

3 + 1 and 3 + 2 sterile neutrino fits

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We present the results of fits of short-baseline neutrino-oscillation data in 3 + 1 and 3 + 2 neutrino-mixing schemes. In spite of the presence of a tension in the interpretation of the data, 3 + 1 neutrino mixing is attractive for its simplicity and for the natural correspondence of one new entity (a sterile neutrino) with a new effect (short-baseline oscillations). The allowed regions in the oscillation parameter space can be tested in near-future experiments. In the framework of 3 + 2 neutrino mixing, there is less tension in the interpretation of the data, at the price of introducing a second sterile neutrino. Moreover, the improvement of the parameter goodness of fit is mainly a statistical effect due to an increase in the number of parameters. The CP violation in short-baseline experiments allowed in 3 + 2 neutrino mixing can explain the positive $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal and the negative $\nu_\mu \rightarrow \nu_e$ measurement in the MiniBooNE experiment. For the CP -violating phase, we obtained two minima of the marginal χ^2 close to the two values where CP violation is maximal.

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I. INTRODUCTION

The recent agreement of MiniBooNE antineutrino data [1] with the short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation signal observed several years ago in the Liquid Scintillator Neutrino Detector (LSND) experiment [2] has opened intense theoretical and experimental activity aimed at clarifying the explanation of these observations in a framework compatible with the data of other neutrino-oscillation experiments. Several short-baseline neutrino-oscillation experiments did not observe neutrino oscillations and their data constrain the interpretation of the LSND and MiniBooNE antineutrino signal. However, there are other positive indications of short-baseline neutrino oscillations that may be taken into account: the reactor antineutrino anomaly [3], in favor of a small short-baseline disappearance of $\bar{\nu}_e$, the gallium neutrino anomaly [3–12], in favor of a short-baseline disappearance of ν_e , and the MiniBooNE low-energy anomaly [3,7,9,10]. In this paper, we consider only the reactor antineutrino anomaly, by taking into account the new calculation of reactor antineutrino fluxes in Ref. [13]. We leave the discussion of the effects of the more controversial gallium anomaly and MiniBooNE low-energy anomaly to a following article [14].

The results of solar, atmospheric and long-baseline neutrino-oscillation experiments led us to the current standard three-neutrino-mixing paradigm, in which the three active neutrinos ν_e , ν_μ , ν_τ are superpositions of three

massive neutrinos ν_1 , ν_2 , ν_3 with respective masses m_1 , m_2 , m_3 . The measured solar (SOL) and atmospheric (ATM) squared-mass differences can be interpreted as

$$\Delta m_{\text{SOL}}^2 = \Delta m_{21}^2 = (7.6 \pm 0.2) \times 10^{-5} \text{ eV}^2, \quad (1)$$

$$\Delta m_{\text{ATM}}^2 = |\Delta m_{31}^2| = 2.32_{-0.08}^{+0.12} \times 10^{-3} \text{ eV}^2 \quad (2)$$

(see Refs. [15,16] for Eqs. (1) and (2), respectively), with $\Delta m_{kj}^2 = m_k^2 - m_j^2$.

The completeness of the three-neutrino-mixing paradigm has been challenged by the LSND [2] and MiniBooNE [1] observations of short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions at different values of distance (L) and energy (E), but at approximately the same L/E . Since the distance and energy dependencies of neutrino oscillations occur through this ratio, the agreement of the MiniBooNE and LSND signals raised interest in the possible existence of one or more squared-mass differences larger than about 0.5 eV, which is much bigger than the values of Δm_{SOL}^2 and Δm_{ATM}^2 . Hence, we are led to the extension of three-neutrino mixing with the introduction of one or more sterile neutrinos which do not have weak interactions and do not contribute to the invisible width of the Z boson [17]. In this paper we consider the simplest possibilities: 3 + 1 mixing with one sterile neutrino and 3 + 2 mixing with two sterile neutrinos.

The existence of sterile neutrinos which have been thermalized in the early Universe is compatible with big-bang nucleosynthesis data [18,19] (with the indication, however, that schemes with more than one sterile neutrino are disfavored [20]) and with the cosmological measurements of the cosmic microwave background and large-scale structures if the neutrino masses are limited below

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about 1 eV [21–25]. Therefore, in this paper we consider squared-mass differences smaller than 10 eV².

II. 3 + 1 NEUTRINO MIXING

In this section we consider the simplest extension of three-neutrino mixing with the addition of one massive neutrino. In such a four-neutrino-mixing framework, the flavor neutrino basis is composed of the three active neutrinos ν_e, ν_μ, ν_τ and a sterile neutrino ν_s .

So-called 2 + 2 four-neutrino-mixing schemes are strongly disfavored by the absence of any signal of sterile-neutrino effects in solar and atmospheric neutrino data [26]. Hence, we consider only the so-called 3 + 1 four-neutrino schemes depicted in Fig. 1. Since the inverted scheme has three massive neutrinos at the eV scale, it is disfavored by cosmological data [21,22] over the normal scheme. In both 3 + 1 schemes, the effective flavor transition and survival probabilities in short-baseline (SBL) experiments are given by

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{SBL}(-)} = \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \quad (\alpha \neq \beta), \quad (3)$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha}^{\text{SBL}(-)} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right), \quad (4)$$

for $\alpha, \beta = e, \mu, \tau, s$, with

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2, \quad (5)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2). \quad (6)$$

Therefore:

- (1) All effective SBL oscillation probabilities depend only on the absolute value of the largest squared-mass difference Δm_{41}^2 .
- (2) All oscillation channels are open, each one with its own oscillation amplitude.
- (3) The oscillation amplitudes depend only on the absolute values of the elements in the fourth column of

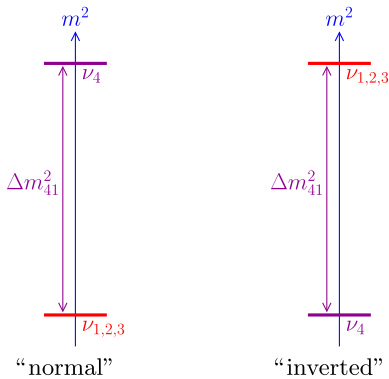


FIG. 1 (color online). Schematic description of the two possible 3 + 1 schemes that we are considering, taking into account that $|\Delta m_{21}^2| \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2|$.

the mixing matrix, i.e. on three real numbers with sum less than unity, since the unitarity of the mixing matrix implies $\sum_\alpha |U_{\alpha 4}|^2 = 1$.

