

Rapid thermalization in field theory from gravitational collapse

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(Received 21 June 2011; published 8 September 2011)

Motivated by the duality with thermalization in field theory, we study gravitational collapse of a minimally coupled massless scalar field in Einstein gravity with a negative cosmological constant. We investigate the system numerically and establish that for small values of the initial amplitude of the scalar field there is no black hole formation, rather, the scalar field performs an oscillatory motion typical of geodesics in Anti de Sitter. For large enough values of the amplitude of the scalar field, we find black hole formation, which we detect numerically as the emergence of an apparent horizon. Using the time of formation as an estimate for thermalization in the field theory, we conclude that thermalization occurs very rapidly, close to the causal bound for a very wide range of black hole masses. We further study the thermalization time in more detail as a function of the amplitude and the width of the initial Gaussian scalar field profile and detect a rather mild structure.

DOI: [10.1103/PhysRevD.84.066006](https://doi.org/10.1103/PhysRevD.84.066006)

PACS numbers: 11.25.Tq, 04.25.D-

I. INTRODUCTION

Understanding cooperative phenomena far from equilibrium poses one of the most challenging problems of present physics. One source of methods is provided by the AdS/CFT correspondence [1], which identifies a field theory without gravity with a string theory with gravity. In the classical limit of string theory, one can use the appropriate generalization of Einstein's equations to follow the evolution in time of the fields in the dual field theory. In this context, the gauge/gravity duality opens a particularly important window in the absence of regular field theoretic methods: to study far-from-equilibrium phenomena, one needs to study the evolution of Einstein's equation with appropriate boundary conditions.

In the framework of the AdS/CFT correspondence, a field theory in equilibrium at finite temperature is dual to a black hole in asymptotically Anti de Sitter (AdS) spacetime [2]. One very important development has been the establishment of the correspondence for applications to linear response theory for the near equilibrium region, that is, in the regime of long wave lengths and low energies with local fluid variables varying very slowly compared to microscopic scales (see [3] for a review). The next frontier comes from the fact that the evolution of spacetimes with the formation of black hole horizons is equivalent to non-

equilibrium dynamics and evolution towards thermalization in field theory.

Besides the general reasons to study far-from-equilibrium phenomena using the gauge/gravity correspondence, one practical motivation comes from the Relativistic Heavy Ion Collider (RHIC) and LHC experiments. Two crucial points are worth highlighting: (i) the observed quark-gluon plasma (QGP) is strongly coupled and (ii) hydrodynamical description fits in a wide range of processes. Theoretical and experimental developments indicate that the QGP produced at RHIC is a strongly interacting liquid rather than the weakly interacting gas of quarks and gluons that was previously expected [4–6]. Another piece of evidence pointing to the need for methods applicable to strong coupled theories comes from the fact that the produced plasma locally isotropizes over a time scale of $\tau_{\text{iso}} \leq 1$ fm/c. The dynamics of such rapid isotropization in a far-from-equilibrium non-Abelian plasma can not be described with the standard methods of field theory or hydrodynamics [7]. In this letter, we propose to study such rapid thermalization via its gravity dual—gravitational collapse.

The question of thermalization is also crucial in condensed matter systems. There is an active interest in the understanding of the time evolution of a system following a quench [8–11]. A holographic approach to this area is showing promise in lower dimensions [12]. String theory also provides a framework for understanding some superconductors holographically [13,14]. Moreover, some hard to understand properties of condensed matter system like

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the structure of non-Fermi liquids have been recently described using holographic models [15,16]. These models always involve a black hole, and we believe that the formation of such a black hole will help our understanding of those systems at a more fundamental level.

In this paper, we present a full numerical analysis of the gravitational collapse of a massless minimally coupled scalar field in the presence of a negative cosmological constant. We go beyond previous attempts within the AdS/CFT approach that relied on perturbation theory [17] or toy models for quench [18–20]. Our focus is on properties of the thermalization time.

An intuitive explanation for the rapid thermalization time at RHIC within the context of the gauge/gravity correspondence is as follows. Local gauge-invariant operators are mostly sensitive to geometry near the boundary. As explained in [17], in the large N limit which is the region explored by the gauge/gravity correspondence, trace factorization ensures that the expectation value of products equals the product of expectation values. Only one-point functions of gauge-invariant operators survive. Other studies supporting rapid thermalization in the context of the gauge/gravity correspondence include [18,20–22].

