PHYSICAL REVIEW D **84,** 071303(R) (2011)

Magnetic moments of the ground-state $J^P = \frac{3}{2}^+$ baryon decuplet

Milton Dean Slaughter*

Department of Physics, Florida International University, Miami, Florida 33199, USA

(Received 15 September 2011; published 20 October 2011)

The magnetic moment—a function of the electric charge form factor $F_1(q^2)$ and the magnetic dipole form factor $F_2(q^2)$ at zero four-momentum transfer q^2—of the ground-state $J^P = \frac{3}{2}^+$ baryon decuplet magnetic moments have been studied for many years with limited success. At present, only the magnetic moment of the Ω^- has been accurately determined. We calculate nonperturbatively the magnetic moments of the *physical baryon decuplet* $J^P = \frac{3}{2}^+$ members and, in particular, we obtain $\mu_{\Delta^{++}} = (+3.67 \pm 0.07)\mu_N$, $\mu_{\Delta^+} = (+1.83 \pm 0.04)\mu_N$, $\mu_{\Delta^0} = (0)\mu_N$, and the magnetic moments of their U-Spin partners in terms of Ω^- magnetic moment data.

DOI: 10.1103/PhysRevD.84.071303 PACS numbers: 13.40.Em, 12.38.Lg, 13.40.Gp, 14.20.Jn

I. INTRODUCTION

The properties of the ground-state $J^P = \frac{3}{2}^+$ baryon decuplet magnetic moments Δ, Ξ^*, Σ^* and Ω^- have been studied for many years with limited success. Although the masses (pole or otherwise) and decay aspects and other physical observables of some of these particles have been ascertained, the magnetic moments of many are yet to be determined. From the Particle Data Group [1], only the magnetic moment of the Ω^- [2] has been accurately determined. The magnetic moment is a function of the electric charge form factor $F_1(q^2)$ and the magnetic dipole form factor $F_2(q^2)$ at zero four-momentum transfer $q^2 \equiv -Q^2$. The lack of experimental data for the decuplet particle members is associated with their very short lifetimes (many available strong interaction decay channels) and the existence of nearby particles with quantum numbers that allow for configuration mixing greatly increasing the difficulty of experimental determination of physical observables. The Ω^- (strangeness $S = -3$) is an exception in that it is composed of three valence s quarks that make its lifetime substantially longer (weak interaction decay) than any of its decuplet partners. However, even for the Ω^-, away from the static ($q^2 = 0$) limit, the electric charge and magnetic dipole form factors are not known. Theoretical models abound: Beg *et al.* [3] and Gerasimov [4], and Lichtenberg [5] provide excellent sources of methodological information.

In Ref. [6], we illustrated how one may calculate the magnetic moments of the *physical decuplet* U-spin $= \frac{3}{2}$ *quartet members* (the Δ^-, Σ^{*-}, and Ξ^{*-}) in terms of that of the Ω^- (U-spin $= \frac{3}{2}$ as well) without ascribing any specific form to their quark structure or intraquark interactions [6–11]. Theoretical and computational investigations and reviews involving the magnetic moments of the Ω^- and the Δ^- and lattice quantum chromodynamics (LQCD) (quenched and unquenched, unphysical pion mass) techniques are also available [12–16].

In this article all equal-time commutation relations (ETCRs) involve at most one current density, thus, problems associated with Schwinger terms are avoided. ETCRs involve the vector and axial-vector charge generators (the V_α and $A_\alpha\{\alpha = \pi, K, D, F, B, \ldots\}$) of the symmetry groups of QCD. They are valid even though these symmetries are broken [7–10,17–20] and *even when the Lagrangian is not known or cannot be constructed*.

