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Dressed Polyakov loop and flavor dependent phase transitions

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The chiral condensate and dressed Polyakov loop at finite temperature and density have been investigated in the framework of the $N_f = 2 + 1$ Nambu-Jona-Lasinio (NJL) model with two degenerate u, d quarks and one strange quark. In the case of explicit chiral symmetry breaking with physical quark masses, it is found that the phase transitions for light u, d, and s quark are sequentially happened, and the separation between the transition lines for different flavors becomes wider and wider with the increase of baryon density. For each flavor, the pseudocritical temperatures for chiral condensate and dressed Polyakov loop differ in a narrow transition range in the lower baryon density region, and the two transitions coincide in the higher baryon density region.

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I. INTRODUCTION

A QCD vacuum is characterized by spontaneous chiral symmetry breaking and color confinement. The dynamical chiral symmetry breaking is due to a nonvanishing quark antiquark condensate, $\langle\bar{q}q\rangle \simeq (250\ \mathrm{MeV})^3$ in the vacuum, which induces the presence of the light Nambu-Goldstone particles, the pions and kaons in the hadron spectrum. The confinement represents that only colorless states are observed in the spectrum, which is commonly described by the linearly rising potential between two heavy quarks at large distances, $V_{\bar{Q}Q}(R) = \sigma_s R$, where $\sigma_s \simeq (425\ \mathrm{MeV})^2$ is the string tension.

It is expected that chiral symmetry can be restored and color degrees of freedom can be freed at high temperature and/or density. The interplay between chiral and deconfinement phase transitions at finite temperature and density are of continuous interests for studying the QCD phase diagram [1–9]. The chiral restoration is characterized by the restoration of chiral symmetry, and the deconfinement phase transition is characterized by the breaking of center symmetry, which are only well defined in two extreme quark mass limits, respectively. In the chiral limit when the current quark mass is zero $m = 0$, the chiral condensate $\langle\bar{q}q\rangle$ is the order parameter for the chiral phase transition. When the current quark mass goes to infinity $m \to \infty$, QCD becomes pure gauge $SU(3)$ theory, which is center symmetric in the vacuum, and the usually used order parameter is the Polyakov loop expectation value $\langle P\rangle$ [1], which is related to the heavy quark free energy. At zero density and chiral limit, lattice QCD results show that the chiral and deconfinement phase transitions occur at the same critical temperature, e.g. see Refs. [10–14] and also review papers [15,16]. This result is highly nontrivial because these two distinct phase transitions involve different mechanisms at different energy scales. It has been largely believed for a long time that chiral symmetry restoration always coincides with deconfinement phase transition in the whole (T, μ) plane.

However, for the case of finite physical quark mass, neither the chiral condensate nor the Polyakov loop is a good order parameter. For heavy quark, there is no dynamical chiral symmetry breaking (e.g. see [17]) thus no chiral restoration. On the other hand, the linear potential description for confinement property is not suitable for a light quark system. In recent years, several lattice groups have made much effort on investigating the chiral and deconfinement phase transition temperatures with almost physical quark masses, e.g. RBC-Bielefeld group [18], which later merged with part of the MILC group [19] and formed the hotQCD group [20,21], and Wuppertal-Budapest group [22–26]. The result from the RBC-Bielefeld group in 2006 [18] found that the two pseudocritical temperatures for $N_f = 2 + 1$ coincide at $T_c = 192(7)(4)$ MeV. The Wuppetal-Budapest group found that for the case of $N_f = 2 + 1$, there are three pseudocritical temperatures, the transition temperature for chiral restoration of u, d quarks $T_c^{\chi(ud)} = 151(3)(3)$ MeV, the transition temperature for s quark number susceptibility $T_c^s = 175(2)(4)$ MeV, and the deconfinement transition temperature $T_c^d = 176(3)(4)$ MeV from the Polyakov loop. Recently, it is shown in [27,28], by using an improved HISQ action, hotQCD Collaboration results are close to the Wuppetal-Budapest Collaboration results.

The relation between the chiral and deconfinement phase transitions has also attracted more interest recently in studying the phase diagram at high baryon density region [29]. It is conjectured in Ref. [30] that in the large N_c limit, a confined but chiral symmetric phase, which is called quarkyonic phase, can exist in the high baryon

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density region. The quarkyonic phase or chiral density wave state is due to the quark-hole pairing near the Fermi surface. Nevertheless, it attracts a lot of interest to study whether such a chiral symmetric but confined phase can survive in a real QCD phase diagram, and how it competes with nuclear matter and the color superconducting phase [31].

In the framework of QCD effective models, there is still no dynamical model that can describe the chiral symmetry breaking and confinement simultaneously. The main difficulty of an effective QCD model to include confinement mechanism lies in that it is difficult to calculate the Polyakov loop analytically. Currently, the popular models used to investigate the chiral and deconfinement phase transitions are the Polyakov Nambu-Jona-Lasinio model [32–39] and Polyakov linear sigma model [40,41]. However, the shortcoming of these models is that the temperature dependence of the Polyakov loop potential is put in by hand from lattice result, which cannot be self-consistently be extended to finite baryon density. Recently, efforts have been made in Refs. [42,43] to derive a low-energy effective theory for confinement-deconfinement and chiral symmetry breaking/restoration.