- (4) *CP* violation cannot be observed in SBL oscillation experiments, even if the mixing matrix contains *CP*-violation phases. In other words, neutrinos and antineutrinos have the same effective SBL oscillation probabilities.

Before the recent indication of an antineutrino $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal consistent with the LSND antineutrino signal, the MiniBooNE Collaboration published the results of neutrino data which do not show a corresponding $\nu_\mu \rightarrow \nu_e$ signal [27]. This difference between the MiniBooNE neutrino and the antineutrino data may be due to *CP* violation.

The absence of any difference in the effective SBL oscillation probabilities of neutrinos and antineutrinos in 3 + 1 four-neutrino-mixing schemes implies that these schemes cannot explain the difference between neutrino and antineutrino oscillations observed in the MiniBooNE experiment. Moreover, the dependence of all the oscillation amplitudes in Eqs. (5) and (6) on three independent absolute values of the elements in the fourth column of the mixing matrix implies that the amplitude of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions is limited by the absence of large SBL disappearance of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ observed in several experiments.

The results of reactor neutrino experiments constrain the value $|U_{e4}|^2$ through the measurement of $\sin^2 2\vartheta_{ee}$. The calculation of the reactor $\bar{\nu}_e$ flux has been recently improved in Ref. [13], resulting in an increase of about 3% with respect to the previous value adopted by all experiments for the comparison with the data (see Ref. [28]). Since the measured reactor rates are in approximate agreement with those derived from the old $\bar{\nu}_e$ flux, they show a deficit with respect to the rates derived from the new $\bar{\nu}_e$ flux. This is the “reactor antineutrino anomaly” [3], which is quantified by the value

$$\bar{R}_{\text{reactor anomaly}} = 0.946 \pm 0.024 \quad (7)$$

for the average of the ratios of measured event rates and those expected in absence of $\bar{\nu}_e$ transformations into other states. We considered the integral rates of the Bugey-3 [29], Bugey-4 [30], Rovno91 [31], Gosgen [32], ILL [33] and Krasnoyarsk [34] short-baseline reactor antineutrino experiments using the information in Table II of Ref. [3].¹

¹We do not use the two rates of the Savannah River (SRPI and SRPII) experiment [35] from Table II of Ref. [3], $R_{\text{SRPI}} = 0.952 \pm 0.006 \pm 0.037$ and $R_{\text{SRPII}} = 1.018 \pm 0.010 \pm 0.037$ because they are about 5.5σ apart, taking into account that their difference 0.066 ± 0.012 is independent of the correlated systematic uncertainty (0.037). Such a large difference cannot be due to neutrino oscillations averaged over the whole energy spectrum because the two measurements have been made at distances which are not different enough (18 m and 24 m). We also do not use the Rovno88 [36] rates from Table II of Ref. [3] because the correlation with Bugey-4 and Rovno91 is not clear.

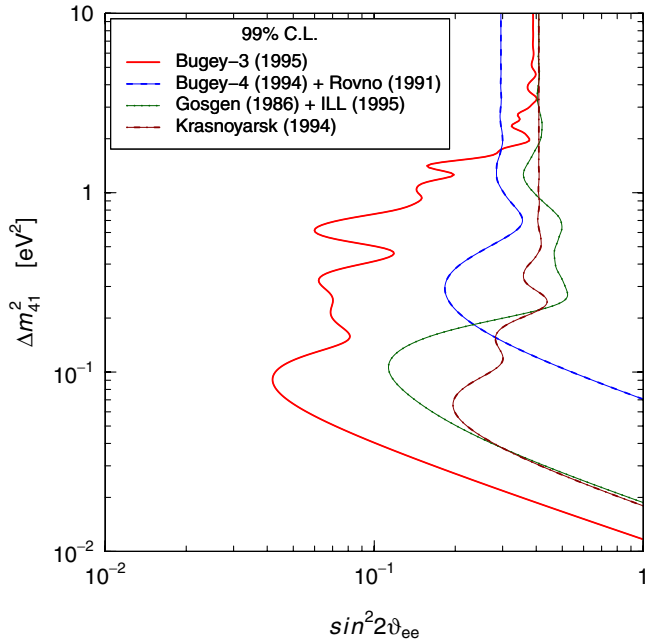


FIG. 2 (color online). Exclusion curves obtained from the data of reactor $\bar{\nu}_e$ disappearance experiments (see Ref. [3]).

Hence, the reactor antineutrino anomaly is a 2.2σ indication that there is a small short-baseline disappearance of $\bar{\nu}_e$ which may correspond to the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal observed in the LSND and MiniBooNE experiments. However, the $\bar{\nu}_e$ disappearance is small and large values of $\sin^2 2\vartheta_{ee}$ are constrained by the exclusion curves in Fig. 2. (As in Ref. [3], the Bugey-3 exclusion curve has been obtained by fitting the three integral rates measured at $L = 15, 40, 95$ m and the 40 m/15 m spectral ratio in Fig. 15 of Ref. [29].) Since values of $|U_{e4}|^2$ close to unity are excluded by solar neutrino oscillations (which require large $|U_{e1}|^2 + |U_{e2}|^2$), for small $\sin^2 2\vartheta_{ee}$ we have

$$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2. \quad (8)$$

The value of $\sin^2 2\vartheta_{\mu\mu}$ is constrained by the curves in Fig. 3, which have been obtained from the lack of ν_μ disappearance in the CDHSW ν_μ experiment [37] and from the requirement of large $|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2$ for atmospheric neutrino oscillations [38]. Hence, $|U_{\mu 4}|^2$ is small and

$$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu 4}|^2. \quad (9)$$

From Eqs. (5), (8), and (9), for the amplitude of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions, we obtain [39,40]