The electromagnetic current $j^\mu_{em}(0)$ obeys the *double ETCRs* $[[j^\mu_{em}(0), V_{\pi^+}], V_{\pi^-}] = [[j^\mu_{em}(0), A_{\pi^+}], A_{\pi^-}] = 2j^\mu_{em3}(0)$ and $[[j^\mu_{em}(0), V_{\pi^+}], V_{\pi^-}] = [[j^\mu_{em3}(0), V_{\pi^+}], V_{\pi^-}]$ [11]—V_{π^+} and V_{π^-} are vector charge generators, A_{π^+} and A_{π^-} are axial-vector charge generators, and $j^\mu_{em3}(0)$ is the isovector part of $j^\mu_{em}(0)$—even in the presence of symmetry breaking. The double ETCRs, in addition to ETCRs involving axial-vector charges [18–20], allow us to relate form factors—$F_1(q^2)$ and $F_2(q^2)$ where U-Spin *is not restricted to* $\frac{3}{2}$—associated with the U-spin $= \frac{3}{2}$ Δ^- (and hence the Σ^{*-} and Ξ^{*-}, and the Ω^-) with those associated with decuplet members having U-spin $= 1$ (the Δ^0, Σ^{*0}, and Ξ^{*0}), U-spin $= \frac{1}{2}$ (the Δ^+, and Σ^{*+}), and U-spin $= 0$ (the Δ^{++}).

In the infinite-momentum frame broken symmetry is characterized by the existence of physical on-mass-shell hadron annihilation operators $a_\alpha(\vec{k}, \lambda)$ (momentum $\vec{k}(|\vec{k}| \to \infty)$, helicity λ, and $SU_F(N)$ flavor index α) and their creation operator counterparts which produce physical states when acting on the vacuum. Indeed, the physical on-mass-shell hadron annihilation operator $a_\alpha(\vec{k}, \lambda)$ is related linearly under flavor transformations to the *representation* annihilation operator $a_j(\vec{k}, \lambda)$. Thus, in the infinite-momentum frame, physical states denoted by $|\alpha, \vec{k}, \lambda\rangle$ (which do not belong to irreducible representations) are linear combinations of representation states denoted by $|j, \vec{k}, \lambda\rangle$ (which do belong to irreducible representations) plus nonlinear corrective terms that are best calculated in a frame where mass differences are de-emphasized such as in the infinite-momentum frame.

*slaughtm@FIU.Edu, Slaughts@PhysicsResearch.Net

Mathematically [7–10], this is expressed by: $|\alpha, \vec{k}, \lambda\rangle = \sum_j C_{\alpha j}|j, \vec{k}, \lambda\rangle$, $|\vec{k}| \to \infty$, where the orthogonal matrix $C_{\alpha j}$ depends on physical $SU_F(N)$ mixing parameters, is defined only in the ∞-momentum frame, and can be constrained directly by ETCRs.

The particular Lorentz frame that one might utilize when analyzing current-algebraic sum rules does not matter when flavor symmetry is exact and is strictly a matter of taste and calculational convenience, whereas when one uses current-algebraic sum rules in broken symmetry, the choice of frame is paramount since one wishes to emphasize the calculation of leading order contributions while simultaneously simplifying the calculation of symmetry breaking corrections [6–11,17].

II. ETCRS IN THE INFINITE-MOMENTUM FRAME AND FLAVOR BROKEN SYMMETRY

The physical vector charge V_{K^0} is $V_{K^0} = V_6 + iV_7$, the physical vector charge $V_{\pi^\pm} = V_1 \pm iV_2$. The λ_a, $a = 1, 2, \cdots, 35$ satisfy the Lie algebra $[(\lambda_a/2), (\lambda_b/2)] = i\sum_c f_{abc}(\lambda_c/2)$, where the f_{abc} are structure constants of the flavor group $SU_F(6)$ and $V_a^\mu(x) = \bar{q}^i(x)(\lambda_a/2)_{ij}\times \gamma^\mu q^j(x)$. The physical electromagnetic current $j_{em}^\mu(0)$ may be written (u, d, s, c, b, t quark system) as $j_{em}^\mu(0) = V_3^\mu(0) + (1/3)^{1/2}V_8^\mu(0) - (2/3)^{1/2}V_{15}^\mu(0) + (2/5)^{1/2}\times V_{24}^\mu(0) - (3/5)^{1/2}V_{35}^\mu(0) + (1/3)^{1/2}(\text{singlet current}) = j_V^\mu(0) + j_S^\mu(0)$, where $j_V^\mu(0) \equiv j_{em3}^\mu(0) =$ the isovector part of the electromagnetic current, $j_S^\mu(0) \equiv$ the isoscalar part of the electromagnetic current. The flavor $U(6)$ singlet current $V_0^\mu(x) = \bar{q}^i(x)(\lambda_0/2)_{ij}\gamma^\mu q^j(x)$ where $\lambda_0 \equiv \sqrt{1/3}I$, I is the identity, so that $\mathrm{Tr}(\lambda_a\lambda_b) = 2\delta_{ab}$ holds for all $\lambda_{a'}(a' = 0, 1, 2, \cdots, 35)$. The $U(6)$ singlet charge V_0 commutes with all of the V_a. One may verify that the ETCR $[V_{K^0}, j_{em}^\mu(0)] = 0$ and the double ETCRs [11] mentioned in the Introduction hold.

