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## Charged bottomoniumlike states  $Z_b(10610)$  and  $Z_b(10650)$ and the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$  decay

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Inspired by the newly observed two charged bottomoniumlike states, we consider the possible contribution from the intermediate  $Z_b(10610)$  and  $Z_b(10650)$  states to the  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$  decay process, which naturally explains Belle's previous observation of the anomalous  $\Upsilon(2S)\pi^+\pi^-$  production near the peak of  $\Upsilon(5S)$  at  $\sqrt{s} = 10.87$  GeV [K.F. Chen *et al.* (Belle Collaboration),<br>Phys Rev Lett 100, 112001 (2008)] The resulting  $d\Gamma(\Upsilon(5S) \rightarrow \Upsilon(2S) \pi^+ \pi^-)/dm$ , and  $d\Gamma(\Upsilon(5S) \rightarrow$ Phys. Rev. Lett. 100[, 112001 \(2008\)](http://dx.doi.org/10.1103/PhysRevLett.100.112001)]. The resulting  $d\Gamma(Y(5S) \to Y(2S)\pi^+\pi^-)/dm_{\pi^+\pi^-}$  and  $d\Gamma(Y(5S) \to Y(2S)\pi^+\pi^-)/d\cos\theta$  distributions agree with Balle's measurement after inclusion of these  $Z$ , states. This  $\frac{\gamma(2S)\pi^+\pi^-}{\gamma}$  dcos $\theta$  distributions agree with Belle's measurement after inclusion of these  $Z_b$  states. This formalism also reproduces the Belle observation of the double-peak structure and its reflection in the  $\Upsilon(2S)\pi^+$  invariant mass spectrum of the  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$  decay.

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Very recently, the Belle Collaboration announced the first observation of two charged bottomoniumlike states  $Z_b(10610)$  and  $Z_b(10650)$  in the hidden-bottom decay channels  $\Upsilon(nS)\pi^{\pm}$   $(n = 1, 2, 3)$  and  $h_b(mP)\pi^{\pm}$   $(m = 1, 2)$ <br>of  $\Upsilon(5S)$  [1]. The measured parameters of Z. (10610) and of  $\Upsilon(5S)$  [\[1\]](#page-5-0). The measured parameters of  $Z_b(10610)$  and  $Z_b(10650)$  are

$$
M_{Z_b(10610)}/\Gamma_{Z_b(10610)} = 10608.4 \pm 2.0/15.6 \pm 2.5 \text{ MeV},
$$
  

$$
M_{Z_b(10650)}/\Gamma_{Z_b(10650)} = 10653.2 \pm 1.5/14.4 \pm 3.2 \text{ MeV}.
$$

The analysis of the angular distribution indicates that the quantum numbers of both  $Z_b(10610)$  and  $Z_b(10650)$ are  $I^G(J^P) = 1^+(1^+)$ . Both  $Z_b(10610)$  and  $Z_b(10650)$  are charged hidden-bottom states. Moreover, they are very close to the thresholds of  $B\overline{B}^*$  and  $B^*\overline{B}^*$  [\[2](#page-5-1)], respectively. Thus,  $Z_b(10610)$  and  $Z_b(10650)$  are ideal candidates of the  $B\overline{B}^*$  and  $B^*\overline{B}^*$  S-wave molecular states, which were studied extensively in Refs. [[3](#page-5-2)[,4\]](#page-5-3).

On the other hand, a new puzzle arises in the theoretical study [\[5,](#page-5-4)[6](#page-5-5)] of the dipion invariant mass distribution and the  $\cos\theta$  distribution of the anomalous  $\Upsilon(2S)\pi^+\pi^-$  produc-<br>tion near the peak of  $\Upsilon(5S)$  [7]. While all the other calcution near the peak of  $\Upsilon(5S)$  [[7](#page-5-6)]. While all the other calculations are well in accord with the Belle data, the predicted differential width  $d\Gamma(Y(5S) \to Y(2S)\pi^+\pi^-)/d\cos\theta$  dis-<br>agrees with the Belle measurement [5]. In this work, we agrees with the Belle measurement [[5\]](#page-5-4). In this work, we will illustrate that the inclusion of these two  $Z_b$  states in the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decays explains the puzzling line shape of  $d\Gamma(Y(5S) \to \Upsilon(2S)\pi^+\pi^-)/d\cos\theta$  very naturally naturally.

In general, there exist three mechanisms for the  $\Upsilon(5S)$ hidden-bottom decays with the dipion emission

$$
Y(5S) \to Y(2S)(p_1)\pi^+(p_2)\pi^-(p_3).
$$

The first one is the  $\Upsilon(2S)\pi^+\pi^-$  direct production by  $\Upsilon(5S)$  decay (see Fig. [1\(a\)](#page-0-4)). The so-called direct production of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  denotes that there does not exist the contribution from the intermediate mesons (such as  $\sigma$ (600),  $f_0(980)$ , hadronic loop constructed by  $B^{(*)}$  or  $B_s^{(*)}$  mesons,  $Z_b$ ) to  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ . Thus,<br>the direct production of  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$  provides the direct production of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$  provides<br>the background contribution the background contribution.

