Quasiparticle description of (2 + 1)- flavor lattice QCD equation of state

Vinod Chandra[1,*] and V. Ravishankar[2,†]

[1]*Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai-400005, India*
[2]*Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208 016, India*

A quasiparticle model has been employed to describe the (2 + 1)-flavor lattice QCD equation of state with physical quark masses. The interaction part of the equation of state has been mapped to the effective fugacities of otherwise noninteracting quasigluons and quasiquarks. The mapping is found to be exact for the equation of state. The model leads to nontrivial dispersion relations for quasipartons. The dispersion relations, effective quasiparticle number densities, and trace anomaly have been investigated employing the model. A virial expansion for the equation of state has further been obtained to investigate the role of interactions in quark-gluon plasma. Finally, Debye screening in quark-gluon plasma has been studied employing the model.

I. INTRODUCTION

The main purpose of this article is to explore the quasiparticle picture of quantum-chromodynamics (QCD) at high temperature. In particular, we wish to describe recent lattice data on (2 + 1)-flavor QCD equation of state (EOS) by employing a quasiparticle model [1–3]. The EOS is an important quantity to study the properties of hot QCD matter which is commonly known as quark-gluon plasma (QGP) in relativistic heavy ion collisions at BNL, RHIC and CERN, LHC. This study is mandated by the strongly interacting nature of QGP which has been inferred from the recent experimental observations at RHIC [4–7]. This observation is consistent with the lattice simulations of the EOS [8–14], which predict a strongly interacting behavior even at temperatures which are a few T_c (QCD transition temperature).

The most striking features of RHIC results [4] are the large collective flow and strong jet quenching of high transverse momentum jets shown by QGP. Similar conclusions have been drawn from the very recent preliminary results for Pb-Pb collisions at LHC at $\sqrt{s} = 2.76$ TeV [15–18]. This has led to a tiny value of the shear viscosity to entropy density ratio for QGP and near perfect fluid picture of QGP [3,19–26] (except near the QCD transition temperature where the bulk viscosity of QGP is equally important as shear viscosity [27–29]). In an attempt to appreciate this result, interesting analogies have been drawn with ADS/CFT correspondence [30] as well as with some strongly coupled classical systems [31]. In any case, the emergence of strongly interacting behavior puts into doubt the reliability of a large body of analyses which are based on ideal or nearly ideal behavior of QGP in heavy ion collisions.

In the light of these observations, it would be right to state that QGP may lie in the strongly interacting domain (nonperturbative) of QCD. Therefore, lattice gauge theory [32–34] would be the best approach to address the physics of QGP in RHIC in terms of a reliable EOS which is very precisely evaluated [8–11]. The lattice EOS is far from being ideal. The EOS is $\approx 10\%$ away from its ideal behavior even at $4T_c$. However, in several works devoted to QGP ideal EOS is employed. This is certainly not desirable for QGP in RHIC. Therefore, there is an urgent need to address this issue by developing models to employ realistic QGP equations of state to investigate the bulk and transport properties of QGP. This could be done by casting hot QCD medium effects in terms of effective quasiparticle degrees of freedom.

There have been several attempts to describe QCD medium effects at high temperature in terms of quasiparticle degrees of freedom. These attempts include, (i) the effective mass approaches to study QCD thermodynamics [35–39], and (ii) approaches based on the Polyakov loop [40–44]. A different approach in terms of quasiparticles, inspired by Landau theory of Fermi liquids has been proposed recently both for EOS based on pQCD [1,2,26,45] and pure lattice gauge theory [3]. This model is fundamentally different from the above two approaches, and quite powerful: it reproduces the EOS with remarkable accuracy, especially in the case of lattice EOS—where it is exact; the collective nature of the quasigluons is manifest in the single particle dispersion relations. It is also successful in terms of predictions regarding the bulk and transport properties of QGP [1–3,26]. References [3,26] showed that the shear viscosity and its ratio with the entropy density (η, η/s) are highly sensitive to the interactions. They could be taken of as good diagnostics to distinguish various EOS at RHIC.

The model was tested only against the pure $SU(3)$ gauge theory EOS [3], where the description was, as we mentioned, exact. It was not employed for extracting a quasiparticle description in the case of full QCD, by the inclusion of quark sector. In this paper, we remedy this

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*vinodc@theory.tifr.res.in
†vravi@iitk.ac.in

draw back and extend the model to the matter sector, by taking up a recently computed $(2 + 1)$-flavor lattice QCD equation of state with physical quark masses [10]. It is to be noted that this EOS has further been refined by improving the accuracy in [11,12]. Here, we adopt the philosophy same as in [3]. Again, we map $(2 + 1)$-flavor lattice QCD data for the EOS [10] in terms of quasiparticle degrees of freedom which are free up to effective gluon fugacity, z_g and effective quark fugacity, z_q. In this model, the strange-quark sector is different from light quark sector due to contributions coming from the strange-quark mass. They are otherwise characterize by the same effective fugacity z_q. Such a characterization could only be possible because the mass corrections from light quark-sector in the deconfined phase of QCD are very very small. So, the model will be more realistic at higher temperatures.

Here, it is worth mentioning that such a quasiparticle description of the lattice QCD EOS could be thought of as a first step towards an effective field theory/effective kinetic theory to explore complicated nature of strong interaction in QGP. Leaving these ambitious investigations for future studies, here, we have attempted to understand the role of QCD interactions in terms of a Virial expansion for QGP employing the quasiparticle model. The Virial expansion has been obtained in terms of effective quasiparticle number densities which are, in turn, expressed in terms of $z_{g/q}$. As we shall see that the Virial expansion of the EOS is very helpful to understand the role of strong interaction in QGP and may perhaps play crucial role in developing the effective models.

The paper is organized as follows. In Sec. II, we introduce the quasiparticle model and study its features and physical significance. Here, we discuss the viability of the model by studying the temperature dependence of the quasiparticle pressure, and trace anomaly in terms of effective fugacities. We find that the model reproduces the EOS almost exactly. In Sec. III, we discuss the physical significance and viability of the quasiparticle model. In Sec. IV, we discuss the implications of the model. Here, we propose a Virial expansion for QCD at high temperature, in terms of effective quasiparticle number densities and explore the role of interaction in hot QCD. We further study the Debye screening and charge renormalization in hot QCD. In Sec. V, we present the conclusions and the future direction of the work.

II. THE QUASIPARTICLE MODEL

Before, we introduce the quasiparticle description, let us define the notations. The quantities, z_g, and z_q will denote effective gluon and quark/antiquark fugacities, respectively. The quasigluon equilibrium distribution function will be denoted by f_{eq}^g, quasiquark/antiquark distribution function by f_{eq}^q for light quarks (u,d), and f_{eq}^s for strange quark. The respective dispersions (single quasiparticle energy) will be denoted as ω_p^g, ω_p^q and ω_p^s. $n_{g,q,s}$ denotes the

effective quasiparticle number densities. In all the physical quantities that will be discussed in the paper, the subscript g will denote the gluonic contribution while q and s denote the contributions from the light-quark sector and strange-quark sector, respectively.