III. THE ELECTROMAGNETIC CURRENT MATRIX ELEMENT

For the on-mass shell $J^P = 3/2^+$ ground-state decuplet baryon B with mass m_B, the Lorentz-covariant and gauge-invariant electromagnetic current matrix element in momentum space with four-momentum vectors $P \equiv p_1 + p_2$, $q \equiv p_2 - p_1$ (λ_1 and λ_2 denote helicity) is given by:

$$\langle B(p_2, \lambda_2)|j_{em}^\mu(0)|B(p_1, \lambda_1)\rangle \equiv \frac{e}{(2\pi)^3}\sqrt{\frac{m_B^2}{E_B^t E_B^s}}\bar{u}_B^\alpha(p_2, \lambda_2)[\Gamma_{\alpha\beta}^\mu]u_B^\beta(p_1, \lambda_1), \quad (1)$$

$$\Gamma_{\alpha\beta}^\mu = g_{\alpha\beta}\left\{F_1^B(q^2)\gamma^\mu + \frac{F_2^B(q^2)i\sigma^{\mu\nu}}{2m_B}q_\nu\right\} + \frac{q_\alpha q_\beta}{2m_B^2}\left\{F_3^B(q^2)\gamma^\mu + \frac{F_4^B(q^2)i\sigma^{\mu\nu}}{2m_B}q_\nu\right\}, \quad (2)$$

where $e = +\sqrt{4\pi\alpha}$, $\alpha =$ the fine structure constant, the F_i^B are the four $\gamma^* BB$ form factors [$F_1^B(0) \sim$ electric charge in units of e, $(F_1^B(0) + F_2^B(0)) \sim$ magnetic dipole moment in units of $e/(2m_B)$] and $\Gamma_{\alpha\beta}^\mu$ is written in standard form [21]. The electric charge multipole amplitude $G_E^B(q^2) = [F_1^B(q^2)(3 - 2\eta) + \eta\{F_2^B(q^2)(3 - 2\eta) - 2(-1 + \eta)\times (F_3^B(q^2) + \eta F_4^B(q^2))\}]/3[\text{units of } e]$, the magnetic dipole multipole amplitude $G_M^B(q^2) = [(5 - 4\eta)(F_1^B(q^2) + F_2^B(q^2)) - 4\eta(-1 + \eta)(F_3^B(q^2) + F_4^B(q^2))]/5[\text{units of } e/(2m_B)]$, the electric quadrupole multipole amplitude $G_Q^B(q^2) = F_1^B(q^2) + F_3^B(q^2)(-1 + \eta) + \eta\{F_2^B(q^2) + F_4^B(q^2)(-1 + \eta)\}\times [\text{units of } e/m_B^2]$, and the magnetic octupole multipole amplitude $G_O^B(q^2) = [F_1^B(q^2) + F_2^B(q^2) + (-1 + \eta)\{F_3^B(q^2) + F_4^B(q^2)\}]\sqrt{6}[\text{units of } e/(2m_B^3)]$ where $\eta \equiv q^2/(4m_B^2)$. $Q_B =$ charge of decuplet baryon B in units of e, μ_B is the magnetic moment (measured in nuclear magneton units $\mu_N = e/(2m)$, $m =$ proton mass) of baryon B.

In Eq. (1), $u_B^\beta(\nu_B, \theta, \lambda)$ is a spin 3/2 baryon Rarita-Schwinger [22] spinor with helicity λ, three-momentum $\vec{p}$ with angle θ referred to the $\hat{z}$-axis, energy E_B^p, and velocity parameter $\nu_B = \sinh^{-1}(|\vec{p}|/m_B)$ [6].