II. FIELD THEORY SETUP

Our field theory setup is rather specific and we clarify it now. The overall field theoretic question one is interested in answering is: How does a field theory react to a rapid injection of energy? This is precisely what the RHIC experiment is all about for large values of the energy of the colliding particles—How does QCD matter behave under such collision? Now, the collision is clearly anisotropic as one has two gold atoms colliding. A gravity approximation to such experiment has been developed in the context of numerical relativity by Chesler and Yaffe in [23–25]. A natural time scale in such collision process is the isotropization time which in the case of RHIC is $\tau_{\text{iso}} \leq 1 \text{ fm}/c$ [7]. Our setup has one major difference. Namely, our injection of energy is spatially homogeneous, that is, it is the same in all points of the field theory space at a given time. We thus study a situation that is similar but not exactly equal to a quench. In this sense, our setup is perhaps less directly applicable to RHIC-type experiments of collision, but rather speaks loudly about more universal properties of strongly coupled field theories at large N , which are dual to gravity theories. What we set out to study in this paper, using gravity methods, is the time in field theory between the injection of energy at $t = 0$ and the formation of the quark gluon plasma. We call this time the thermalization time. Of course, other interesting time scales might be present in a given experimental setup. One example is the above mentioned isotropization time, another natural time is the time at which the hydrodynamical approximation becomes a valid description of the quark gluon plasma. The thermalization time, as defined in our context, is ex-

pected to be of the same order but slightly smaller than these other two.

III. COLLAPSE IN ASYMPTOTICALLY AdS₅ SPACES

We consider a minimal Einstein-scalar field action with a negative cosmological constant $\Lambda = -6/L^2$:

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa} \left(R + \frac{12}{L^2} \right) - \frac{1}{2} (\partial\phi)^2 - U(\phi) \right]. \quad (1)$$

We tackle this system using numerical methods developed and tested in [26,27], which we will describe in more detail below. We choose the metric Ansatz to be

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 (d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta d\varphi^2)), \quad (2)$$

where α and a are each functions of only t and r . For convenience, let us define

$$X = \partial_r \phi, \quad Y = \frac{a}{\alpha} \partial_t \phi. \quad (3)$$

The Einstein equations can be written as:

$$\partial_r a = \frac{a}{r} (1 - a^2) + \frac{ra}{6} \left[X^2 + Y^2 + a^2 \left(U - \frac{12}{L^2} \right) \right], \quad (4)$$

$$\partial_r \ln(\alpha a) = \frac{r}{3} (X^2 + Y^2), \quad (5)$$

$$\frac{3}{r} \partial_t a = \alpha XY. \quad (6)$$

Here, we are using units where $\kappa = 1$. It is worth pointing out that Eq. (6) can be shown to be automatically satisfied when the other equations are satisfied.

The Klein-Gordon equation takes the form

$$\partial_t Y = \frac{1}{r^3} \partial_r \left(r^3 \frac{\alpha}{a} X \right) - \alpha a \partial_\phi U. \quad (7)$$

The main difference of the system given by Eqs. (4), (5), and (7) with respect to the treatment presented in [26,27] lies in the powers of r that appear. For example, in Eq. (7) we have a cubic power of r rather than a square one. This higher power of r leads to a more singular behavior near $r = 0$ which presents a numerical challenge.

IV. RESULTS

We consider a massless scalar field, *i.e.*, $U = 0$. We choose initial data of the form $\phi = A \exp(-(r - r_0)^2/\sigma^2)$ where the amplitude A , center of the scalar field profile r_0 and profile width σ are constants. This determines X through $X = \partial_r \phi$ and we choose $Y = X$ initially so that the wave starts out purely ingoing.

