sf}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} (g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{2}^{PV}m_{\kappa}f_{\kappa}F_{1}^{-D\eta_{v}}(m_{\kappa}^{2}) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{\kappa}^{0}\phi) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{E}^{PV}g_{sf}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} (g_{s}^{*}\sin\phi) - h_{0}^{2}h_{0}^{*}(m_{\kappa}^{2}) \right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{*} \to \pi^{*}\phi) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{A}^{PV}g_{sf}f_{T}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{\pi}^{2}}} + a_{2}^{PV}m_{\kappa}f_{\kappa}f_{\kappa}F_{1}^{D}(m_{\kappa}^{2}) \right) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D_{s}^{*} \to \pi^{*}\phi) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ud} \left(a_{A}^{PV}g_{sf}f_{T}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{\pi}^{2}}} + a_{2}^{PV}m_{\kappa}f_{\kappa}f_{\kappa}F_{1$$

where the effective strong coupling constants between the intermediate state and the final states for $D \rightarrow PV$ are $g_q = 4.2$, $g_s = 4.6$, and $g_{ss} = 4.5$. The formulas for singly Cabibbo-suppressed decays of $D \rightarrow PV$ are shown as

$$\begin{split} \mathcal{A}(D^{0} \to \pi^{+}\rho^{-}) &= -G_{F}V_{cd}^{*}V_{ad}f_{\pi} \left(a_{E}^{FV}g_{d}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}} - \sqrt{2}a_{1}^{V}m_{\rho}A_{0}^{h\rho}(m_{\pi}^{2})\right)(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{-}\rho^{+}) &= -G_{F}V_{cd}^{*}V_{ad}d_{\pi}^{EV}g_{d}f_{\pi}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}} - \sqrt{2}a_{1}^{*}m_{\rho}f_{\rho}F_{D}^{1m}(m_{\rho}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{0}\rho^{0}) &= -\frac{G_{F}}{\sqrt{2}}V_{cd}V_{ad}d_{\pi}^{EV}m_{\rho}(f_{\rho}F_{P}^{1m}(m_{\rho}^{2}) + f_{\pi}A_{0}^{h\rho}(m_{\pi}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{-}K^{++}) &= \sqrt{2}G_{F}V_{cs}V_{ad}(a_{E}^{EV}g_{sss}f_{D}m_{D}^{2}\left(\frac{f_{\eta}}{m_{D}^{2}} - \frac{g_{\eta}}{m_{\eta}^{2}} + \frac{f_{\eta}}{m_{D}^{2}}\right) + a_{1}^{VW}m_{K}f_{K}F_{D}^{0K}(m_{K}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{-}K^{+-}) &= \sqrt{2}G_{F}V_{cs}^{*}V_{ad}(a_{E}^{EV}g_{sss}f_{D}m_{D}^{2}\left(\frac{f_{\eta}}{m_{D}^{2}} - \frac{f_{\eta}}{m_{\eta}^{2}} + \frac{f_{\eta}}{m_{D}^{2}}\right) + a_{1}^{VW}m_{K}f_{K}A_{0}^{0K^{*}}(m_{K}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{K}^{0}K^{*0}) &= \sqrt{2}G_{F}a_{E}^{EV}f_{D}m_{D}^{2}\left(V_{*d}^{*}V_{ad}g_{s}\frac{f_{\pi}}{m_{D}^{2}} + V_{*s}V_{sd}g_{sl}\left[\frac{f_{\eta}}{m_{D}^{2}} - \frac{f_{\eta}}{m_{D}^{2}} + \frac{f_{\eta}}{m_{D}^{2}} - \frac{f_{\eta}}{m_{D}^{2}}\right) + a_{1}^{VW}m_{K}f_{K}A_{0}^{0K^{*}}(m_{K}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \bar{K}^{0}K^{*0}) &= \sqrt{2}G_{F}a_{E}^{FV}f_{D}m_{D}^{2}\left(V_{*d}^{*}V_{ad}g_{s}\frac{f_{\pi}}{m_{D}^{2}} - \frac{f_{\pi}}{m_{D}^{2}} + V_{*s}V_{sd}g_{sl}\left[\frac{f_{\eta}}{m_{D}^{2}} - \frac{f_{\eta}}{m_{D}^{2}} + \frac{f_{\eta}}{m_{D}^{2}} - \frac{f_{\eta}}{m_{H}^{2}}\right])(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{0}) &= \int_{Q}^{2}V_{*d}v_{d}ad_{E}^{V}m_{w}(f_{w}F_{1}^{0}m_{w}^{2})(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{0}\phi) &= G_{F}V_{cs}V_{uad}a_{s}^{0}m_{\phi}f_{\phi}F_{D}^{D}m_{(\eta}\phi)(e^{*}\phi) - f_{\eta}\partial_{\phi}\partial_{\phi}(m_{\eta}^{2}))(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \eta\phi) &= G_{F}a_{E}^{2}W_{m}\left(\left[V_{*d}^{*}V_{sd}f_{\eta}^{*} + V_{*s}^{*}V_{sd}f_{\eta}^{*}\right]A_{0}^{D}m_{\eta}(m_{\eta}^{2}) - \frac{1}{\sqrt{2}}V_{*}V_{sd}d_{g}f_{\mu}f_{1}^{D}m_{(\eta}\phi)(e^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \eta\phi) &= G_{F}a_{E}^{0}W_{m}\left(\left[V_{*d}^{*}V_{sd}f_{\eta}^{*} + V_{*s}^{*}V_{sd}f_{\eta}^{*}\right)A_{0}^{D}m_{\eta}^{*}\phi) + \frac{1}{\sqrt{2}}}\cos\phi V_{sd}^{*}V_{sd}f_{\mu}f_{$$