Specifically:

$$u_B^\beta(\nu_B, \theta, \lambda) = \sum_{m_1=-(1/2)}^{+(1/2)}\sum_{m_2=-1}^{+1}\langle 1/2, 1, 3/2|m_1, m_2, \lambda\rangle \times u_B(\nu_B, \theta, m_1)\epsilon_B^\beta(\nu_B, \theta, m_2), \quad (3)$$

$$u_B(\nu_B, \theta, m_1) = \begin{pmatrix} \cosh\left(\frac{\nu_B}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)\delta_{m_1,(1/2)} - \sin\left(\frac{\theta}{2}\right)\delta_{m_1,-(1/2)}\right] \\ \cosh\left(\frac{\nu_B}{2}\right)\left[\sin\left(\frac{\theta}{2}\right)\delta_{m_1,(1/2)} + \cos\left(\frac{\theta}{2}\right)\delta_{m_1,-(1/2)}\right] \\ \sinh\left(\frac{\nu_B}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)\delta_{m_1,(1/2)} + \sin\left(\frac{\theta}{2}\right)\delta_{m_1,-(1/2)}\right] \\ \sinh\left(\frac{\nu_B}{2}\right)\left[\sin\left(\frac{\theta}{2}\right)\delta_{m_1,(1/2)} - \cos\left(\frac{\theta}{2}\right)\delta_{m_1,-(1/2)}\right] \end{pmatrix}, \quad (4)$$

$$\epsilon_B^\beta(\nu_B, \theta, m_2) = \begin{pmatrix} \sinh(\nu_B)\delta_{m_2,0} \\ -\frac{m_2}{\sqrt{2}}\cos(\theta)\delta_{|m_2|,1} + \cosh(\nu_B)\sin(\theta)\delta_{m_2,0} \\ -\frac{i}{\sqrt{2}}\delta_{|m_2|,1} \\ \frac{m_2}{\sqrt{2}}\sin(\theta)\delta_{|m_2|,1} + \cosh(\nu_B)\cos(\theta)\delta_{m_2,0} \end{pmatrix}. \quad (5)$$

$\epsilon_B^\beta(\nu_B, \theta, m_2)$ is the baryon polarization (m_2) four-vector where $\epsilon_B^{\beta^*}(\nu_B, \theta, m')\epsilon_{B\beta}(\nu_B, \theta, m) = -\delta_{m'm}$. $u_B(\nu_B, \theta, m_1)$ is a Dirac spinor with helicity index m_1, and $\langle 1/2, 1, 3/2|m_1, m_2, \lambda\rangle$ is a Clebsh-Gordan coefficient where our conventions are those of Rose [23].

Physical states are normalized with $\langle\vec{p}'|\vec{p}\rangle = \delta^3(\vec{p}' - \vec{p})$ and Dirac spinors are normalized by $\bar{u}^{(r)}(p)u^{(s)}(p) = \delta_{rs}$, Dirac matrices are $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ with $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, where $g^{\mu\nu} = \mathrm{Diag}(1, -1, -1, -1)$ [24]. In addition to obeying the Dirac equation, the Rarita-Schwinger spinors satisfy the subsidiary conditions $\gamma_\mu u_B^\mu(p, \lambda) = p_\mu u_B^\mu(p, \lambda) = 0$ and the normalization condition $\bar{u}_B^\alpha(p, \lambda')g_{\alpha\beta}u_B^\alpha(p, \lambda) = -\delta_{\lambda'\lambda}$. Associated with baryon B are the four-momentum vectors p_1 (three-momentum $\vec{t}(\vec{t} = t_z\hat{z})$, energy E_B^t) and p_2 (three-momentum $\vec{s}$ at angle $\theta(0 \le \theta < \pi/2)$ with the $\hat{z}$ axis, energy E_B^s, with $s_z = rt_z$ and r(constant) ≥ 1).