A. Underlying distribution functions and effective fugacity

We initiate the model with the ansatz that the Lattice QCD EOS can be interpreted in terms of noninteracting quasipartons having effective fugacities which encodes all the interaction effects. In the present case, we have three sector, *viz.*, the effective gluonic sector, the light , and the strange-quark sector. Here, the effective gluon sector refers to the contribution of gluonic action to the pressure which also involves contributions from internal fermion lines. Because of purely phenomenological reason, this sector can be recast in terms of effective gluon quasiparticles (which are free gluons with effective fugacity). Similarly the other two sectors also involve interactions among quark, antiquarks, as well as their interactions with gluons. The effective gluon fugacity, z_g is introduced to capture the interaction in the effective gluonic sector. On the other hand, z_q captures interactions in other two sectors. The ansatz can be translated to the form of the equilibrium distribution functions, f_{eq}^g, f_{eq}^q, and f_{eq}^s as follows,

$$
\begin{aligned}
f_{eq}^g &= \frac{z_g \exp(-\beta p)}{(1 - z_g \exp(-\beta p))}, \\[6pt]
f_{eq}^q &= \frac{z_q \exp(-\beta p)}{(1 + z_q \exp(-\beta p))}, \\[6pt]
f_{eq}^s &= \frac{z_q \exp(-\beta\sqrt{p^2 + m^2})}{(1 + z_q \exp(-\beta\sqrt{p^2 + m^2}))},
\end{aligned}
\tag{1}
$$

where m denotes the mass of the strange quark, which we choose to be 0.1 GeV. $\beta = T^{-1}$ denotes inverse of the temperature. Note that we are working in the units where Boltzmann constant, $K_B = 1$, $c = 1$, and $h/2\pi = 1$. We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 2$ for light quarks, $\nu_s = 2 \times 2 \times N_c \times 1$ for the strange quark for $SU(N_c)$. Here, we are dealing with $SU(3)$, so $N_c = 3$. Since the model is valid in the deconfined phase of QCD (beyond T_c, T_c is the QCD transition temperature), the masses of the light quarks can be neglected. Therefore, in our model we only consider the mass for the strange quarks.

As it is well known that QCD thermodynamics at high temperature is described in terms of a Grand canonical ensemble. Now, it is straightforward to write down an effective Grand canonical partition function for hot QCD which yields the forms of the distribution function given in Eq. (1). We denote the partition function by

$\mathbf{Z} = (Z_g \times Z_q \times Z_s)$. The corresponding expressions in terms of z_g and z_q are as follows,

$$\ln(Z_g) = -\nu_g V \int \frac{d^3 p}{8\pi^3} \ln(1 - z_g \exp(-\beta p)) \quad (2)$$

$$\ln(Z_q) = \nu_q V \int \frac{d^3 p}{8\pi^3} \ln(1 + z_q \exp(-\beta p)) \quad (3)$$

$$\ln(Z_s) = \nu_s V \int \frac{d^3 p}{8\pi^3} \ln(1 + z_q \exp(-\beta(\sqrt{p^2 + m^2}))) \quad (4)$$

$$\ln(\mathbf{Z}) = \ln(Z_g) + \ln(Z_q) + \ln(Z_s). \quad (5)$$

Now using the well known thermodynamic relation, $P\beta V = \ln(Z)$, we can match the right-hand side of Eq. (5), with the lattice data for the pressure for $(2 + 1)$-flavor QCD [10], where P denotes the pressure and V denotes the volume. From this relation, we can in principle determine the temperature dependence of z_g and z_q. As emphasized earlier, z_g is determined from the contribution to the lattice pressure purely from gluonic action. This particular contribution to the pressure is denoted as P_g. Remaining part of the pressure is utilized to fix the temperature dependence of z_q. Now, we have two relations and two unknowns. Next, we discuss the determination of z_g and z_q.

1. Determination of z_g and z_q

We determine z_g and z_q numerically. z_g has been determined using the relation,

$$P_g = \frac{-\beta^{-4}\nu_g}{2\pi^2} \int_0^\infty du u^2 \ln(1 - z_g \exp(-u)). \quad (6)$$

On the other hand, z_q has been determined numerically using the following relation,

$$(P - P_g) = \frac{\beta^{-4}}{2\pi^2} \int_0^\infty du u^2 \{ \nu_q \ln(1 + z_q \exp(-u))$$
$$+ \nu_s \ln(1 + z_q \exp(-\sqrt{u^2 + \tilde{m}^2})) \}, \quad (7)$$

where $\tilde{m} = \beta m$ and u ($u = \beta p$) is a dimensionless quantity. We have recorded those values of z_g and z_q which satisfy Eqs. (6) and (7). Next, we discuss their behavior with temperature.

2. Behavior of z_g and z_q

The determination of the quasiparton distribution functions given in Eq. (1) is complete once the temperature dependence of z_g and z_q is fixed. The behavior of z_g and z_q as a function of temperature is shown in Fig. 1 and in Fig. 2 respectively. Clearly, both of them acquire their ideal values (unity) only asymptotically. At lower temperatures,
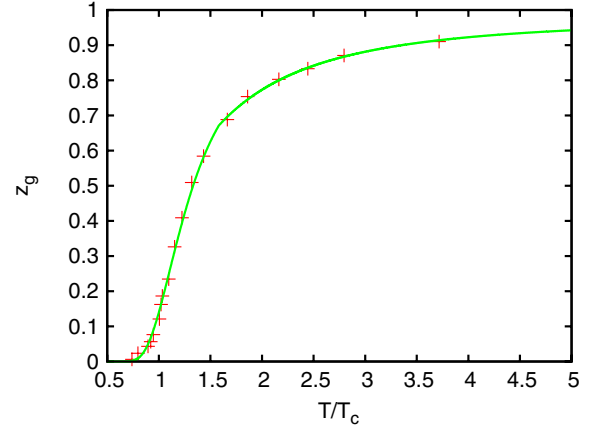

FIG. 1 (color online). Behavior of z_g as a function of T/T_c. The points denote the values obtained from lattice data and solid line denote the fitting function. The fitting parameters are listed in Table. I.
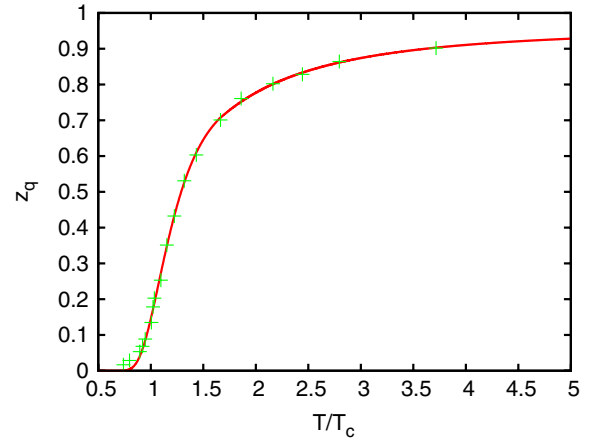

FIG. 2 (color online). Behavior of z_q as a function of T/T_c. The points denote the values obtained from lattice data and solid line denote the fitting function. The fitting parameters are listed in Table. I.

the magnitude of both z_g and z_q is smaller indicating the larger strength of interactions there.

For further analysis, we seek analytic forms for z_g and zq as a function of temperature, which would render the computation more amenable. At this juncture, we note that there are limited number of lattice data for the pressure for a huge range of temperature $((0.5 - 4.0)T_c)$. Since the effective fugacities have been determined from the lattice pressure, the limitations get inherited by them as well, which further passed on to other thermodynamic quantities such as energy density, entropy density and trace anomaly. We hope that the functional form is not drastically altered by future refinements in lattice data.