We choose $L = 1$ and for the spatial grid to be $0 \leq r \leq r_{\text{max}}$ where r_{max} is a constant. For the simulations done in

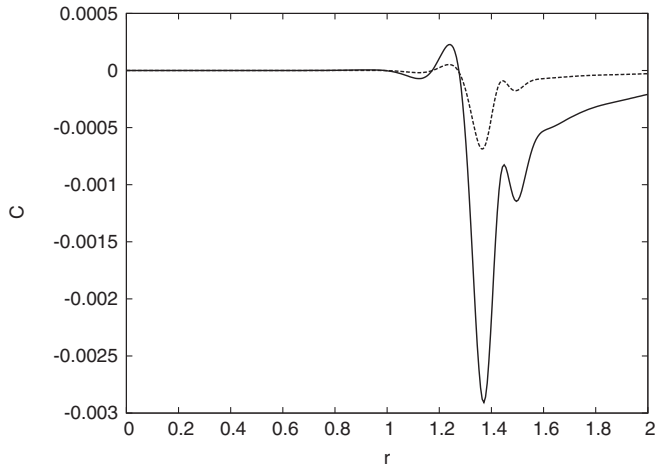


FIG. 1. The constraint quantity at coarse resolution (solid line) and fine resolution (dotted line).

this work we use $r_{\max} = 10$. Since in anti-deSitter spacetime, more resolution is needed at small r we do not choose the spatial points to be evenly spaced. Rather, for a simulation with n spatial points, the value of r at grid point i is $r_i = \sinh(k(i-1))$ where k is a constant chosen so that $r_n = r_{\max}$.

We tested that the code is working properly by considering the constraint quantity $\mathcal{C} \equiv \partial_t a - r\alpha XY/3$. Note that it follows from Eq. (6) that \mathcal{C} vanishes. However, the finite size of the grid spacing means that \mathcal{C} will not vanish exactly in the simulation, but that instead in a properly working simulation, a smaller grid spacing should lead to a smaller \mathcal{C} . In Fig. 1 the result of a test of this sort is shown. The solid line represents \mathcal{C} for a simulation with 800 grid points, while the dotted line is a simulation with 1600 grid points. In both cases the simulations have $A = 0.02$, $r_0 = 4.0$, and $\sigma = 1.5$ and the simulations are run to a time of $t = 0.454$.

For collapse in asymptotically flat spacetimes, small amplitude initial data leads to a wave that is initially ingoing, then undergoes interference near the center and becomes an outgoing wave. For collapse in asymptotically anti-deSitter spacetimes, we would expect similar behavior, except that the outgoing wave should then reach anti-deSitter infinity, bounce and become ingoing again, leading to another bounce, and so on. And indeed, this is what we find. Fig. 2 shows the value of the scalar field at the center as a function of time. ($\phi(0, t)$). The parameters for this simulation are $A = 0.0002$, $r_0 = 4.0$, and $\sigma = 1.5$. This simulation was done with 6400 grid points. Note that there are particular periods of time where the scalar field at the center is non-negligible and that the middle of each such time period is separated from the next one by approximately π . This is exactly what we would expect if the dynamics is mostly that of a massless scalar field on a background anti-deSitter spacetime. In the geometric optics limit, such a scalar field propagates along null

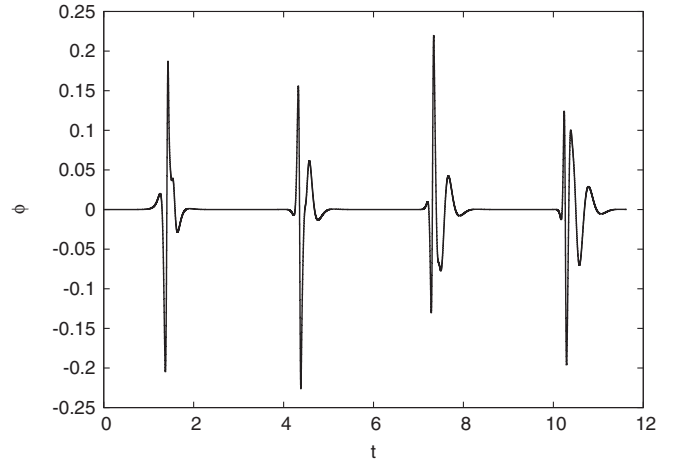


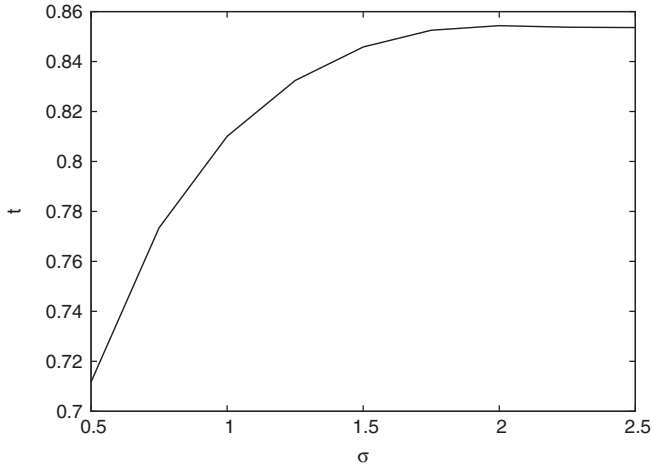
FIG. 2. $\phi(0, t)$ for a small amplitude simulation.