$$\begin{aligned} \mathcal{A}(D_{S}^{+} \to \pi^{+}K^{*0}) &= \sqrt{2}G_{F} \Big(V_{cs}^{*}V_{us}a_{A}^{PV}g_{s}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} + V_{cd}^{*}V_{ud}a_{1}^{PV}m_{K^{*}}f_{\pi}A_{0}^{D_{s}K^{*}}(m_{\pi}^{2}) \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to \pi^{0}K^{*+}) &= G_{F} \Big(V_{cs}^{*}V_{us}a_{A}^{PV}g_{s}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} - V_{cd}^{*}V_{ud}a_{2}^{PV}m_{K^{*}}f_{\pi}A_{0}^{D_{s}K^{*}}(m_{\pi}^{2}) \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to K^{+}\rho^{0}) &= G_{F} \Big(V_{cs}^{*}V_{us}a_{A}^{PV}g_{s}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} - V_{cd}^{*}V_{ud}a_{2}^{PV}m_{\rho}f_{\rho}F_{1}^{D_{s}K}(m_{\rho}^{2}) \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to K^{0}\rho^{+}) &= \sqrt{2}G_{F} \Big(V_{cs}^{*}V_{us}a_{A}^{PV}g_{s}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} + V_{cd}^{*}V_{ud}a_{1}^{PV}m_{\rho}f_{\rho}F_{1}^{D_{s}K}(m_{\rho}^{2}) \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to \eta K^{*+}) &= G_{F} \Big(V_{cs}^{*}V_{us}\Big[a_{A}^{PV}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} (g_{s}\cos\phi - \sqrt{2}g_{ss}\sin\phi) - \sqrt{2}a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D_{s}\eta_{s}}\sin\phi \Big] \\ &+ \sqrt{2}a_{2}^{PV}m_{K^{*}}A_{0}^{D_{S}K^{*}}(m_{\eta}^{2}) \Big[V_{cd}^{*}V_{ud}f_{\eta}^{\eta} + V_{cs}^{*}V_{us}f_{\eta}^{*} \Big] \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to \eta'K^{*+}) &= G_{F} \Big(V_{cs}^{*}V_{us}\Big[a_{A}^{PV}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} (g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + \sqrt{2}a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D_{s}\eta_{s}}\cos\phi \Big] \\ &+ \sqrt{2}a_{2}^{PV}m_{K^{*}}A_{0}^{D_{S}K^{*}}(m_{\eta}^{2}) \Big[V_{cd}^{*}V_{ud}f_{\eta}^{d} + V_{cs}^{*}V_{us}f_{\eta}^{*} \Big] \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to \kappa^{+}\phi) &= \sqrt{2}G_{F} V_{cs}^{*}V_{us}\Big[a_{A}^{PV}g_{ss}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} + a_{2}^{PV}m_{\phi}f_{\phi}F_{1}^{D_{s}K}(m_{\phi}^{2}) + a_{1}^{PV}m_{\phi}f_{K}A_{0}^{D_{s}\phi}(m_{K}^{2}) \Big) (\varepsilon^{*} \cdot p_{D_{s}}), \\ \mathcal{A}(D_{S}^{+} \to K^{+}\phi) &= G_{F} \Big(V_{cs}^{*}V_{us}a_{A}^{PV}g_{ss}f_{K}f_{D_{s}}\frac{m_{D_{s}}^{2}}{m_{D_{s}}^{2} - m_{K}^{2}} + V_{cd}^{*}V_{ud}a_{2}^{PV}m_{\omega}f_{\omega}f_{\omega}f_{\omega}^{$$

