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Dynamical origin of the refinement of the Gribov-Zwanziger theory

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In recent years, the Gribov-Zwanziger action was refined by taking into account certain dimension 2 condensates. In this fashion, one succeeded in bringing the gluon and the ghost propagator obtained from the Gribov-Zwanziger model in qualitative and quantitative agreement with the lattice data. In this paper, we shall elaborate further on this aspect. First, we shall show that more dimension 2 condensates can be taken into account than considered so far and, in addition, we shall give firm evidence that these condensates are in fact present by discussing the effective potential. It follows thus that the Gribov-Zwanziger action dynamically transforms itself into the refined version, thereby showing that the continuum nonperturbative Landau gauge fixing, as implemented by the Gribov-Zwanziger approach, is consistent with lattice simulations.

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I. INTRODUCTION

The infrared behavior of the gluon and ghost propagator has received a lot of interest in recent years, in particular, in the Landau gauge. Many of the discussions were evolved around the zero momentum value of the gluon propagator and the infrared enhancement of the ghost. The common belief is now that in four dimensions and three dimensions the ghost propagator displays no enhanced behavior, while the gluon propagator exhibits positivity violation, being suppressed in the infrared. Moreover, the latter propagator attains a nonvanishing value at zero momentum. These results are supported by many lattice data [1–8] as by many analytical approaches [9–17]. The Landau gauge propagators can then be used to extract results on physical (spectrum) quantities; see e.g. [18–20]. In particular, in the Gribov-Zwanziger (GZ) framework, which accounts for the existence of (most of) the Gribov copies in the path integral [21,22], this behavior of the ghost and gluon propagator was explained by taking into account the existence of a certain Becchi-Rouet-Stora-Tyutin (BRST) invariant dimension 2 condensate [23,24]. This was called the refined Gribov-Zwanziger framework. This particular condensate was investigated as it corresponds to a BRST invariant operator. However, one could go one step further. The Gribov-Zwanziger action has a softly broken BRST symmetry [21,23]. Fortunately, the soft breaking can be taken in due account at the quantum level through a suitable set of extended Ward identities, which can be written down by coupling the composite operator defining the breaking to a suitable set of external sources. The resulting Ward identities enable us to prove the algebraic renormalizability of the starting action and of the softly broken BRST. Given that the BRST is already softly broken from the beginning, one could then ask why one would only investigate $d = 2$ BRST invariant condensates.

In fact, there exists a whole range of $d = 2$ condensates overlooked so far, which might be taken into account. In this paper, we shall first explore these condensates and show that they affect the gluon and the ghost propagator, although not altering their qualitative behavior. The gluon propagator is still suppressed and nonzero at zero momentum, and the ghost propagator is not enhanced. Second, we shall also be able, for the first time, to calculate the effective action with the help of the local composite operator (LCO) formalism at lowest order and give arguments that there is in fact condensation. We shall motivate that the minimum of the effective potential including the condensates is a nontrivial minimum, i.e. in this minimum the condensates are present, leading to a dynamical transformation of the GZ action into the refined GZ action.

This paper is organized as follows. In Sec. II, we shall briefly review the construction of the Gribov-Zwanziger action. The first main point of this paper shall be proven in Sec. III, i.e. there can be more $d = 2$ condensates affecting the GZ action than considered so far. The second main point of this paper is presented in Sec. IV, namely, the construction of the effective action with the help of the local composite operator formalism [25,26]. We first explain the LCO formalism and then apply it to the GZ action with the inclusion of the set of $d = 2$ condensates. We then show that searching for extrema of the effective action automatically leads to nonvanishing condensates, i.e. to the refining of the GZ action. In Sec. V, we present the form of the gluon and the ghost propagator and discuss that these can be in qualitative agreement with the current lattice data. In Sec. VI we collect our conclusions. Technical details are provided in a series of appendixes.

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II. SUMMARY OF THE GRIBOV-ZWANZIGER FORMALISM

The Gribov-Zwanziger action takes into account the existence of Gribov copies by restricting the domain of integration in the functional integral to the Gribov region Ω, which is defined as the set of field configurations fulfilling the Landau gauge condition and for which the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu(\partial_\mu \delta^{ab} + g f_{abc} A^c_\mu), \tag{1}$$

is strictly positive. In [27] it has been first shown that this restriction to the Gribov region Ω can be established by considering the following (local) action:

$$S_{\rm GZ} = S_0 + S_\gamma \tag{2}$$

with

$$\begin{aligned} S_0 &= S_{\rm YM} + S_{\rm gf} + \int d^d x (\bar\varphi^{ac}_\mu \partial_\nu D^{am}_\nu \varphi^{mc}_\mu \\ &\quad - \bar\omega^{ac}_\mu \partial_\nu D^{am}_\nu \omega^{mc}_\mu - g(\partial_\nu \bar\omega^{ac}_\mu) f^{abm} (D_\nu c)^b \varphi^{mc}_\mu), \\ S_\gamma &= -\gamma^2 g \int d^d x \Big(f^{abc} A^a_\mu \varphi^{bc}_\mu + f^{abc} A^a_\mu \bar\varphi^{bc}_\mu \\ &\quad + \frac{d}{g}(N^2-1)\gamma^2 \Big). \end{aligned} \tag{3}$$

with $S_{\rm YM}$ the classical Yang-Mills action and $S_{\rm gf}$ the Landau gauge fixing

$$\begin{aligned} S_{\rm YM} &= \frac{1}{4}\int d^d x F^a_{\mu\nu} F^a_{\mu\nu}, \\ S_{\rm gf} &= \int d^d x (b^a \partial_\mu A^a_\mu + \bar c^a \partial_\mu D^{ab}_\mu c^b). \end{aligned} \tag{4}$$

The fields $(\bar\varphi^{ac}_\mu, \varphi^{ac}_\mu)$ are a pair of complex conjugate bosonic fields, while $(\bar\omega^{ac}_\mu, \omega^{ac}_\mu)$ are anticommuting fields. We recall that we can simplify the notation of the additional fields $(\bar\varphi^{ac}_\mu, \varphi^{ac}_\mu, \bar\omega^{ac}_\mu, \omega^{ac}_\mu)$ in S_0 as S_0 displays a symmetry with respect to the composite index $i = (\mu, c)$. Therefore, we can set

$$(\bar\varphi^{ac}_\mu, \varphi^{ac}_\mu, \bar\omega^{ac}_\mu, \omega^{ac}_\mu) = (\bar\varphi^a_i, \varphi^a_i, \bar\omega^a_i, \omega^a_i), \tag{5}$$

and thus

$$\begin{aligned} S_0 &= S_{\rm YM} + S_{\rm gf} + \int d^d x (\bar\varphi^a_i \partial_\mu (D^{ab}_\mu \varphi^b_i) \\ &\quad - \bar\omega^a_i \partial_\mu (D^{ab}_\mu \omega^b_i) - g f^{abc} \partial_\mu \bar\omega^a_i (D^{bd}_\mu c^d) \varphi^c_i). \end{aligned} \tag{6}$$

The BRST variations of all the fields are given by

$$\begin{aligned} &sA^a_\mu = -(D_\mu c)^a, \qquad sc^a = \frac{1}{2} g f^{abc} c^b c^c, \qquad s\bar c^a = b^a, \\ &sb^a = 0, \qquad s\varphi^a_i = \omega^a_i, \qquad s\omega^a_i = 0, \\ &s\bar\omega^a_i = \bar\varphi^a_i, \qquad s\bar\varphi^a_i = 0. \end{aligned} \tag{7}$$

The massive parameter γ, called the Gribov parameter, is not an independent parameter of the theory, being determined in a self-consistent way by the following gap equation, commonly known as the horizon condition:

$$\langle g f^{abc} A^a_\mu \varphi^{bc}_\mu \rangle + \langle g f^{abc} A^a_\mu \bar\varphi^{bc}_\mu \rangle + 2\gamma^2 d (N^2 - 1) = 0, \tag{8}$$

which ensures the restriction to the Gribov region. This gap equation can also be written as

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0, \tag{9}$$

with Γ the quantum action defined as

$$e^{-\Gamma} = \int [d\Phi] e^{-S_{\rm GZ}}, \tag{10}$$

where $\int [d\Phi]$ stands for the integration over all the fields. The action $S_{\rm GZ}$ is renormalizable. For the benefit of the reader, we have presented the full algebraic proof of the renormalization of this action in Appendix A, since we have to built on this later on. Let us also mention that, recently, an alternative approach was worked out to study the renormalizability of the GZ action [28,29]. In this paper, we shall however follow the original approach of e.g. [30].

We recall that the GZ action breaks the BRST symmetry explicitly [21,23]. This is due to the γ-dependent term, S_γ, and one can easily check from (7) and (2) that

$$\begin{aligned} sS_{\rm GZ} &= s(S_0 + S_\gamma) = s(S_\gamma) \\ &= -g\gamma^2 \int d^d x f^{abc} (A^a_\mu \omega^{bc}_\mu - (D^{am}_\mu c^m)(\bar\varphi^{bc}_\mu + \varphi^{bc}_\mu)). \end{aligned} \tag{11}$$

III. FURTHER REFINING OF THE GRIBOV-ZWANZIGER ACTION

A. Introduction

So far, the GZ action has been refined [23] by investigating the BRST invariant $d = 2$ condensate $\langle \bar\varphi^a_i \varphi^a_i - \bar\omega^a_i \omega^a_i \rangle$ and the well-known condensate $\langle A^a_\mu A^a_\mu \rangle$. The first condensate assures that the gluon propagator is nonzero at zero momentum [23], while the second condensate is indispensable in order to find a good quantitative agreement with the lattice data; see [4,31]. The resulting action, called the refined Gribov-Zwanziger action (RGZ), gives rise to a ghost propagator which behaves like $1/p^2$ for small p^2, and to the tree-level gluon propagator given by

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \frac{1}{p^2+m^2+\frac{2g^2N\gamma^4}{p^2+M^2}}\left[\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab} = \frac{p^2+M^2}{\underbrace{p^4+(M^2+m^2)p^2+2g^2N\gamma^4+M^2m^2}_{\mathcal{D}(p^2)}} \times\left[\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]\delta^{ab}, \tag{12}$$

where M^2 is the mass related to the condensate $\langle\bar\varphi^a_i\varphi^a_i-\bar\omega^a_i\omega^a_i\rangle$ and m^2 to $\langle A^a_\mu A^a_\mu\rangle$. We clearly observe that this propagator is nonvanishing at zero momentum due to the presence of the mass M^2.

However, as the GZ action breaks the BRST symmetry anyhow [see expression (11)], there is *a priori* no need to keep the operators $\bar\varphi^a_i\varphi^a_i$ and $\bar\omega^a_i\omega^a_i$ in a BRST invariant combination, i.e. $(\bar\varphi^a_i\varphi^a_i-\bar\omega^a_i\omega^a_i)=s(\bar\omega^a_i\varphi^a_i)$. In fact, we can split the operator into two separate operators, coupled to different sources. Moreover, there are also other $d=2$ operators, which were overlooked so far. In fact, all possible renormalizable $d=2$ operators $\mathcal{O}_i$ in the GZ action, which have ghost number zero, are given by[1]

$$\mathcal{O}_i=\{A_\mu A_\mu,\ \varphi^a_i\varphi^a_i,\ \varphi^a_i\bar\varphi^a_i,\ \bar\varphi^a_i\bar\varphi^a_i,\ \bar\omega^a_i\omega^a_i\}. \tag{13}$$

We shall only investigate condensates that are fully contracted over the indices (a,i), e.g. like $\varphi^a_i\bar\varphi^a_i=\varphi^{ac}_\mu\bar\varphi^{ac}_\mu$. However, it is possible to make different contractions over the color indices as is shown in [32]. Therefore, if one wants to be absolutely complete, one would have to take into account all possible color contractions. Unfortunately, this would be hopelessly complicated. Though, we hope that a good description of the IR behavior of the gluon and ghost propagator has been captured by taking into account only one color combination. Comparison with lattice data in three and four dimensions seems to confirm this, at least so far [4,31].

We also wish to point out that by including the possibility of condensation of certain operators, we are looking at the GZ dynamics with respect to a dynamically improved vacuum, in particular, an improved calculation of the effective action, and thus of the horizon condition via (9), becomes possible.

B. The action with inclusion of $d=2$ condensates

We propose to study the following extended action:

$$\Sigma_{\rm CGZ}=S_{\rm GZ}+S_{A^2}+S_{\varphi\bar\varphi}+S_{\bar\omega\omega}+S_{\overline{\varphi\varphi},\overline{\omega\varphi}} + S_{\varphi\varphi,\omega\varphi}+S_{\rm vac}, \tag{14}$$

where $S_{\rm GZ}$ is given by Eq. (2) and

$$\begin{aligned}
S_{A^2}&=\int d^dx\left(\frac{\tau}{2}A^a_\mu A^a_\mu-\frac{\zeta}{2}\tau^2\right),\qquad S_{\varphi\bar\varphi}=\int d^4x s(P\bar\varphi^a_i\varphi^a_i)=\int d^4x[Q\bar\varphi^a_i\varphi^a_i-P\bar\varphi^a_i\omega^a_i],\\
S_{\bar\omega\omega}&=\int d^4x s(V\bar\omega^a_i\omega^a_i)=\int d^4x[W\bar\omega^a_i\omega^a_i-V\bar\varphi^a_i\omega^a_i],\\
S_{\overline{\varphi\varphi},\overline{\omega\varphi}}&=\frac12\int d^4x s(\bar G^{ij}\bar\omega^a_i\bar\varphi^a_j)=\int d^4x\left[\bar H^{ij}\bar\omega^a_i\bar\varphi^a_j+\frac12\bar G^{ij}\bar\varphi^a_i\bar\varphi^a_j\right],\\
S_{\varphi\varphi,\omega\varphi}&=\frac12\int d^4x s(H^{ij}\varphi^a_i\varphi^a_j)=\int d^4x\left[\frac12 G^{ij}\varphi^a_i\varphi^a_j-H^{ij}\omega^a_i\varphi^a_j\right],\\
S_{\rm vac}&=\int d^4x\left[\kappa(G^{ij}\bar G^{ij}-2H^{ij}\bar H^{ij})+\lambda(G^{ii}\bar G^{jj}-2H^{ii}\bar H^{jj})\right]-\int d^4x[\alpha(QQ+QW)+\beta(QW+WW)\\
&\quad+\chi Q\tau+\delta W\tau].
\end{aligned} \tag{15}$$

We have introduced a source τ and 4 new doublets of sources, i.e.

$$\begin{gathered}
s\tau=0,\qquad sP=Q,\qquad sQ=0,\qquad sV=W,\qquad sW=0\\
s\bar G^{ij}=2\bar H^{ij},\qquad s\bar H^{ij}=0\qquad sH^{ij}=G^{ij},\qquad sG^{ij}=0,
\end{gathered} \tag{16}$$

where τ is a bosonic source and P, V, H^{ij}, and $\bar H^{ij}$ are Grassmann quantities. For consistency, the sources with double index ij are symmetric in these indices. In this light, we use the following definition for the derivative with respect to a symmetric source $\Lambda_{k\ell}$:

[1]We are not considering the operator $\bar c^a c^a$ here. A $\langle\bar c^a c^a\rangle$ condensate would result in massive ghosts, something which is clearly excluded by lattice simulations. If $\bar c^a c^a$ is not directly coupled to the theory, it can neither radiatively appear due to a shift symmetry of the underlying action, viz. $\bar c^a\to\bar c^a+cte$, with cte a constant Grassmann parameter.

$$\frac{\delta \Lambda_{ij}}{\delta \Lambda_{k\ell}} = \frac{1}{2}(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}). \tag{17}$$

Notice that some sources have double indices, e.g. H^{ij}, while other sources have no indices, e.g. P. The reason for this is only related to the algebraic proof of the renormalization in order to keep certain symmetries, and has no further meaning.

We have also introduced a vacuum term, $S_{\rm vac}$, which shall be important for the renormalization of the vacuum energy. As shown in [25,26], the dimensionless LCO parameters α, β, χ, δ, and ζ of the quadratic terms in the sources are needed to account for the divergences present in the correlation functions like $\langle \mathcal{O}_i(k)\mathcal{O}_j(-k)\rangle$, with $\mathcal{O}_i$ one of the operators given in expression (13).

Now we can prove that the action (14) is renormalizable to all orders. The proof is very similar to that of the renormalizability of the GZ action; the only difficulty is that the mixing between different sources and parameters is now allowed. We refer to Appendixes B and C for all the details.

For the rest of the work, we are only interested in a restricted number of condensates. Therefore, we first set the source $W = 0$, which is coupled to $\bar{\omega}\omega$, as this is not of our current interest,[2] and we also set $P = V = \eta = 0$, as we have only introduced these sources to study the renormalizability in an algebraic fashion. Second, we also take $H^{ij} = \bar{H}^{ij} = 0$ and we set $G^{ij} = \delta^{ij}G$ and $\bar{G}^{ij} = \delta^{ij}\bar{G}$. The action (14) becomes

$$\Sigma_{\rm CGZ} = S_{\rm GZ} + \int d^4x\Big[Q\bar{\varphi}_i^a\varphi_i^a + \frac{1}{2}\tau A_\mu^a A_\mu^a - \frac{1}{2}\zeta\tau^2 - \alpha QQ - \chi Q\tau\Big] + \int d^4x\Big[\frac{1}{2}\bar{G}\bar{\varphi}_i^a\bar{\varphi}_i^a + \frac{1}{2}G\varphi_i^a\varphi_i^a + \rho G\bar{G}\Big], \tag{18}$$

where $(\kappa d(N^2-1) + \lambda d^2(N^2-1)^2)$ was replaced by one parameter ρ.