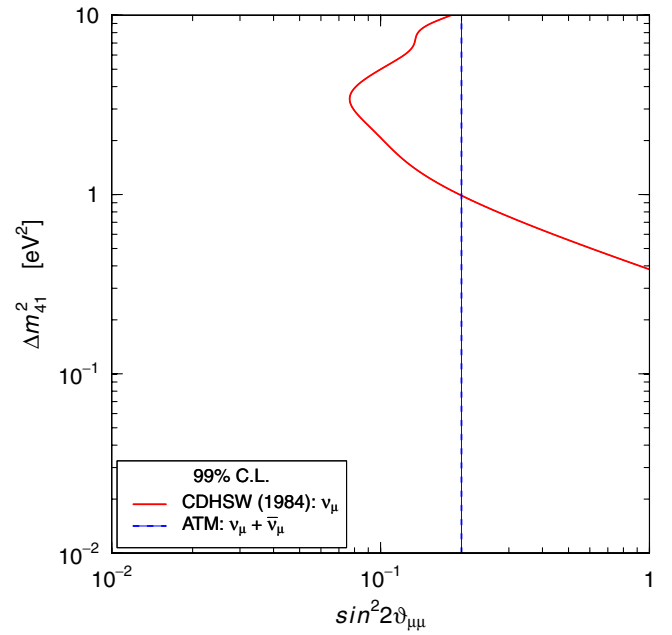


FIG. 3 (color online). Exclusion curves obtained from the data of the CDHSW ν_μ disappearance experiment [37], and from atmospheric neutrino data (extracted from the analysis in Ref. [38]).

$$\sin^2 2\vartheta_{e\mu} \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}. \quad (10)$$

Therefore, if $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ are small, $\sin^2 2\vartheta_{e\mu}$ is quadratically suppressed. This is illustrated in Fig. 4, where one can see that the separate effects of the constraints on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ exclude only the large $\sin^2 2\vartheta_{e\mu}$ part of the region allowed by LSND and MiniBooNE antineutrino data, whereas most of this region is excluded by the combined constraint in Eq. (10). As shown in Fig. 5, the constraint becomes stronger by including the data of the KARMEN [41], NOMAD [42] and MiniBooNE neutrino [27] experiments, which did not observe a short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal. Since the parameter goodness of fit [43] is 6×10^{-6} , 3 + 1 schemes are disfavored by the data. This conclusion has also been reached recently in Refs. [38,44–46] and confirms the pre-MiniBooNE results in Refs. [26,40,47,48].

However, in spite of the low value of the parameter goodness of fit it is not inconceivable to refuse to reject the 3 + 1 schemes for the following reasons:

- (1) It is the simplest scheme beyond the standard three-neutrino mixing which can partially explain the data.
- (2) It corresponds to the natural addition of one new entity (a sterile neutrino) to explain a new effect (short-baseline oscillations). Better fits of the data require the addition of at least another new entity.

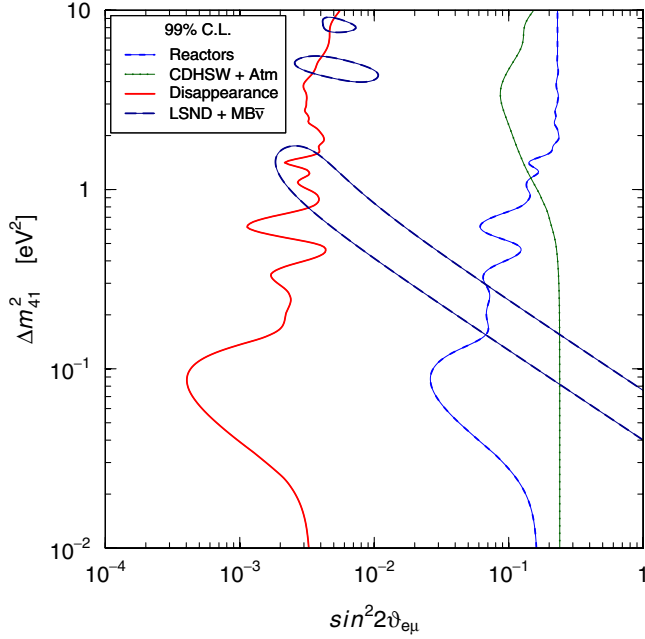


FIG. 4 (color online). Exclusion curves in the $\sin^2 2\vartheta_{e\mu}-\Delta m_{41}^2$ plane obtained from the separate constraints in Figs. 2 and 3 ([blue] short-dashed line and [green] dotted line) and the combined constraint given by Eq. (10) ([red] solid line) from disappearance experiments. The regions allowed by LSND and MiniBooNE antineutrino data are delimited by [dark-blue] long-dashed curves.

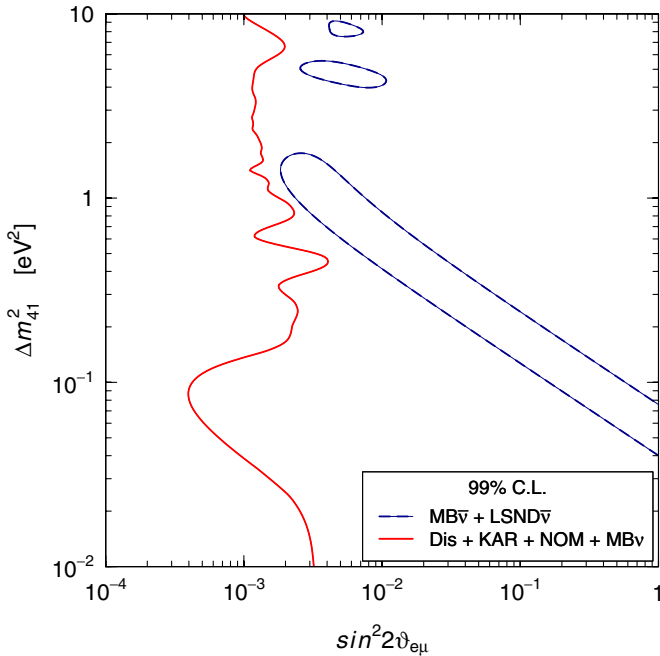


FIG. 5 (color online). Exclusion curve in the $\sin^2 2\vartheta_{e\mu}-\Delta m_{41}^2$ plane obtained with the addition to the disappearance constraint in Fig. 4 of the constraints obtained from KARMEN [41] (KAR), NOMAD [42] (NOM) and MiniBooNE neutrino [27] ($MB\nu$) data ([red] solid line). The regions allowed by LSND and MiniBooNE antineutrino data are delimited by [dark-blue] long-dashed lines.