IV. *U*-SPIN 1, $\frac{1}{2}$, AND 0 DECUPLET BARYON MAGNETIC MOMENT RELATIONSHIPS

Previously [6,25] (***U*-spin $\frac{3}{2}$ *quartet only***), we investigated magnetic moment relationships by utilizing the commutator $[V_{K^0}, j_{em}^\mu(0)] = 0$ inserted between the baryon pairs $(\langle\Xi^{*-}s^\sigma|, |\Omega^-t^\sigma\rangle)$, $(\langle\Sigma^{*-}s^\sigma|, |\Xi^{*-}t^\sigma\rangle)$, and $(\langle\Delta^-s^\sigma|, |\Sigma^{*-}t^\sigma\rangle)$ where each baryon ($B = \Delta^-$, Σ^{*-}, Ξ^{*-}, or Ω^-) had $Q_B = -e$, helicity $+3/2$ and $t_z \to \infty$ and $s_z \to \infty$, and where

$$q_B^2 = -\frac{(1-r)^2}{r}m_B^2 - \frac{s_x^2}{r} \equiv -Q_B^2, \qquad q^2_{B|s_x=0} = -\frac{(1-r)^2}{r}m_B^2. \tag{6}$$

We found that:

$$F_2^B(q_B^2) = \frac{m_B^2}{m_{\Omega^-}^2}F_2^{\Omega^-}(q_{\Omega^-}^2), \tag{7}$$

$$F_1^B(q_B^2) = F_1^{\Omega^-}(q_{\Omega^-}^2). \tag{8}$$

Clearly, if one knows $F_1^{\Omega^-}(q_{\Omega^-}^2)$ for some range $0 \ge q_{\Omega^-}^2 \ge q_K^2$, then one knows the value of $r_K \ge r \ge 1$ and thus q_B^2 (from Eq. (6)) for this same range and hence one can infer $F_1^B(q_B^2)$ and $F_2^B(q_B^2)$ from Eqs. (7) and (8). We illustrate this in Fig. 1 where B is the Δ^- (or the Δ^+—see Eq. (11) below) and $F_1^{\Delta^-}(q_{\Delta^-}^2)$ is predicted using lattice calculations from Ref. [26] for the Ω^- electric charge form factor (dipole fit).

To obtain the magnetic moments of the U-Spin 1, $\frac{1}{2}$, and 0 decuplet baryons, *one must find a way to quantitatively connect* the decuplet U-Spin multiplets. We proceed to do this by first defining $\langle Bs^\sigma, 3/2|j_{em}^\mu(0)|Bt^\sigma, 3/2\rangle \equiv \langle B\rangle$, $\langle Bs^\sigma, 3/2|j_V^\mu(0)|Bt^\sigma, 3/2\rangle \equiv \langle B\rangle_3$, and $\langle Bs^\sigma, 3/2|j_S^\mu(0)|Bt^\sigma, 3/2\rangle \equiv \langle B\rangle_S$ so that $\langle B\rangle = \langle B\rangle_3 + \langle B\rangle_S$ where B is now any decuplet baryon). With that notation, in Ref. [6], we found that $\langle\Delta^-\rangle = \langle\Omega^-\rangle$. Second, we utilize the double ETCRs to relate the matrix elements $\langle\Delta^-\rangle$, $\langle\Delta^0\rangle$, $\langle\Delta^+\rangle$, and $\langle\Delta^{++}\rangle$ (a U-Spin singlet) to each other and to that of the Ω^-. We can—for example—use $[V_{K^0}, j_{em}^\mu(0)] = 0$ to obtain the magnetic moment of the Σ^{*+} from that of the Δ^+ (U-Spin doublet) and the magnetic moments of the Σ^{*0} and Ξ^{*0} from that of the Δ^0 (U-Spin triplet).