C. A diagrammatical look at the potential mixing and at the vacuum divergences

Before starting the calculation of the effective action, we can provide some simplification with the help of a diagrammatical argument. First, looking at the action (18), we see that a term $\chi Q\tau$ is present. This term is responsible for killing the divergences in the vacuum correlators $\langle A^2(x)\bar{\varphi}\varphi(y)\rangle$ for $x \to y$. However, we can prove that there are no divergences of this kind in the one-loop diagrams. Let us start by considering these one-loop diagrams. There is only one possible type of diagram for $\langle A^2(x)\bar{\varphi}\varphi(y)\rangle$, as displayed in Fig. 1.

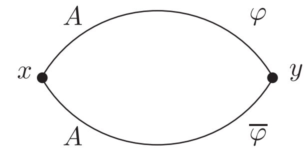
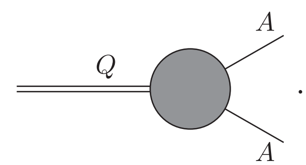


FIG. 1. One-loop diagram for $\langle A^2(x)\bar{\varphi}\varphi(y)\rangle$.

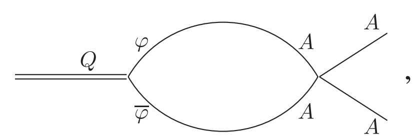
The UV behavior of this diagram is finite, as can be extracted from the list of propagators (D1). Indeed, for large momenta, the corresponding integral of the diagram 1 behaves like $\sim \int d^4p \frac{1}{p^4}\frac{1}{p^4}$, which is perfectly finite in the UV. Therefore, $\langle \lim_{x\to y} A^2(x)\bar{\varphi}\varphi(y)\rangle$ is not divergent at one loop. In the next section, we shall explicitly prove this.

At two loops, it is not possible to present the same argument as there exists a diagram which can be logarithmically divergent: as can be checked from the list of propagators (D1).

Second, we can also have a look at the mixing of the operators A^2 and $\bar{\varphi}\varphi$. In the algebraic analysis (see Appendix C), we have found that a mixing is possible between the different operators; see Eq. (C26). This means that algebraically, a counterterm of the type $QA_\mu A_\mu$ is allowed. This counterterm is needed to cancel the infinities of the following type of diagram:



However, we can prove that there are no infinities at one loop, as the only possible diagram is given by



which is similar to the diagram in Fig. 1. We can thus conclude that the mixing can only start at two loops. Again, we cannot exclude divergences at two loops, due to a similar diagram as in Fig. 2.

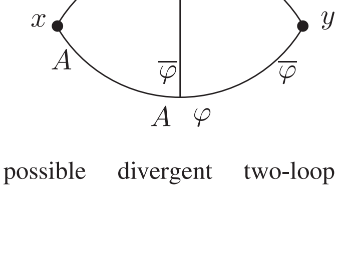


FIG. 2. A possible divergent two-loop diagram for $\langle A^2(x)\bar{\varphi}\varphi(y)\rangle$.

[2]There is no quadratic coupling of ω and $\bar{\omega}$ to the gluon sector, thus such a condensate would not directly influence the gluon propagator.

IV. THE EFFECTIVE ACTION

In this section, we shall try to calculate the effective action. The calculation is quite technical and shall therefore be split in different steps, although the result is reasonably compact and can be immediately found in expression (96).

The energy functional can be written as

$$e^{-W(Q,\tau,G,\bar{G})} = \int [dA_\mu][dc][d\bar{c}][db][d\varphi][d\bar{\varphi}][d\omega] \times [d\bar{\omega}]e^{-\Sigma_{\rm CGZ}}, \tag{19}$$

with $\Sigma_{\rm CGZ}$ given by Eq. (18). We recall that in $d = 4 - \epsilon$ dimensions, we have the following dimensionalities:

$$[A_\mu] = [\varphi] = \frac{d-2}{2} = 1 - \frac{\epsilon}{2}, \qquad [g] = \frac{4-d}{2} = \frac{\epsilon}{2},$$
$$[\tau] = [Q] = [G] = [\bar{G}] = 2,$$
$$[\zeta] = [\alpha] = [\chi] = [\rho] = d - 4 = -\epsilon. \tag{20}$$

A. The LCO formalism

In order to calculate the effective action, we shall follow the local composite operator formalism developed in [25,26]. Let us outline the main idea. We start from a LCO $\mathcal{O}$, in our case a local dimension-two operator within a dimension-four theory. As done several times, we couple the operator(s) of interest to (an) appropriate source(s) J, and add the term $J\mathcal{O}$ to the Lagrangian. This gives rise to a functional $W(J)$ which we need to Legendre transform to find the effective potential. However, as already observed, novel infinities shall arise, which are proportional to J^2. These infinities are due to the divergences in the correlator $\langle \lim_{x\to y}\mathcal{O}(x)\mathcal{O}(y)\rangle$, as explained in Sec. III C. Therefore, in general, a term proportional to J^2 is always needed in the counterterm, and the starting action needs to display a term[3] ζJ^2. The novel parameter ζ, called the LCO parameter, is needed to absorb the divergences in J^2, i.e. $\delta\zeta J^2$. With the inclusion of the term ζJ^2, the functional $W(J)$ obeys the following homogeneous renormalization group equation (RGE):

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g^2)\frac{\partial}{\partial g^2} - \gamma_J(g^2)\int d^4x J\frac{\delta}{\delta J} + \eta(g^2,\zeta)\frac{\partial}{\partial\zeta}\right)W(J) = 0, \tag{21}$$

with $\eta(g^2,\zeta)$ the running of ζ,

$$\mu\frac{\partial}{\partial\mu}\zeta = \eta(g^2,\zeta). \tag{22}$$

Notice that it is necessary to include the running of ζ at this point.

Now the question is, how can we determine this seemingly arbitrary parameter ζ? This is possible by employing the renormalization group equations. We can write

$$\zeta_0 J_0^2 = \mu^{-\epsilon}(\zeta J^2 + \delta\zeta J^2), \tag{23}$$

where the second term of the right-hand side represents the counterterm. As the left-hand side is independent from μ, we can derive both sides with respect to μ to find

$$-\epsilon(\zeta+\delta\zeta) + \left(\mu\frac{\partial}{\partial\mu}\zeta + \mu\frac{\partial}{\partial\mu}(\delta\zeta)\right) - 2\gamma_J(g^2)(\zeta+\delta\zeta) = 0, \tag{24}$$

where $\gamma_J(g^2)$ is the anomalous dimension of J. As we can consider ζ to be a function of g^2, and by evoking the β function

$$\beta(g^2) = \mu\frac{\partial}{\partial\mu}g^2, \tag{25}$$

the Eq. (24) becomes

$$\beta(g^2)\frac{\partial}{\partial g^2}\zeta(g^2) = 2\gamma_J(g^2)\zeta + f(g^2), \tag{26}$$

with $f(g^2) = \epsilon\delta\zeta - \beta(g^2)\frac{\partial}{\partial g^2}(\delta\zeta) + 2\gamma_G(g^2)\delta\zeta$. The general solution of this differential equation reads

$$\zeta(g^2) = \zeta_p(g^2) + \alpha\exp\left(2\int_1^{g^2}\frac{\gamma_J(z)}{\beta(z)}dz\right), \tag{27}$$

with $\zeta_p(g^2)$ a particular solution of (26). A possible particular solution is given by

$$\zeta_p(g^2) = \frac{c_0}{g^2} + c_1\hbar + c_2 g^2\hbar^2 + \cdots. \tag{28}$$

where we have temporarily introduced the dependence on $\hbar$. Notice therefore that the n-loop result for $\zeta(p^2)$ will require the $(n+1)$ loop results of $\beta(g^2)$, $\gamma_J(g^2)$, and $f(g^2)$. As we would like ζ to be multiplicatively renormalizable, we set $\alpha = 0$. In this case we have that

$$\zeta(g^2) + \delta\zeta(g^2) = \zeta_0 = Z_\zeta\zeta(g^2), \tag{29}$$

and we have removed the independent parameter α. Also, now that ζ is a function of g^2, the RGE (21) becomes

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g^2)\frac{\partial}{\partial g^2} - \gamma_J(g^2)\int d^4x J\frac{\delta}{\delta J}\right)W(J) = 0, \tag{30}$$

as deriving with respect to ζ is now incorporated in deriving with respect to g^2.

After determining the LCO parameter ζ, the next step is to calculate the effective action by doing a Legendre

[3] For an example, see the action (18), where the term $-\frac{1}{2}\zeta\tau^2 - \alpha QQ - \chi Q\tau$ is needed in the starting action. The sources Q and τ are coupled to the LCO operators $\mathcal{O}_1 = \bar{\varphi}_i\varphi_i$ and $\mathcal{O}_2 = A_\mu A_\mu$. Note that here, also a mixing term $\chi Q\tau$ accounting for the divergences in $\lim_{x\to y}\langle\mathcal{O}_1(x)\mathcal{O}_2(y)\rangle$ is present.

transformation. However, it shall be easier to perform a Hubbard-Stratonovich transformation on $W(J)$, where we introduce an auxiliary field σ describing the composite operator $\mathcal{O}$. In this way, we can get rid of the quadratic term in J^2 and a clear relation with the effective action emerges, as will be shown later on in this section. We only need to mention that the case we are handling here is a bit more complicated due to the mixing of the operators $\mathcal{O}_1 = \bar{\varphi}_i\varphi_i$ and $\mathcal{O}_2 = A_\mu A_\mu$, and to the mixing of the vacuum divergences. However, the basic principles remain the same.

B. Differential equation for the LCO parameters ζ, α, χ, and ρ

We shall try to determine the four LCO parameters ζ, α, χ, and ρ. We shall first derive a differential equation for these parameters, in a way analogous to [25,33]. As there can be mixing, we shall define $\delta\zeta$, $\delta\omega$, and $\delta\chi$ as follows:

$$-\frac{1}{2}\zeta_0\tau_0^2 - \alpha_0 Q_0^2 - \chi_0 Q_0\tau_0 = -\mu^{-\epsilon}\left(\frac{1}{2}\zeta\tau^2 + \alpha Q^2 + \chi Q\tau + \frac{1}{2}\delta\zeta\tau^2 + \delta\alpha Q^2 + \delta\chi Q\tau\right), \tag{31}$$

while $\delta\rho$ can be defined independently,

$$\rho_0 G_0\bar{G}_0 = \mu^{-\epsilon} Z_\rho Z_G Z_{\bar{G}}\rho G\bar{G} = \mu^{-\epsilon}\left(1 + \frac{\delta\rho}{\rho}\right)\rho G\bar{G}. \tag{32}$$

We further define the anomalous dimension of G,

$$\mu\frac{\partial}{\partial\mu}\ln Z_G = \gamma_G(g^2) \Rightarrow \mu\frac{\partial}{\partial\mu}G = -\gamma_G(g^2)G, \tag{33}$$

which is exactly the same as the anomalous dimension of $\bar{G}$ as $Z_G = Z_{\bar{G}}$. To define the anomalous dimensions of Q and τ, we start from Eq. (C26),

$$\underbrace{\begin{bmatrix} Q_0 \\ \tau_0 \end{bmatrix}}_{X_0} = \underbrace{\begin{bmatrix} Z_{QQ} & 0 \\ Z_{\tau Q} & Z_{\tau\tau} \end{bmatrix}}_{Z} \underbrace{\begin{bmatrix} Q \\ \tau \end{bmatrix}}_{X}, \tag{34}$$

a relation stemming from the algebraic renormalization. To the matrix Z, we can associate the anomalous dimension matrix Γ,

$$\mu\frac{\partial}{\partial\mu}Z = Z\Gamma, \tag{35}$$

and thus

$$\begin{aligned}\Gamma &= Z^{-1}\mu\frac{\partial}{\partial\mu}Z \\ &= \begin{bmatrix} Z_{QQ}^{-1}\mu\frac{\partial}{\partial\mu}Z_{QQ} & 0 \\ -Z_{\tau Q}\mu\frac{\partial}{\partial\mu}Z_{QQ} + Z_{\tau\tau}^{-1}\mu\frac{\partial}{\partial\mu}Z_{\tau Q} & Z_{\tau\tau}^{-1}\mu\frac{\partial}{\partial\mu}Z_{\tau\tau} \end{bmatrix} \\ &= \begin{bmatrix} \gamma_{QQ} & 0 \\ \Gamma_{21} & \gamma_{\tau\tau} \end{bmatrix}. \end{aligned} \tag{36}$$

This matrix is then related to the anomalous dimension of the operators,

$$X_0 = ZX \Rightarrow 0 = \mu\frac{\partial Z}{\partial\mu}X + Z\mu\frac{\partial X}{\partial\mu} \Rightarrow \mu\frac{\partial X}{\partial\mu} = -\Gamma X, \tag{37}$$

so the anomalous dimensions of the sources Q and τ are given by

$$\mu\frac{\partial}{\partial\mu}\begin{bmatrix} Q \\ \tau \end{bmatrix} = \begin{bmatrix} -\gamma_{QQ} & 0 \\ -\Gamma_{21} & -\gamma_{\tau\tau} \end{bmatrix}\begin{bmatrix} Q \\ \tau \end{bmatrix}. \tag{38}$$

With these definitions in mind, we can derive a differential equation for $\delta\zeta$, $\delta\omega$, $\delta\chi$, and $\delta\rho$. We start with that of $\delta\rho$. Starting from expression (32) and deriving with respect to μ, we find

$$-\epsilon(\rho + \delta\rho) + \left(\mu\frac{\partial}{\partial\mu}\rho + \mu\frac{\partial}{\partial\mu}(\delta\rho)\right) - 2\gamma_G(g^2)(\rho + \delta\rho) = 0. \tag{39}$$

As we can consider ρ to be a function of g^2, according to the standard LCO formalism, we can rewrite this equation as

$$\beta(g^2)\frac{\partial}{\partial g^2}\rho(g^2) = \epsilon(\rho + \delta\rho) - \beta(g^2)\frac{\partial}{\partial g^2}(\delta\rho) + 2\gamma_G(g^2)(\rho + \delta\rho). \tag{40}$$

As ρ is finite, we can even further simplify this into

$$\beta(g^2)\frac{\partial}{\partial g^2}\rho(g^2) = 2\gamma_G(g^2)\rho + \epsilon\delta\rho - \beta(g^2)\frac{\partial}{\partial g^2}(\delta\rho) + 2\gamma_G(g^2)\delta\rho. \tag{41}$$

In an analogous fashion, we can find the differential equations for $\delta\zeta$, $\delta\omega$, and $\delta\chi$. If we derive (31) with respect to μ, we find the following set of coupled differential equations:

$$\beta(g^2)\frac{\partial}{\partial g^2}\frac{\zeta(g^2)}{2}=\frac{\epsilon}{2}\delta\zeta-\frac{1}{2}\beta(g^2)\frac{\partial}{\partial g^2}(\delta\zeta)+\gamma_{\tau\tau}(g^2)(\zeta+\delta\zeta),$$
$$\beta(g^2)\frac{\partial}{\partial g^2}\alpha(g^2)=\epsilon\delta\alpha-\beta(g^2)\frac{\partial}{\partial g^2}(\delta\alpha)+2\gamma_{QQ}(g^2)(\alpha+\delta\alpha)+\Gamma_{21}(g^2)(\chi+\delta\chi), \tag{42}$$
$$\beta(g^2)\frac{\partial}{\partial g^2}\chi(g^2)=\epsilon\delta\chi-\beta(g^2)\frac{\partial}{\partial g^2}(\delta\chi)+\gamma_{QQ}(g^2)(\chi+\delta\chi)+\gamma_{\tau\tau}(g^2)(\chi+\delta\chi)+\Gamma_{21}(g^2)(\zeta+\delta\zeta).$$

C. Determination of the LCO parameters $\delta\zeta$, $\delta\alpha$, $\delta\chi$, and $\delta\rho$

In order to determine the counterterm parameters $\delta\zeta$, $\delta\alpha$, $\delta\chi$, and $\delta\rho$ at one loop, we need to calculate the one-loop divergence of the energy functional $W(Q,\tau,G,\bar{G})$. The details of these calculations can be found in Appendix E. From Sec. III C, we know that at one loop $\delta\chi$ should be zero. This observation shall serve as a check of our computations.