TABLE I. Values of χ^2 , number of degrees of freedom (NDF), goodness-of-fit (GoF) and best-fit values of the mixing parameters obtained in our 3 + 1 and 3 + 2 fits of short-baseline oscillation data. The last three rows give the results of the parameter-goodness-of-fit test [43]: $\Delta\chi_{\text{PG}}^2$, number of degrees of freedom (NDF_{PG}) and parameter goodness of fit (PGoF).

	3 + 1	3 + 2
χ_{min}^2	100.2	91.6
NDF	104	100
GoF	59%	71%
Δm_{41}^2 [eV ²]	0.89	0.90
$ U_{e4} ^2$	0.025	0.017
$ U_{\mu 4} ^2$	0.023	0.019
Δm_{51}^2 [eV ²]		1.61
$ U_{e5} ^2$		0.017
$ U_{\mu 5} ^2$		0.0061
η		1.51π
$\Delta\chi_{\text{PG}}^2$	24.1	22.2
NDF_{PG}	2	5
PGoF	6×10^{-6}	5×10^{-4}

(In any case, at least one sterile neutrino is needed to generate short-baseline oscillations.)

- (3) The minimum value of the global χ^2 is rather good: $\chi_{\text{min}}^2 = 100.2$ for 104 degrees of freedom.

Note also that 3 + 1 schemes are favored with respect to 3 + 2 schemes by the big-bang nucleosynthesis limit $N_{\text{eff}} \leq 4$ at 95% C.L. obtained in Ref. [20]. Therefore, we consider the global fit of all data in 3 + 1 schemes, which yields the best-fit values of the oscillation parameters listed in Table I.

Figures 6 and 7 show the allowed regions in the $\sin^2 2\vartheta_{e\mu}-\Delta m_{41}^2$, $\sin^2 2\vartheta_{ee}-\Delta m_{41}^2$ and $\sin^2 2\vartheta_{\mu\mu}-\Delta m_{41}^2$ planes and the marginal $\Delta\chi^2$'s for Δm_{41}^2 , $\sin^2 2\vartheta_{e\mu}$, $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$. The best-fit values of the oscillation amplitudes are

$$\sin^2 2\vartheta_{e\mu} = 0.0023, \quad (11)$$

$$\sin^2 2\vartheta_{ee} = 0.098, \quad (12)$$

$$\sin^2 2\vartheta_{\mu\mu} = 0.091. \quad (13)$$

From Fig. 6 one can see that the allowed regions are compatible with those allowed by appearance data (the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ data of the LSND [2], KARMEN [41] and MiniBooNE [1] experiments and the $\nu_\mu \rightarrow \nu_e$ data of the NOMAD [42] and MiniBooNE [27] experiments) and are slightly pushed toward the left by the disappearance constraints. Future experiments aimed at checking the LSND

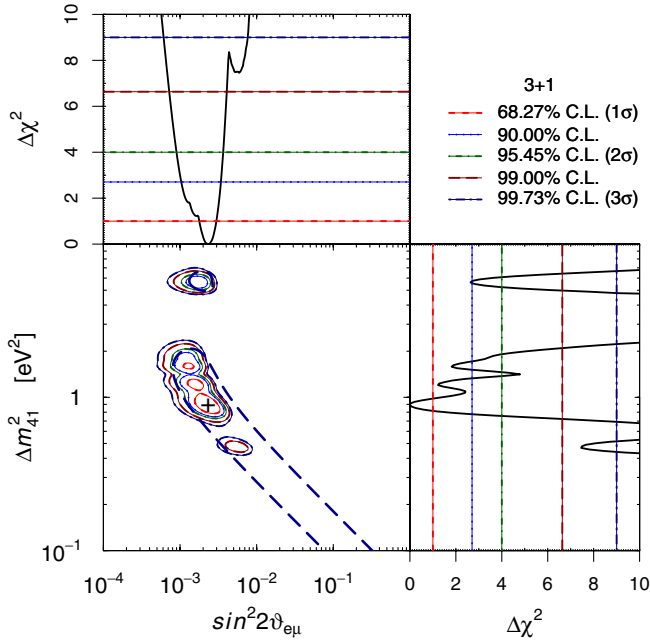


FIG. 6 (color online). Allowed regions in the $\sin^2 2\vartheta_{e\mu}-\Delta m_{41}^2$ plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta_{e\mu}$ and Δm_{41}^2 obtained from the global fit of all the considered data in 3 + 1 schemes. The best-fit point corresponding to χ_{\min}^2 is indicated by a cross. The isolated [dark-blue] dash-dotted contours enclose the regions allowed at 3 σ by the analysis of appearance data (the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ data of the LSND [2], KARMEN [41] and MiniBooNE [1] experiments and the $\nu_\mu \rightarrow \nu_e$ data of the NOMAD [42] and MiniBooNE [27] experiments).

[2] and MiniBooNE [1] $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation signal (such as those in Refs. [49–51]) should aim at exploring these regions.