We find that there is no universal functional form that describes the data in the full range of temperatures, either in the gluonic sector or the quark sector. There are some

TABLE I. Fitting parameters for z_g and z_q

$z_{g,q}$	$a_{g,q}$	$b_{g,q}$	$a'_{g,q}$	$b'_{g,q}$
Gluon	0.803 ± 0.009	1.837 ± 0.039	0.978 ± 0.007	0.942 ± 0.035
Quark	0.810 ± 0.010	1.721 ± 0.040	0.960 ± 0.007	0.846 ± 0.033

common features though. Both the sectors are characterized by a ''low temperature'' and a ''high temperature'' regime, with the cross over temperatures given by $x_{g,q} \equiv T_{g,q}/T_c \sim 1.68$, 1.70, respectively. The functional forms on either side of $x_{g,q}$ are the same for both the sectors, but with different parameters. Thus, when $x < x_{q,g}$, the good fitting function has the form $a'_{g,q} \exp(-b'_{g,q}/x^5)$. In the complementary case $x > x_{q,g}$, it has the form $a'_{g,q} \exp(-b'_{g,q}/x^2)$. The latter form is mandated by the fact that at high temperature the trace anomaly, $\frac{\epsilon - 3P}{T^4}$ predominantly goes as $1/T^2$ [10] in lattice QCD. We list the fitting parameters for z_g and z_q in Table I.

We shall utilize these forms to study temperature dependence of the trace anomaly in later part of the paper. We shall see that these forms correctly reproduce the high and low temperature behavior of the trace anomaly.

Temperature dependences of effective fugacities, in Figs. 1 and 2 reveal that the effective gluon and quark fugacities are of same order of magnitude for the whole range of temperature. This indicates that effective gluons and quarks contribute equally in our description. Its possible physical consequences, and an understanding from basic calculations in QCD is beyond the scope of the present work and will be a matter of future investigations.

It is worth noting that effective fugacity descriptions have been earlier employed in condensed matter systems in the last decade. To study the nature of Bose-Einstein (BE) condensation transition in interacting Bose gases, a parametric EOS in terms of the effective fugacity has been proposed by Li *et al.* [46]. This provides a scheme to explore the quantum-statistical nature of the BEC transition. There have been other works to study the noninteracting BE systems in harmonic trap [47] as well interacting bosonic systems [48]. Moreover, effective fugacity description has been used for a unitary fermion gas by Chen *et al.* [49] for studying thermodynamics with non-Gaussian correlations. None of them employed the effective dispersion relations which we obtain naturally in this work. We shall now proceed to discuss the physical significance of the quasiparticle model and its viability.

III. PHYSICAL SIGNIFICANCE AND VIABILITY OF THE MODEL

A. The modified dispersion relations

It has been emphasized in Ref. [2] that the physical significance of effective fugacity could be seen in terms of modified dispersion relations. The effective fugacities modify the single quasiparton energy as follows,

$$\begin{aligned}
\omega_p^g &= p + T^2 \partial_T ln(z_g) \\
\omega_p^q &= p + T^2 \partial_T ln(z_q) \\
\omega_p^s &= \sqrt{p^2 + m^2} + T^2 \partial_T ln(z_q).
\end{aligned} \tag{8}$$

These dispersion relations can be interpreted as follows. The single quasiparton energy not only depends upon the momentum but also gets contribution from the collective excitations of the quasipartons. The second terms is like the gap in the energy due to the presence of quasiparticle excitations. This immediately reminds us of Landau's theory of Fermi liquids. Therefore, it is safe to say that the present quasiparticle model is in the spirit of Landau theory of Fermi liquids. These modified dispersion relations in Eq. (8) have emerged from the thermodynamic definition of the average energy of the system, due to the temperature dependent fugacities, $z_{g/q}$. Let us consider the expression for the energy density, ϵ obtained in terms of the grand canonical partition function, **Z** as,

$$\epsilon = -\frac{1}{V}\frac{\partial \ln(\mathbf{Z})}{\partial \beta}. \tag{9}$$

Substituting for the effective partition function ($\mathbf{Z} = Z_g \times Z_q \times Z_s$), we obtain,

$$\begin{aligned}
\epsilon &\equiv \frac{\nu_g}{8\pi^3} \int d^3p(p + T^2\partial_T \ln(z_g))f_{eq}^g + \frac{1}{8\pi^3} \int d^3p\left\{\left(p + T^2\partial_T \ln(z_q)\right)\nu_q f_{eq}^q + \left(\sqrt{p^2 + m^2} + T^2\partial_T \ln(z_q)\right)\nu_s f_{eq}^s\right)\right\} \\
&\equiv 3(P_g + P_q + P_s) + \frac{T^2\partial_T \ln(z_g)}{2\pi^2}\int d^3p\nu_g f_{eq}^g + \frac{T^2\partial_T \ln(z_q)}{2\pi^2}\int d^3p(\nu_q f_{eq}^q + \nu_s f_{eq}^s).
\end{aligned} \tag{10}$$

The above equation can be recast employing the expression for the pressure in terms of the temperature dependent, $z_{g/q}$ as,

$$\frac{(\epsilon - 3P)}{T^4} = T \frac{\partial}{\partial T}\left(\frac{P}{T^4}\right) \tag{11}$$

Therefore, these modified dispersion relations naturally ensure the thermodynamic consistency condition in high temperature QCD, and lead to the trace anomaly, which we have discussed, in detail, in the next subsection. Moreover, these effective fugacities can be expressed in terms of effective quasiparticle number densities. These number densities leads to a simple Virial expansion for the EOS which we shall discuss in the next section.

Next, we look at the group velocity of quasipartons. The group velocity can be obtained as $\vec{v}_p = \partial_{\vec{p}}\omega_p$. It is easy to see that the modified term in the dispersion relations is purely temperature dependent, therefore it will not change the group velocity of a quasiparton. $\vec{v}_p = \hat{p}$ for quasigluons and quasiquarks (u, d) and $\vec{v}_p = \frac{\vec{p}}{\sqrt{p^2 + m^2}}$. for strange quarks. The dispersion relation in Eq. (8) contributes to the trace anomaly in hot QCD which we shall discuss soon.

Let us now discuss the significance of gluon condensate, in the hot QCD thermodynamics. In this context, D'Elia, Giacomo, and Meggiolaro [50] have studied the electric and magnetic contributions to the condensate, and shown that near T_c the former vanishes, however the latter remains unchanged. These authors investigated such effects by analyzing the two-point correlation functions both in pure-gauge sector, and the full QCD [50,51]. It has been shown in [52] that the effects of the gluon condensate are significant for $T \gtrsim T_c$, and becomes vanishingly small beyond $2T_c$. Therefore, in the effective mass description of hot QCD for the temperatures, $T = 1 - 2T_c$, one needs to consider the contributions of the condensate, while comparing the predictions on thermodynamic observables with the lattice QCD data. However, in our model, z_g, z_q capture these effects, and we do not need to incorporate the contributions separately. This can be understood as follows, In the lattice data employed here, normalization of the pressure and energy density were chosen such that at $T = 0$, these quantities vanish [10], and the effects of the gluon condensate may be significant at higher temperatures. These effects are well captured in the trace anomaly in lattice QCD, which is the basic quantity computed in the lattice. And, all other thermodynamic quantities have been derived from the trace anomaly. In turn, these effects are automatically encoded in the pressure, the energy density, etc. In our study, such effects have been captured in the effective fugacities, $z_{g/q}$ from the beginning, since, we have determined them from the lattice data on pressure. The gluon condensate contribute significantly to the energy density, entropy density, and the trace anomaly through temperature derivatives of the $z_{g/q}$, in terms of modified dispersion relations.