The formulas for doubly Cabibbo-suppressed decays of $D \rightarrow PV$ are shown as

$$\begin{split} \mathcal{A}(D^{0} \to \pi^{-}K^{*+}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us} \Big(a_{E}^{PV}g_{s}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}} + a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\pi}(m_{K^{*}}^{2})\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \pi^{0}K^{*0}) &= G_{F}V_{cd}^{*}V_{us}(a_{2}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\pi}(m_{K^{*}}^{2}) - a_{E}^{PV}g_{s}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{+}\rho^{-}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}f_{K}\Big(a_{1}^{PV}m_{\rho}A_{0}^{D\rho}(m_{K}^{2}) - a_{E}^{PV}g_{s}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{0}\rho^{0}) &= G_{F}V_{cd}^{*}V_{us}f_{K}\Big(a_{2}^{PV}m_{\rho}A_{0}^{D\rho}(m_{K}^{2}) - a_{E}^{PV}g_{s}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{0}\rho) &= G_{F}V_{cd}^{*}V_{us}f_{K}\Big(a_{2}^{PV}m_{\rho}A_{0}^{D\omega}(m_{K}^{2}) - a_{E}^{PV}g_{s}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{0}\phi) &= G_{F}V_{cd}^{*}V_{us}f_{K}\Big(a_{2}^{PV}m_{\omega}A_{0}^{D\omega}(m_{K}^{2}) - a_{E}^{PV}g_{s}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to K^{0}\phi) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}a_{E}^{PV}g_{s}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \eta K^{*0}) &= G_{F}V_{cd}^{*}V_{us}\Big(a_{E}^{PV}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}(g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{2}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\eta_{q}}(m_{K^{*}}^{2})\cos\phi\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{0} \to \eta' K^{*0}) &= G_{F}V_{cd}^{*}V_{us}\Big(a_{E}^{PV}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}(g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{2}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\eta_{q}}(m_{K^{*}}^{2})\sin\phi\Big)(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to \pi^{+}K^{*0}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}\Big(a_{A}^{PV}g_{s}f_{K}f_{D}\frac{m_{D}^{2}}{m_{D}^{2}-m_{K}^{2}}} + a_{2}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\eta}(m_{K^{*}}^{2})\Big)(\varepsilon^{*} \cdot p_{D}), \end{split}$$

$$\begin{aligned} \mathcal{A}(D^{+} \to \pi^{0}K^{*+}) &= G_{F}V_{cd}^{*}V_{us} \Big(a_{A}^{PV}g_{s}f_{K}f_{D} \frac{m_{D}^{*}}{m_{D}^{*} - m_{K}^{*}} - a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\pi}(m_{K^{*}}^{*}) \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to K^{+}\rho^{0}) &= G_{F}V_{cd}^{*}V_{us}f_{K} \Big(a_{A}^{PV}g_{s}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} - a_{1}^{PV}m_{\rho}A_{0}^{D\rho}(m_{K}^{2}) \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to K^{0}\rho^{+}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}f_{K} \Big(a_{A}^{PV}g_{s}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} + a_{2}^{PV}m_{\rho}A_{0}^{D\rho}(m_{K}^{2}) \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to K^{+}\omega) &= G_{F}V_{cd}^{*}V_{us}f_{K} \Big(a_{A}^{PV}g_{s}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} + a_{1}^{PV}m_{\omega}A_{0}^{D\omega}(m_{K}^{2}) \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to K^{+}\phi) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}d_{A}^{PV}g_{ss}f_{L}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to \eta K^{*+}) &= G_{F}V_{cd}^{*}V_{us}\Big(a_{A}^{PV}f_{K}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} (g_{s}\cos\phi - \sqrt{2}g_{ss}\sin\phi) + a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\eta_{q}}(m_{K^{*}}^{2})\cos\phi \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to \eta' K^{*+}) &= G_{F}V_{cd}^{*}V_{us}\Big(a_{A}^{PV}f_{K}f_{D} \frac{m_{D}^{2}}{m_{D}^{2} - m_{K}^{2}} (g_{s}\sin\phi + \sqrt{2}g_{ss}\cos\phi) + a_{1}^{PV}m_{K^{*}}f_{K^{*}}F_{1}^{D\eta_{q}}(m_{K^{*}}^{2})\sin\phi \Big) (\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D^{+} \to \eta' K^{*+}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}m_{K^{*}}(a_{2}f_{K^{*}}F_{1}^{D^{K}}(m_{K^{*}}^{2}) + a_{1}^{PV}f_{K}A_{0}^{D_{S}K^{*}}(m_{K}^{2}))(\varepsilon^{*} \cdot p_{D}), \\ \mathcal{A}(D_{S}^{+} \to K^{+}K^{*0}) &= \sqrt{2}G_{F}V_{cd}^{*}V_{us}m_{K^{*}}(a_{1}^{PV}f_{K^{*}}F_{1}^{D^{K}}(m_{K^{*}}^{2}) + a_{2}^{PV}f_{K}A_{0}^{D_{S}K^{*}}(m_{K}^{2}))(\varepsilon^{*} \cdot p_{D}). \end{aligned}$$