In Appendix E, Eq. (E16), we have found

$$\delta\zeta=-\frac{1}{\epsilon}\frac{3}{16\pi^2}(N^2-1),\qquad \delta\alpha=-\frac{1}{\epsilon}\frac{1}{4\pi^2}(N^2-1)^2,$$
$$\delta\chi=0,\qquad \delta\rho=\frac{1}{\epsilon}\frac{1}{4\pi^2}(N^2-1)^2. \tag{43}$$

The value of $\delta\zeta$ provides already a first check of our results. In fact, this quantity has been calculated up to three loops; see [25,34]. Our one-loop value for $\delta\zeta$ coincides with that reported in [25,34]. Second, we also see that indeed $\delta\chi=0$ at one loop, which nicely confirms our diagrammatical power counting argument.

D. Solving the differential equations for ζ, α, χ, and ρ

In this section, we shall try to solve the differential equations (41) and (42) when possible. For these calculations, it is useful to keep in mind the β function, here given up to two loops,

$$\beta(g^2)=-\epsilon g^2-2(\beta_0 g^4+\beta_1 g^6+O(g^8)), \tag{44}$$

with

$$\beta_0=\frac{11}{3}\left(\frac{N}{16\pi^2}\right),\qquad \beta_1=\frac{34}{3}\left(\frac{N}{16\pi^2}\right)^2, \tag{45}$$

in order to keep track of the orders. We start with (41),

$$\beta(g^2)\frac{\partial}{\partial g^2}\rho(g^2)=2\gamma_G(g^2)\rho+\epsilon\delta\rho-\beta(g^2)\frac{\partial}{\partial g^2}(\delta\rho)+2\gamma_G(g^2)\delta\rho. \tag{46}$$

In order to solve this differential equation, we need to parametrize ρ as follows:

$$\rho=\frac{\rho_0}{g^2}+\rho_1+\rho_2 g^2+\mathcal{O}(g^4). \tag{47}$$

We also need the explicit value of the anomalous dimension γ_G. We have from the definition (33) that

$$\gamma_G(g^2)=\mu\frac{\partial}{\partial\mu}\ln Z_G, \tag{48}$$

and thus we need the value of Z_G. From the renormalization factors (C20) and (A41), we find that

$$\gamma_G(g^2)=-\mu\frac{\partial}{\partial\mu}\ln Z_\varphi=-\mu\frac{\partial}{\partial\mu}\ln(Z_g^{-1}Z_A^{-1/2}). \tag{49}$$

In [35], the factors Z_g and Z_A have been calculated up to three loops,

$$Z_A=1+\frac{13}{6}\frac{1}{\epsilon}\frac{Ng^2}{16\pi^2}+\left(\frac{-13}{8}\frac{1}{\epsilon^2}+\frac{59}{16}\frac{1}{\epsilon}\right)\left(\frac{Ng^2}{16\pi^2}\right)^2+\cdots,$$
$$Z_g=1-\frac{11}{6}\frac{1}{\epsilon}\frac{Ng^2}{16\pi^2}+\left(\frac{121}{24}\frac{1}{\epsilon^2}-\frac{17}{6}\frac{1}{\epsilon}\right)\left(\frac{Ng^2}{16\pi^2}\right)^2+\cdots. \tag{50}$$

So one can calculate $\gamma_G(g^2)$ up to three loops if necessary. Here only the first loop shall be useful for our calculations, i.e.

$$\gamma_G(g^2)=\frac{3}{4}\frac{Ng^2}{16\pi^2}+\cdots, \tag{51}$$

as $\delta\rho$[see Eq. (E16)] is only known up to lowest order. With this information, we can solve the differential equation (46) up to lowest order, by matching the corresponding orders in g^2

$$\rho=\frac{24}{53}\frac{(N^2-1)^2}{Ng^2}+\rho_1+\cdots. \tag{52}$$

Unfortunately, we cannot solve the differential equation for ρ_1 as we would require the two-loop value of $\delta\rho$, which is however not easily computed. Therefore, in the current work, we leave this value as a parameter to be determined.

Let us now turn to the set of differential equations (42). We can do a similar analysis as above for the first differential equation, namely

$$\beta(g^2)\frac{\partial}{\partial g^2}\frac{\zeta(g^2)}{2}=\frac{\epsilon}{2}\delta\zeta-\frac{1}{2}\beta(g^2)\frac{\partial}{\partial g^2}(\delta\zeta)+\gamma_{\tau\tau}(g^2)(\zeta+\delta\zeta). \tag{53}$$

We shall again parametrize ζ as follows:

$$\zeta=\frac{\zeta_0}{g^2}+\zeta_1+\zeta_2 g^2+\mathcal{O}(g^4). \tag{54}$$

In fact, we can even solve this differential equation to two loops. From [25,33,34], we know that

$$\delta\zeta = \frac{N^2-1}{16\pi^2}\Big[-\frac{3}{\epsilon} + \Big(\frac{35}{2}\frac{1}{\epsilon^2} - \frac{139}{6}\frac{1}{\epsilon}\Big)\Big(\frac{g^2N}{16\pi^2}\Big) + \Big(-\frac{665}{6}\frac{1}{\epsilon^3} + \frac{6629}{36}\frac{1}{\epsilon^2} - \Big(\frac{71551}{432} + \frac{231}{16}\zeta(3)\Big)\frac{1}{\epsilon}\Big)\Big(\frac{g^2N}{16\pi^2}\Big)^2\Big], \tag{55}$$

and

$$Z_{\tau\tau} = 1 - \frac{35}{6}\frac{1}{\epsilon}\Big(\frac{g^2N}{16\pi^2}\Big) + \Big[\frac{2765}{72}\frac{1}{\epsilon^2} - \frac{449}{48}\frac{1}{\epsilon}\Big]\Big(\frac{g^2N}{16\pi^2}\Big)^2 + \Big[-\frac{113365}{432}\frac{1}{\epsilon^3} + \frac{41579}{576}\frac{1}{\epsilon^2} + \Big(-\frac{75607}{2592} - \frac{3}{16}\zeta(3)\Big)\frac{1}{\epsilon}\Big]\Big(\frac{g^2N}{16\pi^2}\Big)^3, \tag{56}$$

so that from (36)

$$\gamma_{\tau\tau}(g^2) = \frac{35}{6}\Big(\frac{g^2N}{16\pi^2}\Big) + \frac{449}{24}\Big(\frac{g^2N}{16\pi^2}\Big)^2 + \Big(\frac{94363}{864} + \frac{9}{16}\zeta(3)\Big)\Big(\frac{g^2N}{16\pi^2}\Big)^3. \tag{57}$$

By solving the differential equation for ζ, we can determine ζ to one-loop order. In principle, we can even go one loop further with the known results. However, as we shall only determine the effective potential to one-loop order, we do not need this next loop result. We find

$$\zeta = \frac{N^2-1}{16\pi^2}\Big[\frac{9}{13}\frac{16\pi^2}{g^2N} + \frac{161}{52}\Big] \tag{58}$$

(see also [33]).

The second and third differential equation of (42) are coupled. However, they can be simplified and decoupled as $\delta\chi = 0$,

$$\beta(g^2)\frac{\partial}{\partial g^2}\alpha(g^2) = 2\gamma_{QQ}(g^2)\alpha + \epsilon\delta\alpha - \beta(g^2)\frac{\partial}{\partial g^2}(\delta\alpha) + 2\gamma_{QQ}(g^2)\delta\alpha + \Gamma_{21}(g^2)\chi,$$
$$\beta(g^2)\frac{\partial}{\partial g^2}\chi(g^2) = \gamma_{QQ}(g^2)\chi + \gamma_{\tau\tau}(g^2)\chi + \Gamma_{21}(g^2)(\zeta + \delta\zeta). \tag{59}$$

Fortunately, we know that $\Gamma_{21} = 0$ at lowest order, from the diagrammatical argument in Sec. III C. Therefore, we can set $\Gamma_{21} = 0 + O(g^4)$. When parametrizing as usual

$$\alpha = \frac{\alpha_0}{g^2} + \alpha_1 + \alpha_2 g^2 + \mathcal{O}(g^4),$$
$$\chi = \frac{\chi_0}{g^2} + \chi_1 + \chi_2 g^2 + \mathcal{O}(g^4), \tag{60}$$

we find for the solution of the differential equations

$$\alpha_0 = -\frac{24(N^2-1)^2}{35N}, \qquad \chi_0 = 0. \tag{61}$$

E. Hubbard-Stratonovich transformations

In this section, we shall get rid of the unwanted quadratic source dependence by the introduction of multiple Hubbard-Stratonovich (HS) fields. We can then rewrite the relevant part of the action in terms of finite fields and sources:

$$\int d^4x\Big[\underbrace{Z_{QQ}Z_{\varphi}}_{c}Q\bar{\varphi}_i^a\varphi_i^a + \frac{1}{2}\underbrace{Z_AZ_{\tau\tau}}_{b}\tau A_\mu^aA_\mu^a + \frac{1}{2}\underbrace{Z_AZ_{\tau Q}}_{a}QA_\mu^aA_\mu^a - \frac{1}{2}\underbrace{Z_{\zeta\zeta}Z_{\tau\tau}^2\zeta}_{\zeta'}\mu^{-\epsilon}\tau^2 - \underbrace{Z_{QQ}^2Z_{\alpha\alpha}\alpha}_{\alpha'}\mu^{-\epsilon}QQ - \underbrace{Z_{QQ}Z_{\chi\chi}Z_{\tau\tau}\chi}_{\chi'}\mu^{-\epsilon}Q\tau\Big] + \int d^4x\Big[Z_GZ_{\varphi}\frac{1}{2}\bar{G}\bar{\varphi}_i^a\bar{\varphi}_i^a + Z_GZ_{\varphi}\frac{1}{2}G\varphi_i^a\varphi_i^a + Z_\rho Z_G^2\rho G\bar{G}\Big].$$

We shall now perform the following HS transformations by multiplying expression (19) with the following unities:[4]

[4]We dropped irrelevant normalization factors.

$$1=\int[d\sigma_1]e^{-(1/4\zeta')\int d^dx((\sigma_1/g)+b\mu^{\epsilon/2}A^2-2\zeta'\mu^{-\epsilon/2}\tau-\chi'\mu^{-\epsilon/2}Q)^2},$$
$$1=\int[d\sigma_2]e^{-(1/4\zeta'[4\alpha'\zeta'-\chi'^2])\int d^dx((\sigma_2/g)+(b\chi'-2a\zeta')\mu^{\epsilon/2}A^2-2c\zeta'\mu^{\epsilon/2}\bar\varphi\varphi+(4\alpha'\zeta'-\chi'^2)\mu^{-\epsilon/2}Q)^2},$$
$$1=\int[d\sigma_3]e^{-(1/4Z_\rho Z_G^2\rho)\int d^dx((\sigma_3/g)+(1/2)\mu^{\epsilon/2}Z_GZ_\varphi\overline{\varphi\varphi}+(1/2)\mu^{\epsilon/2}Z_GZ_\varphi\varphi\varphi+Z_G^2Z_\rho\rho\mu^{-\epsilon/2}\bar G+Z_G^2Z_\rho\rho\mu^{-\epsilon/2}G)^2},$$
$$1=\int[d\sigma_4]e^{-(1/4Z_\rho Z_G^2\rho)\int d^dx((\sigma_4/g)+(i/2)\mu^{\epsilon/2}Z_GZ_\varphi\overline{\varphi\varphi}-(i/2)\mu^{\epsilon/2}Z_GZ_\varphi\varphi\varphi-iZ_G^2Z_\rho\rho\mu^{-\epsilon/2}\bar G+iZ_G^2Z_\rho\rho\mu^{-\epsilon/2}G)^2}, \quad (62)$$

where we have introduced four new fields, σ_1,σ_2, σ_3, and σ_4. By doing these HS transformations, we can remove the quadratic sources and rewrite the functional energy as

$$e^{-W(Q,\tau,G,\bar G)}=\int[dA_\mu][dc][d\bar c][db][d\sigma_1][d\sigma_2][d\sigma_3][d\sigma_4][d\varphi][d\bar\varphi][d\omega][d\bar\omega]\times e^{[-\int d^dx(\mathcal{L}(\phi,\sigma_1,\dots,\sigma_4)-\mu^{-\epsilon/2}(\sigma_1/g)(2\zeta'\tau+\chi'Q/2\zeta')+\mu^{-\epsilon/2}(\sigma_2/g)(Q/2\zeta')+(1/2)\mu^{-\epsilon/2}(\sigma_3-i\sigma_4/g)\bar G+(1/2)(\sigma_3+i\sigma_4/g)\mu^{-\epsilon/2}G)]}, \quad (63)$$

with $\phi=(A_\mu, c, \bar c, b, \varphi, \bar\varphi, \omega, \bar\omega)$ and

$$\int d^dx\mathcal{L}(\phi,\sigma_1,\dots,\sigma_4)=S_{\rm GZ}+\int d^dx\bigg(\frac{1}{4\zeta'}\frac{\sigma_1^2}{g^2}+\frac{b}{2\zeta'}\frac{\sigma_1}{g}\mu^{\epsilon/2}A^2+\frac{b^2}{4\zeta'}\mu^\epsilon(A_\mu^aA_\mu^a)^2+\frac{1}{4\zeta'[4\alpha'\zeta'-\chi'^2]}\frac{\sigma_2^2}{g^2}$$
$$+\frac{b\chi'-2a\zeta'}{2\zeta'[4\alpha'\zeta'-\chi'^2]}\mu^{\epsilon/2}\frac{\sigma_2}{g}A^2-\frac{c}{4\alpha'\zeta'-\chi'^2}\mu^{\epsilon/2}\frac{\sigma_2}{g}\bar\varphi\varphi+\frac{(b\chi'-2a\zeta')^2}{4\zeta'[4\alpha'\zeta'-\chi'^2]}\mu^\epsilon(A_\mu^aA_\mu^a)^2$$
$$+\frac{c^2\zeta'}{[4\alpha'\zeta'-\chi'^2]}\mu^\epsilon(\bar\varphi_i^a\varphi_i^a)^2-\frac{c(b\chi'-2a\zeta')}{4\alpha'\zeta'-\chi'^2}\mu^\epsilon A_\mu^aA_\mu^a\bar\varphi_i^b\varphi_i^b+\frac{1}{4Z_\rho Z_G^2\rho}\bigg(\frac{\sigma_3^2}{g^2}+\frac{\sigma_4^2}{g^2}\bigg)$$
$$+\mu^{\epsilon/2}\frac{Z_\varphi}{4Z_\rho Z_G\rho}\frac{\sigma_3}{g}(\overline{\varphi\varphi}+\varphi\varphi)+\mu^{\epsilon/2}\frac{Z_\varphi}{4Z_\rho Z_G\rho}\frac{i\sigma_4}{g}(\overline{\varphi\varphi}-\varphi\varphi)+\mu^\epsilon\frac{Z_\varphi^2}{4Z_\rho\rho}\bar\varphi_i^a\bar\varphi_i^a\varphi_j^b\varphi_j^b\bigg). \quad (64)$$

As these HS transformations do not put everything in the right form yet, we propose the following *extra* transformation:

$$\sigma_1\frac{\chi'}{2\zeta'}-\frac{\sigma_2}{2\zeta'}=\sigma_2'. \quad (65)$$

So (63) becomes

$$e^{-W(Q,\tau,G,\bar G)}=\int[dA_\mu][dc][d\bar c][db][d\sigma_1][d\sigma_2][d\sigma_3][d\sigma_4][d\varphi][d\bar\varphi][d\omega][d\bar\omega]\times e^{[-\int d^dx(\mathcal{L}(\phi,\sigma_1,\dots,\sigma_4)-\mu^{-\epsilon/2}(\sigma_1/g)\tau-\mu^{-\epsilon/2}(\sigma_2'/g)Q+(1/2)\mu^{-\epsilon/2}(\sigma_3-i\sigma_4/g)\bar G+(1/2)(\sigma_3+i\sigma_4/g)\mu^{-\epsilon/2}G)]}, \quad (66)$$

where

$$\int d^dx\mathcal{L}(\phi,\sigma_1,\dots,\sigma_4)=S_{\rm GZ}+\int d^dx\bigg(\frac{\alpha'}{4\alpha'\zeta'-\chi'^2}\frac{\sigma_1^2}{g^2}+\frac{\zeta'}{4\alpha'\zeta'-\chi'^2}\frac{\sigma_2^2}{g^2}-\frac{\chi'}{4\alpha'\zeta'-\chi'^2}\frac{\sigma_1\sigma_2}{g^2}+\frac{2b\alpha'-a\chi'}{4\alpha'\zeta'-\chi'^2}\frac{\sigma_1}{g}\mu^{\epsilon/2}A^2$$
$$-\frac{b\chi'-2a\zeta'}{[4\alpha'\zeta'-\chi'^2]}\mu^{\epsilon/2}\frac{\sigma_2}{g}A^2-\frac{c\chi'}{4\alpha'\zeta'-\chi'^2}\mu^{\epsilon/2}\frac{\sigma_1}{g}\bar\varphi\varphi+\frac{2c\zeta'}{4\alpha'\zeta'-\chi'^2}\mu^{\epsilon/2}\frac{\sigma_2}{g}\bar\varphi\varphi$$
$$+\frac{b^2}{4\zeta'}\mu^\epsilon(A_\mu^aA_\mu^a)^2+\frac{(b\chi'-2a\zeta')^2}{4\zeta'[4\alpha'\zeta'-\chi'^2]}\mu^\epsilon(A_\mu^aA_\mu^a)^2+\frac{c^2\zeta'}{[4\alpha'\zeta'-\chi'^2]}\mu^\epsilon(\bar\varphi_i^a\varphi_i^a)^2$$
$$-\frac{c(b\chi'-2a\zeta')}{4\alpha'\zeta'-\chi'^2}\mu^\epsilon A_\mu^aA_\mu^a\bar\varphi_i^b\varphi_i^b+\frac{1}{4Z_\rho Z_G^2\rho}\bigg(\frac{\sigma_3^2}{g^2}+\frac{\sigma_4^2}{g^2}\bigg)+\mu^{\epsilon/2}\frac{Z_\varphi}{4Z_\rho Z_G\rho}\frac{\sigma_3}{g}(\overline{\varphi\varphi}+\varphi\varphi)$$
$$+\mu^{\epsilon/2}\frac{Z_\varphi}{4Z_\rho Z_G\rho}\frac{i\sigma_4}{g}(\overline{\varphi\varphi}-\varphi\varphi)+\mu^\epsilon\frac{Z_\varphi^2}{4Z_\rho\rho}\bar\varphi_i^a\bar\varphi_i^a\varphi_j^b\varphi_j^b\bigg). \quad (67)$$