Recent investigation revealed that quark propagator heat kernels can also act as an order parameter as they transform nontrivially under the center transformation related to deconfinement transition [44–46]. The exciting result is the behavior of the spectral sum of the Dirac operator under center transformation [45,47–49]. A new order parameter, called dressed Polyakov loop has been defined that can be represented as a spectral sum of the Dirac operator [49]. It has been found the infrared part of the spectrum particularly plays a leading role in confinement [45]. This result is encouraging since it gives a hope to relate the chiral phase transition with the confinement-deconfinement phase transition. The order parameter for chiral phase transition is related to the spectral density of the Dirac operator through Banks-Casher relation [4]. Therefore, both the dressed Polyakov loop and the chiral condensate are related to the spectral sum of the Dirac operator. Behavior of the dressed Polyakov loop is mainly studied in the framework of Lattice gauge theory [50–52]. Apart from that, studies based on Dyson-Schwinger equations [53–55] and the PNJL model [56,57] have been carried out. In those studies the role of dressed Polyakov loop as an order parameter is discussed at zero chemical potential. The dressed Polyakov loop at finite temperature and density has been investigated in the two-flavor NJL model in Ref. [58]. In this paper, we show the phase diagram in the framework of a three-flavor NJL model by using the dressed Polyakov loop as an equivalent order parameter.

This paper is organized as follows. We introduce the dressed Polyakov loop as an equivalent order parameter of confinement-deconfinement phase transition and the NJL model in Sec. II. Then in Sec. III, we show the results of the three-flavor QCD phase diagram in the T-μ plane in the chiral limit and in the case of explicit chiral symmetry breaking, respectively. At the end, we give the conclusion and discussion.

II. DRESSED POLYAKOV LOOP AND THE THREE-FLAVOR NJL MODEL

We first introduce the dressed Polyakov loop. To do this we have to consider a $U(1)$ valued boundary condition for the fermionic fields in the temporal direction instead of the canonical choice of antiperiodic boundary condition,

$$\psi(x, \beta) = e^{-i\phi}\psi(x, 0), \tag{1}$$

where $0 \leq \phi < 2\pi$ is the phase angle and β is the inverse temperature.

Dual quark condensate Σ_n is then defined by the Fourier transform (with respect to the phase ϕ) of the general boundary condition dependent quark condensate [49–51],

$$\Sigma_n = -\int_0^{2\pi}\frac{d\phi}{2\pi}e^{-in\phi}\langle\bar{\psi}\psi\rangle_\phi, \tag{2}$$

where n is the winding number.

The particular case of $n = 1$ is called the dressed Polyakov loop which transforms in the same way as the conventional thin Polyakov loop under the center symmetry and hence is an order parameter for the deconfinement transition [49–51]. It reduces to the thin Polyakov loop and to the dual of the conventional chiral condensate in infinite and zero quark mass limits, respectively, i.e., in the chiral limit $m \to 0$ we get the dual of the conventional chiral condensate and in the $m \to \infty$ limit we have a thin Polyakov loop [49–51].

The Lagrangian of the three-flavor NJL model [59] is given as

$$\begin{aligned}\mathcal{L} = {}& \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + G_s\sum_a\{(\bar{\psi}\tau_a\psi)^2 \\ & + (\bar{\psi}i\gamma_5\tau_a\psi)^2\} - K\{\mathrm{Det}_f[\bar{\psi}(1+\gamma_5)\psi] \\ & + \mathrm{Det}_f[\bar{\psi}(1-\gamma_5)\psi]\},\end{aligned} \tag{3}$$

where $\psi = (u, d, s)^T$ denotes the transpose of the quark field, and $m = \mathrm{Diag}(m_u, m_d, m_s)$ is the corresponding mass matrix in the flavor space. τ_a with $a = 1, \cdots, N_f^2 - 1$ are the eight Gell-Mann matrices, and Det_f means determinant in flavor space. The last term is the standard form of the 't Hooft interaction, which is invariant under $SU(3)_L \times SU(3)_R \times U(1)_B$ symmetry, but breaks down the $U_A(1)$ symmetry.