Figure 7 shows that the allowed regions in the $\sin^2 2\vartheta_{ee}-\Delta m_{41}^2$ and $\sin^2 2\vartheta_{\mu\mu}-\Delta m_{41}^2$ planes lie just on the left of the disappearance constraints, as expected. From the left panel in Fig. 7, one can see that the allowed regions in the $\sin^2 2\vartheta_{ee}-\Delta m_{41}^2$ plane are compatible with the area indicated by the gallium anomaly [12]. The allowed region around the best-fit point and the isolated region at $\Delta m_{41}^2 \simeq 6$ eV 2 are also compatible with the recent results in Ref. [52]. If the 3 + 1 neutrino-mixing scheme is realized in nature, future experiments searching for short-baseline $\bar{\nu}_e$ disappearance (such as those in Refs. [11,49,51,53–58]) should find a disappearance compatible with the reactor antineutrino anomaly in Eq. (7). Future experiments searching for short-baseline $\bar{\nu}_\mu$ disappearance (such as those in Refs. [49,59]) should find a disappearance just below the current bound, for $0.4 \lesssim \Delta m_{41}^2 \lesssim 7$ eV 2 . Short-baseline $\bar{\nu}_e$ and $\bar{\nu}_\mu$ disappearance can also have observable effects, respectively, in solar neutrino experiments [60,61], long-baseline neutrino os-

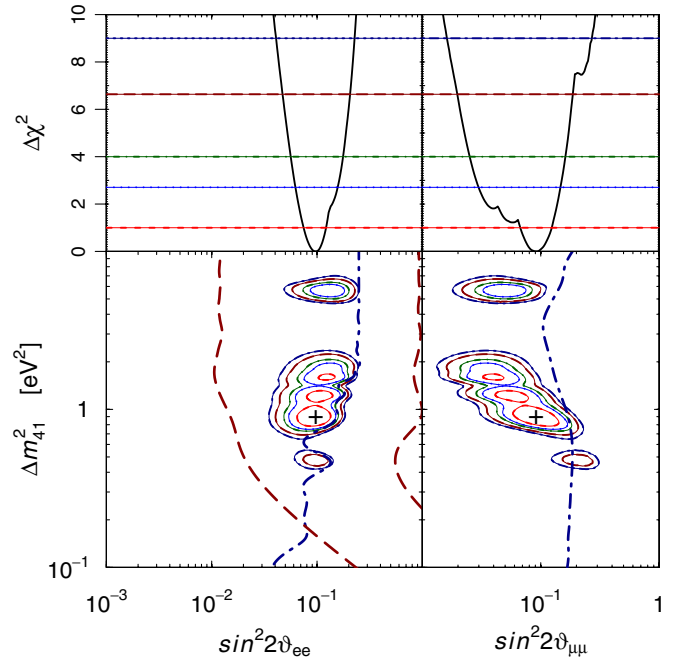


FIG. 7 (color online). Allowed regions in the $\sin^2 2\vartheta_{ee}-\Delta m_{41}^2$ and $\sin^2 2\vartheta_{\mu\mu}-\Delta m_{41}^2$ planes and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ obtained from the global fit of all the considered data in 3 + 1 schemes. The best-fit point corresponding to χ_{\min}^2 is indicated by a cross in both panels. The line types [and colors] have the same meanings as in Fig. 6. The isolated [dark-blue] dash-dotted lines are the 3 σ exclusion curves obtained from reactor neutrino data in the left plot (corresponding to the [blue] dashed line in Fig. 4) and from CDHSW and atmospheric neutrino data in the right plot (corresponding to the green dotted line in Fig. 4). The isolated [dark-red] long-dashed lines delimit the region allowed at 99% C.L. by the gallium anomaly [12].

cillation experiments [62,63] and atmospheric neutrino experiments [64–67].

III. 3 + 2 NEUTRINO MIXING

The CP -violating difference between MiniBooNE neutrino and antineutrino data can be explained by introducing another physical effect in addition to a sterile neutrino: a second sterile neutrino in 3 + 2 schemes [38,44,46,68–70], nonstandard interactions [44] and CPT violation [45,71]. In this section we discuss the possibility of 3 + 2 neutrino mixing according to the possible schemes illustrated schematically in Fig. 8. The inverted and perverted schemes have been called, respectively, 2 + 3 and 1 + 3 + 1 in Ref. [72]. Since the inverted and perverted schemes have three or four massive neutrinos at the eV scale, they are disfavored by cosmological data [21,22] over the normal scheme. Note also that all 3 + 2 schemes are disfavored by the big-bang nucleosynthesis limit $N_{\text{eff}} \leq 4$ at 95% C.L. obtained in Ref. [20].

In 3 + 2 schemes the relevant effective oscillation probabilities in short-baseline experiments are given by

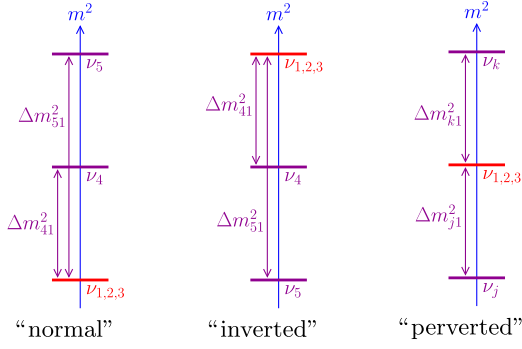


FIG. 8 (color online). Schematic description of the three possible 3 + 2 schemes that we are considering, taking into account that $|\Delta m_{21}^2| \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2| < |\Delta m_{51}^2|$. In the perverted scheme, the identification of the labels k and j is chosen in order to satisfy the inequality $|\Delta m_{41}^2| < |\Delta m_{51}^2|$.

$$P_{\nu_{\mu}^{(-)} \rightarrow \nu_e^{(-)}}^{\text{SBL}} = 4|U_{\mu 4}|^2|U_{e 4}|^2 \sin^2 \phi_{41} + 4|U_{\mu 5}|^2|U_{e 5}|^2 \sin^2 \phi_{51} + 8|U_{\mu 4} U_{e 4} U_{\mu 5} U_{e 5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54}^{(+)} \eta), \quad (14)$$

$$P_{\nu_{\alpha}^{(-)} \rightarrow \nu_{\alpha}^{(-)}}^{\text{SBL}} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \phi_{41} + |U_{\alpha 5}|^2 \sin^2 \phi_{51}) - 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2 \phi_{54}, \quad (15)$$

for $\alpha, \beta = e, \mu$, with

$$\phi_{kj} = \Delta m_{kj}^2 L / 4E, \quad \eta = \arg[U_{e 4}^* U_{\mu 4} U_{e 5} U_{\mu 5}^*]. \quad (16)$$