geodesics, and for anti-deSitter spacetime (with $L = 1$) it takes a null geodesic a time of $\pi/2$ to propagate from the center to infinity.

For sufficiently large amplitudes, the collapse process results in the formation of a black hole. This is signalled by the formation of a marginally outer-trapped surface (also called an apparent horizon), where the outgoing null geodesics cease to diverge from each other and instead begin to converge. In spherically symmetric spacetimes, an apparent horizon has the property that $\nabla^a r \nabla_a r = 0$. In our coordinate system, this would mean $a^{-2} \rightarrow 0$. More precisely, our coordinate system breaks down when an apparent horizon forms, and so our signal that a black hole is forming is that $a \rightarrow \infty$ in the simulations. Actually, it suffices to set a fairly moderate value for the maximum allowed value of a , which we denote a_{\max} and to stop the simulation whenever a reaches a_{\max} , noting that a black hole has formed at that time. The position r at which $a = a_{\max}$ gives us the size of the black hole and allows us to estimate its mass.

We would like to know how long the process of black hole formation takes, *i.e.*, we would like to know how much time elapses between the initial time and the time a marginally outer trapped surface forms. For simulations, it is natural to use the coordinate t as a measure of time, and this is the notion of time that we will use here. In particular, we denote by t_{AH} the coordinate time t at which an apparent horizon first forms. Note that our coordinate t has geometric meaning in that it is the coordinate that is orthogonal to the area coordinate r . However, one might also want to know how long the process takes in terms of an ingoing null coordinate v as used in [17]. Such a coordinate is constant along ingoing null geodesics and can be normalized by *e.g.* choosing it to be equal to the anti-deSitter time at infinity.

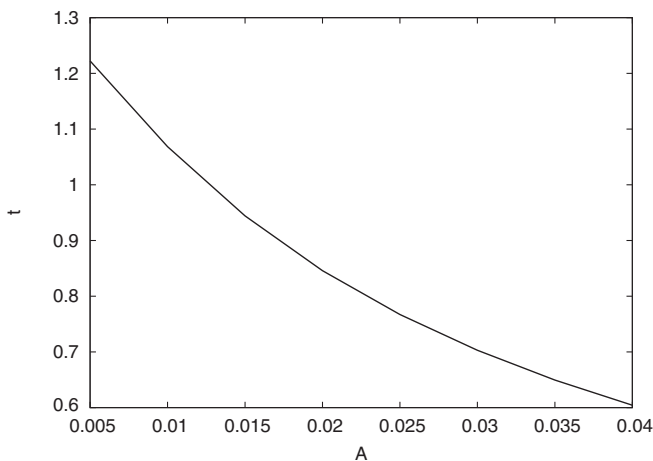
The time of black hole formation t_{AH} depends on the choice of initial data. In particular, we want to know how this time depends on the initial width of the pulse σ . This

FIG. 3. The dependence of t_{AH} on σ .

dependence is shown in Fig. 3, which gives t_{AH} for several simulations with different values of σ . For each of these simulations, we have $A = 0.02$ and $r_0 = 4.0$. These simulations were done with 6400 grid points.

We also want to know how the time of apparent horizon formation depends on the initial amplitude of the pulse A . This dependence is shown in Fig. 4, which gives t_{AH} for several simulations with different values of A . For each of these simulations, we have $\sigma = 1.5$ and $r_0 = 4.0$. These simulations were done with 6400 grid points. The masses of the black holes formed in the simulations range from 1.0 for the smallest amplitude to 62.1 for the largest amplitude, that is, over 2 orders of magnitude.