The ϕ dependent thermodynamic potential in the mean field level is given as the following:

$$\Omega_\phi = \sum_f \Omega_{\phi,M_f} + 2G_s\sum_f\langle\sigma\rangle_{\phi,f}^2 - 4K\langle\sigma\rangle_{\phi,u}\langle\sigma\rangle_{\phi,d}\langle\sigma\rangle_{\phi,s}, \tag{4}$$

with

$$\Omega_{\phi,M_f} = -2N_c \int_\Lambda \frac{d^3p}{(2\pi)^3}\Big[E_{p,f} + \frac{1}{\beta} ln(1 + e^{-\beta E^-_{p,f}}) + \frac{1}{\beta} ln(1 + e^{-\beta E^+_{p,f}})\Big], \tag{5}$$

where the sum is in the flavor space, $E_{p,f} = \sqrt{p^2 + M^2_{\phi,f}}$ and $E^{\pm}_{p,f} = E_{p,f} \pm [\mu + i(\phi - \pi)T]$, with the constituent quark mass

$$M_{\phi,i} = m_i - 4G_s\langle\sigma\rangle_{\phi,i} + 2K\langle\sigma\rangle_{\phi,j}\langle\sigma\rangle_{\phi,k}, \tag{6}$$

where (i, j, k) is the quark flavor indices (u, d, s), and $\langle\sigma\rangle_{\phi,f} = \langle\bar{\psi}_f\psi_f\rangle_\phi$. We will only consider isospin symmetric quark matter and define a uniform chemical potential μ for u, d, and s.

It is known that the NJL model lacks of confinement and the gluon dynamics is encoded in a static coupling constant for four point contact interaction. However, assuming that we can read the information of confinement from the dual chiral condensate, it would be interesting to see the behavior of the dressed Polyakov loop in a scenario without any explicit mechanism for confinement.

The thermodynamic potential contains an imaginary part. We take only the real part of the potential and the imaginary phase factor is not considered in this work. The mean field $\langle\sigma\rangle_\phi$ is obtained by minimizing the potential for each value of $\phi \in [0, 2\pi)$ for fixed T and μ. The conventional chiral condensate is $\langle\sigma\rangle_\pi = \langle\bar{\psi}\psi\rangle_\pi$. For brevity, from here on we will represent the conventional chiral condensate as $\langle\sigma\rangle$. The dressed Polyakov loop Σ_1 is obtained by integrating over the angle.

III. PHASE DIAGRAM FOR THREE FLAVORS

We investigate phase transitions for two cases, i.e., in the chiral limit and in the case of explicit chiral symmetry breaking with physical quark mass, and the corresponding parameters shown in Table_I are taken from Refs. [60–62]:

A. Phase diagram in the chiral limit

We first consider the case of chiral limit, i.e. $m_u = m_d = m_s = 0$. In Fig. 1, we show the behavior of the conventional chiral condensate $-\langle\sigma\rangle$ and the corresponding

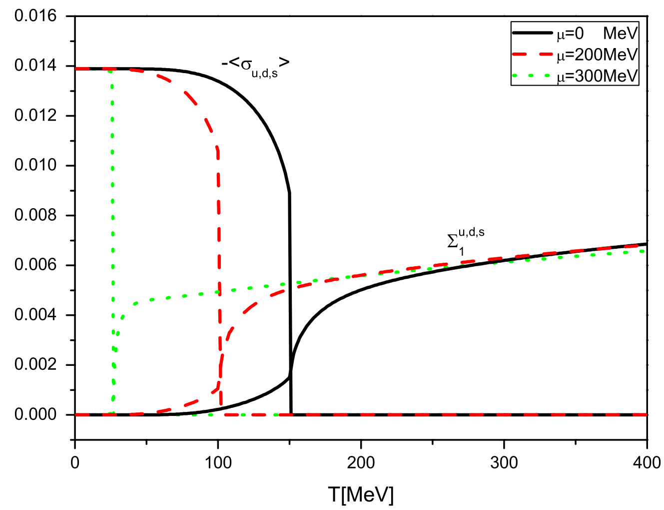

FIG. 1 (color online). The conventional chiral condensate $-\langle\sigma\rangle_{u,d,s}$ and the dressed Polyakov loop $\Sigma_1^{u,d,s}$ of u, d, s quarks as functions of temperature for different values of the chemical potentials. Here, $-\langle\sigma\rangle$ and Σ_1 both are measured in [GeV^3].

dressed Polyakov loop Σ_1 for u, d, and s quarks at different chemical potentials as functions of temperature. For both order parameters, it is observed that there are three temperature regions for $-\langle\sigma\rangle$ and Σ_1. For $-\langle\sigma\rangle$, at smaller temperatures it remains constant at a value corresponding to the value of the conventional chiral condensate in the vacuum, then it drops to zero at the critical temperature T_c, and eventually keeps zero above the critical temperature. The critical temperature decreases with the increase of the chemical potential. It is noticed that, in order to guide eyes, we have connected the two end points of the order parameter at the jump.

On the other hand the behavior for the dressed Polyakov loop is just the opposite. It remains zero for small temperatures and then jumps at the critical temperature, and finally saturates to a high value that varies very slowly with temperatures. The almost zero value of Σ_1 for small temperatures is due to the fact that the $U(1)$ boundary condition dependent general quark condensate nearly does not vary with the angle ϕ for small temperatures [see Eq. (2)].