The QCD Multipole Expansion method [\[8\]](#page-5-7) is generally applied to deal with the dipion transition between heavy quarkonia. So far, there exist many theoretical efforts study the dipion transitions between the bottomonia [[8](#page-5-7)[–15\]](#page-5-8) (see Refs. [[16](#page-5-9)[–18\]](#page-5-10) for a detailed review). In this work, we do



<span id="page-0-4"></span>FIG. 1 (color online). The diagrams in the  $\Upsilon(5S)$  hiddenbottom decay. Here, Fig.  $1(a)$  represents the  $Y(5S)$  direct decay into  $\Upsilon(2S)\pi^+\pi^-$ , while Fig. [1\(b\)](#page-0-4) denotes the intermediate hadronic loop contribution to  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ . (c) and (d) describe the intermediate  $Z_b^{\pm}$  contribution to  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  where  $Z^{\pm} = \{7, (10610)^{\pm} \}$   $\Upsilon(10650)^{\pm}$  $\Upsilon(2S)\pi^+\pi^-$ , where  $Z_b^{\pm} = \{Z_b(10610)^{\pm}, Z_b(10650)^{\pm}\}.$ 

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not intend to calculate the contribution from the direct transition under the framework of the QCD Multipole Expansion method, but alternatively follow the effective Lagrangian approach to describe  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$ transitions. The transition amplitude of the direct production of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  can be written as

<span id="page-1-0"></span>
$$
\mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-]_{\text{Direct}}
$$
\n
$$
= \frac{\mathcal{F}^{(n)}}{f_\pi^2} \epsilon_{\Upsilon(5S)} \cdot \epsilon_{\Upsilon(2S)} \left\{ \left[ q^2 - \kappa^{(n)}(\Delta M)^2 \left( 1 + \frac{2m_\pi^2}{q^2} \right) \right]_{S-\text{wave}} + \left[ \frac{3}{2} \kappa^{(n)}((\Delta M)^2 - q^2) \left( 1 - \frac{4m_\pi^2}{q^2} \right) \left( \cos \theta^2 - \frac{1}{3} \right) \right]_{D-\text{wave}} \right\},\tag{1}
$$

which was suggested by Novikov and Shifman in the study of the  $\psi' \rightarrow J/\psi \pi^+ \pi^-$  decay [[19](#page-5-11)], where the subscripts S-wave and D-wave denote the S-wave and D-wave contributions, respectively.  $\Delta M$  is the mass difference between  $\Upsilon(5S)$  and  $\Upsilon(2S)$ .  $q^2 = (p_2 + p_3)^2 \equiv m_{\pi^+\pi^-}^2$  denotes the invariant mass of  $\pi^+\pi^-$ , while  $\theta$  is the angle between  $\Upsilon(5S)$  and  $\pi^-$  in the  $\pi^+\pi^-$  rest frame. The pion decay constant and mass are taken as  $f_{\pi} = 130 \text{ MeV}$  and  $m_{\pi}$  = 140 MeV, respectively. In Eq. [\(1](#page-1-0)),  $\kappa$  and  $\mathcal{F}$  are free parameters to be determined when fitting the experimental data.

Different from the other low-lying bottomonia with  $J^{PC} = 1^{--}$ ,  $\Upsilon(5S)$  is above the  $B^{(*)} \overline{B}^{(*)}$  thresholds and predominantly decays into  $B^{(*)}\bar{B}^{(*)}$  pair, which may render the coupled channel effect quite important [[20](#page-5-12)–[22](#page-5-13)]. When exploring the  $Y(5S)$  hidden-bottom decay, the coupled channel effect has to be taken into account. In other words, there also exists the second mechanism contributing to the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  transitions as shown in Fig. [1\(b\)](#page-0-4), where the intermediate  $B^{(*)}$  and  $\bar{B}^{(*)}$  hadronic loop is the bridge to connect the initial state  $\Upsilon(5S)$  and final state  $\Upsilon(2S)\pi^+\pi^-$ . Furthermore,  $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$  can be approximately expressed as a sequential decay process.  $\Upsilon(5S)$  first transits into  $\Upsilon(2S)$  and the scalar meson  $\sigma$ (600). Then,  $\sigma$ (600) couples with the dipion. Choosing  $\sigma$ (600) as the intermediate state contribution to the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  process is not only consistent with the Belle data [[1](#page-5-0)[,7\]](#page-5-6), but also allowed by the phase space of the decay channel.

If comparing the dipion invariant mass spectrum of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$  in Refs. [[1](#page-5-0),[7\]](#page-5-6), the data in Ref. [\[1\]](#page-5-0) at the higher end of  $m_{\pi^+\pi^-}$  are qualitatively different from those in Ref. [\[7\]](#page-5-6), where the total events in Ref. [\[7](#page-5-6)] are at least one order of magnitude less than those in Ref. [[1\]](#page-5-0). Such a large accumulation of events at  $m_{\pi^+ \pi^-} > 700 \text{ MeV}$ [\[1\]](#page-5-0) might be due to the contribution from the tail of the intermediate  $f_0(980)$ . We did not include the  $f_0(980)$  contribution when we analyzed the data in [\[7\]](#page-5-6). Considering the situation of the new data of the dipion invariant mass spectrum of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$  [\[1](#page-5-0)], we also include the  $f_0(980)$  contribution to the analysis of  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^{+}\pi^{-}$  in the following.