B. Viability of the model

As it has been already emphasized in the previous section that the model yields (2 + 1)-flavor lattice QCD EOS almost perfectly. To check further the viability of the model, we study the temperature dependence of the quasiparticle pressure, energy density, and the trace anomaly and check them against the direct lattice results. Lets us first discuss the temperature dependence of the pressure. We have plotted the quasiparticle pressure along with the lattice data in Fig. 3. We find that the agreement between the lattice data and quasiparticle model for the EOS is almost perfect beyond T_c. The temperature dependence of the energy density as a function of temperature is shown in Fig. 4. The quasiparticle results agree well with the lattice data beyond T_c.

Encouraged from the crucial observation that lattice and quasiparticle model predictions are the same, we now proceed to study the trace anomaly as a function of temperature obtained from the quasiparticle model.

1. The trace anomaly

Trace anomaly gets contribution from all the three sectors. The gluonic and light quarks contributions come purely from the modified part of the dispersion relations. On the other hand, in the strange-quark sector trace anomaly gets additional contribution from the mass. We denote the trace anomaly by $\Delta = (\epsilon - 3P) \equiv \Delta_g + \Delta_q + \Delta_s$.

$$\begin{aligned}
\frac{\Delta_g}{T^4} &= T\partial_T \ln(z_g)\frac{n_g}{T^3} \\
\frac{\Delta_q}{T^4} &= T\partial_T \ln(z_q)\frac{n_q}{T^3} \\
\frac{\Delta_s}{T^4} &= T\partial_T \ln(z_g)\frac{n_s}{T^3}
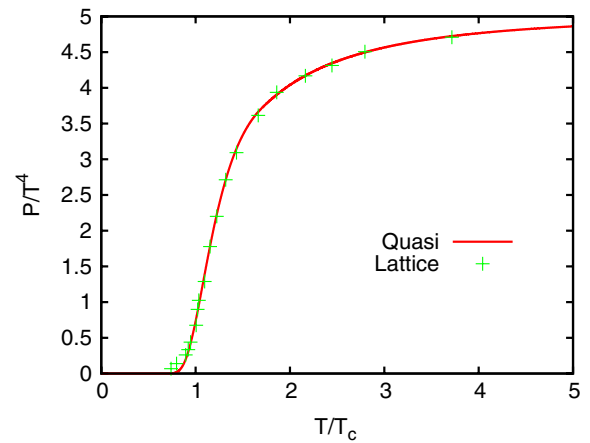\end{aligned} \tag{12}$$

FIG. 3 (color online).   Behavior of P/T^4 as a function of T/T_c. The quasiparticle pressure is obtained by using the fitting parameters for z_g and z_q listed in Table I. Lattice Data are also shown as points.
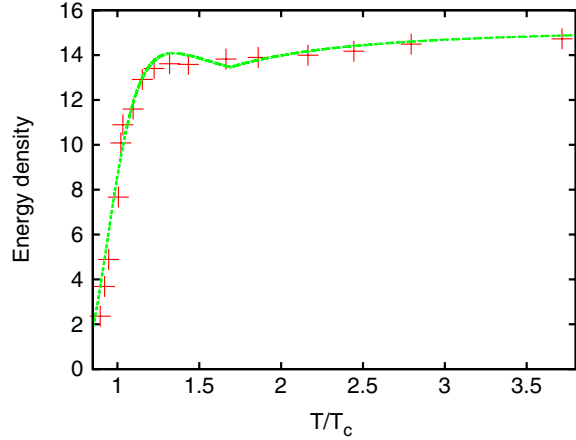
FIG. 4 (color online). Behavior of ϵ/T^4 as a function of T/T_c in the quasiparticle model. The lattice results are shown as points and the solid line shows the quasiparticle result obtained by utilizing the fitting parameters, $a_{g,q}$, $b_{g,q}$, and $a'_{g,q}$, $b'_{g,q}$ listed in Table. I.

where n_g, n_q and n_s are the effective number densities for the quasipartons and are defined by,

$$n_g = \frac{\nu_g}{2\pi^2} \int_0^\infty dp\, p^2 f_{eq}^g \equiv T^3 \frac{\nu_g \text{PolyLog}[3, z_g]}{\pi^2}$$

$$n_q = \frac{\nu_q}{2\pi^2} \int_0^\infty dp\, p^2 f_{eq}^q \equiv T^3 \frac{-\nu_q \text{PolyLog}[3, -z_q]}{\pi^2}$$

$$n_s = \frac{\nu_s}{2\pi^2} \int_0^\infty dp\, p^2 f_{eq}^s \equiv T^3 \frac{-\nu_s \text{PolyLog}[3, -z_q]}{\pi^2}$$

$$- \frac{3\tilde{m}^2}{\pi^2} \ln(1 + z_q). \tag{13}$$

We shall first discuss the behavior of these effective number densities as a function of temperature and thereby the temperature dependence of the trace anomaly in hot QCD.
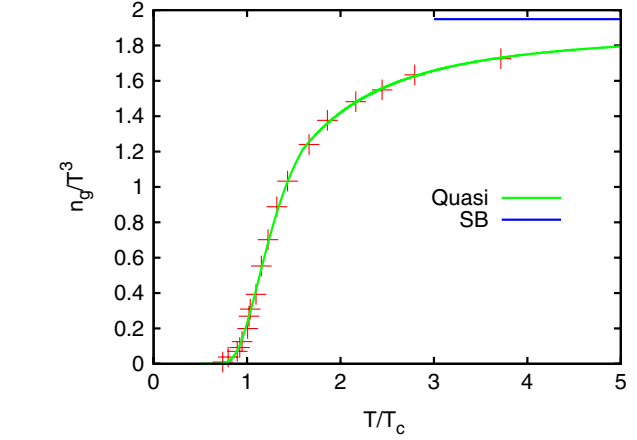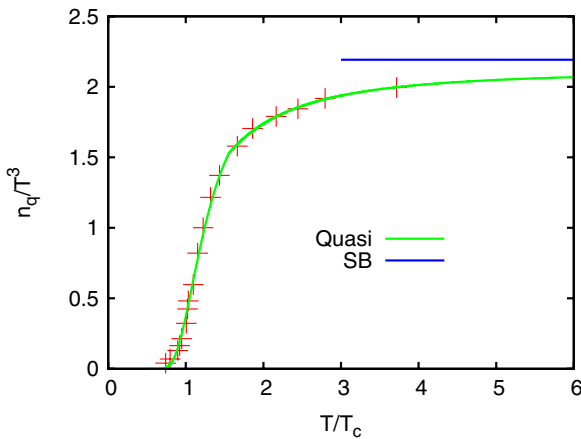

FIG. 5 (color online). Behavior of n_g as a function of T/T_c. N_g is obtained by employing the discrete data points for z_g as well as fitting parameter for z_g listed in Table. I. The Stefan-Boltzmann (SB) limit of n_g is also shown.

We determine the effective number densities exactly by the numerical evolutions of the integrals in Eq. (13). They are represented in terms of PolyLog functions merely to understand the nontrivial temperature dependence of the effective number densities. Here, we see that n_g/T^3, and $n_{q/s}/T^3$ scales with T/T_c in a nontrivial way. Their behaviors with temperature are shown in Figs. 5 and 6.