It is seen that the phase transitions for chiral restoration and dressed Polyakov loop are of first order in the whole T-μ plane. For the two-flavor case, it was found that these two phase transitions are of second order. The N_f dependent result is in agreement with the results given by Pisarski and Wilczek in Ref. [63]. The first-order phase transition in the three-flavor case is due to the 't Hooft interaction in Eq. (3), which contributes a cubic term in the thermodynamical potential in Eq. (4).

Figure 2 shows the phase diagram of three-flavor in the chiral limit. We find almost exact matching for the transition temperatures calculated from these two quantities in the whole T-μ plane.

TABLE I. Two sets of parameters in yhree-flavor NJL model: the current quark mass m_q for up and down quark and m_s for strange quark, coupling constants G and K, with a spatial momentum cutoff $\Lambda = 602.3$ MeV.

	m_q[MeV]	m_s[MeV]	$G_s\Lambda^2$	$K\Lambda^5$
Chiral limit	0	0	1.926	12.36
Physical mass	5.5	140.7	1.835	12.36

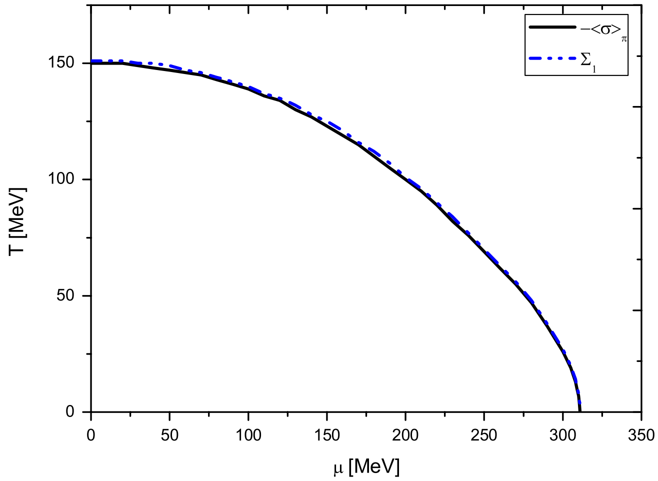

FIG. 2 (color online). Three-flavor phase diagram in the T-μ plane for the case of chiral limit. The solid line is the critical line for Σ_1, and the dashed line is the critical line for conventional chiral phase transition.

B. Phase diagram with physical quark mass

For the case of finite quark mass $m_u = m_d = 5.5$ MeV and $m_s = 140.7$ MeV, we have chosen the model parameters of $G_s\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$ with $\Lambda = 602.3$ MeV as in Ref. [60] to fit $m_\pi = 135.0$ MeV, $f_\pi = 92.4$ MeV, $m_K = 497.7$ MeV, and $m_{\eta'} = 957.8$ MeV.

In Figs. 3 and 4, we show the behavior of the conventional chiral condensate $-\langle\sigma\rangle$ and the dressed Polyakov loop Σ_1 at different chemical potentials as functions of temperature for u, d, and s quarks, respectively.

For both cases, it is observed that there are three temperature regions for $-\langle\sigma\rangle$ and Σ_1. For $-\langle\sigma\rangle$, at smaller temperatures it remains constant at a value corresponding

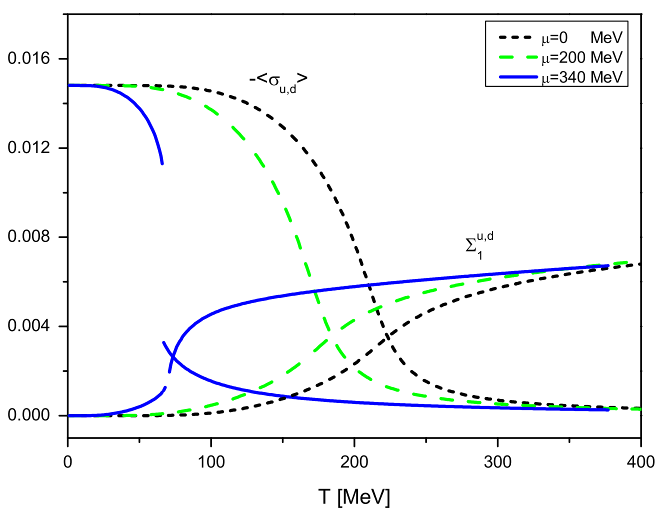

FIG. 3 (color online). The conventional chiral condensate $-\langle\sigma\rangle_{u,d}$ and the dressed Polyakov loop $\Sigma_1^{u,d}$ of u, d quarks as functions of temperature for different values of the chemical potentials. Here, $-\langle\sigma\rangle$ and Σ_1 both are measured in [GeV^3].