The effective Lagrangians relevant to Fig. [1\(b\)](#page-0-4) include

$$
\mathcal{L}_{YBB} = ig_{YBB} Y_{\mu} (\partial^{\mu} B B^{\dagger} - B \partial^{\mu} B^{\dagger}), \qquad (2)
$$

$$
\mathcal{L}_{Y\mathcal{B}^*\mathcal{B}} = -ig_{Y\mathcal{B}^*\mathcal{B}}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu Y_\nu (\partial_\alpha \mathcal{B}^*_{\beta} \mathcal{B}^\dagger + \mathcal{B}\partial_\alpha \mathcal{B}^{*\dagger}_{\beta}),
$$
\n(3)

$$
\mathcal{L}_{Y\mathcal{B}^*\mathcal{B}^*} = -ig_{Y\mathcal{B}^*\mathcal{B}^*} \{ Y^\mu (\partial_\mu \mathcal{B}^{*\nu} \mathcal{B}^{*\dagger}_\nu - \mathcal{B}^{*\nu} \partial_\mu \mathcal{B}^{*\dagger}_\nu) \n+ (\partial_\mu \gamma_\nu \mathcal{B}^{*\nu} - Y_\nu \partial_\mu \mathcal{B}^{*\nu}) \mathcal{B}^{*\mu\dagger} \n+ \mathcal{B}^{*\mu} (Y^\nu \partial_\mu \mathcal{B}^{*\dagger}_\nu - \partial_\mu Y_\nu \mathcal{B}^{*\nu\dagger}) \},
$$
\n(4)

and

$$
\mathcal{L}_{\mathcal{S}\mathcal{B}^{(*)}B^{(*)}} = g_{\mathcal{B}\mathcal{B}\mathcal{S}} \mathcal{S}\mathcal{B}\mathcal{B}^{\dagger} - g_{\mathcal{B}^{*}\mathcal{B}^{*}\mathcal{S}} \mathcal{S}\mathcal{B}^{*}\mathcal{B}^{* \dagger}, \quad (5)
$$

where  $B = (\bar{B}^0, B^-, B_s^-)$  and  $(\bar{B}^{\dagger})^T = (B^0, B^+, B_s^+)$ .<br>There are 4 diagrams Thus the concrete expressions of There are 4 diagrams. Thus, the concrete expressions of decay amplitudes are written as

$$
\mathcal{M}_{B\bar{B}}^{B} = (i)^{3} \int \frac{d^{4}q}{(2\pi)^{4}} [ig_{Y(5S)BB} \epsilon_{Y(5S)}^{\mu} (ip_{2\mu} - ip_{1\mu})] [ig_{Y(nS)BB} \epsilon_{Y(nS)}^{\rho} (-ip_{1\rho} - iq_{\rho})] [g_{BBS}]
$$
  

$$
\times \frac{1}{p_{1}^{2} - m_{B}^{2}} \frac{1}{p_{2}^{2} - m_{B}^{2}} \frac{1}{q^{2} - m_{B}^{2}} \mathcal{F}(q^{2}),
$$
 (6)

$$
\mathcal{M}_{B\bar{B}^*}^{B^*} = (i)^3 \int \frac{d^4q}{(2\pi)^4} \left[ -g_{\gamma(5S)BB^*} \varepsilon_{\mu\nu\alpha\beta} (-ip_0^{\mu}) \varepsilon_{\gamma(5S)}^{\nu} (ip_2^{\alpha}) \right] \left[ -g_{\gamma(nS)BB} \varepsilon_{\delta\tau\theta\phi} (ip_3^{\delta}) \varepsilon_{\gamma(nS)}^{\tau} (iq^{\theta}) \right] \left[ -g_{B^*B^*S} \right] \times \frac{1}{p_1^2 - m_B^2} \frac{-g^{\beta\rho} + p_2^{\beta} p_2^{\rho} / m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{-g^{\phi\rho} + q^{\phi} q^{\rho} / m_{B^*}^2}{q_2^2 - m_{B^*}^2} \mathcal{F}(q^2),
$$
\n(7)

$$
\mathcal{M}_{B\bar{B}^*}^s = (i)^3 \int \frac{d^4q}{(2\pi)^4} \left[ -g_{\Upsilon(5S)B^*B} \varepsilon_{\mu\nu\alpha\beta} (-ip_0^{\mu}) \varepsilon_{\Upsilon(5S)}^{\nu} (ip_1^{\alpha}) \right] \left[ -g_{\Upsilon(nS)B^*B} \varepsilon_{\delta\tau\theta\phi} (ip_3^{\delta}) \varepsilon_{\Upsilon(nS)}^{\tau} (-ip_1^{\theta}) \right] \left[ g_{BBS} \right]
$$
  
 
$$
\times \frac{-g^{\beta\phi} + p_1^{\beta} p_1^{\phi} / m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}(q^2), \tag{8}
$$

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$$
\mathcal{M}_{B^{*}\bar{B}^{*}}^{B^{*}} = (i)^{3} \int \frac{d^{4}q}{(2\pi)^{4}} \left[ -ig_{Y(5S)B^{*}B^{*}} \epsilon_{Y(5S)}^{\mu}((ip_{2\mu} - ip_{1\mu})g_{\nu\rho} + (-ip_{0\rho} - ip_{2\rho})g_{\mu\nu} + (ip_{1\nu} + ip_{0\nu})g_{\mu\rho} \right] \times \left[ -ig_{Y(nS)B^{*}B^{*}} \epsilon_{Y(nS)}^{\phi}((-ip_{1\phi} - iq_{\phi})g_{\alpha\beta} + (ip_{3\beta} + ip_{1\beta})g_{\alpha\phi} + (iq_{\alpha} - ip_{3\alpha})g_{\beta\phi} \right) \left[ -g_{B^{*}B^{*}S} \right] \times \frac{-g^{\rho\alpha} + p_{1}^{\rho}p_{1}^{\alpha}/m_{B^{*}}^{2}}{p_{1}^{2} - m_{B^{*}}^{2}} \frac{-g^{\nu\tau} + p_{2}^{\nu}p_{2}^{\tau}/m_{B^{*}}^{2}}{p_{2}^{2} - m_{B^{*}}^{2}} \frac{-g^{\beta\tau} + q^{\beta}q^{\tau}/m_{B^{*}}^{2}}{q^{2} - m_{B^{*}}^{2}} \mathcal{F}(q^{2}). \tag{9}
$$