Note the change in sign of the contribution of the CP -violating phase η going from neutrinos to antineutrinos, which allows us to explain the CP -violating difference between the MiniBooNE neutrino and antineutrino data. In our analysis we consider $\Delta m_{41}^2 > 0$ and $\Delta m_{51}^2 > 0$, with $\Delta m_{51}^2 > \Delta m_{41}^2$, which implies $\Delta m_{54}^2 > 0$. These assumptions correspond to the normal scheme in Fig. 8, which is favored by cosmological data, as noted above. In any case, the results of our analysis can also be applied to the inverted scheme ($\Delta m_{41}^2 < 0$, $\Delta m_{51}^2 < 0$, $\Delta m_{54}^2 < 0$) with the change $\eta \rightarrow 2\pi - \eta$. Instead the perverted schemes, which have been considered in the fit of Ref. [46], require a separate treatment because in these schemes $|\Delta m_{54}^2| = |\Delta m_{31}^2| + |\Delta m_{41}^2|$. For simplicity we do not consider them here, because they are strongly disfavored by cosmological data, having four massive neutrinos at the eV scale.

Figures 9–12 show the marginal allowed regions in two-dimensional planes of interesting combinations of the oscillation parameters and the corresponding marginal $\Delta\chi^2$'s obtained in our 3 + 2 global fit of the same set of data used in Fig. 5. The best-fit values of the mixing parameters are shown in Table I.

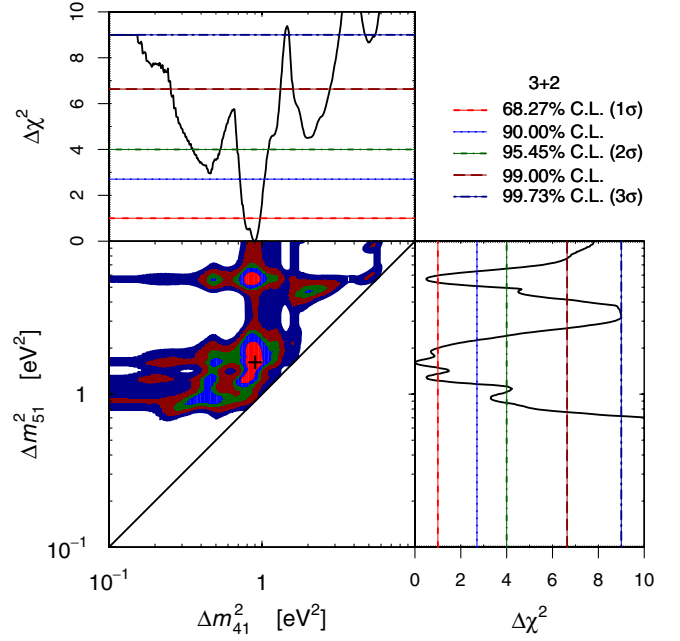


FIG. 9 (color online). Allowed regions in the $\Delta m_{41}^2 - \Delta m_{51}^2$ plane and corresponding marginal $\Delta\chi^2$'s obtained from the global fit of all the considered data in 3 + 2 schemes. The best-fit point corresponding to χ_{\min}^2 is indicated by a cross.

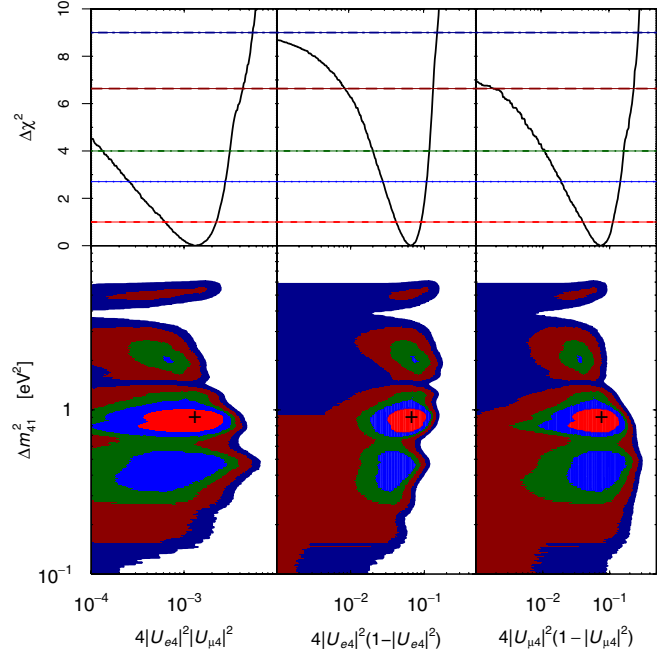


FIG. 10 (color online). Allowed regions in the $4|U_{e 4}|^2|U_{\mu 4}|^2 - \Delta m_{41}^2$, $4|U_{e 4}|^2(1 - |U_{e 4}|^2) - \Delta m_{41}^2$ and $4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2) - \Delta m_{41}^2$ planes and marginal $\Delta\chi^2$'s for $4|U_{e 4}|^2|U_{\mu 4}|^2$, $4|U_{e 4}|^2(1 - |U_{e 4}|^2)$ and $4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$ obtained from the global fit of all the considered data in 3 + 2 schemes. The line types [and colors] have the same meanings as in Fig. 9. The best-fit point corresponding to χ_{\min}^2 is indicated by a cross in each panel.