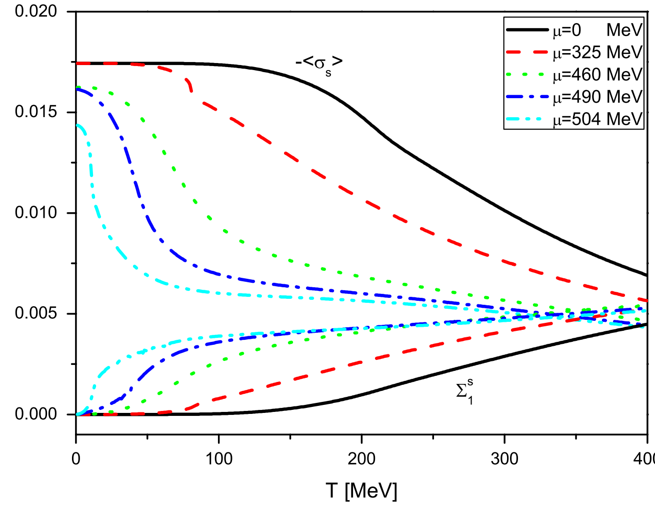

FIG. 4 (color online). The conventional chiral condensate $-\langle\sigma\rangle_s$ and the dressed Polyakov loop Σ_1^s of s quark as functions of temperature for different values of the chemical potentials. Here, $-\langle\sigma\rangle$ and Σ_1 both are measured in [GeV^3].

to the value of the conventional chiral condensate in the vacuum, then it rapidly decreases in a small window of temperature and eventually almost saturates to a lower value. The decreasing occurs at different temperatures for different values of the chemical potentials. On the other hand the behavior for the dressed Polyakov loop is just the opposite. It remains almost zero for small temperatures and then rises rapidly, finally saturating to a high value that varies very slowly with temperatures. The almost zero value of Σ_1 for small temperatures is due to the fact that the $U(1)$ boundary condition dependent general quark condensate nearly does not vary with the angle ϕ for small temperatures [see Eq. (2)].

The critical temperature for a real phase transition or the pseudocritical temperature for a crossover is extracted from the susceptibility of the order parameter or the temperature derivative of the order parameter. For example, for a chiral phase transition of the strange quark, the (pseudo) critical temperature is extracted from the temperature derivative $\partial(-<\sigma_s>)/(\partial T)$. This quantity describes how fast the order parameter changes with temperature. Normally the critical temperature corresponds to the fastest change of the order parameter, and the temperature derivative of the order parameter shows a peak at the critical point. However, there are some subtleties to determine the pseudocritical temperature for the chiral restoration of the strange quark. We show how we determine the pseudocritical temperature of the crossover by using Fig. 5, which is the temperature derivative of the chiral condensate of the strange quark corresponding to Fig. 4.

For $\mu = 0$, from Fig. 5 one can observe that the temperature derivative of the strange quark condensate shows a peak at $T = 196$ MeV; correspondingly, from Fig. 4, one can see that the strange quark condensate changes fast at

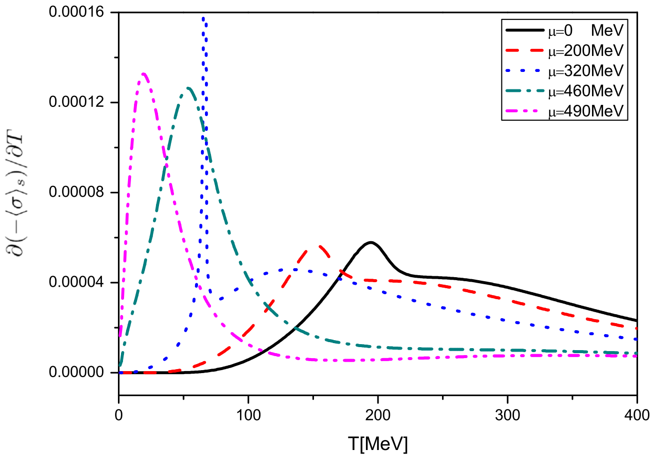

FIG. 5 (color online). The derivative of strange chiral condensate $\partial(-\langle\sigma\rangle_s)/\partial T$ as functions of T for different values of μ.

$T = 196$ MeV, which is the critical temperature for a chiral phase transition of the u, d quarks at zero chemical potential. However, the value of the strange quark condensate at $T_{c,\chi}^{u,d} = 196$ MeV is still around its vacuum value; one cannot locate the pseudocritical temperature of the strange quark at $T_{c,\chi}^{u,d} = 196$ MeV even though there is a peak for the temperature derivative of the strange quark condensate. The reasonable explanation of the fast change of the strange quark condensate at $T_{c,\chi}^{u,d} = 196$ MeV is that the strange quark feels the chiral phase transition of u, d quarks due to the flavor mixing effect. For $\mu = 0$, from Fig. 5 one can also observe a bump region of the temperature derivative of the strange quark condensate around $T = 250$ MeV; however, there is no obvious peak that shows up. Therefore, we cannot extract an explicit pseudocritical temperature from the chiral phase transition of the strange quark. Correspondingly, we find that the strange quark condensate at $\mu = 0$ changes smoothly with temperature.