The amplitude  $\mathcal{M}_{AB}^C$  indicates that the initial  $\Upsilon(5S)$  dissolves into intermediate AB which transit into the final solves into intermediate AB, which transit into the final  $\Upsilon(2S)$  and scalar meson by exchanging meson C. In the above expressions, the form factor is introduced by  $\mathcal{F}(q^2) = (\Lambda^2 - m_E^2)/(q^2 - m_E^2)$ <br>the exchanged  $R^{(*)}$  meson in the  $\mathcal{F}(q^2) = (\Lambda^2 - m_E^2)/(q^2 - m_E^2)$ . And  $m_E$  is the mass of<br>the exchanged  $B^{(*)}$  meson in the  $B^{(*)}\bar{B}^{(*)} \rightarrow \Upsilon(2S)S$  tran-<br>sitions shown in Fig. 1(b) and  $\Lambda = m_E + \alpha \Lambda_{\text{QCD}}$  with sitions shown in Fig. [1\(b\)](#page-0-4) and  $\Lambda = m_E + \alpha \Lambda_{\text{QCD}}$  with  $\Lambda_{\text{QCD}} = 220$  MeV. As indicated in Ref. [\[5\]](#page-5-4), we can parameterize the decay amplitude of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$ corresponding to Fig. [1\(b\)](#page-0-4) as

<span id="page-2-0"></span>
$$
\mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)\sigma(600) \to \Upsilon(2S)\pi^{+}\pi^{-}]
$$
  
= 
$$
\frac{\epsilon_{\Upsilon(5S)} \cdot \epsilon_{\Upsilon(2S)}^{*} F_{\sigma}}{(p_{2} + p_{3})^{2} - m_{\sigma}^{2} + i m_{\sigma} \Gamma_{\sigma}},
$$
 (10)

if only considering the S-wave contribution. Here, we introduce  $F_{\sigma}$  as the fitting parameter.

Similar to Eq. ([10](#page-2-0)), the parameterized decay amplitude of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  with  $f_0(980)$  as the intermediate state can be expressed as

$$
\mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)f_0(980) \to \Upsilon(2S)\pi^+\pi^-]
$$
  
= 
$$
\frac{\epsilon_{\Upsilon(5S)} \cdot \epsilon_{\Upsilon(2S)}^* F_{f_0}}{(p_2 + p_3)^2 - m_{f_0}^2 + i m_{f_0} \Gamma_{f_0}},
$$
 (11)

which corresponds to Fig. [1\(b\)](#page-0-4) with the replacement  $\sigma \rightarrow f_0(980)$ .

Regarding the contribution of these two newly observed  $Z_b$  states to the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  process, we introduce the third mechanism depicted in Fig. [1\(c\)](#page-0-4) and [1\(d\)](#page-0-4), where  $Z_b^{\pm}$ s are the intermediate states and interact with  $\Upsilon(5S)\pi^{\pm}$  and  $\Upsilon(2S)\pi^{\pm}$ . The general expressions of the amplitudes of Fig. 1(c) and 1(d) are amplitudes of Fig.  $1(c)$  and  $1(d)$  are

$$
\mathcal{M}[\Upsilon(5S) \to Z_b^+ \pi^- \to \Upsilon(2S) \pi^+ \pi^-]_{Z_b^+}
$$
  
=  $F_{Z_b^+} \epsilon_{\Upsilon(5S)}^{\mu} \epsilon_{\Upsilon(2S)}^{*\nu} \frac{-g_{\mu\nu} + (p_1^{\mu} + p_2^{\mu})(p_1^{\nu} + p_2^{\nu})/m_{Z_b}^2}{(p_1 + p_2)^2 - m_{Z_b}^2 + im_{Z_b} \Gamma_{Z_b}}$  (12)

$$
\mathcal{M}[\Upsilon(5S) \to Z_b^- \pi^+ \to \Upsilon(2S) \pi^- \pi^+]_{Z_b^-}
$$
  
=  $F_{Z_b^-} \epsilon_{\Upsilon(5S)}^{\mu} \epsilon_{\Upsilon(2S)}^{*\nu} \frac{-g_{\mu\nu} + (p_1^{\mu} + p_3^{\mu})(p_1^{\nu} + p_3^{\nu})/m_{Z_b}^2}{(p_1 + p_3)^2 - m_{Z_b}^2 + im_{Z_b} \Gamma_{Z_b}}$ , (13)

respectively, where we define  $F_{Z_b^+} = g_{Y(5S)Z_b^+ \pi} g_{Z_b^+ Y(2S) \pi^+}$ and  $F_{Z_p^-} = g_{Y(5S)Z_p^-\pi}g_{Z_p^-\gamma(2S)\pi^-}$ . Since Fig. [1\(c\)](#page-0-4) and [1\(d\)](#page-0-4) are related to each other by charge conjugation, thus  $F_{Z_b^-} = F_{Z_b^+} = F_{Z_b}$ .<br>Thus the total

Thus, the total decay amplitude of the  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^+\pi^-$  decay is