Now acting with $\frac{\delta}{\delta Q}|_{Q,\tau=0}$ and $\frac{\delta}{\delta \tau}|_{Q,\tau=0}$ on the energy functional, before and after the HS transformation, gives us the following two relations,:

$$Z_{QQ}Z_\varphi\langle\bar{\varphi}_i^a\varphi_i^a\rangle + \frac{1}{2}Z_AZ_{\tau W}\langle A_\mu^a A_\mu^a\rangle = -\mu^{-\epsilon/2}\frac{\langle\sigma_2\rangle}{g},$$
$$\frac{1}{2}Z_AZ_{\tau\tau}\langle A_\mu^a A_\mu^a\rangle = -\mu^{-\epsilon/2}\frac{\langle\sigma_1\rangle}{g}, \quad (68)$$

while acting with $\frac{\delta}{\delta G}|_{G,\bar{G}=0}$ and $\frac{\delta}{\delta \bar{G}}|_{G,\bar{G}=0}$,

$$Z_GZ_\varphi\langle\varphi\varphi\rangle = \mu^{-\epsilon/2}\frac{\langle\sigma_3 + i\sigma_4\rangle}{g},$$
$$Z_GZ_\varphi\langle\overline{\varphi\varphi}\rangle = \mu^{-\epsilon/2}\frac{\langle\sigma_3 - i\sigma_4\rangle}{g}, \quad (69)$$

or equivalently

$$Z_GZ_\varphi\frac{1}{2}\langle\varphi\varphi + \overline{\varphi\varphi}\rangle = \mu^{-\epsilon/2}\frac{\langle\sigma_3\rangle}{g},$$
$$Z_GZ_\varphi\frac{i}{2}\langle\overline{\varphi\varphi} - \varphi\varphi\rangle = \mu^{-\epsilon/2}\frac{\langle\sigma_4\rangle}{g}. \quad (70)$$

F. The effective action

If we introduce the parameters

$$\frac{m^2}{2} = \frac{1}{4\alpha_0\zeta_0 - 2\chi_0^2}(2\alpha_0 g\sigma_1 - \chi_0 g\sigma_2),$$
$$M^2 = \frac{1}{2\alpha_0\zeta_0 - \chi_0^2}(\chi_0 g\sigma_1 - \zeta_0 g\sigma_2),$$
$$\rho = -\frac{53N}{48(N^2-1)^2}(\sigma_3 + i\sigma_4)g,$$
$$\rho^\dagger = -\frac{53N}{48(N^2-1)^2}(\sigma_3 - i\sigma_4)g, \quad (71)$$

with α_0, ζ_0, χ_0 given in Eqs. (58)–(61), then the quadratical part of the Lagrangian (67) is given by

$$\int d^dx\mathcal{L}(\phi,\sigma_1,\ldots,\sigma_4) = S_{\rm GZ}^{\rm quad} + \int d^dx\left(\frac{\alpha'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_1^2}{g^2} + \frac{\zeta'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_2^2}{g^2} - \frac{\chi'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_1\sigma_2}{g^2}\right.$$
$$\left. + \frac{1}{4Z_\rho Z_G^2\rho}\left(\frac{\sigma_3^2}{g^2} + \frac{\sigma_4^2}{g^2}\right) + \frac{m^2}{2}\mu^{\epsilon/2}A^2 - M^2\mu^{\epsilon/2}\bar{\varphi}\varphi + \mu^{\epsilon/2}\frac{\rho}{2}\overline{\varphi\varphi} + \mu^{\epsilon/2}\frac{\rho^\dagger}{2}\varphi\varphi\right). \quad (72)$$

We have left out the higher order terms as we shall only calculate the one-loop effective potential $\Gamma^{(1)}$.

All details of the calculations of the effective potential have been collected in Appendix E. The final result for the effective potential $\Gamma^{(1)}$ is given by

$$\Gamma^{(1)} = \frac{(N^2-1)^2}{16\pi^2}\left[(M^2 - \sqrt{\rho\rho^\dagger})^2\ln\frac{M^2 - \sqrt{\rho\rho^\dagger}}{\bar{\mu}^2} + (M^2 + \sqrt{\rho\rho^\dagger})^2\ln\frac{M^2 + \sqrt{\rho\rho^\dagger}}{\bar{\mu}^2} - 2(M^2 + \rho\rho^\dagger)\right]$$
$$+ \frac{3(N^2-1)}{64\pi^2}\left[-\frac{5}{6}(m^4 - 2\lambda^4) + y_1^2\ln\frac{(-y_1)}{\bar{\mu}} + y_2^2\ln\frac{(-y_2)}{\bar{\mu}} + y_3^2\ln\frac{(-y_3)}{\bar{\mu}} - y_4^2\ln\frac{(-y_4)}{\bar{\mu}} - y_5^2\ln\frac{(-y_5)}{\bar{\mu}}\right]$$
$$- 2(N^2-1)\frac{\lambda^4}{Ng^2} + \frac{3}{2}\frac{\lambda^4}{32\pi^2}(N^2-1) + \frac{1}{2}\frac{48(N^2-1)^2}{53N}\left(1 - Ng^2\frac{53}{24}\frac{\rho_1}{(N^2-1)^2}\right)\frac{\rho\rho^\dagger}{g^2} + \frac{9}{13}\frac{N^2-1}{N}\frac{m^4}{2g^2}$$
$$- \frac{24}{35}\frac{(N^2-1)^2}{N}\frac{M^4}{g^2} - \frac{161}{52}\frac{N^2-1}{16\pi^2}\frac{m^4}{2} - M^4\alpha_1 + M^2m^2\chi_1, \quad (73)$$

where y_1, y_2, and y_3 are the solutions of the equation

$$y^3 + (m^2 + 2M^2)y^2 + (\lambda^4 + M^4 - \rho\rho^\dagger + 2M^2m^2)y$$
$$+ M^2\lambda^4 + 1/2(\rho + \rho^\dagger)\lambda^4 + M^4m^2 - m^2\rho\rho^\dagger = 0$$

and y_4 and y_5 of the equation $y^2 + 2M^2y + M^4 - \rho\rho^\dagger = 0$. We employed the $\overline{\rm MS}$ scheme.

G. A firm indication that the condensates are nonvanishing by minimizing the effective potential

To simplify the calculations, let us set $\rho = \rho^\dagger = 0$, which corresponds to the case of not considering the condensates $\langle\overline{\varphi\varphi}\rangle$ and $\langle\varphi\varphi\rangle$. For the moment, we are only considering $\langle\varphi\bar{\varphi}\rangle$, which already has the desired influence on the propagators; see the next section. In this case, the effective action simplifies and becomes

$$\Gamma^{(1)} = \frac{(N^2-1)^2}{16\pi^2}\left[2M^4\ln\frac{M^2}{\bar{\mu}^2} - 2M^2\right] + \frac{3(N^2-1)}{64\pi^2}\left[-\frac{5}{6}(m^4-2\lambda^4) + M^4\ln\frac{(M^2)}{\bar{\mu}} + y_2^2\ln\frac{(-y_2)}{\bar{\mu}} + y_3^2\ln\frac{(-y_3)}{\bar{\mu}} - 2M^4\ln\frac{M^2}{\bar{\mu}}\right] - 2(N^2-1)\frac{\lambda^4}{Ng^2} + \frac{3}{2}\frac{\lambda^4}{32\pi^2}(N^2-1) + \frac{9}{13}\frac{N^2-1}{N}\frac{m^4}{2g^2} - \frac{24}{35}\frac{(N^2-1)^2}{N}\frac{M^4}{g^2} - \frac{161}{52}\frac{N^2-1}{16\pi^2}\frac{m^4}{2} - M^4\alpha_1 + M^2m^2\chi_1, \tag{74}$$

where y_2 and y_3 are given by $\frac{1}{2}(-m^2 - M^2 \pm \sqrt{m^4 - 2M^2m^2 + M^4 - 4\lambda^4})$.

In order to find the minimum, we should derive this action with respect to m^2 and M^2 and set the equations equal to zero. In addition, we should also impose the horizon condition (9). Therefore, we have the following three conditions:

$$\frac{\partial\Gamma}{\partial M^2} = 0, \qquad \frac{\partial\Gamma}{\partial m^2} = 0, \qquad \frac{\partial\Gamma}{\partial\lambda^4} = 0, \tag{75}$$

which have to be solved for M^2, m^2, and λ^4. Unfortunately, it is impossible to solve these equations exactly due to the two unknown parameters α_1 and χ_1. However, we would like to know if the condensate $\langle\bar{\varphi}\varphi\rangle$ is present or not. For this, we need to uncover if $M^2 = 0$ can be a solution of the above expression. We can strongly argue that this is not the case, and thus that $M^2 \neq 0$.

We shall start from expression (74) and derive with respect to M^2, m^2, and λ^4. As we would like to know if $M^2 = 0$ can be a minimum of the potential, we further set $M^2 = 0$. We then obtain the following equations:

$$\begin{aligned}
&\frac{3(\ln(m^2-\sqrt{m^4-4\lambda^4}) - \ln(m^2+\sqrt{m^4-4\lambda^4}))\lambda^4}{4\pi^2\sqrt{m^4-4\lambda^4}} + m^2\chi_1 - \frac{8}{\pi^2} = 0,\\
&\frac{1}{\sqrt{m^4-4\lambda^4}}\bigg[11\sqrt{m^4-4\lambda^4}(24\ln 2-17)m^2 + 39(-m^4+\sqrt{m^4-4\lambda^4}m^2+2\lambda^4)\ln\left(\frac{1}{8}(m^2-\sqrt{m^4-4\lambda^4})\right)\\
&\quad + 39(m^4+\sqrt{m^4-4\lambda^4}m^2-2\lambda^4)\ln\left(\frac{1}{8}(m^2+\sqrt{m^4-4\lambda^4})\right)\bigg] = 0,\\
&\frac{\lambda^2}{\sqrt{m^4-4\lambda^4}}\bigg[9(\sqrt{m^4-4\lambda^4}-m^2)\ln\left(\frac{1}{8}(m^2-\sqrt{m^4-4\lambda^4})\right) + 9(m^2+\sqrt{m^4-4\lambda^4})\ln\left(\frac{1}{8}(m^2+\sqrt{m^4-4\lambda^4})\right)\\
&\quad + \sqrt{m^4-4\lambda^4}(-15+176\ln 2)\bigg] = 0,
\end{aligned} \tag{76}$$

where we have chosen to set[5] $\bar{\mu} = 2$ and $N = 3$. Now looking at these equations, we see that the second and third equation can be solved exactly for m^2 and λ. There are even multiple solutions possible. We take the solution which has the lowest value for the effective action with $M^2 = 0$. However, for this solution to be also a solution of the first equation, these values should be very specific and the chance that they will also satisfy the first equation is practically nonexistent, with a certain value of χ_1. Moreover, at a different scale $\bar{\mu}$, the three equations will look slightly different. However, χ_1 is a number and stays the same. Therefore, it would be necessary that at all different scales these three equations can be solved exactly for only two parameters. This is practically impossible, leading to the conclusion that $M^2 \neq 0$. A similar reasoning can be worked out if ρ and/or $\rho^\dagger$ would be allowed. The main result is that it appears impossible for all these condensates to be zero, making the associated refinement inevitable.

[5]We work in units $\Lambda_{\overline{\mathrm{MS}}} = 1$.

In conclusion, we have a firm indication that the condensate $\langle\bar{\varphi}\varphi\rangle$ is indeed present, thereby suggesting the dynamical transformation of the GZ framework into a refined GZ framework.

V. THE GLUON AND THE GHOST PROPAGATOR

A. The gluon propagator

Let us now discuss that the gluon propagator can be infrared suppressed and nonzero at zero momentum. Indeed, starting from the further refined action (14), the quadratic action is given by

$$\begin{aligned}
S_{\rm quad} = &\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + b\partial_\mu A_\mu + \bar{c}\partial^2 c + \bar{\varphi}\partial^2\varphi\\
&- \bar{\omega}\partial^2\omega - \gamma^2 g f^{abc}A_\mu^b(\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) + \gamma^4 d(N^2-1)\\
&- M^2\bar{\varphi}\varphi + \frac{m^2}{2}A_\mu A_\mu - \frac{\rho}{2}\overline{\varphi\varphi} - \frac{\rho^\dagger}{2}\varphi\varphi,
\end{aligned} \tag{77}$$

where we have replaced the source τ with m^2, Q with $-M^2$, $\bar{G}^{ij}$ with $-\delta^{ij}\rho$, and G^{ij} with $-\delta^{ij}\rho^\dagger$ and set all other sources equal to zero. From this, we can easily deduce the gluon propagator

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right]\delta^{ab} \times \underbrace{\frac{2(M^2+p^2)^2 - 2\rho\rho^\dagger}{2M^4p^2 + 2p^6 + 2M^2(2p^4+\lambda^4) - \lambda^4(\rho+\rho^\dagger) + 2m^2((M^2+p^2)^2 - \rho\rho^\dagger) + 2p^2(\lambda^4 - \rho\rho^\dagger)}}_{D(p^2)}, \tag{78}$$

with $\lambda^4 = 2g^2N\gamma^4$. If we assume that $\rho = \rho^\dagger$, we then find the following gluon propagator:

$$D(p^2) = \frac{M^2 + p^2 + \rho}{p^4 + M^2p^2 + p^2(\rho + m^2) + m^2(M^2+\rho) + \lambda^4}, \tag{79}$$

which has exactly the same form as the refined gluon propagator (12). However, for the moment we cannot say whether $\rho = \rho^\dagger$ is the case or not. Notice that ρ, $\rho^\dagger$, as well as M^2 can provide in an independent way that $D(0) \neq 0$. However, in principle, it could occur that $M^4 = \rho\rho^\dagger$, giving $D(0) = 0$, at least at tree level, as is clear from (78). For the moment, we cannot say much about the precise values of the condensates behind the parameters M^2, ρ, and $\rho^\dagger$. This is currently being further investigated using lattice data input in [31].