We have also shown the Stefan-Boltzmann value for the number densities in Figs. 5 and 6. $(n_g/T^3)|_{SB} = \nu_g \zeta(3)/\pi^2$; $(n_q/T^3)|_{SB} = 3\nu_q \zeta(3)/4\pi^2$; and $(\nu_s/T^3)|_{SB} = 3\nu_s \zeta(3)/4\pi^2$. Note that the SB limit has not been obtained even at $T = 5T_c$. The effective number densities are roughly $\sim 10\%$ away at this temperature in all the three sectors. This is just the reflection of the fact that lattice EOS itself is away from SB limit there.

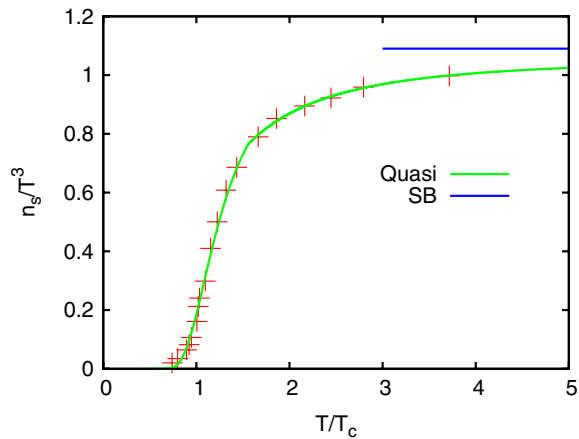Using Table. I, the quantities $T\partial_T \ln(z_g)$ and $T\partial \ln(z_q)$ can easily be obtained as,

FIG. 6 (color online). Behavior of n_q (left) and n_s (right) as a function of T/T_c. They are obtained by employing the discrete data points for z_q as well as fitting parameter for of z_q listed in Table. I including the strange-quark mass dependence. The Stefan-Boltzmann (SB) value is also shown beyond $3T_c$.

$$T\partial_T \ln(z_g) = \begin{cases} \frac{5b_g}{x^5}; & x \le x_g \\ \frac{2b_g'}{x^2}; & x > x_g \end{cases}$$

$$T\partial_T \ln(z_q) = \begin{cases} \frac{5b_q}{x^5}; & x \le x_q \\ \frac{2b_q'}{x^2}; & x > x_q \end{cases}. \qquad (14)$$

It is now straightforward to compute the trace anomaly employing the fitting functions for the effective fugacities; and effective quasiparticle number densities using Eq. (12). Behavior of Δ along with corresponding lattice values has been shown in Fig. 7. As it is clear from Fig. 3 and 7, pressure and trace anomaly computed by employing the quasiparticle model show good agreement with the lattice data of the same. This sets the utility of the model. Once these two quantities are known, it is straightforward to determine the energy density (e) and the entropy density (s) by the standard thermodynamic relations,

$$e = 3P_{\text{quasi}} + \Delta, \quad s = \frac{e + P_{\text{quasi}}}{T} \equiv 4P_{\text{quasi}} + \Delta.$$

If we see, closely the behavior of energy density or the trace anomaly as a function of temperature in Figs. 4 and 7, we observe that around $T = 1.7T_c$, both the quantities are not showing smooth behavior. There is no physical reason associated with this. This is merely the artifact of the two distinct fitting functions for z_g and z_q below and above this temperature. At this temperature these functions have different slopes. This problem may not be present, if we could have found a single fitting functions for z_g and z_q for the whole range of temperatures. As emphasized earlier, these fitting functions were needed to compute, the temperature derivatives of z_g and z_q. Moreover, these points merely reflect the fact that at this point, perhaps various thermodynamic quantities are changing their slopes, but smoothly, which is not captured appropriately in the fitting functions. This may not be thought of as a serious problem since, one could compute the temperature derivatives of z_g and z_q,
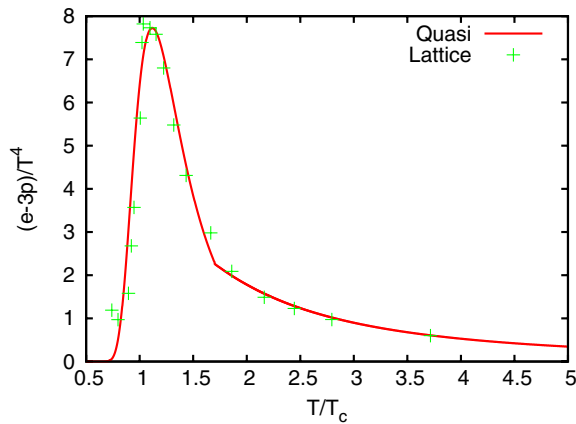

FIG. 7 (color online).   Behavior of trace anomaly as a function of T/T_c. The solid line denotes the values obtained from the quasiparticle description and the points denote the lattice results.

directly by inverting relations in Eq. (12). In that case, we shall not get a continuous curve for energy density, rather discrete points and they will perfectly match with the lattice predictions.

IV. IMPLICATIONS OF THE MODEL

A. Virial expansion for hot QCD

To translate QCD interactions in RHIC era in terms of a Virial expansion is quite a nontrivial task. There are very few attempts in this direction [53]. Here, we see that the quasiparticle understanding of equation of state for QGP plays very crucial role to obtain a very simple Virial expansion of the EOS in terms of effective quasiparticle number densities. The model tells us that the Virial expansion in $(2 + 1)$-flavor QCD could be subdivided in three sectors, *viz.*, gluonic, light-quark, and strange-quark sector and one can define the Virial expansion in each sector and finally combine them.

We begin with the purely gluonic sector first and subsequently discuss the matter sector (light quarks and strange quarks).

1. Virial expansion in purely gluonic sector

To obtain the Virial expansion in this sector, we Taylor expand the pressure, P_g and the effective gluon number density, n_g in gluonic sector in the powers of z_g (assuming $z_g < 1$) and eliminate the explicit dependence of z_g from both the expressions. This technique which is the standard way to obtain the Virial expansion of a ideal Bose/Fermi gas with fixed number of particles [54], is equally applicable here, although the systems are physically distinct, since z_g and z_q do not correspond to the particle conservation. The procedure is straightforward. For the sake of completeness, we shall write a few steps. The expressions for P_g and n_g in the powers of z_g are obtained as,

$$\frac{P_g}{\nu_g T} = \frac{1}{\lambda_{\text{th}}^3} \sum_{l=1}^{\infty} b_l z_g^l. \qquad (15)$$

$$\frac{n_g}{\nu_g} = \frac{1}{\lambda_{\text{th}}^3} \sum_{l=1}^{\infty} l b_l z_g^l. \qquad (16)$$

The quantity $\lambda_{\text{th}} \equiv 1/T$ is the thermal wave length of gluons and the coefficient b_l is given as,

$$b_l = \frac{1}{2\pi^2 l} \int_0^{\infty} u^2 \exp(-lu) du \equiv \frac{1}{\pi^2 l^4}. \qquad (17)$$