<span id="page-2-2"></span>
$$
\mathcal{M}_{\text{total}} = \mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-]_{\text{Direct}} + e^{i\phi_\sigma} \mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)\sigma(600) \to \Upsilon(2S)\pi^+\pi^-] \n+ e^{i\phi_{f_0}} \mathcal{M}[\Upsilon(5S) \to \Upsilon(2S)f_0(980) \to \Upsilon(2S)\pi^+\pi^-] + \sum_{Z_b} e^{i\phi_{Z_b}} \{\mathcal{M}[\Upsilon(5S) \to Z_b^+\pi^- \to \Upsilon(2S)\pi^+\pi^-]_{Z_b^+} \n+ \mathcal{M}[\Upsilon(5S) \to Z_b^-\pi^+ \to \Upsilon(2S)\pi^+\pi^-]_{Z_b^-} \}
$$
\n(14)

where we have introduced the phase angles  $\phi_{\sigma}$ ,  $\phi_{f_0}$ ,  $\varphi_{Z_b(10610)}$ , and  $\varphi_{Z_b(10650)}$ .

As a three-body decay, the differential decay width for  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  reads as,

$$
d\Gamma = \frac{1}{3} \frac{1}{(2\pi)^3} \frac{1}{32m_{\gamma(5S)}^3} |\mathcal{M}_{\text{total}}|^2 dm_{\gamma(2S)\pi}^2 dm_{\pi\pi}^2, \quad (15)
$$

with  $m_{Y(2S)\pi^+}^2 = (p_1 + p_2)^2$  and  $m_{\pi^+\pi^-}^2 = (p_2 + p_3)^2$ .<br>The relevent recononce perceptors are listed in Table I. The relevant resonance parameters are listed in Table [I](#page-2-1).

If considering only the contributions from Fig.  $1(a)$  and [1\(b\)](#page-0-4) in our present scenario, we have four free parameters as listed in Table [II](#page-3-0), where the  $\sigma$ (600) contribution is included to fit the Belle data [\[7](#page-5-6)]. With the help of the MINUIT package, we perform the global fit to the experimental data of the dipion invariant mass spectrum distribution and the  $\cos\theta$  distribution of the  $\Upsilon(2S)\pi^+\pi^-$ 

<span id="page-2-1"></span>TABLE I. The resonance parameters adopted in our calculation [[1,](#page-5-0)[2,](#page-5-1)[23\]](#page-5-14).

<b>State</b>	Mass (GeV)	<b>State</b>	Mass (GeV)	Width (GeV)
Y(5S)	10.87	$\sigma$ (600)	0.478	0.324
		$f_0(980)$	0.98	0.1
Y(2S)	10.023	$Z_h(10610)$	10.608	0.0156
		$Z_h(10650)$	10.653	0.0144

<span id="page-3-0"></span>TABLE II. The values of the fitting parameters for the best fit to the Belle data of  $\Upsilon(2S)\pi^+\pi^-$  production near the peak of  $\Upsilon(5S)$  [\[7\]](#page-5-6) without considering the contributions from  $Z_b(10610)$ and  $Z_b(10650)$ . For the obtained central values of the parameters, the corresponding partial decay width of  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^{+}\pi^{-}$  is 0.836 MeV.

Parameter	Value	Parameter	Value
$\mathcal{F}$	$0.943 \pm 0.071$	к	$0.739 \pm 0.034$
F	$25.603 \pm 2.175 \text{ GeV}^2$	$\phi_{\sigma}$	$2.623 \pm 0.132$ Rad

production near the peak of  $\Upsilon(5S)$  [\[7](#page-5-6)]. The best fit to the dipion invariant mass spectrum distribution is shown in the left panel in Fig. [2.](#page-3-1) Unfortunately, the corresponding  $\cos\theta$ distribution of the  $\Upsilon(2S)\pi^+\pi^-$  production strongly deviates from the Belle data, as shown in the right panel of Fig. [2.](#page-3-1) The values of the obtained fitting parameters are presented in Table [II](#page-3-0). Such discrepancy between theoretical and experimental results stimulates a New Puzzle first indicated in Ref. [[5\]](#page-5-4). At present, solving this new puzzle becomes an important and intriguing research topic, which will be helpful to the underlying mechanism behind the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decay.

In contrast, we consider the contribution from  $Z_b(10610)$  and  $Z_b(10650)$  in the following and discuss the dependence of  $d\Gamma/dm_{\pi^+\pi^-}$  and  $d\Gamma/d\cos\theta$  of  $\Upsilon(5S) \rightarrow$ <br> $\Upsilon(2S)\pi^+\pi^-$  on  $m_{\pi^+}$  and  $\cos\theta$  respectively Under this  $\Upsilon(2S)\pi^+\pi^-$  on  $m_{\pi^+\pi^-}$  and cos $\theta$ , respectively. Under this scheme, we refit the Belle data [7] with Eq. (14). There are scheme, we refit the Belle data [[7](#page-5-6)] with Eq. [\(14\)](#page-2-2). There are 10 fitting parameters as listed in Table [III.](#page-3-2) In Fig. [3](#page-3-3), we present a comparison between the Belle data (dots with error bars) and our best fit (histograms) to the Belle data [[1](#page-5-0)], which indicates that the line shapes of the invariant mass spectra of  $\pi^+\pi^-$  and  $\Upsilon(2S)\pi^+$  for  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^+\pi^-$  describe the Belle data [[1](#page-5-0)] well. The double-peak structure around 10.6 GeV and its reflection around 10.25 GeV are reproduced by our model well. With the central values of these parameters in Table [III](#page-3-2), we obtain the partial decay width of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^{+}\pi^{-}$ 