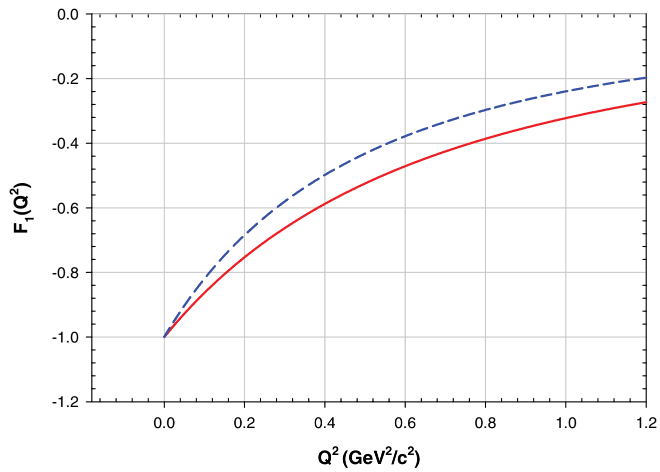

FIG. 1 (color online). The Solid curve is a dipole fit $[(-1) \times (1 + Q^2/\Lambda_{E_0}^2)^{-2}]$ with $\Lambda_{E_0} = 1.146$ GeV$/c$ to lattice calculations for the Ω^- electric charge multipole form factor G_{E0} taken from Table III of Ref. [26]. The Dashed curve is the Δ^- electric charge form factor calculated using Eq. (8) and (6) and the above Ω^- lattice dipole fit using $\Lambda_{E_0} = 1.146$ GeV$/c$ which is independent of Q^2. The $\Delta^+ F_1(Q^2)$ electric charge form factor as a function of Q^2 is just (-1) times that of the Dashed curve according to Eq. (11) and the assumption that $m_\Delta = 1.22 \pm 0.01$ GeV$/c^2$ for all Δ charge states.

The double ETCRs $[[j_{em}^\mu(0), V_{\pi^+}], V_{\pi^-}] = [[j_{em3}^\mu(0), V_{\pi^+}], V_{\pi^-}] = 2j_{em3}^\mu(0)$ [11] sandwiched between the pair states $\langle\Delta^{++}|, |\Delta^{++}\rangle$, $\langle\Delta^+|, |\Delta^+\rangle$, $\langle\Delta^0|, |\Delta^0\rangle$, and $\langle\Delta^-|, |\Delta^-\rangle$ can be used to determine the $SU(2)$ parametrization of $j_{em}^\mu(0)$ for the Δ states in the infinite-momentum frame. This produces six equations:

$$\langle\Delta^{++}\rangle = \langle\Delta^-\rangle - 2\langle\Delta^-\rangle_3, \qquad \langle\Delta^{++}\rangle_3 = -\langle\Delta^-\rangle_3 \tag{9a}$$

$$\langle\Delta^+\rangle = \langle\Delta^-\rangle - \frac{4}{3}\langle\Delta^-\rangle_3, \qquad 3\langle\Delta^+\rangle_3 = -\langle\Delta^-\rangle_3 \tag{9b}$$

$$\langle\Delta^0\rangle = \langle\Delta^-\rangle - \frac{2}{3}\langle\Delta^-\rangle_3, \qquad 3\langle\Delta^0\rangle_3 = \langle\Delta^-\rangle_3. \tag{9c}$$

Third, the axial-vector matrix elements (in the infinite-momentum frame) [11] $\langle\Delta^+, 3/2|A_{\pi^-}|\Delta^{++}, 3/2\rangle = \langle\Delta^-, 3/2|A_{\pi^-}|\Delta^0, 3/2\rangle \equiv -\sqrt{3/2}\tilde{g}$, and $\langle\Delta^0, 3/2|A_{\pi^-}|\Delta^+, 3/2\rangle = -\sqrt{2}\tilde{g}$ and the double ETCR $[[j_{em}^\mu(0), A_{\pi^+}], A_{\pi^-}] = 2j_{em3}^\mu(0)$ sandwiched between the same pair Δ states allow us to write the following four equations:

$$3\tilde{g}^2[\langle\Delta^-\rangle - \langle\Delta^0\rangle] = 4\langle\Delta^-\rangle_3, \tag{10a}$$

$$3\tilde{g}^2[7\langle\Delta^0\rangle - 3\langle\Delta^-\rangle - 4\langle\Delta^+\rangle] = 4\langle\Delta^-\rangle_3, \tag{10b}$$

$$3\tilde{g}^2[-7\langle\Delta^+\rangle + 4\langle\Delta^0\rangle + 3\langle\Delta^{++}\rangle] = 4\langle\Delta^-\rangle_3, \tag{10c}$$

$$3\tilde{g}^2[\langle\Delta^+\rangle - \langle\Delta^{++}\rangle] = 4\langle\Delta^-\rangle_3. \tag{10d}$$

Finally, Eqs. (10) in conjunction with Eqs. (9) imply in broken symmetry that:

TABLE I. ground-state baryon decuplet magnetic moment μ_B in units of μ_N.