B. The ghost propagator

The one-loop ghost propagator is given by

$$\begin{aligned} \mathcal{G}^{ab}(k^2) = \delta^{ab}\mathcal{G}(k^2) &= \delta^{ab}\left(\frac{1}{k^2} + \frac{1}{k^2}\left[g^2\frac{N}{N^2-1}\int\frac{d^4q}{(2\pi)^4}\frac{(k-q)_\mu k_\nu}{(k-q)^2}\langle A^a_\mu A^a_\nu\rangle\right]\frac{1}{k^2}\right) + \mathcal{O}(g^4) \\ &= \delta^{ab}\frac{1}{k^2}(1+\sigma(k^2)) + \mathcal{O}(g^4) = \delta^{ab}\frac{1}{k^2(1-\sigma(k^2))} + \mathcal{O}(g^4), \end{aligned} \tag{80}$$

with

$$\begin{aligned} \sigma(k^2) &= \frac{N}{N^2-1}\frac{g^2}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{(k-q)_\mu k_\nu}{(k-q)^2}\langle A^a_\mu A^a_\nu\rangle \\ &= Ng^2\frac{k_\mu k_\nu}{k^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(k-q)^2}\left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] \\ &\quad \times \frac{2(M^2+q^2)^2 - 2\rho\rho^\dagger}{2M^4q^2 + 2q^6 + 2M^2(2q^4+\lambda^4) - \lambda^4(\rho+\rho^\dagger) + 2m^2((M^2+q^2)^2-\rho\rho^\dagger) + 2q^2(\lambda^4-\rho\rho^\dagger)}. \end{aligned}$$

As we are interested in the infrared behavior of this propagator, we expand the previous expression for small k^2,

$$\begin{aligned} \sigma(k^2 \approx 0) &= Ng^2\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2} \\ &\quad \times \frac{2(M^2+q^2)^2 - 2\rho\rho^\dagger}{2M^4q^2 + 2q^6 + 2M^2(2q^4+\lambda^4) - \lambda^4(\rho+\rho^\dagger) + 2m^2((M^2+q^2)^2-\rho\rho^\dagger) + 2q^2(\lambda^4-\rho\rho^\dagger)} + O(k^2). \end{aligned} \tag{81}$$

Let us now have a look at the gap equation. For this we can start from the (one-loop) effective action which can be written as (see Appendix E)

$$\Gamma^{(1)}_\gamma = -d(N^2-1)\gamma^4 + \frac{(N^2-1)}{2}(d-1)\int\frac{d^dq}{(2\pi)^d}\ln A + \cdots,$$

with

$$A = \frac{2M^4q^2 + 2q^6 + 2M^2(2q^4 + \lambda^4) - \lambda^4(\rho + \rho^\dagger) + 2m^2((M^2 + q^2)^2 - \rho\rho^\dagger) + 2q^2(\lambda^4 - \rho\rho^\dagger)}{2(M^2 + q^2)^2 - 2\rho\rho^\dagger},$$

and the $\cdots$ indicating parts independent from λ. Setting $\lambda^4 = 2g^2N\gamma^4$, we rewrite the previous expression,

$$\mathcal{E}^{(1)} = \frac{\Gamma_\gamma^{(1)}}{N^2 - 1}\frac{2g^2N}{d} = -\lambda^4 + g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\ln A + \cdots.$$

The gap equation is given by $\frac{\partial\mathcal{E}^{(1)}}{\partial\lambda^2} = 0$,

$$1 = g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{2M^2 + 2q^2 - \rho - \rho^\dagger}{2M^4q^2 + 2q^6 + 2M^2(2q^4 + \lambda^4) - \lambda^4(\rho + \rho^\dagger) + 2m^2((M^2 + q^2)^2 - \rho\rho^\dagger) + 2q^2(\lambda^4 - \rho\rho^\dagger)}, \tag{82}$$

where we have excluded the solution $\lambda = 0$. With the help of this gap equation, we can rewrite Eq. (81),

$$\sigma(k^2 \approx 0) = 1 + Ng^2\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d} \times \frac{2M^4/q^2 + 2M^2 - 2\rho\rho^\dagger/q^2 + \rho + \rho^\dagger}{2M^4q^2 + 2q^6 + 2M^2(2q^4 + \lambda^4) - \lambda^4(\rho + \rho^\dagger) + 2m^2((M^2 + q^2)^2 - \rho\rho^\dagger) + 2q^2(\lambda^4 - \rho\rho^\dagger)} + O(k^2). \tag{83}$$

From the representation (80), it is clear that an infrared ghost enhancement can only take place if $\sigma(0) = 1$, meaning that the integral appearing in (83) should vanish. This integral is finite. We can rewrite it as ($d = 4$)

$$I = Ng^2\frac{3}{32\pi^2}\int_0^\infty dq\frac{q(M^4 - (r^2 + s^2)) + q^3(M^2 + r)}{M^4q^2 + q^6 + M^2(2q^4 + \lambda^4) - r\lambda^4 + m^2((M^2 + q^2)^2 - (r^2 + s^2)) + q^2(\lambda^4 - (r^2 + s^2))},$$

with $I = \sigma(k^2 \approx 0) - 1$, where we have parametrized

$$\rho = r + is, \qquad \rho^\dagger = r - is. \tag{84}$$

We further write

$$I = \frac{3Ng^2}{64\pi^2}\int_0^\infty dx(M^4 - (r^2 + s^2) + x(M^2 + r))/(x^3 + x^2(2M^2 + m^2) + x(M^4 + 2m^2M^2 + \lambda^4 - (r^2 + s^2)) + \lambda^4(M^2 - r) + m^2(M^4 - (r^2 + s^2))). \tag{85}$$

Solution of cubic equation.—The next step would be to solve the cubic equation in the denominator of the equation above,

$$x^3 + x^2\underbrace{(2M^2 + m^2)}_{a} + x\underbrace{(M^4 + 2m^2M^2 + \lambda^4 - (r^2 + s^2))}_{b} + \underbrace{\lambda^4(M^2 - r) + m^2(M^4 - (r^2 + s^2))}_{c} = 0. \tag{86}$$

In general, the roots are given by

$$x_1 = \frac{-1}{3}\left(a + \sqrt[3]{\frac{m + \sqrt{n}}{2}} + \sqrt[3]{\frac{m - \sqrt{n}}{2}}\right),$$
$$x_2 = \frac{-1}{3}\left(a + \frac{-1 + i\sqrt{3}}{2}\sqrt[3]{\frac{m + \sqrt{n}}{2}} + \frac{-1 - i\sqrt{3}}{2}\sqrt[3]{\frac{m - \sqrt{n}}{2}}\right),$$
$$x_3 = \frac{-1}{3}\left(a + \frac{-1 - i\sqrt{3}}{2}\sqrt[3]{\frac{m + \sqrt{n}}{2}} + \frac{-1 + i\sqrt{3}}{2}\sqrt[3]{\frac{m - \sqrt{n}}{2}}\right), \tag{87}$$

with

$$m = 2(m^2 - M^2)((m^2 - M^2)^2 - 9(r^2 + s^2)) - 9(m^2 - M^2 + 3r)\lambda^4,$$
$$n = [2(m^2 - M^2)((m^2 - M^2)^2 - 9(r^2 + s^2)) - 9(m^2 - M^2 + 3r)\lambda^4]^2 - 4[(m^2 - M^2)^2 + 3(r^2 + s^2 - \lambda^4)]^3. \quad (88)$$

Of course, it is possible that two (or three) solutions coincide. This can be checked by calculating the discriminant

$$\Delta = -4a^3c + a^2b^2 - 4b^3 + 18abc - 27c^2. \quad (89)$$

If $\Delta = 0$, then the equation has three real roots and at least two are equal.

Case 1: $x_1 \neq x_2 \neq x_3$. If $x_1 \neq x_2 \neq x_3$, we can rewrite the integral I as

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[\int_0^\infty dx \frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_2)(x_1-x_3)}\frac{1}{x-x_1} + \int_0^\infty dx \frac{M^4 - (r^2+s^2) + (M^2+r)x_2}{(x_2-x_3)(x_2-x_1)(x-x_2)} + \int_0^\infty dx \frac{M^4 - (r^2+s^2) + (M^2+r)x_3}{(x_3-x_1)(x_3-x_1)(x-x_3)}\Bigg]. \quad (90)$$

These integrals are now easy to solve; they all are of the type $\int dx \frac{1}{x} = \ln x$.

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[\underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_2)(x_1-x_3)}}_{u_1}\ln(x-x_1)\Big|_0^\infty + \underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_2}{(x_2-x_3)(x_2-x_1)}}_{v_1}\ln(x-x_2)\Big|_0^\infty + \underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_3}{(x_3-x_1)(x_3-x_1)}}_{w_1}\ln(x-x_3)\Big|_0^\infty\Bigg].$$

One could expect there is a problem at infinity, in contrast with what we have concluded before. However, as $u_1 + v_1 + w_1 = 0$, the infinities cancel. We obtain,

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[\frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_2)(x_1-x_3)}\ln(-x_1) + \frac{M_4 - (r^2+s^2) + (M^2+r)x_2}{(x_2-x_3)(x_2-x_1)}\ln(-x_2) + \frac{M^4 - (r^2+s^2) + (M^2+r)x_3}{(x_3-x_1)(x_3-x_1)}\ln(-x_3)\Bigg]. \quad (91)$$

Case 2: $x_1 = x_2 \neq x_3$. In this case, we can rewrite the integral I as

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[\int_0^\infty dx \underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_3)^2}}_{u_2}\frac{1}{x-x1} - \int_0^\infty dx \underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_3)^2}}_{v_2}\frac{1}{x-x_3} + \int_0^\infty dx \underbrace{\frac{M^4 - (r^2+s^2) + (M^2+r)x_3}{x_1-x_3}}_{w_2}\frac{1}{(x-x_3)^2}\Bigg]. \quad (92)$$

One can check that $u_2 + v_2 = 0$, so we can perform the integrations

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[\frac{M^4 - (r^2+s^2) + (M^2+r)x^1}{(x_1-x_3)^2}\ln(-x1) - \frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{(x_1-x_3)^2}\ln(-x_3) - \frac{M^4 - (r^2+s^2) + (M^2+r)x_3}{x_1-x_3}\frac{1}{x_3^2}\Bigg]. \quad (93)$$

Case 3: $x_1 = x_2 = x_3$. Finally, in this case we can write

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[(M^2+r)\int_0^\infty dx \frac{1}{(x-x_1)^2} + (M^4 - (r^2+s^2) + (M^2+r)x_1)\int_0^\infty dx \frac{1}{(x-x_1)^3}\Bigg], \quad (94)$$

so after integration

$$I = Ng^2 \frac{3}{64\pi^2}\Bigg[-\frac{M^2+r}{x_1} + \frac{M^4 - (r^2+s^2) + (M^2+r)x_1}{2x_1^2}\Bigg]. \quad (95)$$

Now we can try to draw some conclusions about the ghost propagator. Looking at the different cases, it is clear that only for special values of the mass parameters (condensates) I could be zero, which would correspond to the case of an infrared ghost enhancement. This appears to be a strong indication that the ghost propagator is not enhanced, in addition to the nonvanishing gluon propagator at zero momentum. We shall further investigate this in [31]. It is perhaps interesting to note that, for $M^4 = \rho\rho^\dagger = r^2 + s^2$, we do not necessarily find $\sigma(0) = 1$, or said otherwise, a vanishing gluon propagator at zero momentum does not necessarily correspond to an infrared enhanced ghost and vice versa.

VI. CONCLUSION

Although this paper is quite technical, the conclusions are quite simple. First, we have shown that, using the GZ action, more condensates can influence the dynamics. We have investigated in detail the following condensates: $\langle A^a_\mu A^a_\mu\rangle$, $\langle\bar\varphi^a_i\varphi^a_i\rangle$, $\langle\bar\varphi^a_i\bar\varphi^a_i\rangle$, and $\langle\varphi^a_i\varphi^a_i\rangle$ where the latter two were never investigated before. We have proven that we can renormalize the GZ action in the presence of these condensates. In particular, a renormalizable effective potential, compatible with the renormalization group, can be constructed for the associated local composite operators. Second, for the first time, we were able to calculate the one-loop effective potential in the LCO formalism,

$$\begin{aligned}\Gamma^{(1)} &= \frac{(N^2-1)^2}{16\pi^2}\left[(M^2-\sqrt{\rho\rho^\dagger})^2\ln\frac{M^2-\sqrt{\rho\rho^\dagger}}{\bar\mu^2}+(M^2+\sqrt{\rho\rho^\dagger})^2\ln\frac{M^2+\sqrt{\rho\rho^\dagger}}{\bar\mu^2}-2(M^2+\rho\rho^\dagger)\right]\\&\quad+\frac{3(N^2-1)}{64\pi^2}\left[-\frac{5}{6}(m^4-2\lambda^4)+y_1^2\ln\frac{(-y_1)}{\bar\mu}+y_2^2\ln\frac{(-y_2)}{\bar\mu}+y_3^2\ln\frac{(-y_3)}{\bar\mu}-y_4^2\ln\frac{(-y_4)}{\bar\mu}-y_5^2\ln\frac{(-y_5)}{\bar\mu}\right]\\&\quad-2(N^2-1)\frac{\lambda^4}{Ng^2}+\frac{3}{2}\frac{\lambda^4}{32\pi^2}(N^2-1)+\frac{1}{2}\frac{48(N^2-1)^2}{53N}\left(1-Ng^2\frac{53}{24}\frac{\rho_1}{(N^2-1)^2}\right)\frac{\rho\rho^\dagger}{g^2}\\&\quad+\frac{9}{13}\frac{N^2-1}{N}\frac{m^4}{2g^2}-\frac{24}{35}\frac{(N^2-1)^2}{N}\frac{M^4}{g^2}-\frac{161}{52}\frac{N^2-1}{16\pi^2}\frac{m^4}{2}-M^4\alpha_1+M^2m^2\chi_1,\end{aligned}\tag{96}$$

where y_1, y_2, and y_3 are the solutions of the equation

$$\begin{aligned}&y^3+(m^2+2M^2)y^2+(\lambda^4+M^4-\rho\rho^\dagger+2M^2m^2)y\\&\quad+M^2\lambda^4+1/2(\rho+\rho^\dagger)\lambda^4+M^4m^2-m^2\rho\rho^\dagger=0\end{aligned}$$

and y_4 and y_5 of the equation $y^2+2M^2y+M^4-\rho\rho^\dagger=0$. Unfortunately, due to the existence of yet unknown higher loop parameters, i.e. α_1, ρ_1, and χ_1, in the one-loop effective action, we are unable to give an estimate for the different condensates. Nevertheless, we have been able to already provide strong indications that some condensates are in fact nonzero and shall lower the effective action. We hope to come back to the explicit computation of the parameters α_1, ρ_1, and χ_1 in the future. In particular, one should compute the divergences of the vacuum diagram in Fig. 2, the similar one for the mixing, and other divergent two-loop diagrams stemming from the operators $\varphi\varphi$ and $\overline{\varphi\varphi}$. Once this task will be executed, all information is available to actually work out the one-loop effective potential and to investigate its structure and the associated formation of the RGZ condensates.

Third, we have also shown that in this further refined framework, the gluon propagator can be nonzero at zero momentum, and the ghost propagator can be nonenhanced.

A complementary approach to the current one is to find out to what extent a gluon propagator of the type (78) or ghost propagator of the type (80) could describe the lattice data, not only qualitatively, but also quantitatively. This is currently under investigation in [31] for different spacetime dimensions. In [4] it was already shown that a RGZ propagator (78) reproduces the SU(3) data very well.

Another question, which was not answered here, is whether $\sigma(k^2)$ [see Eq. (80)] is in fact smaller than 1. This is necessary in order to assure staying within the Gribov horizon. However, this question shall also be addressed in [31], and we refer to that paper for further details on this matter.

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APPENDIX A: RECAPITULATION OF THE GRIBOV-ZWANZIGER ACTION AND OF ITS RENORMALIZABILITY

In this appendix, we shall repeat the complete proof of the renormalization of the Gribov-Zwanziger action [30].

1. The Gribov-Zwanziger action and the BRST symmetry

We start with the Gribov-Zwanziger action,

$$S_{\rm GZ} = S_0 + S_\gamma, \tag{A1}$$

with

$$\begin{aligned} S_0 &= S_{\rm YM} + S_{\rm gf} + \int d^dx(\bar\varphi_i^a \partial_\mu(D_\mu^{ab}\varphi_i^b) - \bar\omega_i^a\partial_\mu(D_\mu^{ab}\omega_i^b) \\ &\quad - gf^{abc}\partial_\mu\bar\omega_i^a D_\mu^{bd}c^d\varphi_i^c), \\ S_\gamma &= -\gamma^2 g\int d^dx\left(f^{abc}A_\mu^a\varphi_\mu^{bc} + f^{abc}A_\mu^a\bar\varphi_\mu^{bc} + \frac{d}{g}(N^2-1)\gamma^2\right). \end{aligned} \tag{A2}$$

We recall that we have simplified the notation of the additional fields $(\bar\varphi_\mu^{ac}, \varphi_\mu^{ac}, \bar\omega_\mu^{ac}, \omega_\mu^{ac})$ in S_0 as S_0 displays a symmetry with respect to the composite index $i = (\mu, c)$. Therefore, we have set

$$(\bar\varphi_\mu^{ac}, \varphi_\mu^{ac}, \bar\omega_\mu^{ac}, \omega_\mu^{ac}) = (\bar\varphi_i^a, \varphi_i^a, \bar\omega_i^a, \omega_i^a). \tag{A3}$$

The BRST variations of all the fields are given by

$$\begin{aligned} &sA_\mu^a = -(D_\mu c)^a, \qquad sc^a = \frac{1}{2}gf^{abc}c^bc^c, \qquad s\bar c^a = b^a, \\ &sb^a = 0, \qquad s\varphi_i^a = \omega_i^a, \qquad s\omega_i^a = 0, \\ &s\bar\omega_i^a = \bar\varphi_i^a, \qquad s\bar\varphi_i^a = 0. \end{aligned} \tag{A4}$$

However, due to the γ dependent term, S_γ, the Gribov-Zwanziger action breaks the BRST symmetry softly [21,23]; see Eq. (11). In order to discuss the renormalizability of $S_{\rm GZ}$, we should treat the breaking as a composite operator to be introduced into the action by means of a suitable set of external sources. This procedure can be done in a BRST invariant way, by embedding $S_{\rm GZ}$ into a larger action, namely

$$\Sigma_{\rm GZ} = S_{\rm YM} + S_{\rm gf} + S_0 + S_s, \tag{A5}$$

where

$$\begin{aligned} S_s &= s\int d^dx(-U_\mu^{ai}D_\mu^{ab}\varphi_i^b - V_\mu^{ai}D_\mu^{ab}\bar\omega_i^b - U_\mu^{ai}V_\mu^{ai} + T_\mu^{ai}gf_{abc}D_\mu^{bd}c^d\bar\omega_i^c) \\ &= \int d^dx(-M_\mu^{ai}D_\mu^{ab}\varphi_i^b - gf^{abc}U_\mu^{ai}D_\mu^{bd}c^d\varphi_i^c + U_\mu^{ai}D_\mu^{ab}\omega_i^b - N_\mu^{ai}D_\mu^{ab}\bar\omega_i^b - V_\mu^{ai}D_\mu^{ab}\bar\varphi_i^b + gf^{abc}V_\mu^{ai}D_\mu^{bd}c^d\bar\omega_i^c \\ &\quad - M_\mu^{ai}V_\mu^{ai} + U_\mu^{ai}N_\mu^{ai} + R_\mu^{ai}gf^{abc}D_\mu^{bd}c^d\bar\omega_i^c + T_\mu^{ai}gf_{abc}D_\mu^{bd}c^d\bar\varphi_i^c). \end{aligned} \tag{A6}$$