<span id="page-3-1"></span>

FIG. 2 (color online). (Color online). The dipion invariant mass  $(m_{\pi\pi})$  distribution (left panel) and the cos $\theta$  distribution (right panel) of the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decay. The dots with error bars are the results measured by Belle [[7\]](#page-5-6), while the green histograms are the best fit from our model without including the intermediate  $Z_b(10610)^{\pm}$  and  $Z_b(10650)^{\pm}$  contribution to  $Y(5S) \rightarrow Y(2S)\pi^{+}\pi^{-}$  When fitting the experimental data [7]  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ . When fitting the experimental data [[7\]](#page-5-6), we only include the  $\sigma$ (600) contribution.

<span id="page-3-2"></span>TABLE III. The values of the fitting parameters for the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decay after including the contributions from  $Z_b(10610)$  and  $Z_b(10650)$ .

Parameter	Value	Parameter	Value
F	$1.404 \pm 0.068$	к	$0.301 \pm 0.013$
$F_{\sigma}$	$20.037 \pm 0.423 \text{ GeV}^2$	$\phi_{\sigma}$	$0.907 \pm 0.132$ rad
$F_{f_0}$	$17.076 \pm 3.563 \text{ GeV}^2$	$\phi_{f_0}$	$-0.753 \pm 0.14$ rad
$F_{Z_h(10610)}$	$3.412 \pm 0.385 \text{ GeV}^2$	$\varphi_{Z_h(10610)}$	$-3.135 \pm 0.03$ rad
$F_{Z_h(10650)}$	$2.994 \pm 0.261 \text{ GeV}^2$	$\varphi_{Z_h(10650)}$	$-2.836 \pm 0.165$ rad

 $\Gamma = 0.915$  MeV, which is consistent with the Belle<br>measurement  $\Gamma = 0.85 \pm 0.07(\text{stat}) \pm 0.16(\text{syst})$  MeV [7] measurement  $\Gamma = 0.85 \pm 0.07$ (stat)  $\pm 0.16$ (syst) MeV [[7\]](#page-5-6).<br>Thus the contribution from these charged Z, resonances Thus, the contribution from these charged  $Z_b$  resonances provides a possible solution to the puzzle of why the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decay width is abnormally large [[7\]](#page-5-6).

From Table [III](#page-3-2), we notice that the uncertainty of  $F_{f_0}$  is one order of magnitude larger than that of  $F_{\sigma}$ , which means the fit is less sensitive to the  $f_0(980)$  than to the  $\sigma(600)$ . Using Eq. [\(14\)](#page-2-2), we reanalyze the new Belle data in Ref. [\[1\]](#page-5-0) with the obtained fitting parameters in Table [IV,](#page-4-0) where we do not include the  $f_0(980)$  contribution. The comparison between our fitting result and the experimental data are given in Fig. [4.](#page-4-1) By the scenario in Eq. [\(14\)](#page-2-2), we reproduce the Belle data well, which confirms that the intermediate  $f_0(980)$  contribution to  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$  is small. If comparing the obtained values of the fitting parameter in Tables [III](#page-3-2) and [IV,](#page-4-0) we notice that the eight common parameters do not change much in the two schemes. With the parameters listed in Tables [III](#page-3-2) and [IV,](#page-4-0) we also present the  $\cos\theta$  distribution with and without the intermediate  $f_0$ contribution. The experimental measurement of the  $\cos\theta$ distribution for  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  [[7\]](#page-5-6) can be described well with the scenarios in this work. This fact indicates that the two  $Z_b$  structures play an important role in the understanding of the Belle data, especially the  $\cos\theta$  distribution of  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ .

In summary, the Belle Collaboration announced an exciting observation of two charged bottomoniumlike states

<span id="page-3-3"></span>

FIG. 3 (color online). (Color online). The invariant mass spectra of  $\pi^+ \pi^-$  and  $\Upsilon(2S)\pi^+$  for  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+ \pi^-$ . Here, the histograms are theoretical results obtained in our scenario including the intermediate  $f_0(980)$  contribution, while dots with error bars are the Belle data in Ref. [\[1\]](#page-5-0). We also plot the  $f_0(980)$ contribution separately (red solid lines).

## CHARGED BOTTOMONIUMLIKE STATES  $Z_b(10610)$  ... PHYSICAL REVIEW D 84, 074016 (2011)

<span id="page-4-0"></span>TABLE IV. The values of the fitting parameters for the  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$  decay after including the contributions from  $Z_b(10610)$  and  $Z_b(10650)$ . These parameters are obtained by fitting the new experimental data without including the contributions from  $f_0(980)$ .

Parameter	Value	Parameter	Value
$\mathcal F$	$1.073 \pm 0.064$	к	$0.379 \pm 0.034$
$F_{\sigma}$	$23.833 \pm 2.503 \text{ GeV}^2$	$\phi_{\sigma}$	$1.127 \pm 0.128$ rad
$F_{Z_h(10610)}$	$3.200 \pm 0.345 \text{ GeV}^2$		$\varphi_{Z_b(10610)}$ -3.141 ± 0.076 rad
$F_{Z_h(10650)}$	$2.686 \pm 0.306 \text{ GeV}^2$		$\varphi_{Z_b(10650)}$ -2.703 ± 0.225 rad

 $Z_b(10610)$  and  $Z_b(10650)$ . These  $Z_b$  states are good candidates of exotic states, which calls for theoretical efforts in revealing their underlying structures. Carrying out the phenomenological study relevant to  $Z_b(10610)$  and  $Z_b(10650)$  is one of the important and valuable issues of heavy quarkonium physics, which is full of challenges and opportunities [[24](#page-5-15),[25](#page-5-16)].