Consequently the ratio of Eq. (15) with Eq. (16) and expanding it as,

$$\frac{P_g}{n_g T} = \sum_{k=1}^{\infty} a_k (\tilde{N}_g \lambda_{\text{th}}^3)^{k-1} \equiv \frac{\sum_{l=1}^{\infty} b_l z_g^l}{\sum_{l=1}^{\infty} l b_l z_g^l}, \qquad (18)$$

where $\tilde{N}_g = \frac{n_g}{\nu_g}$ and a_k's are the Virial coefficients. Using, Eq. (15) and (16) in Eq. (18) and comparing the terms of order z_g, z_g^2 and z_g^3, we obtain,

$$a_1 = 1;$$
$$a_2 = -\frac{b_2}{b_1^2} \equiv -\frac{\pi^2}{2^4};$$
$$a_3 = \frac{\left(-2b_3 + \frac{4b_2^2}{b_1}\right)}{b_1^3} \qquad (19)$$
$$a_4 = \frac{1}{b_1^4}\left(-3b_4 - \frac{20b_2^3}{b_1^2} + \frac{18b_2b_3}{b_1}\right).$$

One can, in principle obtain all the Virial coefficients comparing various order coefficients of z_g in Eq. (18). Now, the Virial expansion up to $O((N_g'\lambda_{\text{th}}^3)^3)$ can be written as,

$$\frac{P_g}{n_gT} = 1 + a_2(N_g'\lambda_{\text{th}}^3) + a_3(N_g'\lambda_{\text{th}}^3)^2 + a_4(N_g'\lambda_{\text{th}}^3)^3. \quad (20)$$

If we exploit the temperature dependence of effective gluon number density shown in Fig. 5. We can compare the strength of various order terms in Eq. (20). We find that third term is $\ll$ second term and fourth term is $\ll$ third term. Remember that the Virial coefficients in Eq. (19) do not acquire any temperature dependence and are same as those for an ideal gluonic plasma with temperature independent fugacity. The interactions merely renormalizes the number density of quasigluons. This confirms our view point that hot QCD medium effects can entirely be mapped in to the noninteracting/weakly interacting quasiparticle degrees of freedom. The validity of the Virial expansion in this sector is ensured by the fact that $N_g'\lambda_{\text{th}}^3 \ll 1$.

Let us now move to the matter sector and first discuss the light quark sector followed by the strange-quark sector.

2. Virial expansion in the matter sector

We shall exactly follow the same procedure discussed earlier to obtain the Virial expansion. We denote the light quarks contribution to pressure as P_q and contribution from strange quarks to P_s. Expanding these quantities along with effective number densities n_q and n_s in the power of z_q (assuming $z_q < 1$), we obtain the following expressions,

$$\frac{P_q}{\nu_qT} = \frac{1}{\lambda_{\text{th}}^3}\sum_{l=1}^{\infty}(-1)^{l-1}b_lz_q^l \qquad (21)$$

$$\frac{P_s}{\nu_sT} = \frac{1}{\lambda_{\text{th}}^3}\sum_{l=1}^{\infty}(-1)^{l-1}b_l'z_q^l \qquad (22)$$

$$\frac{n_q}{\nu_q} = \frac{1}{\lambda_{\text{th}}^3}\sum_{l=1}^{\infty}(-1)^{l-1}lb_lz_q^l \qquad (23)$$

$$\frac{n_s}{\nu_s} = \frac{1}{\lambda_{\text{th}}^3}\sum_{l=1}^{\infty}(-1)^{l-1}lb_l'z_q^l. \qquad (24)$$

Where the coefficient b_l' is defined as,

$$b_l' = \frac{1}{2\pi^2 l}\int_0^{\infty} u^2 \exp(-l\sqrt{u^2 + \tilde{m}^2})du = b_l - \frac{\tilde{m}^2}{4\pi^2 l^3} \qquad (25)$$

Repeating the analysis same as for gluons, and denoting the Virial coefficients in light quark sector as a_l^q and strange-quark sector as a_l^s, the Virial expansion for P_q and P_s would have the following forms,

$$\frac{P_q}{n_qT} = a_1^q + a_2^q(N_q'\lambda_{\text{th}}^3) + a_3^q(N_q'\lambda_{\text{th}}^3)^2 + a_4^q(N_q'\lambda_{\text{th}}^3)^3$$
$$\frac{P_s}{n_sT} = a_1^s + a_2^s(N_s'\lambda_{\text{th}}^3) + a_3^s(N_s'\lambda_{\text{th}}^3)^2 + a_4^s(N_s'\lambda_{\text{th}}^3)^3. \qquad (26)$$

where $N_q' = \frac{n_q}{\nu_q}$ and $N_s' = \frac{n_s}{\nu_s}$. In principle, all the Virial coefficients (a_l^q and a_l^s) are possible to compute in terms of b_l and b_l'. We shall only discuss up to the fourth Virial coefficient. These coefficients are obtained as follows,

$$a_1^q = 1; \qquad a_2^q = \frac{b_2}{b_1^2} = \frac{\pi^2}{2^4}; \qquad a_3^q = \frac{\left(-2b_3 + \frac{4b_2^2}{b_1}\right)}{b_1^3}$$

$$a_4^q = \frac{1}{b_1^4}\left(3b_4 + \frac{20b_2^3}{b_1^2} - \frac{18b_2b_3}{b_1}\right);$$

$$a_1^s = 1; \qquad a_2^s = \left(b_2 - \frac{\tilde{m}^2}{2^5\pi^2}\right)\Big/\left(b_1 - \frac{\tilde{m}^2}{2^2\pi^2}\right)^2; \qquad (27)$$

$$a_3^s = \frac{\left(-2b_3' + \frac{4(b_2')^2}{b_1'}\right)}{(b_1')^3};$$

$$a_4^s = \frac{1}{(b_1')^4}\left(3b_4' + \frac{20(b_2')^3}{(b_1')^2} - \frac{18b_2'b_3'}{b_1'}\right).$$

Again the dominant contribution is from the second terms in the Virial expansion of P_q and P_s in Eq. (26). This we have observed by exploiting the temperature dependence of the effective quasiparticle number densities shown in Fig. 6.

Since the total quasiparticle pressure is $P = P_g + P_q + P_s$, Virial expansion of the full EOS can be obtained by using the individual Virial expansions obtained in the effective gluonic sector (EGS), and the matter sector [Eqs. (20) and (26)]. Note that all the Virial coefficients in EGS as well as in the matter sector are independent of temperature. The information about the interaction has been captured in the effective quasiparton number densities. The second Virial coefficient is negative in the gluonic sector and positive in the quark sector. This is expected from the quantum statistics of quasigluons and quasiquarks. It is straightforward an exercise to determine the other thermodynamic observables in term of effective

quasiparton number densities by using the well known thermodynamic relations. These expressions will also contain the temperature derivatives of effective number densities in addition. In other words, both z_g and the modification factor to the dispersion relations, $T\partial_T \ln(z_g)$, $T\partial_T \ln(z_q)$ will appear in their expressions. Finally, the validity of the Virial expansion in the matter sector is ensured by the fact that $N'_{q,s}\lambda^3_{th} << 1$.

This is perhaps the first time, we have obtained such a simple Virial expansion for hot QCD where interactions appear as suppression factors through the effective quasiparton number densities. This has only been possible due to the quasiparticle description of hot QCD. Interesting enough, such a description works well down to temperatures which are of the order $1.0T_c$. The Virial expansion here highlights the role of interactions in hot QCD. The Virial expansion may possibly play crucial role to explore a quantitative understanding of Fermi liquid like picture of hot QCD interactions as indicated by our quasiparticle description and also play important role to develop effective field theory and effective kinetic theory for such a quasiparticle model. We shall leave these interesting issues for the future investigations.