We have introduced three new doublets (U_μ^{ai}, M_μ^{ai}), (V_μ^{ai}, N_μ^{ai}), and (T_μ^{ai}, R_μ^{ai}) with the following BRST transformations:

$$\begin{aligned} &sU_\mu^{ai} = M_\mu^{ai}, \qquad sM_\mu^{ai} = 0, \qquad sV_\mu^{ai} = N_\mu^{ai}, \\ &sN_\mu^{ai} = 0, \qquad sT_\mu^{ai} = R_\mu^{ai}, \qquad sR_\mu^{ai} = 0. \end{aligned} \tag{A7}$$

We have therefore a BRST symmetry at our disposal, made possible by introducing new sources which also have a BRST variation. However, we do not want to alter our original theory (A1). Therefore, at the end, we have to set the sources equal to the following values:

$$\begin{aligned} &U_\mu^{ai}|_{\rm phys} = N_\mu^{ai}|_{\rm phys} = T_\mu^{ai}|_{\rm phys} = 0, \\ &M_{\mu\nu}^{ab}|_{\rm phys} = V_{\mu\nu}^{ab}|_{\rm phys} = -R_{\mu\nu}^{ab}|_{\rm phys} = \gamma^2\delta^{ab}\delta_{\mu\nu}. \end{aligned} \tag{A8}$$

2. The Ward identities

Following the procedure of the algebraic renormalization outlined in [36], we should try to find as many Ward identities as possible. Before doing this, in order to be able to write the Slavnov-Taylor identity, we first have to couple all nonlinear BRST transformations to a new source. Looking at (A4), we see that only A_μ^a and c^a transform nonlinearly under the BRST s. Therefore, we add the following term to the action $\Sigma_{\rm GZ}$:

$$S_{\rm ext} = \int d^dx\left(-K_\mu^a(D_\mu c)^a + \frac{1}{2}gL^af^{abc}c^bc^c\right), \tag{A9}$$

with K_μ^a and L^a two new sources which shall be put to zero at the end,

$$K_\mu^a|_{\rm phys} = L^a|_{\rm phys} = 0. \tag{A10}$$

These sources are invariant under the BRST transformation,

$$sK_\mu^a = 0, \qquad sL^a = 0. \tag{A11}$$

The new action is therefore given by

$$\Sigma'_{\rm GZ} = \Sigma_{\rm GZ} + S_{\rm ext}. \tag{A12}$$

The next step is now to find the Ward identities obeyed by the action $\Sigma'_{\rm GZ}$. We have enlisted all the identities below:

(1) The Slavnov-Taylor identity is given by

$$\mathcal{S}(\Sigma'_{\rm GZ}) = 0, \tag{A13}$$

with

$$S(\Sigma'_{\rm GZ}) = \int d^d x\bigg(\frac{\delta\Sigma'_{\rm GZ}}{\delta K^a_\mu}\frac{\delta\Sigma'_{\rm GZ}}{\delta A^a_\mu} + \frac{\delta\Sigma'_{\rm GZ}}{\delta L^a}\frac{\delta\Sigma'_{\rm GZ}}{\delta c^a} + b^a\frac{\delta\Sigma'_{\rm GZ}}{\delta\bar c^a} + \bar\varphi^a_i\frac{\delta\Sigma'_{\rm GZ}}{\delta\bar\omega^a_i} + \omega^a_i\frac{\delta\Sigma'_{\rm GZ}}{\delta\varphi^a_i} + M^{ai}_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta U^{ai}_\mu} + N^{ai}_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta V^{ai}_\mu} + R^{ai}_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta T^{ai}_\mu}\bigg).$$

(2) The $U(f)$ invariance is given by

$$U_{ij}\Sigma'_{\rm GZ} = 0,$$
$$U_{ij} = \int d^d x\bigg(\varphi^a_i\frac{\delta}{\delta\varphi^a_j} - \bar\varphi^a_j\frac{\delta}{\delta\bar\varphi^a_i} + \omega^a_i\frac{\delta}{\delta\omega^a_j} - \bar\omega^a_j\frac{\delta}{\delta\bar\omega^a_i} - M^{aj}_\mu\frac{\delta}{\delta M^{ai}_\mu} - U^{aj}_\mu\frac{\delta}{\delta U^{ai}_\mu} + N^{ai}_\mu\frac{\delta}{\delta N^{aj}_\mu} + V^{ai}_\mu\frac{\delta}{\delta V^{aj}_\mu} + R^{aj}_\mu\frac{\delta}{\delta R^{ai}_\mu} + T^{aj}_\mu\frac{\delta}{\delta T^{ai}_\mu}\bigg). \tag{A14}$$

By means of the diagonal operator $Q_f = U_{ii}$, the i-valued fields and sources can be assigned an additional charge. One can find all quantum numbers in Tables I and II.

(3) The Landau gauge condition reads

$$\frac{\delta\Sigma'_{\rm GZ}}{\delta b^a} = \partial_\mu A^a_\mu. \tag{A15}$$

(4) The antighost equation yields

$$\frac{\delta\Sigma'_{\rm GZ}}{\delta\bar c^a} + \partial_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta K^a_\mu} = 0. \tag{A16}$$

TABLE I. Quantum numbers of the fields.

	A^a_μ	c^a	$\bar c^a$	b^a	φ^a_i	$\bar\varphi^a_i$	ω^a_i	$\bar\omega^a_i$
Dimension	1	0	2	2	1	1	1	1
Ghost number	0	1	−1	0	0	0	1	−1
Q_f charge	0	0	0	0	1	−1	1	−1

TABLE II. Quantum numbers of the sources.

U^{ai}_μ	M^{ai}_μ	N^{ai}_μ	V^{ai}_μ	R^{ai}_μ	T^{ai}_μ	K^a_μ	L^a
2	2	2	2	2	2	3	4
−1	0	1	0	0	−1	−1	−2
−1	−1	1	1	1	1	0	0

(5) The linearly broken local constraints yield

$$\frac{\delta\Sigma'_{\rm GZ}}{\delta\bar\varphi^a_i} + \partial_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta M^{ai}_\mu} + gf_{dba}T^{di}_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta K^{bi}_\mu} = gf^{abc}A^b_\mu V^{ci}_\mu,$$
$$\frac{\delta\Sigma'_{\rm GZ}}{\delta\omega^a_i} + \partial_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta N^{ai}_\mu} - gf^{abc}\bar\omega^b_i\frac{\delta\Sigma'_{\rm GZ}}{\delta b^c} = gf^{abc}A^b_\mu U^{ci}_\mu. \tag{A17}$$

(6) The exact $\mathcal{R}_{ij}$ symmetry reads

$$\mathcal{R}_{ij}\Sigma'_{\rm GZ} = 0, \tag{A18}$$

with

$$\mathcal{R}_{ij} = \int d^d x\bigg(\varphi^a_i\frac{\delta}{\delta\omega^a_j} - \bar\omega^a_j\frac{\delta}{\delta\bar\varphi^a_i} + V^{ai}_\mu\frac{\delta}{\delta N^{aj}_\mu} - U^{aj}_\mu\frac{\delta}{\delta M^{ai}_\mu} + T^{ai}_\mu\frac{\delta}{\delta R^{aj}_\mu}\bigg). \tag{A19}$$

.

(7) The integrated Ward identity is given by

$$\int d^d x\bigg(c^a\frac{\delta\Sigma'_{\rm GZ}}{\delta\omega^a_i} + \bar\omega^a_i\frac{\delta\Sigma'_{\rm GZ}}{\delta\bar c^a} + U^{ai}_\mu\frac{\delta\Sigma'_{\rm GZ}}{\delta K^a_\mu}\bigg) = 0. \tag{A20}$$

Here we should add that due to the presence of the sources T^{ai}_μ and R^{ai}_μ, the powerful ghost Ward identity [36] is broken, and we are unable to restore this identity. For the standard Yang-Mills theory, this identity has the following form:

$$\mathcal{G}^a(S_{\rm YM} + S_{\rm gf}) = \Delta^a_{\rm cl}, \tag{A21}$$

with

$$\mathcal{G}^a = \int d^d x\bigg(\frac{\delta}{\delta c^a} + gf^{abc}\bar c^b\frac{\delta}{\delta b^c}\bigg), \tag{A22}$$

and

$$\Delta^a_{\rm cl} = g\int d^4 x f^{abc}(K^b_\mu A^c_\mu - L^b c^c), \tag{A23}$$

i.e. a linear breaking. However, it shall turn out that this is not a problem for the renormalization procedure being undertaken; see later.

3. The counterterm

The next step in the algebraic renormalization is to translate all these symmetries, which are not anomalous, into constraints on the counterterm $\Sigma^c_{\rm GZ}$. This expression is an integrated polynomial of dimension four and with ghost number zero and is constructed from the fields and sources. The classical action $\Sigma'_{\rm GZ}$ changes under quantum corrections according to

$$\Sigma'_{\rm GZ} \to \Sigma'_{\rm GZ} + h\Sigma^c_{\rm GZ}, \tag{A24}$$

where h is the perturbation parameter. Demanding that the perturbed action $(\Sigma'_{\rm GZ} + h\Sigma^c_{\rm GZ})$ fulfills the same set of Ward identities obeyed by $\Sigma'_{\rm GZ}$, it follows that the counterterm $\Sigma^c_{\rm GZ}$ is constrained by the following identities:

(1) The linearized Slavnov-Taylor identity yields

$$\mathcal{B}\Sigma^c_{\rm GZ} = 0, \tag{A25}$$

with $\mathcal{B}$ the nilpotent linearized Slavnov-Taylor operator,

$$\mathcal{B} = \int d^4x\bigg(\frac{\delta\Sigma'_{\rm GZ}}{\delta K^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma'_{\rm GZ}}{\delta A^a_\mu}\frac{\delta}{\delta K^a_\mu} + \frac{\delta\Sigma'_{\rm GZ}}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma'_{\rm GZ}}{\delta c^a}\frac{\delta}{\delta L^a} + b^a\frac{\delta}{\delta\bar{c}^a} + \bar{\varphi}^a_i\frac{\delta}{\delta\bar{\omega}^a_i} + \omega^a_i\frac{\delta}{\delta\varphi^a_i} + M^{ai}_\mu\frac{\delta}{\delta U^{ai}_\mu} + N^{ai}_\mu\frac{\delta}{\delta V^{ai}_\mu} + R^{ai}_\mu\frac{\delta}{\delta T^{ai}_\mu}\bigg), \tag{A26}$$

and

$$\mathcal{B}^2 = 0. \tag{A27}$$

(2) The $U(f)$ invariance gives

$$U_{ij}\Sigma^c_{\rm GZ} = 0. \tag{A28}$$

(3) The Landau gauge condition

$$\frac{\delta\Sigma^c_{\rm GZ}}{\delta b^a} = 0. \tag{A29}$$

(4) The antighost equation

$$\frac{\delta\Sigma^c_{\rm GZ}}{\delta\bar{c}^a} + \partial_\mu\frac{\delta\Sigma^c_{\rm GZ}}{\delta K^a_\mu} = 0. \tag{A30}$$

(5) The linearly broken local constraints yield

$$\bigg(\frac{\delta}{\delta\bar{\varphi}^a_i} + \partial_\mu\frac{\delta}{\delta M^{ai}_\mu} + \partial_\mu\frac{\delta}{\delta M^{ai}_\mu} + gf_{abc}T^{bi}_\mu\frac{\delta}{\delta K^{ci}_\mu}\bigg)\Sigma^c_{\rm GZ} = 0,$$
$$\bigg(\frac{\delta}{\delta\omega^a_i} + \partial_\mu\frac{\delta}{\delta N^{ai}_\mu} - gf^{abc}\bar{\varphi}^b_i\frac{\delta}{\delta b^c}\bigg)\Sigma^c_{\rm GZ} = 0. \tag{A31}$$

(6) The exact $\mathcal{R}_{ij}$ symmetry imposes

$$\mathcal{R}_{ij}\Sigma^c_{\rm GZ} = 0, \tag{A32}$$

with $\mathcal{R}_{ij}$ given in (A19).

(7) Finally, the integrated Ward identity becomes

$$\int d^dx\bigg(c^a\frac{\delta\Sigma^c_{\rm GZ}}{\delta\omega^a_i} + \bar{\omega}^a_i\frac{\delta\Sigma^c_{\rm GZ}}{\delta\bar{c}^a} + U^{ai}_\mu\frac{\delta\Sigma^c_{\rm GZ}}{\delta K^a_\mu}\bigg) = 0. \tag{A33}$$

The most general counterterm $\Sigma^c_{\rm GZ}$ of $d = 4$, which obeys the linearized Slavnov-Taylor identity, has ghost number zero, and vanishing Q_f number, can be written as

$$\Sigma^c_{\rm GZ} = a_0S_{\rm YM} + \mathcal{B}\int d^dx\{a_1K^a_\mu A^a_\mu + a_2\partial_\mu\bar{c}^aA^a_\mu + a_3L^ac^a + a_4U^{ai}_\mu\partial_\mu\varphi^a_i + a_5V^{ai}_\mu\partial_\mu\bar{\omega}^a_i + a_6\bar{\omega}^a_i\partial^2\varphi^a_i + a_7U^{ai}_\mu V^{ai}_\mu + a_8gf^{abc}U^{ai}_\mu\varphi^b_iA^c_\mu + a_9gf^{abc}V^{ai}_\mu\bar{\omega}^b_iA^c_\mu + a_{10}gf^{abc}\bar{\omega}^a_iA^c_\mu\partial_\mu\varphi^b_i + a_{11}gf^{abc}\bar{\omega}^a_i(\partial_\mu A^c_\mu)\varphi^b_i + b_1R^{ai}_\mu U^{ai}_\mu + b_2T^{ai}_\mu M^{ai}_\mu + b_3gf_{abc}R^{ai}_\mu\bar{\omega}^b_iA^c_\mu + b_4gf_{abc}T^{ai}_\mu\bar{\varphi}^b_iA^c_\mu + b_5R^{ai}_\mu\partial_\mu\bar{\omega}^a_i + b_6T^{ai}_\mu\partial_\mu\bar{\varphi}^a_i\}, \tag{A34}$$

with $a_0, \ldots, a_{11}, b_1, \ldots, b_6$ arbitrary parameters. Now we can impose the constraints on the counterterm. First, although the ghost Ward identity (A21) is broken, we know that this is not so in the standard Yang-Mills case. Therefore, we can already set $a_3 = 0$ as this term is not allowed in the counterterm of the standard Yang-Mills action, which is a special case of the action we are studying.[6] Second, due to the Landau gauge condition (3) and the antighost equation (4) we find

$$a_1 = a_2. \tag{A35}$$

Next, the linearly broken constraints (5) give the following relations:

$$a_1 = -a_8 = -a_9 = a_{10} = a_{11} = -b_3 = b_4, \qquad a_4 = a_5 = -a_6 = a_7, \qquad b_1 = b_2 = b_5 = b_6 = 0. \tag{A36}$$

[6]In particular, since we will always assume the use of a mass independent renormalization scheme, we may compute a_3 with all external mass scales (= sources) equal to zero. Said otherwise, a_3 is completely determined by the dynamics of the original Yang-Mills action, in which case it is known to vanish to all orders.