The temperature derivative of the strange quark condensate at $\mu = 200$ MeV is similar to the case at $\mu = 0$. The only difference is that the peak moves to a lower temperature. The small jump at the large strange chiral condensate region is induced by the u, d quark chiral phase transition. It cannot be regarded as the phase transition for the strange quark even though it corresponds to a peak of the strange chiral susceptibility, because the order parameter does not change very much compared to its vacuum value. It should still be regarded as in the chiral symmetry breaking phase. At $\mu = 320$ MeV, it is seen from Fig. 5 that the left peak develops to a sharp peak at $T_{c,\chi}^{u,d}$, and an obvious peak shows up in the right bump region. Therefore, one can extract the pseudocritical temperature for the chiral phase transition of the strange quark. For a higher chemical potential, e.g, $\mu = 460$ MeV or $\mu = 490$ MeV, because u, d quarks are already in chiral symmetric phase, there is only one peak that shows up for the temperature derivative of the strange quark condensate in Fig. 5, and the location of the peak gives the pseudocritical temperature of the phase transition.

As we have discussed in detail above, one has to combine the information from the order parameter itself as well as the temperature derivative of the order parameter in order to determine the pseudocritical temperature of the crossover. This method is also used to determine the dressed Polyakov loop of the strange quark. The critical and pseudocritical temperatures extracted from the temperature derivative of the order parameters are shown in Fig. 6. It is found that the chiral and deconfinement phase transitions are flavor dependent.

At the low baryon chemical potential region when $\mu < 270$ MeV, for light flavors, i.e. for u, d quarks, we observe from Fig. 3 that the conventional chiral condensate and the dressed Polyakov loop change rapidly with the increase of temperature. From the temperature derivative of the order parameters of the chiral condensate and dressed Polyakov loop, we can obtain two separate pseudocritical temperatures T_c^χ and $T_c^\mathcal{D}$ for fixed μ, and we find T_c^χ is always smaller than $T_c^\mathcal{D}$.

However, in the chemical potential region when $\mu < 270$ MeV, for s quark, from Fig. 4 we can see that the conventional chiral condensate and dressed Polyakov loop change smoothly with the increase of temperature. From the temperature derivative of the order parameters, one cannot extract the values of the pseudocritical temperatures as already discussed. Therefore, in Fig. 6 of the three-flavor phase diagram, we can read that in the region around $0 < \mu < 270$ MeV, the phase transitions for u, d are crossover, and different order parameters have different

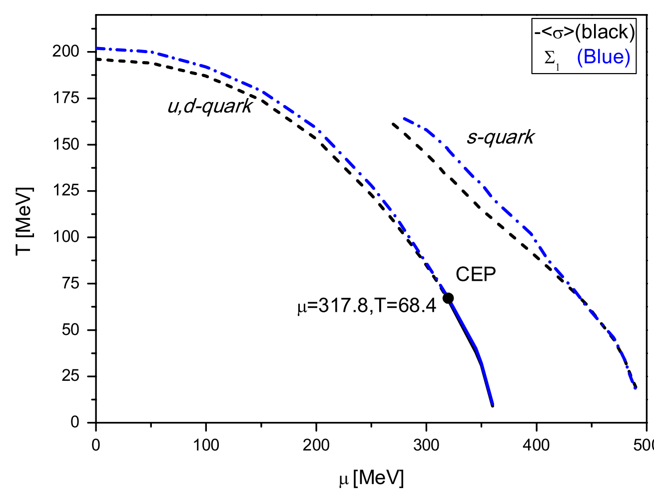

FIG. 6 (color online). Three-flavor phase diagram in the T-μ plane for the case of $m_u = m_d = 5$ MeV and $m_s = 140.7$ MeV. The dashed-dotted lines are the critical line for Σ_1, and the dashed lines are the critical line for conventional chiral phase transition in the region of crossover. The solid lines indicates the first-order phase transitions, and the solid circle indicates the critical end points for chiral phase transitions of u, d quarks.

pseudocritical temperatures. The s flavor experiences a rapid crossover, and no pseudocritical temperatures can be extracted from the order parameters. From the lattice results in Ref. [24] at zero chemical potential, there is also no pseudocritical temperature for the order parameter of the strange quark's chiral condensate.

At a higher baryon chemical potential region, it is observed from Fig. 3 that the conventional chiral condensate and the dressed Polyakov loop change sharply with the increase of temperature. From the temperature derivative of the order parameters, we find that the phase transitions are of first order, and the critical temperatures for chiral and dressed Polyakov loop coincide with each other around the critical end point (CEP).

For s quark, from Fig. 4 we can see that when the chemical potential becomes higher and higher, the conventional chiral condensate and dressed Polyakov loop change more rapidly with the increase of temperature. The temperature derivative of the order parameters gives separate values of the pseudocritical temperatures in the region $270 < \mu < 450$ MeV, and the two pseudocritical temperatures merge in the region of $\mu > 450$ MeV.