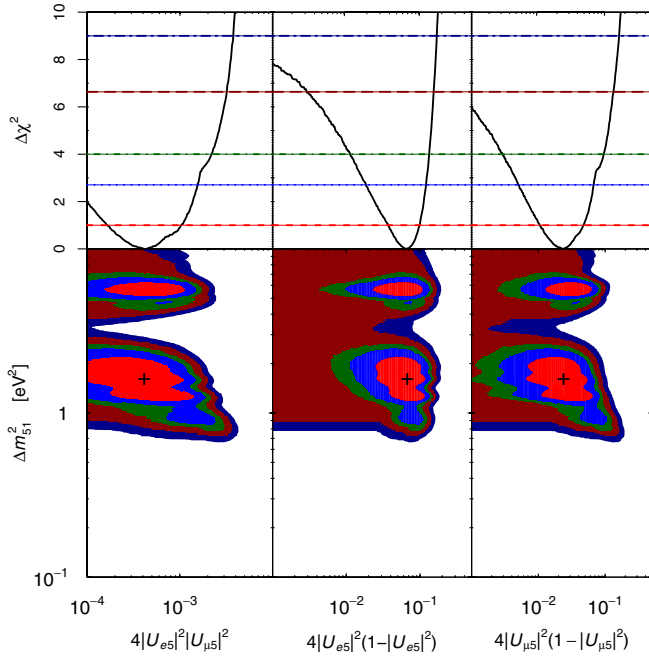


FIG. 11 (color online). Allowed regions in the $4|U_{e5}|^2 |U_{\mu5}|^2 - \Delta m_{51}^2$, $4|U_{e5}|^2(1 - |U_{e5}|^2) - \Delta m_{51}^2$ and $4|U_{\mu5}|^2(1 - |U_{\mu5}|^2) - \Delta m_{51}^2$ planes and marginal $\Delta\chi^2$'s for $4|U_{e5}|^2 |U_{\mu5}|^2$, $4|U_{e5}|^2(1 - |U_{e5}|^2)$ and $4|U_{\mu5}|^2(1 - |U_{\mu5}|^2)$ obtained from the global fit of all the considered data in 3 + 2 schemes. The line types [and colors] have the same meanings as in Fig. 9. The best-fit point corresponding to χ^2_{\min} is indicated by a cross in each panel.

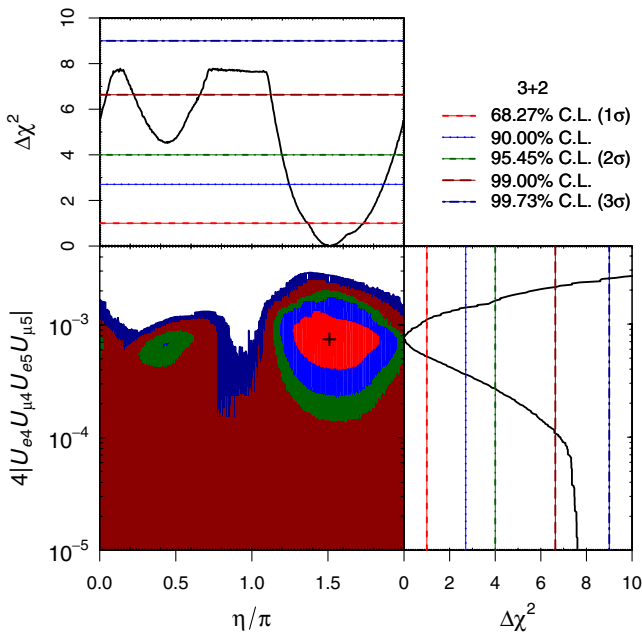


FIG. 12 (color online). Allowed regions in the $\eta - 4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|$ plane and corresponding marginal $\Delta\chi^2$'s obtained from the global fit of all the considered data in 3 + 2 schemes. The best-fit point corresponding to χ^2_{\min} is indicated by a cross.

The correlation of the allowed regions of η and $4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|$ in Fig. 12 is due to their presence in the last term in the effective $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probability in Eq. (14). The marginal $\Delta\chi^2$ for η has two minima close to the two values where CP -violation is maximal ($\eta = \pi/2$ and $\eta = 3\pi/2$), in agreement with what we expected from the need to fit the positive $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal and negative $\nu_\mu \rightarrow \nu_e$ measurement in the MiniBooNE experiment in the same range of L/E . From Fig. 12 one can also see that the marginal $\Delta\chi^2$ for η is always smaller than the $\Delta\chi^2 \simeq 7.8$ corresponding to a negligibly small value of $4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|$. (This value is reached for $\eta \simeq 0.1\pi$ and around $\eta = \pi$.) Such a $\Delta\chi^2$ is smaller than the difference of the χ^2 minima in the 3 + 1 and 3 + 2 schemes because the condition for $4|U_{e4}U_{\mu4}U_{e5}U_{\mu5}|$ to vanish requires that only one of U_{e5} and $U_{\mu5}$ vanishes. In particular, if only $U_{\mu5}$ is practically negligible, the reactor antineutrino data can be fitted slightly better than in 3 + 1 schemes, as already noted in Ref. [46].

The parameter goodness of fit obtained with the comparison of the fit of LSND and MiniBooNE antineutrino data and the fit of all other data is 5×10^{-4} . This is an improvement with respect to the 6×10^{-6} parameter goodness of fit obtained in 3 + 1 schemes. However, the value of the parameter goodness of fit remains low and the improvement is due mainly to the increased number of degrees of freedom, as one can see from Table I. The persistence of a bad parameter goodness of fit is a consequence of the fact that the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions observed in the LSND and MiniBooNE experiments must correspond in any neutrino-mixing schemes to enough short-baseline disappearance of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ which has not been observed and there is an irreducible tension between the LSND and MiniBooNE antineutrino data and the KARMEN antineutrino data. The only benefit of 3 + 2 schemes with respect to 3 + 1 schemes is that they allow us to explain the difference between MiniBooNE neutrino and antineutrino data through CP violation. In fact, neglecting the MiniBooNE neutrino data, we obtain $\Delta\chi^2_{\text{PG}} = 16.6$ with $\text{PGoF} = 3 \times 10^{-4}$ in 3 + 1 schemes and $\Delta\chi^2_{\text{PG}} = 20.4$ with $\text{PGoF} = 1 \times 10^{-3}$ in 3 + 2 schemes. In this case, $\Delta\chi^2_{\text{PG}}$ is even lower in 3 + 1 schemes than in 3 + 2 schemes.

The tension between the LSND and MiniBooNE antineutrino data and the disappearance, KARMEN, NOMAD and MiniBooNE neutrino data is illustrated in Fig. 13, which is the analogue for 3 + 2 schemes of Fig. 5 in 3 + 1 schemes. In practice, in order to show the tension in a two-dimensional figure, we have marginalized the χ^2 over all the other mixing parameters, including the two Δm^2 's.