The R_{ij} symmetry (6) does not give any new information, while the integrated Ward identity (7) relates the two previous strings of parameters,

$$a_1 = -a_8 = -a_9 = a_{10} = a_{11} = -b_3 = b_4 \equiv a_3 = a_4 = -a_5 = a_6. \tag{A37}$$

Taking all this information together, we obtain the following counterterm:

$$\Sigma^c = a_0 S_{\rm YM} + a_1 \int d^d x \bigg(A^a_\mu \frac{\delta S_{\rm YM}}{\delta A^a_\mu} + \partial_\mu \bar{c}^a \partial_\mu c^a + K^a_\mu \partial_\mu c^a + M^{ai}_\mu \partial_\mu \varphi^a_i - U^{ai}_\mu \partial_\mu \omega^a_i + N^{ai}_\mu \partial_\mu \bar{\omega}^a_i + V^{ai}_\mu \partial_\mu \bar{\varphi}^a_i + \partial_\mu \bar{\varphi}^a_i \partial_\mu \varphi^a_i + \partial_\mu \omega^a_i \partial_\mu \bar{\omega}^a_i + V^{ai}_\mu M^{ai}_\mu - U^{ai}_\mu N^{ai}_\mu - g f_{abc} U^{ai}_\mu \varphi^b_i \partial_\mu c^c - g f_{abc} V^{ai}_\mu \bar{\omega}^b_i \partial_\mu c^c - g f_{abc} \partial_\mu \bar{\omega}^a_i \varphi^b_i \partial_\mu c^c - g f_{abc} R^{ai}_\mu \partial_\mu c^b \bar{\omega}^c_i + g f_{abc} T^{ai}_\mu \partial_\mu c^b \bar{\varphi}^c_i \bigg). \tag{A38}$$

4. The renormalization factors

As a final step, we have to show that the counterterm (A38) can be reabsorbed by means of a multiplicative renormalization of the fields and sources. If we try to absorb the counterterm into the original action, we easily find

$$Z_g = 1 - h\frac{a_0}{2}, \qquad Z_A^{1/2} = 1 + h\left(\frac{a_0}{2} + a_1\right) \tag{A39}$$

and

$$Z_{\bar{c}}^{1/2} = Z_c^{1/2} = Z_A^{-1/4} Z_g^{-1/2} = 1 - h\frac{a_1}{2}, \qquad Z_b = Z_A^{-1}, \qquad Z_K = Z_c^{1/2}, \qquad Z_L = Z_A^{1/2}. \tag{A40}$$

The results (A39) are already known from the renormalization of the original Yang-Mills action in the Landau gauge [36]. Further, we also obtain

$$\begin{aligned} &Z_\varphi^{1/2} = Z_{\bar{\varphi}}^{1/2} = Z_g^{-1/2} Z_A^{-1/4} = 1 - h\frac{a_1}{2}, \\ &Z_\omega^{1/2} = Z_A^{-1/2}, \qquad Z_{\bar{\omega}}^{1/2} = Z_g^{-1}, \\ &Z_M = 1 - \frac{a_1}{2} = Z_g^{-1/2} Z_A^{-1/4}, \qquad Z_N = Z_A^{-1/2}, \\ &Z_U = 1 + h\frac{a_0}{2} = Z_g^{-1}, \qquad Z_V = 1 - h\frac{a_1}{2} = Z_g^{-1/2} Z_A^{-1/4}, \\ &Z_T = 1 + h\frac{a_0}{2} = Z_g^{-1}, \qquad Z_R = 1 - h\frac{a_1}{2} = Z_g^{-1/2} Z_A^{-1/4}. \end{aligned} \tag{A41}$$

This concludes the proof of the renormalizability of the action (A1) which is the physical limit of $\Sigma'_{\rm GZ}$. Notice that in the physical limit (A8), we have that

$$Z_{\gamma^2} = Z_g^{-1/2} Z_A^{-1/4}. \tag{A42}$$

APPENDIX B. INCLUSION OF THE OPERATOR A^2 IN THE GRIBOV-ZWANZIGER ACTION

For the benefit of the reader, let us also repeat the renormalization of the operator A^2 in the Gribov-Zwanziger action, which was first tackled in [37]. In that paper, it was shown that the presence of the condensate $\langle A^2 \rangle$ does not spoil the renormalizability of the GZ action. The GZ action with inclusion of the local composite operator $A^a_\mu A^a_\mu$ is given by

$$S_{\rm AGZ} = S_{\rm GZ} + S_{A^2}, \tag{B1}$$

where

$$S_{A^2} = \int d^d x \left(\frac{\tau}{2} A^a_\mu A^a_\mu - \frac{\zeta}{2}\tau^2 \right), \tag{B2}$$

with τ a new source invariant under the BRST transformation s and ζ a new parameter. The renormalization can be done very easily with the help of the previous section.

1. The starting action and the BRST

Again, we shall make $S_{\rm AGZ}$ BRST invariant. We define

$$\Sigma_{\rm AGZ} = \Sigma'_{\rm GZ} + \Sigma_{A^2} \tag{B3}$$

where $\Sigma'_{\rm GZ}$ is given in expression (A12) and

$$\Sigma_{A^2} = \int d^4 x s \left(\frac{\eta}{2} A^a_\mu A^a_\mu - \frac{\zeta}{2}\tau^2 \right) = \int d^4 x \left[\frac{1}{2}\tau A^a_\mu A^a_\mu + \eta A^a_\mu \partial_\mu c^a - \frac{1}{2}\zeta\tau^2 \right], \tag{B4}$$

with η a new source and $s\eta = \tau$, so that (η, τ) forms a doublet. At the end, we replace all the sources with their physical values [see expressions (A8) and (A10)] and in addition

$$\eta|_{\rm phys} = 0, \tag{B5}$$

so one recovers $S_{\rm AGZ}$ again.

2. The Ward identities

It is now easily checked that the Ward identities Eqs. (A13)–(A20) of Appendix A 2 remain preserved. Obviously, the Slavnov-Taylor identity receives an extra term,

$$\mathcal{S}(\Sigma_{\rm AGZ}) = 0, \tag{B6}$$

where

$$\begin{aligned} S(\Sigma_{\rm AGZ}) = \int d^d x\Big(& \frac{\delta\Sigma_{\rm AGZ}}{\delta K^a_\mu}\frac{\delta\Sigma_{\rm AGZ}}{\delta A^a_\mu} + \frac{\delta\Sigma_{\rm AGZ}}{\delta L^a}\frac{\delta\Sigma_{\rm AGZ}}{\delta c^a} \\ & + b^a\frac{\delta\Sigma_{\rm AGZ}}{\delta\bar{c}^a} + \bar{\varphi}^a_i\frac{\delta\Sigma_{\rm AGZ}}{\delta\bar{\omega}^a_i} + \omega^a_i\frac{\delta\Sigma_{\rm AGZ}}{\delta\varphi^a_i} \\ & + M^{ai}_\mu\frac{\delta\Sigma_{\rm AGZ}}{\delta U^{ai}_\mu} + N^{ai}_\mu\frac{\delta\Sigma_{\rm AGZ}}{\delta V^{ai}_\mu} + R^{ai}_\mu\frac{\delta\Sigma_{\rm AGZ}}{\delta T^{ai}_\mu} \\ & + \tau\frac{\delta\Sigma_{\rm AGZ}}{\delta\eta}\Big). \end{aligned}$$

3. The counterterm

As all the Ward identities remain the same, it is easy to check that the counterterm is given by

$$\begin{aligned} \Sigma^c_{\rm AGZ} = \Sigma^c_{\rm GZ} + \int d^4x\Big(& \frac{a_2}{2}\tau A^a_\mu A^a_\mu + \frac{a_3}{2}\zeta\tau^2 \\ & + (a_2 - a_1)\eta A^a_\mu\partial_\mu c^a\Big). \end{aligned} \tag{B7}$$

where $\Sigma^c_{\rm GZ}$ is the counterterm (A38). This counterterm can be absorbed in the original action, $\Sigma_{\rm AGZ}$, leading to the same renormalization factors as in Eqs. (A39)–(A41).

In addition Z_τ is related to Z_g and $Z_A^{1/2}$ [37],

$$Z_\tau = Z_g Z_A^{-1/2}, \tag{B8}$$

and Z_ζ and Z_η are given by

$$\begin{aligned} Z_\zeta &= 1 + h(-a_3 - 2a_2 + 4a_1 - 2a_0), \\ Z_\eta &= 1 + h\Big(\frac{a_0}{2} - \frac{3}{2}a_1 + a_2\Big). \end{aligned} \tag{B9}$$

APPENDIX C: RENORMALIZATION OF THE FURTHER REFINED ACTION

1. The starting action

Let us repeat the starting action (14),

$$\begin{aligned} \Sigma_{\rm CGZ} = {} & \Sigma'_{\rm GZ} + \Sigma_{A^2} + S_{\varphi\bar{\varphi}} + S_{\bar{\omega}\omega} + S_{\overline{\varphi\varphi},\overline{\omega\varphi}} \\ & + S_{\varphi\varphi,\omega\varphi} + S_{\rm vac}, \end{aligned} \tag{C1}$$

where $\Sigma'_{\rm GZ}$ is given by Eq. (A12), Σ_{A^2} by (B4), and

$$\begin{aligned} S_{\varphi\bar{\varphi}} &= \int d^4x s(P\bar{\varphi}^a_i\varphi^a_i) = \int d^4x[Q\bar{\varphi}^a_i\varphi^a_i - P\bar{\varphi}^a_i\omega^a_i], \\ S_{\bar{\omega}\omega} &= \int d^4x s(V\bar{\omega}^a_i\omega^a_i) = \int d^4x[W\bar{\omega}^a_i\omega^a_i - V\bar{\varphi}^a_i\omega^a_i], \\ S_{\overline{\varphi\varphi},\overline{\omega\varphi}} &= \frac{1}{2}\int d^4x s(\bar{G}^{ij}\bar{\omega}^a_i\bar{\varphi}^a_j) = \int d^4x\Big[\bar{H}^{ij}\bar{\omega}^a_i\bar{\varphi}^a_j + \frac{1}{2}\bar{G}^{ij}\bar{\varphi}^a_i\bar{\varphi}^a_j\Big], \\ S_{\varphi\varphi,\omega\varphi} &= \frac{1}{2}\int d^4x s(H^{ij}\varphi^a_i\varphi^a_j) = \int d^4x\Big[\frac{1}{2}G^{ij}\varphi^a_i\varphi^a_j - H^{ij}\omega^a_i\varphi^a_j\Big], \\ S_{\rm vac} &= \int d^4x\Big[\kappa(G^{ij}\bar{G}^{ij} - 2H^{ij}\bar{H}^{ij}) + \lambda(G^{ii}\bar{G}^{jj} - 2H^{ii}\bar{H}^{jj})\Big] - \int d^4x[\alpha(QQ + QW) + \beta(QW + WW) + \chi Q\tau + \delta W\tau]. \end{aligned} \tag{C2}$$

2. The Ward identities

With the help of Appendix A, we can easily summarize all Ward identities obeyed by the action $\Sigma_{\rm CGZ}$:

(1) The Slavnov-Taylor identity reads

$$\mathcal{S}(\Sigma_{\rm CGZ}) = 0, \tag{C3}$$

with

$$\begin{aligned} \mathcal{S}(\Sigma_{\rm CGZ}) = \int d^4x\Big(& \frac{\delta\Sigma_{\rm CGZ}}{\delta K^a_\mu}\frac{\delta\Sigma_{\rm CGZ}}{\delta A^a_\mu} + \frac{\delta\Sigma_{\rm CGZ}}{\delta L^a}\frac{\delta\Sigma_{\rm CGZ}}{\delta c^a} + b^a\frac{\delta\Sigma_{\rm CGZ}}{\delta\bar{c}^a} + \bar{\varphi}^a_i\frac{\delta\Sigma_{\rm CGZ}}{\delta\bar{\omega}^a_i} + \omega^a_i\frac{\delta\Sigma_{\rm CGZ}}{\delta\varphi^a_i} + M^{ai}_\mu\frac{\delta\Sigma_{\rm CGZ}}{\delta U^{ai}_\mu} \\ & + N^{ai}_\mu\frac{\delta\Sigma_{\rm CGZ}}{\delta V^{ai}_\mu} + R^{ai}_\mu\frac{\delta\Sigma_{\rm CGZ}}{\delta T^{ai}_\mu} + Q\frac{\delta\Sigma_{\rm CGZ}}{\delta P} + W\frac{\delta\Sigma_{\rm CGZ}}{\delta V} + \tau\frac{\delta\Sigma_{\rm CGZ}}{\delta\eta} + 2\bar{H}^{ij}\frac{\delta\Sigma_{\rm CGZ}}{\delta\bar{G}^{ij}} + G^{ij}\frac{\delta\Sigma_{\rm CGZ}}{\delta H^{ij}}\Big). \end{aligned}$$

(2) For the $U(f)$ invariance we now have

$$U_{ij}\Sigma_{\rm CGZ} = 0, \tag{C4}$$

where

$$U_{ij} = \int d^4x\Big(\varphi_i^a \frac{\delta}{\delta\varphi_j^a} - \bar{\varphi}_j^a \frac{\delta}{\delta\bar{\varphi}_i^a} + \omega_i^a \frac{\delta}{\delta\omega_j^a} - \bar{\omega}_j^a \frac{\delta}{\delta\bar{\omega}_i^a} - M_\mu^{aj} \frac{\delta}{\delta M_\mu^{ai}} - U_\mu^{aj} \frac{\delta}{\delta U_\mu^{ai}} + N_\mu^{ai} \frac{\delta}{\delta N_\mu^{aj}} + V_\mu^{ai} \frac{\delta}{\delta V_\mu^{aj}} + R_\mu^{aj} \frac{\delta}{\delta R_\mu^{ai}}$$
$$+ T_\mu^{aj} \frac{\delta}{\delta T_\mu^{ai}} + 2\bar{G}^{ki} \frac{\delta}{\delta\bar{G}^{kj}} - 2G^{kj} \frac{\delta}{\delta G^{ki}} + 2\bar{H}^{ki} \frac{\delta}{\delta\bar{H}^{kj}} - 2H^{kj} \frac{\delta}{\delta H^{ki}}\Big).$$

By means of the diagonal operator $Q_f = U_{ii}$, the single i-valued fields and sources still turn out to possess an additional quantum number.

(3) The Landau gauge condition and the antighost equation are given by

$$\frac{\delta\Sigma_{\rm CGZ}}{\delta b^a} = \partial_\mu A_\mu^a, \tag{C5}$$

$$\frac{\delta\Sigma_{\rm CGZ}}{\delta\bar{c}^a} + \partial_\mu \frac{\delta\Sigma_{\rm CGZ}}{\delta K_\mu^a} = 0. \tag{C6}$$

(4) The linearly broken local constraints yield

$$\begin{aligned} &\frac{\delta\Sigma_{\rm CGZ}}{\delta\bar{\varphi}_i^a} + \partial_\mu \frac{\delta\Sigma_{\rm CGZ}}{\delta M_\mu^{ai}} + gf_{dba}T_\mu^{di} \frac{\delta\Sigma_{\rm CGZ}}{\delta K_\mu^{bi}} \\ &\quad = gf^{abc}A_\mu^b V_\mu^{ci} + \cdots, \\ &\frac{\delta\Sigma_{\rm CGZ}}{\delta\omega_i^a} + \partial_\mu \frac{\delta\Sigma_{\rm CGZ}}{\delta N_\mu^{ai}} - gf^{abc}\bar{\omega}_i^b \frac{\delta\Sigma_{\rm CGZ}}{\delta b^c} \\ &\quad = gf^{abc}A_\mu^b U_\mu^{ci} + \cdots. \end{aligned} \tag{C7}$$

where the $\cdots$ are extra linear breaking terms irrelevant for our purposes.

(5) The exact $\mathcal{R}_{ij}$ symmetry is *broken* beyond simple repair.

(6) The integrated Ward Identity is *broken* also beyond simple repair.

(7) There is however a new identity,

$$\frac{\delta\Sigma_{\rm CGZ}}{\delta P} = \frac{\delta\Sigma_{\rm CGZ}}{\delta V}. \tag{C8}$$

TABLE III. Quantum numbers of the fields and sources.