- M. Artuso, B. Meadows, and A. A. Petrov, Annu. Rev. Nucl. Part. Sci. 58, 249 (2008).
- [2] G. Burdman and I. Shipsey, Annu. Rev. Nucl. Part. Sci. 53, 431 (2003); G. Burdman, E. Golowich, J. L. Hewett, and S. Pakvasa, Phys. Rev. D 66, 014009 (2002).
- [3] S. Fajfer, S. Prelovsek, and P. Singer, Phys. Rev. D 64, 114009 (2001).
- [4] A.A. Petrov, Phys. Rev. D 69, 111901 (2004).
- [5] G. D'Ambrosio and D.-N. Gao, Phys. Lett. B 513, 123 (2001).
- [6] B.D. Yabsley, Int. J. Mod. Phys. A 19, 949 (2004).
- [7] F.E. Close and H.J. Lipkin, Phys. Lett. B 551, 337 (2003).
- [8] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B591, 313 (2000).
- [9] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001); C.-D. Lu, K. Ukai, and M.-Z. Yang, Phys. Rev. D 63, 074009 (2001); C.-D. Lu and M.-Z. Yang, Eur. Phys. J. C 23, 275 (2002).
- [10] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. Lett.
 87, 201806 (2001); Phys. Rev. D 65, 054022 (2002).
- [11] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C 29, 637 (1985); M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987).
- [12] A. Ali, G. Kramer, and C.-D. Lu, Phys. Rev. D 58, 094009 (1998); 59, 014005 (1998).
- [13] H.-Y. Cheng, Phys. Lett. B 335, 428 (1994); Z. Phys. C 69, 647 (1996).
- [14] H.-Y. Cheng, Eur. Phys. J. C 26, 551 (2003).

- [15] Y.-L. Wu, M. Zhong, and Y.-F. Zhou, Eur. Phys. J. C 42, 391 (2005).
- [16] J.L. Rosner, Phys. Rev. D 60, 114026 (1999).
- [17] C.-W. Chiang and J. L. Rosner, Phys. Rev. D 65, 054007 (2002); C.-W. Chiang, Z. Luo, and J. L. Rosner, Phys. Rev. D 67, 014001 (2003).
- [18] B. Bhattacharya and J. L. Rosner, Phys. Rev. D 77, 114020 (2008); 79, 034016 (2009); 81, 014026 (2010); 82, 037502 (2010).
- [19] H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D 81, 074021 (2010).
- [20] D.-S. Du, Y. Li, and C.-D. Lu, Chin. Phys. Lett. 23, 2038 (2006).
- [21] J.J. Sakurai, Phys. Rev. 156, 1508 (1967).
- [22] A. K. Das and V. S. Mathur, Mod. Phys. Lett. A 8, 2079 (1993); P. F. Bedaque, A. K. Das, and V. S. Mathur, Phys. Rev. D 49, 269 (1994); 49, 1339 (1994).
- [23] G. Kramer and C.-D. Lu, Int. J. Mod. Phys. A 13, 3361 (1998).
- [24] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
- [25] A. J. Buras, J. M. Gerard, and R. Ruckl, Nucl. Phys. B268, 16 (1986).
- [26] K. Nakamura *et al.* (Particle Data Group Collaboration), J. Phys. G 37, 075021 (2010).
- [27] P. Ball, G.W. Jones, and R. Zwicky, Phys. Rev. D 75, 054004 (2007).
- [28] D. Melikhov and B. Stech, Phys. Rev. D 62, 014006 (2000).