The  $Z_b(10610)$  and  $Z_b(10650)$  states are related to the anomalous phenomena of  $\Upsilon(2S)\pi^+\pi^-$  production near  $\Upsilon(5S)$ , previously reported by Belle [\[7](#page-5-6)]. Comparing the fitting results without and with the contributions from the newly observed states, we notice that the intermediate  $Z_b(10610)$  and  $Z_b(10650)$  play a crucial role in the behavior of  $d\Gamma(Y(5S) \to Y(2S)\pi^+\pi^-)/d\cos\theta$ . The inclusion of<br>the  $Z_{\perp}(10610)$  and  $Z_{\perp}(10650)$  contribution to  $Y(5S) \to$ the  $Z_b(10610)$  and  $Z_b(10650)$  contribution to  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^+\pi^-$  provides a unique mechanism of understand the puzzling  $\cos\theta$  distribution of  $\Upsilon(2S)\pi^+\pi^-$  production<br>near  $\Upsilon(5S)$  [7]. The double-neak structure and its reflection near  $\Upsilon(5S)$  [[7\]](#page-5-6). The double-peak structure and its reflection in the  $\Upsilon(2S)\pi^+$  invariant mass spectrum of  $\Upsilon(5S) \rightarrow$  $\Upsilon(2S)\pi^+\pi^-$  [\[1\]](#page-5-0) are also reproduced by this mechanism. In this work, the values of the fitting parameters in our scenario are obtained by fitting Belle data [\[1](#page-5-0),[7](#page-5-6)]. To some extent, the interpretation of the values of these parameters is related to the understanding of background, the structures of two  $Z_b$  states, etc, which is an interesting research topic.

Besides finding the signals of  $Z_b(10610)$  and  $Z_b(10650)$ in the  $\Upsilon(2S)\pi^{\pm}$  decay channel, Belle's analysis of its remaining four hidden-bottom decay channels  $\Upsilon(nS)\pi^{\pm}$ remaining four hidden-bottom decay channels  $\Upsilon(nS)\pi^{\pm}$ <br>(*n* = 1–3) and *h*.(*mP*) $\pi^{\pm}$  (*m* = 1–2) also indicate the  $(n = 1, 3)$  and  $h_b(mP)\pi^{\pm}$   $(m = 1, 2)$  also indicate the observation of Z<sub>1</sub>(10610) and Z<sub>1</sub>(10650) [1]. The present observation of  $Z_b(10610)$  and  $Z_b(10650)$  [[1\]](#page-5-0). The present formalism can be extended to study the dipion invariant mass distribution and the cos $\theta$  distribution of  $\Upsilon(5S) \rightarrow \Upsilon(1S, 3S)\pi^+\pi^-$  and  $\Upsilon(5S) \rightarrow h_1(1P, 2P)\pi^+\pi^-$  decay  $\Upsilon(1S, 3S)\pi^+\pi^-$  and  $\Upsilon(5S) \rightarrow h_b(1P, 2P)\pi^+\pi^-$  decay (see Fig. [5](#page-4-2) for more details).

Additionally, Belle's measurement favors the  $B\overline{B}^*$  and  $B^* \overline{B}^*$  molecular explanation of the  $Z_b(10610)$  and

<span id="page-4-1"></span>

 0:225 rad FIG. 4 (color online). (Color online). The distribution invariant mass spectra  $m_{\pi\pi}$  and  $m_{\Upsilon(2S)\pi}$  for  $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$  without including the contributions from  $f_0(980)$ . Here, we use Eq. ([14](#page-2-2)) to redo the analysis. The histograms are the fitting results. The dots with errors correspond to the Belle data [\[1\]](#page-5-0).

<span id="page-4-2"></span>

FIG. 5 (color online). (Color online). The  $\cos\theta$  distributions for  $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ . The histograms in the left-hand side and right-hand side diagrams are the fitting results without and with the contribution from  $f_0(980)$ . The dots with errors correspond to the Belle data [[7\]](#page-5-6).

 $Z_b(10650)$  resonances, respectively. The possible S-wave  $B\overline{B}^*$  and  $B^*\overline{B}^*$  molecular states were investigated extensively in Refs. [[3,](#page-5-2)[4\]](#page-5-3). Very recently, the authors in Ref. [\[26\]](#page-5-17) discussed the special decay behavior of the  $J = 1$  S-wave  $B\overline{B}^*$  and  $B^*\overline{B}^*$  molecular states based on the heavy quark symmetry. Future dynamical study of the mass and decay pattern of the S-wave  $B\overline{B}^*$  and  $B^*\overline{B}^*$  molecular states are very desirable.