At this juncture, we wish to mention that there has been a very recent attempt [55] to study the nuclear matter EOS at subnuclear density in a Virial expansion of a non-ideal gas. The Virial expansion is obtained by considering the fugacities for various species such as neutron, proton etc. The method to obtain the Virial coefficients is standard one as employed in the present work. However, the major difference between the two is in the physical meaning of the fugacities.

B. Comparison with other approaches

We now intend to compare our quasiparticle model with other existing models. In the recent past [40–43] and in a very recent work [44], effects of hot QCD medium have been interpreted in terms of single particle states (effective gluons/quark-anti-quarks) via the Polyakov loop. In these approaches, the expectation value of the Polyakov loop appears in the effective gluon/quark-antiquark distribution functions [43]. It provides a suppression factor in the form of a effective fugacity to an isolated particle with color quantum numbers. On the other hand, there have been successful attempts to encode the high temperature QCD medium effects in terms of effective thermal masses for quasipartons [35–39]. In the recent past, effective mass models have been employed to describe (2 + 1)-flavor QCD [56,57] and the agreement was found to be good. The effective mass models, which we discussed so far are based on the lowest order results in perturbative QCD, equivalently leading order hard-thermal-loop (HTL) results. These models are improved by incorporating the next order HTL contributions by Rebhan and Romatschle [58]. Their predictions were shown to be in agreement with

the lattice results including (2 + 1)-flavor QCD. Furthermore, there are other approaches which also involves quasiparticle picture of hot QCD along with the contribution from the gluon condensate [52]. We shall compare our model with these approaches one by one.

Let us consider the Polyakov loop approach first. There are certain similarities and a number of differences between our model and this approach. The similarities are, (i) both the approaches lead to an effective description of hot QCD in terms of free quasiparticles, (ii) the expectation value of the Polyakov loop which plays the role of effective fugacity as well as effective quasiparton fugacities ($z_{g/q}$) in our model appear as the suppression factors in the corresponding quasiparton distribution functions, (iii) in both the approaches the group velocity ($v_{gr} = \partial_{\vec{p}}\omega_p$), remains unchanged, (iv) both the models are quite successful in reproducing the lattice data on thermodynamic observables (For more details on Polyakov loop method see [42]. Our model yield lattice EOS for $SU(3)$ pure-gauge theory almost perfectly which the deviations which are one part in a million [26]), and same is true for the (2 + 1)-flavor lattice EOS, and (v) the effective gluon distribution function in [43,44] has a similar mathematical structure as our model. In spite of these similarities, our model is fundamentally distinct from this approach. Our model is purely phenomenological, and is more in the spirit of Landaus theory of Fermi liquids. We list below the major differences between the two approaches.

(i) The expectation value of the Polyakov loop appearing in the single particle distribution function does not change the dispersion relation for quasipartons. On the other hand, in our model, we obtain nontrivial quasiparton dispersion relations. ($T^2\partial_T ln(z_{g/q})$).

(ii) The Polyakov loop (its phase) appears as an imaginary chemical potential in the single particle distribution functions [43,44]. This is unlikely to happen in our model. The effective fugacities in our model cannot be interpreted as chemical potentials (real/ imaginary) (since there is no conservation of particle number). They are introduced merely to capture all the interaction effects present in hot QCD medium.

(iii) Employing our quasiparticle model, one can study the bulk and transport properties of hot and dense matter (QGP) in RHIC. These studies have been reported for pure $SU(3)$ gauge theory in [3,26] and will be presented separately for full lattice gauge theory in the near future.

Let us now compare our model with the effective mass models. In this approach, lattice QCD data for the EOS had been interpreted in terms of effective thermal gluon mass and effective thermal quark mass. The quasiparticle model proposed in this paper is completely distinct from this model. The major difference is in the philosophy itself.

The effective fugacities are not the effective masses and they can be interpreted as effective mass in some limiting case ($p \ll T^2 \partial_T \ln(z_{g/q})$). Moreover, our approach explores the Fermi liquid like picture of hot QCD. Another major difference in two of the approaches can be realized in terms of group velocity v_{gr}, v_{gr} in two approaches is not the same. In the effective mass approaches v_{gr} depends on thermal mass parameter ($\vec{v}_{gr} = \vec{p}/\sqrt{p^2 + m(T)^2}$). We have obtained a Virial expansion for hot QCD in terms of the quasipartons. This has not been done employing either of these two models.

Let us now discuss the quasiparticle models which incorporate the effects of gluon condensate explicitly in the analysis [52,59]. In these studies, the importance of the gluon condensate is highlighted, and its effect on the thermodynamic observables in hot QCD was studied in detail [52]. In particular, Castorina and Mannarelli [52], have analyzed the thermodynamic properties of hot QCD between $1 - 2T_c$ by explicitly incorporating the gluon condensate along with the gluon, and quark quasiparticles with thermal masses. The results show excellent agreement with the lattice predictions, both in the pure glue sector and the full QCD sector. Apart from the differences in the dispersion relations, and the philosophy with our model, there has been a very crucial difference. As emphasized earlier, in our model the effect of gluon condensate has been incorporated from the beginning, and not treated explicitly as in [52]. However, both the models are equally successful to describe the lattice QCD thermodynamics.

There is an alternate way to interpret the effective fugacities in terms of effective mass, as follows. Let us suppose, $z_{g/q} \equiv \exp(-m_{\text{eff}}|_{g/q}/T)$. The quantity, m_{eff} can be thought of as $m_{\text{eff}} = g'(T)T$, where g' is an effective coupling. It is to be observed that $z_{g/q}$ are of the order of 0.15 around T_c; it leads to an estimate for $g' \sim 2.0$. This indicates the nonperturbative nature of hot QCD matter near T_c. Moreover, g' becomes less than one beyond $1.3T_c$, and this observation is valid in both effective gluon and matter sector.

C. Debye screening mass and charge renormalization

To investigate how the partonic charges modify in the presence of hot QCD medium, we consider the expression for the Debye mass derived in semiclassical transport theory [60] in terms of equilibrium parton distribution functions. The same expression was obtained from the chromo-electric response functions of QGP [2]. The Debye mass in terms of the quasiparton distribution functions, which are obtained from the $(2 + 1)$-lattice QCD EOS, is given by,

$$M_D^2 = -2N_c Q^2 \int \frac{d^3 p}{8\pi^3} \partial_p f_{eq}^g + Q^2 \int \frac{d^3 p}{8\pi^3} \partial_p (4f_{eq}^q + 2f_{eq}^s),$$
(28)

where Q^2 is the effective coupling which appears in the transport equation. If one assumes QGP as an ideal system of massless gluons and quarks, Eq. (28) reproduces the leading order HTL result for the Debye mass; with the identification that $Q^2 \equiv g^2(T)$ ($g(T)$ is QCD running coupling constant at finite temperature).