Baryon B	This research [a]	Particle Data Group; [b]	Lattice QCD [c]
Δ^{++}	$+3.67 \pm 0.07$	$+5.6 \pm 1.9$	$+3.70 \pm 0.12$
Δ^{+}	$+1.83 \pm 0.04$	$+2.7 \pm 3.6$	$+2.40 \pm 0.06$
Δ^{0}	0 ± 0	-	$+0.001 \pm 0.016$
Δ^{-}	-1.83 ± 0.04	-	-1.85 ± 0.06
Σ^{*+}	$+1.89 \pm 0.04$	-	-
Σ^{*0}	0 ± 0	-	-
Σ^{*-}	-1.89 ± 0.04	-	-
Ξ^{*0}	0 ± 0	-	-
Ξ^{*-}	-1.95 ± 0.05	-	-
Ω^{-}	-2.02 ± 0.05	-2.02 ± 0.05	-1.93 ± 0.08

[a] μ_{Ω^-} is input. $m_\Delta = 1.22 \pm 0.01$ GeV$/c^2$ is assumed for all Δ charge states. μ_{Ω^-} and other baryon masses are from the Particle Data Group [1]. Statistical propagation of errors used in calculations.
[b] Δ^{++} estimate from Ref. [1]. Δ^+ error (quadrature calculated) from Ref. [1] (see original Ref. [27]).
[c] Lattice result from Ref. [13].

$$\tilde{g}^2 = 2, \quad \langle\Delta^{++}\rangle = -2\langle\Delta^-\rangle, \quad \langle\Delta^+\rangle = -\langle\Delta^-\rangle, \text{ and } \langle\Delta^0\rangle = 0. \tag{11}$$

Equation (11) *effectively connects the U-Spin* $1, \frac{1}{2}$, *and* 0 *decuplet baryon matrix elements to that of the U-spin* $= \frac{3}{2}\Delta^-$ (*and hence the* Ω^-) and with Eq. (7), (8), and (6) is valid for *all U*-Spin decuplet baryons—allow us to compute the magnetic moments of the Δ^{++}, Δ^+, and Δ^0 and their (strangeness $S \neq 0$) U-Spin partners in terms of Ω^- magnetic moment data by using the ETCR $[V_{K^0}, j^\mu_{em}(0)] = 0$ which results in:

$$\mu_B = -Q_B\left[1 - \left(\frac{m_B^2}{m_{\Omega^-}^2}\right)\left(\frac{m + m_{\Omega^-}(\mu_{\Omega^-}/\mu_N)}{m}\right)\right]\left(\frac{m}{m_B}\right)\mu_N. \tag{12}$$

Equation (12) is the *main result of this work and is valid for all of the ground-state* $J^P = \frac{3}{2}^+$ *baryon decuplet members*. As the values of m_Δ (all Δ charge states) (pole or Breit-Wigner) are not very well established, we assume $m_\Delta = 1.22 \pm 0.01\,\text{GeV}/c^2$. Experimentally, we have [1], $\mu_{\Omega^-} = (-2.02 \pm 0.05)\mu_N = [(-1 + F_2^{\Omega^-}(0)) \times (m/m_{\Omega^-})]\mu_N$ and $m_{\Omega^-} = 1.6724 \pm 0.0003$ GeV$/c^2$. We summarize our results for all of the ground-state baryon decuplet magnetic moments μ_B in Table I.

V. CONCLUSIONS

We have—nonperturbatively—calculated the magnetic moments of all of the ground-state $J^P = 3/2^+$ *physical decuplet* baryons without ascribing any specific form to their quark structure or intraquark interactions or assuming a Lagrangian (effective or otherwise). The Particle Data Group [1] value of μ_{Ω^-} along with other decuplet mass data was used as input except we took $m_\Delta = 1.22 \pm 0.01$ GeV$/c^2$ (all Δ charge states) as the values of m_Δ are not well-enough established [1]. In particular—utilizing Eq. (12)—we obtained $\mu_{\Delta^-} = (-1.83 \pm 0.04)\mu_N$, $\mu_{\Delta^+} = (+1.83 \pm 0.04)\mu_N$, and $\mu_{\Delta^{++}} = (+3.67 \pm 0.07)\mu_N$ and $\mu_{\Delta^0} = (0)\mu_N$. Our results for the magnetic moments (the Ω^- magnetic moment is input) of the ground-state decuplet baryons are summarized in Table I along with a prediction in Fig. 1 for the Δ^- (and the Δ^+) electric charge form factor as a function of Q^2 based upon Ω^- lattice calculated fit data [26]. Similarly—with Eq. (11)—one may predict the electric charge form factor for the Δ^{++} as a function of Q^2 based upon Ω^- lattice calculated fit data. *For all* of the ground-state $J^P = 3/2^+$ baryons B, we have demonstrated how the $F_1^B(q_B^2)$ and $F_2^B(q_B^2)$ form factors can be calculated in terms of Ω^- data. Future experimental measurements of the Ω^- magnetic moment and accessible form factors for $q^2_{\Omega^-} < 0$ will have great importance for viable theoretical models (especially lattice QCD models) of the structure of baryons. Knowledge of the behavior of the decuplet form factors (or corresponding multipole moments) is critical to our understanding of QCD—standard model, enhanced standard model, lattice gauge models, superstring models, or entirely new models—since these models must be capable of yielding already known results at low or medium energy. Equations (7), (8), (6), and (11) explicitly demonstrate that the electromagnetic charge form factors of the decuplet baryons are very closely related to each other and that their magnetic dipole form factors are also very closely related to each other. This may aid experimental and theoretical ground-state decuplet baryon magnetic moment analyses in the future.