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- <span id="page-5-0"></span>[1] I. Adachi et al. (Belle Collaboration), in Flavor Physics and CP Violation 2011, Kibbutz Maale Hachamisha, Israel, 2011 (unpublished).
- <span id="page-5-1"></span>[2] K. Nakamura et al. (Particle Data Group), [J. Phys. G](http://dx.doi.org/10.1088/0954-3899/37/7A/075021) 37, [075021 \(2010\)](http://dx.doi.org/10.1088/0954-3899/37/7A/075021).
- <span id="page-5-2"></span>[3] Y. R. Liu, X. Liu, W. Z. Deng, and S. L. Zhu, [Eur. Phys. J.](http://dx.doi.org/10.1140/epjc/s10052-008-0640-4) C 56[, 63 \(2008\).](http://dx.doi.org/10.1140/epjc/s10052-008-0640-4)
- <span id="page-5-3"></span>[4] X. Liu, Z. G. Luo, Y. R. Liu, and S. L. Zhu, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-009-1020-4) 61[, 411 \(2009\)](http://dx.doi.org/10.1140/epjc/s10052-009-1020-4).
- <span id="page-5-4"></span>[5] D. Y. Chen, J. He, X. Q. Li, and X. Liu, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.84.074006) 84, [074006 \(2011\)](http://dx.doi.org/10.1103/PhysRevD.84.074006).
- <span id="page-5-5"></span>[6] A. Ali, C. Hambrock, and M. J. Aslam, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.104.162001) 104[, 162001 \(2010\)](http://dx.doi.org/10.1103/PhysRevLett.104.162001).
- <span id="page-5-6"></span>[7] K. F. Chen et al. (Belle Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.100.112001) 100[, 112001 \(2008\)](http://dx.doi.org/10.1103/PhysRevLett.100.112001).
- <span id="page-5-7"></span>[8] Y. P. Kuang and T. M. Yan, Phys. Rev. D 24[, 2874 \(1981\).](http://dx.doi.org/10.1103/PhysRevD.24.2874)
- [9] T.M. Yan, Phys. Rev. D 22[, 1652 \(1980\)](http://dx.doi.org/10.1103/PhysRevD.22.1652).
- [10] H.Y. Zhou and Y.P. Kuang, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.44.756) 44, 756 [\(1991\)](http://dx.doi.org/10.1103/PhysRevD.44.756).
- [11] V. V. Anisovich, D. V. Bugg, A. V. Sarantsev, and B. S. Zou, Phys. Rev. D 51[, R4619 \(1995\)](http://dx.doi.org/10.1103/PhysRevD.51.R4619).
- [12] F. K. Guo, P. N. Shen, H. C. Chiang, and R. G. Ping, [Nucl.](http://dx.doi.org/10.1016/j.nuclphysa.2005.07.019) Phys. A 761[, 269 \(2005\)](http://dx.doi.org/10.1016/j.nuclphysa.2005.07.019).
- [13] F. K. Guo, P. N. Shen, H. C. Chiang, and R. G. Ping, [Phys.](http://dx.doi.org/10.1016/j.physletb.2007.10.021) Lett. B 658[, 27 \(2007\)](http://dx.doi.org/10.1016/j.physletb.2007.10.021).
- [14] Yu. A. Simonov and A. I. Veselov, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.79.034024) **79**, [034024 \(2009\).](http://dx.doi.org/10.1103/PhysRevD.79.034024)
- <span id="page-5-8"></span>[15] Yu. A. Simonov and A. I. Veselov, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2009.02.025) 673, 211 [\(2009\)](http://dx.doi.org/10.1016/j.physletb.2009.02.025).
- <span id="page-5-9"></span>[16] M. B. Voloshin and Yu. M. Zaitsev, [Usp. Fiz. Nauk](http://dx.doi.org/10.3367/UFNr.0152.198707a.0361) 152, [361 \(1987\)](http://dx.doi.org/10.3367/UFNr.0152.198707a.0361) [\[Sov. Phys. Usp.](http://dx.doi.org/10.1070/PU1987v030n07ABEH002924) 30, 553 (1987)].
- [17] D. Besson and T. Skwarnicki, [Annu. Rev. Nucl. Part. Sci.](http://dx.doi.org/10.1146/annurev.ns.43.120193.002001) 43[, 333 \(1993\)](http://dx.doi.org/10.1146/annurev.ns.43.120193.002001).
- <span id="page-5-10"></span>[18] Y. P. Kuang, [Front. Phys. China](http://dx.doi.org/10.1007/s11467-005-0012-6) 1, 19 (2006).
- <span id="page-5-11"></span>[19] V. A. Novikov and M. A. Shifman, Z. Phys. C 8[, 43 \(1981\).](http://dx.doi.org/10.1007/BF01429829)
- <span id="page-5-12"></span>[20] C. Meng and K. T. Chao, Phys. Rev. D 77[, 074003 \(2008\).](http://dx.doi.org/10.1103/PhysRevD.77.074003)
- [21] C. Meng and K. T. Chao, Phys. Rev. D **78**[, 034022 \(2008\).](http://dx.doi.org/10.1103/PhysRevD.78.034022)
- <span id="page-5-13"></span>[22] Yu. A. Simonov and A. I. Veselov, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2008.12.001) 671, 55 [\(2009\)](http://dx.doi.org/10.1016/j.physletb.2008.12.001).
- <span id="page-5-14"></span>[23] E.M. Aitala et al. (E791 Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.86.770) 86[, 770 \(2001\)](http://dx.doi.org/10.1103/PhysRevLett.86.770).
- <span id="page-5-15"></span>[24] N. Brambilla et al. (Quarkonium Working Group), CERN Yellow Report CERN-2005-005, 487 (2005).
- <span id="page-5-16"></span>[25] N. Brambilla et al., [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-010-1534-9) 71, 1534 (2011).
- <span id="page-5-17"></span>[26] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, Phys. Rev. D 84[, 054010 \(2011\).](http://dx.doi.org/10.1103/PhysRevD.84.054010)