From Fig. 6 of the three-flavor phase diagram, we can see that the critical end point for u, d flavors lies at $(T^{u,d}_{CEP}, \mu^{u,d}_{CEP}) = (68.4\text{ MeV}, 317.8\text{ MeV})$, which is different from the results in Ref. [58] for pure two-flavor NJL model. The difference result from (1) different model parameters having been used, and (2) the coupling of s quark to u, d quark contributing one extra term in the thermodynamical potential compared to the pure two-flavor case. The location of CEP in this work is in good agreement with that in Ref. [64].

In Figs. 7 and 8, we show the details of locating the CEP. In the first-order phase region, there are two branches of number densities, i.e., for fixed chemical potential, the number density $n_q = -\frac{\partial\Omega}{\partial\mu}$ has a jump at the transition temperature. The two branches of number densities merge at the CEP. This feature is shown in Fig. 7. At the CEP, the phase transition is of second order and this is indicated by the divergent behavior of the number susceptibility. We show the number susceptibility $\chi_q = -\frac{\partial^2\Omega}{\partial\mu^2}$ as functions of the temperature in Fig. 8. It is clearly seen that χ_q develops a sharp peak at CEP.

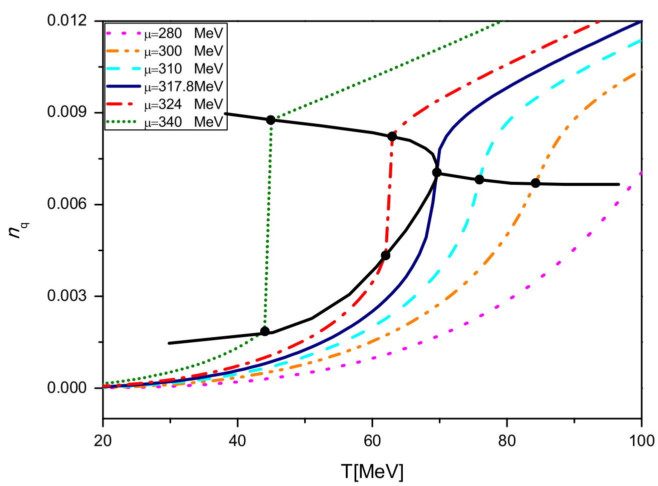

FIG. 7 (color online). The quark number density n_q as functions of the temperature for different chemical potentials, and n_q is in units of [GeV^3].

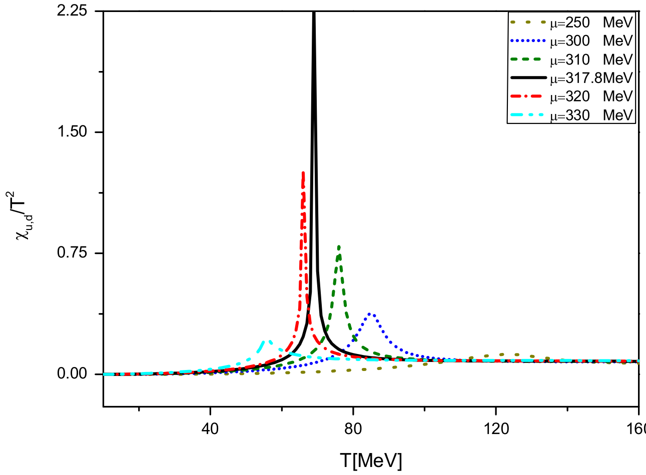

FIG. 8 (color online). The number susceptibility χ_q/T^2 as functions of the temperature for different chemical potentials.

IV. CONCLUSION AND DISCUSSION

We investigate the chiral condensate and the dressed Polyakov loop or dual chiral condensate at finite temperature and density in the three-flavor Nambu-Jona-Lasinio model. It is found that in the chiral limit, the phase transitions are of first order and the critical temperature for chiral phase transition coincides with that of the dressed Polyakov loop. In the case of explicit chiral symmetry breaking, it is found that the phase transitions are flavor dependent, and there is a phase transition range for each flavor. The transition range of the s quark is located at higher temperature and higher baryon density than that of u, d quarks. At low baryon density region, it is found that the transition range of u, d quarks are not separated too much from that of the s quark; however, the separation of the transition ranges for u, d quarks and the s quark becomes wider and wider with the increase of the chemical potential.

For light u, d quarks, the pseudocritical temperature for chiral transition T_c^χ is smaller than that of the dressed Polyakov loop $T_c^{\mathcal{D}}$ in the low baryon density region where the transition is a crossover, and these two phase transitions coincide in the first-order phase transition region at high baryon density. For the s quark, both transitions are of smooth crossover at low baryon density and become rapid crossover at a moderate baryon density region where the pseudocritical temperatures for the chiral condensate and the dressed Polyakov loop are separated. Then, at enough high baryon density, these two transitions coincide with each other.

Our results are based on the NJL model, where the gluon dynamics is encoded in a static coupling constant for four point contact interaction; a quantitative comparison will not match with lattice results. However, we believe the scenario of the sequential phase transitions is physically correct.