[1] K. Nakamura *et al.* (Particle Data Group Collaboration), J. Phys. G **37**, 075021 (2010).
[2] N. B. Wallace, P. M. Border, D. P. Ciampa, G. Guglielmo, K. J. Heller, D. M. Woods, K. A. Johns, Y. T. Gao *et al.*, Phys. Rev. Lett. **74**, 3732 (1995).
[3] M. A. B. Beg, B. W. Lee, and A. Pais, Phys. Rev. Lett. **13**, 514 (1964).
[4] S. B. Gerasimov, Zh. Eksp. Teor. Fiz. **30**, 1559 (1966).[Sov. Phys. JETP **23**, 1040 (1966)].
[5] D. B. Lichtenberg, *Unitary Symmetry And Elementary Particles* (Academic Press, New York, 1978), 2nd ed..
[6] M. D. Slaughter, Phys. Rev. C **83**, 059901(E) (2011); **82**, 015208 (2010).

[7] S. Oneda, H. Umezawa, and S. Matsuda, Phys. Rev. Lett. **25**, 71 (1970).

[8] S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985).

[9] M. D. Slaughter and S. Oneda, Phys. Rev. D **39**, 2062 (1989).

[10] S. Oneda, K. Terasaki, and M. D. Slaughter, University of Maryland, College Park, KEK Library Report No. MDDP-PP-89-130, 1989, http://ccdb4fs.kek.jp/cgi-bin/img/allpdf?198904122.

[11] S. Oneda, T. Tanuma, and M. D. Slaughter, Phys. Lett. B **88**, 343 (1979).

[12] S. Boinepalli, D. B. Leinweber, P. J. Moran, A. G. Williams, J. M. Zanotti, and J. B. Zhang, Phys. Rev. D **80**, 054505 (2009).

[13] C. Aubin, K. Orginos, V. Pascalutsa, and M. Vanderhaeghen, Phys. Rev. D **79**, 051502 (2009).

[14] C. Aubin and K. Orginos, arXiv:1010.0202.

[15] C. Alexandrou, T. Korzec, G. Koutsou, C. Lorce, J. W. Negele, V. Pascalutsa, A. Tsapalis, and M. Vanderhaeghen, Nucl. Phys. A **825**, 115 (2009).

[16] V. Pascalutsa, M. Vanderhaeghen, and S. N. Yang, Phys. Rep. **437**, 125 (2007).

[17] V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Particle Physics* (North-Holland, Amsterdam, 1973).

[18] M. Gell-Mann, Physics **1**, 63 (1964).

[19] S. L. Adler, Phys. Rev. **140**, B736 (1965).

[20] W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

[21] J. G. Korner and M. Kuroda, Phys. Rev. D **16**, 2165 (1977).

[22] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[23] M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).

[24] M. D. Slaughter, Phys. Rev. C **80**, 038201 (2009).

[25] M. D. Slaughter, arXiv:1105.3786.

[26] C. Alexandrou, T. Korzec, G. Koutsou, J. W. Negele, and Y. Proestos, Phys. Rev. D **82**, 034504 (2010).

[27] M. Kotulla (TAPS/A2 Collaboration), Prog. Part. Nucl. Phys. **50**, 295 (2003).