The results of our 3 + 2 global fit are in reasonable agreement with those presented in Ref. [46]. There is a discrepancy in the location of the best-fit point in the $\Delta m_{41}^2 - \Delta m_{51}^2$ plane, but we obtain similar regions for the

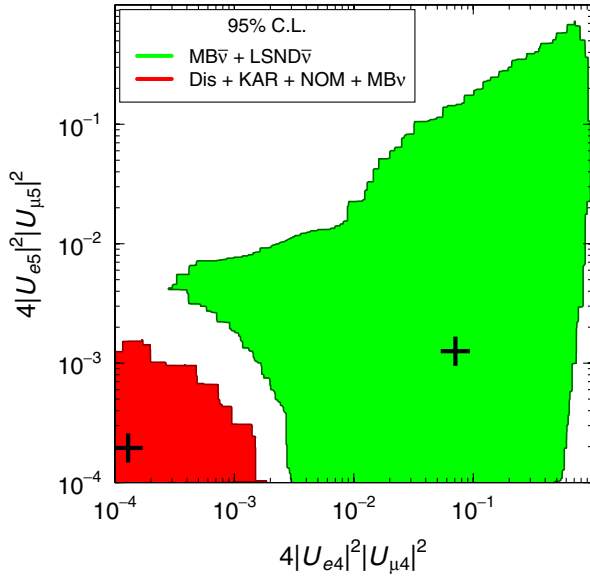


FIG. 13 (color online). Comparison of the 95% C.L. allowed regions in the $4|U_{e4}|^2|U_{\mu4}|^2$ – $4|U_{e5}|^2|U_{\mu5}|^2$ plane obtained from the LSND and MiniBooNE antineutrino data on the right ([green] lighter shaded area) and the disappearance, KARMEN, NOMAD and MiniBooNE neutrino data on the left ([red] darker shaded area). The corresponding best-fit points are indicated by crosses.

local χ^2 minima. Our allowed regions are larger than those presented in Ref. [46]. We think that such difference is probably due to a different treatment of the spectral data of the Bugey-3 reactor experiment [29] which cause the wiggling for $\Delta m^2 \lesssim 1 \text{ eV}^2$ of the disappearance limit in Fig. 4 and the exclusion curve in Fig. 5. Such wiggling is wider in Fig. 3 of Ref. [46], leading to deeper valleys of the χ^2 function and smaller allowed regions. The compatibility with cosmological data of the allowed regions in the Δm_{41}^2 – Δm_{51}^2 plane shown in Fig. 9 will be discussed in a separate article [73]. (An interesting study was presented previously in Ref. [74].)

Figures 10 and 11 show the allowed regions for the amplitudes of the oscillating terms in short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions and $\bar{\nu}_e$ and $\bar{\nu}_\mu$ disappearance, for which we have the following best-fit values:

$$4|U_{e4}|^2|U_{\mu4}|^2 = 0.0013, \quad (17)$$

$$4|U_{e5}|^2|U_{\mu5}|^2 = 0.00042, \quad (18)$$

$$4|U_{e4}|^2(1 - |U_{e4}|^2) = 0.068, \quad (19)$$

$$4|U_{e5}|^2(1 - |U_{e5}|^2) = 0.068, \quad (20)$$

$$4|U_{\mu4}|^2(1 - |U_{\mu4}|^2) = 0.076, \quad (21)$$

$$4|U_{\mu5}|^2(1 - |U_{\mu5}|^2) = 0.024. \quad (22)$$

Comparing the values of $4|U_{e4}|^2|U_{\mu4}|^2$, $4|U_{e4}|^2(1 - |U_{e4}|^2)$ and $4|U_{\mu4}|^2(1 - |U_{\mu4}|^2)$ with those obtained in $3 + 1$ mixing given in Eqs. (11)–(13), one can see that these are lower but keep the same order of magnitude. In the fit of the data, the smaller values of these amplitudes are due to the additional contribution of the amplitudes generated by the mixing of ν_e and ν_μ with ν_5 .

IV. CONCLUSIONS

In this paper we have presented the results of fits of short-baseline neutrino oscillation data in $3 + 1$ and $3 + 2$ neutrino-mixing schemes.

In the framework of $3 + 1$ neutrino-mixing schemes, in Fig. 1, we confirm the strong tension between the LSND and MiniBooNE antineutrino data and the disappearance, KARMEN, NOMAD and MiniBooNE neutrino data discussed recently in Refs. [38,44–46]. Since the minimum value of the global χ^2 is rather good, however, one may choose to consider as possible $3 + 1$ neutrino mixing, which can partially explain the data, taking into account its simplicity and the natural correspondence of one new entity (a sterile neutrino) with a new effect (short-baseline oscillations). Following this approach, we presented the results of the global fit in $3 + 1$ neutrino mixing, which leads to the determination of restricted allowed regions in the mixing parameter space which can be explored in future $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ [49–51], $\bar{\nu}_e$ disappearance [11,49,51,53–58] and $\bar{\nu}_\mu$ disappearance [49,59] experiments.

We have also presented a global fit in the framework of the $3 + 2$ neutrino-mixing schemes in Fig. 8. We have shown that the tension between the LSND and MiniBooNE antineutrino data and the disappearance, KARMEN, NOMAD and MiniBooNE neutrino data is reduced with respect to the $3 + 1$ fit, but is not eliminated (see Fig. 13). Moreover, the improvement of the parameter goodness of fit with respect to that obtained in the $3 + 1$ fit is mainly due to the increase of the number of oscillation parameters, as one can see from Table I. Hence, it seems to be mainly a statistical effect.

The results of our $3 + 2$ fit are compatible with those presented recently in Ref. [46], but we obtain a different indication for the best fit (see Table I). For the CP -violating phase, we obtained two minima of the marginal χ^2 close to the two values where CP violation is maximal.

In conclusion, we think that our results are useful for the discussion and interpretation of current experimental indications in favor of short-baseline neutrino oscillations and for the study of new experiments aimed at a clarification of the validity of these indications.

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