Some aspects of Fig. 4 can be understood using the properties of null geodesics in anti-deSitter spacetime. For a shell starting at infinity and propagating along a null geodesic, it would take a time of $\pi/2$ to reach the center. However, we start our shells at $r_0 = 4$ rather than infinity, and a null geodesic takes a time of $t = 1.32$ to propagate from $r = 4$ to the center. A shell of small mass

FIG. 4. The dependence of t_{AH} on A .

must reach a small radius before it forms an apparent horizon, so it is not surprising that at small amplitude $t_{\text{AH}} \approx 1.32$. However, the more massive a shell is, the larger the radius at which an apparent horizon forms, and therefore, the shorter the time needed to propagate to that radius. Therefore, we would expect t_{AH} to be a decreasing function of A , and Fig. 4 confirms this expectation.

V. THE THIN VAIDYA SHELL APPROXIMATION

Of the many setups discussed in the literature, the one that can be most directly compared to our work here, is the one presented in [17]. This work provided, to our knowledge, the first systematic analysis of the thin shell Vaidya collapse as the dual of rapid thermalization in field theory.

Let us briefly discuss the setup in [17], which considered an action of the form (1). Their metric is spherically symmetric and is given in Vaidya type coordinates

$$ds^2 = 2drdv - g(r, v)dv^2 + f^2(r, v)d\Omega^2 \quad (8)$$

$$\phi = \phi(r, v)$$

where $d\Omega^2$ is the line element of the unit S^3 . The crucial physical information is stored in the scalar field profile, which is $\phi_0(v) < \epsilon$ for $0 < v < T$ and otherwise vanishes. In [17], a perturbation theory in ϵ was developed. The situation we treat is not precisely the same as that of [17]: our initial data is different, and the fact that we use different coordinate systems makes direct comparisons somewhat involved. Nonetheless, there are many similarities. Our Gaussian profile falls off so fast that it might as well be of compact support, and our choice of ingoing waves means that at large r the profile is essentially a function of an ingoing null coordinate v . We can also choose our parameters A and σ (corresponding, respectively, to the ϵ and T of [17]) sufficiently small to be within the perturbative regime of [17] or sufficiently large that the perturbative regime is no longer valid. Furthermore, in both cases many of the main features of the collapse process seem to depend simply on the approximate propagation of the scalar field along null geodesics until the shell becomes sufficiently small that a trapped surface can form.

The main advantage of our method is that we can state rather precisely such things as when a black hole forms and how large it is without having to be in the regime where the perturbation expansion of [17] is well approximated by its first couple of terms.

VI. CONCLUSIONS

Stated in terms of the coordinate time used in our simulations, the time of black hole formation is not instantaneous and depends on the time the shell takes to propagate to a sufficiently small radius that its mass will give rise to a horizon. However, because a sufficiently thin shell propagates along an ingoing null geodesic, this means that if an ingoing null coordinate is chosen as the time, then

black hole formation is essentially instantaneous. Thus, the question of rapid thermalization in the AdS/CFT correspondence seems to hinge on the question of what bulk coordinate is the dictionary translation of time in the boundary CFT. Our explicit simulations show rapid thermalization with times always comparable to the time it takes a null geodesic to travel from the center of the shell profile to the radius at which the shell is sufficiently compact to form a horizon. We obtain our results for masses ranging over 2 orders of magnitude. In a more detailed study we verify that the dependence on the width is milder than the dependence on the amplitude.

We have laid the foundation that will allow us to provide, in full detail, various other properties of the thermalization process by studying its gravitational dual. For example, the role of a mass term in the potential for the scalar field. In the AdS/CFT correspondence, the mass of the scalar field is related to the conformal dimension of the

dual operator. We have considered a massless field corresponding to a dimension four operator in the field theory. Following the discussions presented in [17,18,20,21], we plan to discuss two-point functions, Wilson loops and the entanglement entropy in our collapsing simulations. The study of these quantities naturally allows for a more precise definition of thermalization time than the general one used in this paper. We will present those results elsewhere.

ACKNOWLEDGMENTS

L.P.Z. is grateful to H. de Oliveira and C. Terrero-Escalante for collaboration on similar matters. We thank D. Reichmann and D. Trancanelli for useful conversations. This work is partially supported by Department of Energy under Grant No. DE-FG02-95ER40899 to the University of Michigan and by National Science Foundation Grant No. PHY-0855532 to Oakland University.

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