Employing the distribution functions displayed in Eq. (1) to Eq. (28), we obtain the following expressions for the

$$M_D^2 = Q^2 T^2 \Big(\frac{N_c}{3} \frac{6\text{Polylog}[2, z_g]}{\pi^2} + \frac{1}{2} \times \frac{-12\text{PolyLog}[2, -z_q]}{\pi^2} - \frac{\tilde{m}^2}{4\pi^2} \ln(1 + z_q) \Big).$$
(29)

While determining the Debye mass in Eq. (29) from Eq. (28), we employ Eq. (1) and the following standard integrals,

$$\int_0^\infty p^2 dp \frac{z_{g/q} \exp(-\beta p)}{(1 \mp z_{g/q} \exp(-\beta p))^2} \equiv \pm \frac{2}{\beta^3} \text{PolyLog}[2, \pm z_{g/q}]$$

$$\int_0^\infty p dp \frac{z_q \exp(-\beta p)}{(1 + z_q \exp(-\beta p))^2} \equiv \frac{1}{\beta^3} \ln(1 + z_q).$$
(30)

The Debye mass with the Ideal EOS($z_g = 1, z_q = 1$) will be,

$$(M_D^I)^2 = Q^2 T^2 \Big(\frac{N_c}{3} + \frac{1}{2} - \frac{\tilde{m}^2}{4\pi^2} \ln(2) \Big).$$
(31)

To analyze the role of interactions, we define the effective charges Q_g, Q_q, and Q_s as,

$$Q_g = Q \Big\{ \Big(\frac{6\text{PolyLog}[2, z_g]}{\pi^2} \Big)^{1/2} \Big\}$$

$$Q_q = Q \Big\{ \Big(\frac{-12\text{PolyLog}[2, -z_q]}{\pi^2} \Big)^{1/2} \Big\} \equiv Q_s$$
(32)

Debye mass could be written in terms of these effective charges as,

$$M_D^2 = \Big\{ \frac{N_c}{3} Q_g^2 + \frac{1}{2}(Q_q^2 + Q_s^2) \Big\} T^2 - Q^2 T^2 \frac{\tilde{m}^2}{4\pi^2} \ln(1 + z_q).$$
(33)

As stated earlier, in the ideal limit Eq. (33) will reduce to Eq. (31). The quantities, Q_g, and $Q_{q,s}$ approach to Q. This observation tells us that interactions merely renormalize the effective partonic charges. In fact, the effective charges are reduced as compared to Q and asymptotically approach to the ideal value, Q.

To see, how interactions modify Debye screening mass, we consider the ratio $\mu_d = M_D/M_D^I$. The behavior of μ_d as a function of temperature is shown in Fig. 8. μ_d approaches the ideal value unity only asymptotically and $\mu_d \lesssim 1$. This implies that the presence of interactions
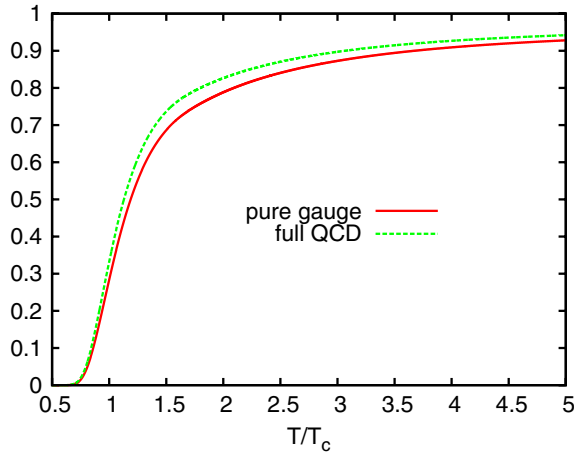
FIG. 8 (color online).   Behavior of μ_d as a function of T/T_c. Note that μ_d approaches to the ideal value only asymptotically.

suppresses the Debye mass as compared to its ideal counterpart.

Next, we compare M_D with the Debye mass obtained in lattice QCD. Let us first discuss the Debye screening mass computed in lattice gauge theory. It has been calculated in pure-gauge theory ($N_f = 0$) [61], in 2-flavor QCD ($N_f = 2$) [62,63], and in $2 + 1$-flavor QCD [64]. The lattice data on Debye mass have been fitted with the simple ansatz motivated by leading order result on Debye mass, $m_D^L \equiv A m_D^{LO}$, where m_D^L denotes the lattice data and, $m_D^{LO} \equiv \sqrt{(1 + \frac{N_f}{6})} g(T) T$ denotes the leading order Debye mass. Here, $g(T)$ is the two loop running coupling constant. This form fits the data quite well if $A \approx 1.4$–1.6 [64].

In our case (see Eq. (33)), Q^2 is a free parameter. We can fix it to match the Debye mass with the lattice result, m_D^L. This leads to,

$$Q^2 = A^2 \frac{g^2(T)}{(6\text{PolyLog}[2, z_g])/\pi^2},$$

$$Q^2 = A^2 g^2(T) \Big\{ ((6\text{PolyLog}[2, z_g] - 12\text{PolyLog}[2, -z_q])/\pi^2) $$
$$- \frac{\tilde{m}^2}{4\pi^2} \ln(1 + z_q) \Big\}^{-1} \qquad (34)$$

in EGS and full QCD with N_f flavors, respectively. In other words, the Debye mass obtained from the quasiparticle model can exactly be matched with the lattice results for the Debye mass. The Debye mass is needed to determine the transport parameters for quark-gluon plasma in RHIC [26]. It is of interest to derive the form of heavy quark potential [65,66] employing the formalism of chromo-electric response functions [2,65]. While deriving the potential, one should keep in mind the fact that hadronic phase to quark-gluon plasma transition is a crossover [67,68] rather than a true phase transition. These issues will be taken up in a separate communication in near future.

V. CONCLUSIONS AND FUTURE PROSPECTS

In conclusion, a quasiparticle model for $(2 + 1)$-flavor lattice QCD has been proposed which is valid in the deconfined phase of QCD. The interactions have been encoded in to the effective gluon and quark fugacities. These effective fugacities nontrivially modify the single quasiparton energies and lead to the trace anomaly in hot QCD. The description accurately reproduces the lattice QCD pressure, energy density, and the trace anomaly. In particular, the model accurately reproduces their low and high temperature behavior. We find that the model is fundamentally distinct from the other quasiparticle models (effective thermal mass, Polyakov loop models, and models with gluon condensate).

Employing the model, temperature dependence of the effective quasiparticle number densities has been obtained. A Virial expansion for QGP has been proposed in terms of effective quasiparticle number densities. The Virial expansion of the quasiparticle equation of state gets contribution from three sectors, *viz.*, the effective gluonic sector, the light-quark-sector, and the strange-quark sector. These sectors were dealt separately and eventually lead to the complete Virial expansion. This is perhaps the first time such a Virial expansion has been proposed for hot QCD. Interestingly, the Virial expansions came out to be mathematically similar as that for an ideal system of gluons, light quarks and, strange quark with temperature dependent fugacities. The Virial expansion has ensured that the quasiparticles are noninteracting. The interactions merely modulate the quasiparticle number densities and modify the single quasiparticle energies in a nontrivial way.

The Virial expansion may play important role to explore the Fermi liquid like picture of hot QCD in the matter sector and, in building effective kinetic theory with the quasiparticle model. The Virial expansion has revealed that the interactions appear to various observables determined by employing the quasiparticle description only in two ways, either through the effective fugacities (act as modulation factors) or through the modified dispersion relation. Finally, Debye mass has been obtained employing the expression obtained from semiclassical transport theory and effective coupling has been determined in terms of effective fugacities. This observation will be required in determining the transport coefficients for quark-gluon plasma. We find the Debye mass obtained from the quasiparticle model can exactly be matched with the lattice results.

The implications of model to study the transport coefficients (shear viscosity, bulk viscosity) will be taken up in the near future. It would also be of interest to extend the present model in the case of finite baryon density, and studying quark-number susceptibilities. It is to be of great interest to establish possible connections of our quasiparticle model with the Polyakov loop models, which is a matter of future investigations.

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