- [29] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D 80, 032005 (2009).
- [30] C.-H. Chen, Y.-L. Shen, and W. Wang, Phys. Lett. B 686, 118 (2010).
- [31] H.-W. Ke, X.-Q. Li, and Z. T. Wei, Eur. Phys. J. C 69, 133 (2010).
- [32] S. Fajfer and J.F. Kamenik, Phys. Rev. D 71, 014020 (2005).
- [33] G. Amoros, S. Noguera, and J. Portoles, Eur. Phys. J. C 27, 243 (2003).
- [34] A. Khodjamirian, C. Klein, T. Mannel, and N. Offen, Phys. Rev. D 80, 114005 (2009).
- [35] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989).
- [36] D. Scora and N. Isgur, Phys. Rev. D 52, 2783 (1995).
- [37] H.-M. Choi and C.-R. Ji, Phys. Lett. B 460, 461 (1999).
- [38] W. Y. Wang, Y. L. Wu, and M. Zhong, Phys. Rev. D 67, 014024 (2003).
- [39] P. Ball, V.M. Braun, and H.G. Dosch, Phys. Rev. D 44, 3567 (1991).
- [40] P. Ball, Phys. Rev. D 48, 3190 (1993).
- [41] K.-C. Yang and W. Y. P. Hwang, Z. Phys. C 73, 275 (1997).
- [42] D.-S. Du, J.-W. Li, and A. M.-Z. Yang, Eur. Phys. J. C 37, 173 (2004).
- [43] T. M. Aliev, A. Ozpineci, and M. Savci, arXiv:hep-ph/ 0401181.
- [44] A. Khodjamirian, R. Ruckl, S. Weinzierl, C. W. Winhart, and O. I. Yakovlev, Phys. Rev. D 62, 114002 (2000).
- [45] M.B. Wise, Phys. Rev. D 45, R2188 (1992).
- [46] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. 281, 145 (1997).

- [47] F. Ambrosino et al., J. High Energy Phys. 07 (2009) 105.
- [48] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D 58, 114006 (1998); Phys. Lett. B 449, 339 (1999).
- [49] K. Maltman, Phys. Lett. B 462, 14 (1999).
- [50] C. M. Shakin and H. Wang, Phys. Rev. D 63, 074017 (2001).
- [51] H. Mendez *et al.* (CLEO Collaboration), Phys. Rev. D 81, 052013 (2010).
- [52] F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, and P. Santorelli, Phys. Rev. D 51, 3478 (1995); F. Buccella, M. Lusignoli, G. Mangano, G. Miele, A. Pugliese, and P. Santorelli, Phys. Lett. B 302, 319 (1993).
- [53] E. Won et al. (Belle Collaboration), arXiv:1107.0553.
- [54] H.-Y. Cheng, arXiv:1011.0790.
- [55] Y.-S. Dai, D.-S. Du, X.-Q. Li, Z.-T. Wei, and B.-S. Zou, Phys. Rev. D 60, 014014 (1999).
- [56] J. O. Eeg, S. Fajfer, and J. Zupan, Phys. Rev. D 64, 034010 (2001).
- [57] S. Fajfer and J. Zupan, Int. J. Mod. Phys. A 14, 4161 (1999).
- [58] F. Buccella, M. Lusignoli, and A. Pugliese, Phys. Lett. B 379, 249 (1996); H.-Y. Cheng and B. Tseng, Phys. Rev. D 59, 014034 (1998).
- [59] P. Colangelo and F. De Fazio, Phys. Lett. B 520, 78 (2001).
- [60] P. Naik *et al.* (CLEO Collaboration), Phys. Rev. D 80, 112004 (2009).
- [61] S. Fajfer, A. Prapotnik, P. Singer, and J. Zupan, Phys. Rev. D 68, 094012 (2003).
- [62] C. P. Jessop *et al.* (CLEO Collaboration), Phys. Rev. D 58, 052002 (1998).