Until now, there have been six quark flavors observed in experiment. These six flavors cover a very wide energy scale, from several MeV to several hundred GeV. Only light quarks experience dynamically chiral symmetry breaking in the vacuum, and chiral phase transition in high temperature and density. However, there are no good order parameters to describe the deconfinement phase transition of light quarks. The conventional Polyakov loop is a good order parameter for confinement-deconfinement phase transition in the limit of infinity heavy quark mass and has the interpretation of the free energy of an infinity heavy quark. In analogy to that we can regard the dressed Polyakov loop as an order parameter for confinement-deconfinement phase transition for a quark with mass m, and interpret the dressed Polyakov loop as the free energy of a quark with any mass m [52]. Therefore, in principle, each flavor can have different critical temperatures for deconfinement phase transition. Lattice results already reflect such properties at zero chemical potential, e.g. the pseudocritical temperatures for order parameters of u, d quarks, s quark and the Polyakov loop are different, and the pseudocritical temperature is higher for heavier quark mass.

It is natural to understand that the separation of the phase transition range for different flavors becomes wider and wider with the increase of the chemical potential. Lattice result at zero chemical potential gives that the pseudocritical temperature for u, d quarks is around 155 MeV, and for s quark is around 175 MeV. The difference is around 20 MeV. However, at zero temperature, the u, d quarks restore chiral symmetry at the chemical potential around their vacuum constituent masses, i.e. $\mu_c^{u,d} \sim M_{u,d} \sim 330$ MeV, and the s quark restores chiral symmetry at the chemical potential around $\mu_c^s \sim M_s \sim 550$ MeV. The difference is around 200 MeV.

Based on the above analysis, in Fig. 9, we show our conjectured three-dimensional (3D) QCD phase diagram for finite temperature T, quark chemical potential μ_q, and isospin chemical potential μ_I.

In the plane of (μ, T), each flavor has its own transition range. The transition range is wider in the low baryon density and becomes narrower and narrower with the increase of the chemical potential, and eventually merges at higher chemical potential. By using the lattice results at zero density, we identify the phase transition range around

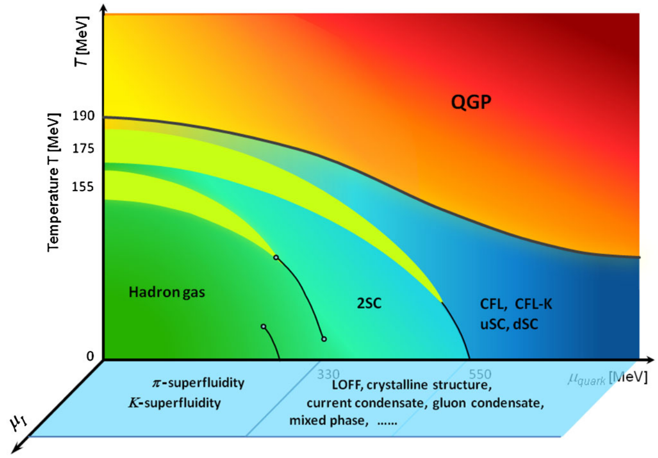

FIG. 9 (color online). Conjectured 3D QCD phase diagram at finite temperature T, quark chemical potential μ_q and isospin chemical potential μ_I.

155 MeV for u, d quark, 175 MeV for s quark, and 190 MeV for heavy flavor. The upper solid line is for the Polyakov loop, which does not change so much with the increase of baryon density. This result agrees with that in any Polyakov loop NJL model and Polyakov loop linear sigma model. Because of the flavor dependent phase transitions, we naturally expect the color superconducting phase for yhe two-flavor quark system and three-flavor quark system in different baryon density regions [31]. Because of the finite mass of the strange quark, the three-flavor color superconducting phase can be in the color flavor locking (CFL) phase [65], CFL-kaon condensate phase (CFL-K) [66], or uSC/dSC phase [67].

When isospin asymmetry is considered, the phase diagram becomes much more complicated. At a low baryon density region, there will be pion superfluidity and kaon superfluidity phases [68]. In the color superconducting phase, because isospin asymmetry induces mismatch between the pairing quarks, there will appear unstable gapless excitations [69,70] when charge neutrality condition is considered. It has been vastly discussed in many literatures about the true ground state of the charge neutral two-flavor and three-flavor cold quark matter, e.g., the Larkin-Ovchinnikov-Fulde-Ferrell sate or other crystalline structure [71], the gluon condensate state [72], the current generation state [73], and so on. The detailed analysis given in Ref. [74] shows that in the gapless color superconducting phase, both the phase part and magnitude part of the order parameter will develop instabilities. The phase part develops into the chromomagnetic instability, which induces the plane-wave state; the magnitude part develops the Sarma instability and Higgs instability; the Sarma instability can be competed with charge neutrality condition. If the Higgs instability cannot be cured by the electric or color Coulomb interaction, it will induce the inhomogeneous state.

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