	A_μ^a	c^a	$\bar{c}^a$	b^a	φ_i^a	$\bar{\varphi}_i^a$	ω_i^a	$\bar{\varphi}_i^a$	U_μ^{ai}	M_μ^{ai}	N_μ^{ai}	V_μ^{ai}
Dimension	1	0	2	2	1	1	1	1	2	2	2	2
Ghost number	0	1	−1	0	0	0	1	−1	−1	0	1	0
Q_f charge	0	0	0	0	1	−1	1	−1	−1	−1	1	1

	R_μ^{ai}	T_μ^{ai}	K_μ^a	L^a	Q	P	W	V	τ	η	G^{ij}	$\bar{G}^{ij}$	H^{ij}	$\bar{H}^{ij}$
Dimension	2	2	3	4	2	2	2	2	2	2	2	2	2	2
Ghost number	0	−1	−1	−2	0	−1	0	−1	0	−1	0	0	−1	1
Q_f charge	1	1	0	0	0	0	0	0	0	0	−2	2	−2	2

3. The counterterm

These identities (C3)–(C8) can be translated into constraints on the counterterm according to the quantum action principle (QAP); see [36]. Unfortunately, many identities are broken due to the introduction of these $d = 2$ operators. However, we are using mass independent renormalization schemes and, therefore, the new massive sources (P, Q, V, W, G^{ij}, $\bar{G}^{ij}$, H^{ij}, $\bar{H}^{ij}$) cannot influence the counterterm of the original GZ action (A38) since they are coupled to $d = 2$ operators. Said otherwise, there are no new vertices capable of destroying the UV structure of the original GZ theory (A38). We only need to check whether these operators themselves are renormalizable. Thus, the counterterm is given by

$$\Sigma_{\rm CGZ}^c = \Sigma_{\rm GZ}^c + \Sigma_A^c + \Sigma_{P\text{-}H}^c, \tag{C9}$$

with $\Sigma_{\rm GZ}^c$ given by Eq. (A38) and Σ_A^c given by

$$\Sigma_A^c = \int d^4x\Big(\frac{a_2}{2}\tau A_\mu^a A_\mu^a + \frac{a_3}{2}\zeta\tau^2 + (a_2 - a_1)\eta A_\mu^a \partial_\mu c^a\Big), \tag{C10}$$

as already determined in (B7). $\Sigma_{P\ldots H}^c$ is dependent of all the sources (P, Q, V, W, G^{ij}, $\bar{G}^{ij}$, H^{ij}, $\bar{H}^{ij}$), is of dimension 4, ghost number -1 and $Q_f = 0$, and obeys the remaining Ward identities. For completeness, we have enlisted relevant quantum numbers of all fields and sources in Table III. Because of the linearly broken constraints we find

$$\begin{aligned} &\frac{\partial\Sigma_{P\text{-}H}^c}{\partial\varphi} = 0, \qquad \frac{\partial\Sigma_{P\text{-}H}^c}{\partial\bar{\varphi}} = 0, \\ &\frac{\partial\Sigma_{P\text{-}H}^c}{\partial\omega} = 0, \qquad \frac{\partial\Sigma_{P\text{-}H}^c}{\partial\bar{\omega}} = 0. \end{aligned} \tag{C11}$$

Therefore,

$$\begin{aligned} \Sigma_{P\text{-}H}^c = \mathcal{B}_\Sigma \int d^4x(&b_1 PA_\mu^a A_\mu^a + b_2 VA_\mu^\alpha A_\mu^\alpha + b_3 QP \\ &+ b_4 QV + b_5 WP + b_6 WV + b_7 P\tau + b_8 V\tau \\ &+ b_9 Q\eta + b_{10} W\eta + c_1 H^{ij}\bar{G}^{ij} + c_2 H^{ii}\bar{G}^{jj}), \end{aligned} \tag{C12}$$

where $b_1, \ldots, c_2$ are arbitrary constants. By invoking the new identity

$$\frac{\delta\Sigma_{P\text{-}H}^c}{\delta P} = \frac{\delta\Sigma_{P\text{-}H}^c}{\delta V}, \tag{C13}$$

we can write

$$\Sigma^c_{P\text{-}H} = \int d^4x b_1[(Q+W)A^a_\mu A^a_\mu + 2(P+V)\partial_\mu c^a A^a_\mu] + b_3 QQ + b_4 QW + b_6 WW + b_7 Q\tau + b_8 W\tau + c_1(G^{ij}\bar{G}^{ij} - 2H^{ij}\bar{H}^{ij}) + c_2(G^{ii}\bar{G}^{jj} - 2H^{ii}\bar{H}^{jj}). \quad \text{(C14)}$$

Let us notice that due to the $U(f)$ constraint, the term in c_2 is only present when

$$G^{ij}\bar{G}^{qq} + 2H^{pp}\bar{H}^{ij} = G^{qq}\bar{G}^{ij} + 2H^{ij}\bar{H}^{qq}, \quad \text{(C15)}$$

which is indeed the case due to Hermiticity.

4. The renormalization factors

Let us now try to reabsorb this counterterm into the starting action (14). We shall split this analysis into three parts, according to

$$\Sigma^c_A + \Sigma^c_{P\text{-}H} = \Sigma^c_I + \Sigma^c_{II} + \Sigma^c_{III}, \quad \text{(C16)}$$

where

$$\Sigma^c_I = \int d^4x c_1(G^{ij}\bar{G}^{ij} - 2H^{ij}\bar{H}^{ij}) + c_2(G^{ii}\bar{G}^{jj} - 2H^{ii}\bar{H}^{jj}),$$
$$\Sigma^c_{II} = \int d^4x b_1[(Q+W)A^a_\mu A^a_\mu + 2(P+V)\partial_\mu c^a A^a_\mu] + \frac{a_2}{2}\tau A^a_\mu A^a_\mu + (a_2 - a_1)\eta A^a_\mu \partial_\mu c^a,$$
$$\Sigma^c_{III} = \int d^4x b_3 QQ + b_4 QW + b_6 WW + b_7 Q\tau + b_8 W\tau + \frac{a_3}{2}\zeta\tau^2 \quad \text{(C17)}$$

are the three parts which we shall try to absorb separately.

First, we start with the vacuum counterterm connected to the arbitrary parameters c_1 and c_2. If we redefine c_1 and c_2, we can write

$$\Sigma^c_I = \int d^4x c_1\kappa(G^{ij}\bar{G}^{ij} - 2H^{ij}\bar{H}^{ij}) + c_2\lambda(G^{ii}\bar{G}^{jj} - 2H^{ii}\bar{H}^{jj}), \quad \text{(C18)}$$

and if we define

$$\bar{H}^{ij}_0 = Z_{\bar{H}}\bar{H}^{ij}, \quad H^{ij}_0 = Z_H H^{ij}, \quad \bar{G}^{ij}_0 = Z_{\bar{G}}\bar{G}^{ij},$$
$$G^{ij}_0 = Z_G\bar{G}^{ij}, \quad \kappa_0 = Z_\kappa\kappa, \quad \lambda_0 = Z_\lambda\lambda, \quad \text{(C19)}$$

we find for the renormalization factors of the new sources and the LCO parameters κ and λ,

$$Z_{\bar{H}} = Z_{\bar{\varphi}}^{-1/2}Z_{\bar{\omega}}^{-1/2}, \quad Z_{\bar{G}} = Z_{\bar{\varphi}}^{-1},$$
$$Z_H = Z_\varphi^{-1/2}Z_\omega^{-1/2}, \quad Z_G = Z_\varphi^{-1},$$
$$Z_\kappa = (1+c_1)Z_{\bar{G}}^{-1}Z_G^{-1} = (1+c_1)Z_{\bar{H}}^{-1}Z_H^{-1},$$
$$Z_\lambda = (1+c_2)Z_{\bar{G}}^{-1}Z_G^{-1} = (1+c_2)Z_{\bar{H}}^{-1}Z_H^{-1}, \quad \text{(C20)}$$

and thus the part Σ^c_I can absorbed in the starting action.

Second, let us focus on Σ^c_{II},

$$\Sigma^c_{II} = \int d^4x b_1[(Q+W)A^a_\mu A^a_\mu + 2(P+V)\partial_\mu c^a A^a_\mu] + \frac{a_2}{2}\tau A^a_\mu A^a_\mu + (a_2 - a_1)\eta A^a_\mu \partial_\mu c^a. \quad \text{(C21)}$$

We propose the following mixing matrix:

$$\begin{pmatrix} Q_0 \\ W_0 \\ \tau_0 \end{pmatrix} = \begin{pmatrix} Z_{QQ} & Z_{QW} & Z_{Q\tau} \\ Z_{WQ} & Z_{WW} & Z_{W\tau} \\ Z_{\tau Q} & Z_{\tau W} & Z_{\tau\tau} \end{pmatrix}\begin{pmatrix} Q \\ W \\ \tau \end{pmatrix}. \quad \text{(C22)}$$

(i) From

$$Q_0\varphi^a_{i,0}\bar{\varphi}^a_{i,0} = [Z_{QQ}Q + Z_{QW}W + Z_{Q\tau}\tau]Z_\varphi\varphi^a_i\bar{\varphi}^a_i = Q\varphi^a_i\bar{\varphi}^a_i, \quad \text{(C23)}$$

we find that $Z_{QQ} = Z_{\bar{\varphi}}^{-1}$, while $Z_{QW} = Z_{Q\tau} = 0$.

(ii) From

$$W_0\bar{\omega}^a_{i,0}\omega^a_{i,0} = [Z_{WQ}Q + Z_{WW}W + Z_{W\tau}\tau]Z_\varphi\bar{\omega}^a_i\omega^a_i = W\varphi^a_i\bar{\varphi}^a_i, \quad \text{(C24)}$$

we find that $Z_{WW} = Z_{\bar{\varphi}}^{-1}$, while $Z_{WQ} = Z_{W\tau} = 0$.

(iii) Finally, from

$$\frac{1}{2}\tau_0 A^a_{\mu,0}A^a_{\mu,0}, = \frac{1}{2}[Z_{\tau Q}Q + Z_{\tau W}W + Z_{\tau\tau}\tau]Z_A A^a_\mu A^a_\mu = \frac{1}{2}(1+a_2)\tau A^a_\mu A^\mu_a + b_1 Q A^a_\mu A^a_\mu + b_1 W A^a_\mu A^a_\mu, \quad \text{(C25)}$$

we obtain $Z_{\tau\tau} = Z_\tau = (1+a_2)Z_A^{-1}$, and $Z_{\tau Q} = Z_{\tau W} = 2b_1$.

In summary, we find the following matrix:

$$\begin{pmatrix} Q_0 \\ W_0 \\ \tau_0 \end{pmatrix} = \begin{pmatrix} Z_\varphi^{-1} & 0 & 0 \\ 0 & Z_\varphi^{-1} & 0 \\ Z_{\tau W} & Z_{\tau W} & Z_{\tau\tau} \end{pmatrix}\begin{pmatrix} Q \\ W \\ \tau \end{pmatrix}. \quad \text{(C26)}$$

Now that we have the mixing matrix at our disposal, we can pass to the corresponding bare operators by taking the inverse of this matrix,

$$\begin{pmatrix} Q \\ W \\ \tau \end{pmatrix} = \begin{pmatrix} Z_\varphi & 0 & 0 \\ 0 & Z_\varphi & 0 \\ -\frac{Z_{\tau W} Z_\varphi}{Z_{\tau\tau}} & -\frac{Z_{\tau W} Z_\varphi}{Z_{\tau\tau}} & \frac{1}{Z_{\tau\tau}} \end{pmatrix} \begin{pmatrix} Q_0 \\ W_0 \\ \tau_0 \end{pmatrix}. \tag{C27}$$

Subsequently, we can derive the corresponding mixing matrix for the operators, since insertions of an operator correspond to derivatives with respect to the appropriate source of the generating functional $Z^c(Q, W, \tau)$. In particular,

$$\begin{aligned} \frac{1}{2} A_0^2 &= \frac{\delta Z^c(Q, W, \tau)}{\delta \tau_0} \bigg|_{\tau_0 = 0} \\ &= \frac{\delta Q}{\delta \tau_0} \frac{\delta Z^c(Q, W, \tau)}{\delta Q} + \frac{\delta W}{\delta \tau_0} \frac{\delta Z^c(Q, W, \tau)}{\delta W} \\ &\quad + \frac{\delta \tau}{\delta \tau_0} \frac{\delta Z^c(Q, W, \tau)}{\delta \tau} \Rightarrow A_0^2 = \frac{1}{Z_{\tau\tau}} A^2, \end{aligned} \tag{C28}$$

and similarly for $\bar{\varphi}^a_{i,0} \varphi^a_{i,0}$ and $\bar{\varphi}^a_{i,0} \omega^a_{i,0}$. We thus need to take the transpose of the previous matrix,

$$\begin{pmatrix} \bar{\varphi}^a_{i,0} \varphi^a_{i,0} \\ \bar{\varphi}^a_{i,0} \omega^a_{i,0} \\ A_0^2 \end{pmatrix} = \begin{pmatrix} Z_\varphi & 0 & -\frac{Z_{\tau W} Z_\varphi}{Z_{\tau\tau}} \\ 0 & Z_\varphi & -\frac{Z_{\tau W} Z_\varphi}{Z_{\tau\tau}} \\ 0 & 0 & \frac{1}{Z_{\tau\tau}} \end{pmatrix} \begin{pmatrix} \bar{\varphi}^a_i \varphi^a_i \\ \bar{\varphi}^a_i \omega^a_i \\ A^2 \end{pmatrix}. \tag{C29}$$

We can make some observations from this matrix. First, we find that A_0^2 does not contain the operators $\bar{\varphi}^a_i \varphi^a_i$ and $\bar{\omega}^a_i \omega^a_i$. This is already a first check on our results as without these latter two operators the GZ action including A^2 is renormalizable, as we have shown already in Appendix B. Second, we observe that

$$\bar{\varphi}\,^a_{i,0} \varphi^a_{i,0} - \bar{\omega}^a_{i,0} \omega^a_{i,0} = Z_\varphi (\bar{\varphi}^a_i \varphi^a_i - \bar{\omega}^a_i \omega^a_i), \tag{C30}$$

meaning that the mixing with A^2 disappears again when recombining the two operators in a certain way. In fact, this is the operator $(\bar{\varphi}^a_i \varphi^a_i - \bar{\omega}^a_i \omega^a_i)$ which we have investigated using the RGZ action [23] and no mixing with A^2 appears for this operator.

We can do a completely analogous reasoning for the part in $\partial_\mu c^a A^a_\mu$. We first set $V + P = X$. We propose

$$\begin{pmatrix} X_0 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} Z_{XX} & Z_{X\eta} \\ Z_{\eta X} & Z_{\eta\eta} \end{pmatrix} \begin{pmatrix} X \\ \eta \end{pmatrix}. \tag{C31}$$

(i) From

$$\begin{aligned} -(X_0)[\bar{\varphi}^a_{i,0} \omega^a_{i,0}] &= -[Z_{XX} X + Z_{X\eta} \eta] Z_{\bar{\varphi}}^{1/2} Z_\omega^{1/2} \bar{\varphi}^a_i \omega^a_i \\ &= -X[\bar{\varphi}^a_i \omega^a_i] \end{aligned}$$

we find that $Z_{XX} = Z_{\bar{\varphi}}^{-1/2} Z_\omega^{-1/2}$, while $Z_{X\eta} = 0$.

(ii) Also, from

$$\begin{aligned} \eta_0 A^a_{\mu,0} \partial_\mu c^a_0 &= [Z_{\eta X} X + Z_{\eta\eta} \eta] Z_A^{1/2} Z_c^{1/2} A^a_\mu \partial_\mu c^a \\ &= (1 + a_2 - a_1) \eta A^a_\mu \partial_\mu c^a \\ &\quad + 2 b_1 X A^a_\mu \partial_\mu c^a, \end{aligned}$$

we obtain $Z_{\eta\eta} = Z_\eta = (1 + a_2 - a_1) Z_A^{-1/2} Z_c^{-1/2}$, and $Z_{\eta X} = 2 b_1$.

Therefore, we find that

$$\begin{pmatrix} \bar{\varphi}^a_{i,0} \omega^a_{i,0} \\ A_{\mu,0} \partial_\mu c_0 \end{pmatrix} = \begin{pmatrix} Z_A^{1/2} Z_c^{1/2} & -2 b_1 \\ 0 & Z_\eta^{-1} \end{pmatrix} \begin{pmatrix} \bar{\varphi}^a_i \omega^a_i \\ A_\mu \partial_\mu c \end{pmatrix}. \tag{C32}$$

Again, we find that $A_{\mu,0} \partial_\mu c_0$ does not contain $\bar{\varphi}^a_{i,0} \omega^a_{i,0}$, which is necessary as the GZ action with the inclusion of A^2 is renormalizable. We also see that, when setting $V = -P$, $X = 0$, the mixing with A^2 disappears again.

Third, the vacuum term Σ^c_{III} has the following form:

$$b_3 QQ + b_4 QW + b_6 WW + b_7 Q\tau + b_8 W\tau + \frac{a_3}{2} \zeta \tau^2. \tag{C33}$$

We know that setting $Q = -W$ has to return the vacuum term from the RGZ action $\sim a_4 Q\tau + \frac{a_3}{2} \zeta \tau^2$. Therefore, we may set

$$b_3 - b_4 + b_6 = 0. \tag{C34}$$

In this case, the vacuum term reduces to

$$\begin{aligned} &- c_1 \alpha (QQ + QW) - c_2 \beta (QW + WW) \\ &\quad - c_3 \chi Q\tau - c_4 \delta W\tau + \frac{a_3}{2} \zeta \tau^2, \end{aligned} \tag{C35}$$

where we have extracted α, β, χ, and δ and some minus signs for convenience. If we allow mixing between the different parameters,

$$\begin{pmatrix} \alpha_0 \\ \beta_0 \\ \chi_0 \\ \delta_0 \\ \zeta_0 \end{pmatrix} = \begin{pmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{\alpha\chi} & Z_{\alpha\delta} & Z_{\alpha\zeta} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{\beta\chi} & Z_{\beta\delta} & Z_{\beta\zeta} \\ Z_{\chi\alpha} & Z_{\chi\beta} & Z_{\chi\chi} & Z_{\chi\delta} & Z_{\chi\zeta} \\ Z_{\delta\alpha} & Z_{\delta\beta} & Z_{\delta\chi} & Z_{\delta\delta} & Z_{\delta\zeta} \\ Z_{\zeta\alpha} & Z_{\zeta\beta} & Z_{\zeta\chi} & Z_{\zeta\delta} & Z_{\zeta\zeta} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \chi \\ \delta \\ \zeta \end{pmatrix}, \tag{C36}$$

when absorbing the counterterm, we find for the mixing matrix of the LCO parameters

$$\begin{pmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{\alpha\chi} & Z_{\alpha\delta} & Z_{\alpha\zeta} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{\beta\chi} & Z_{\beta\delta} & Z_{\beta\zeta} \\ Z_{\chi\alpha} & Z_{\chi\beta} & Z_{\chi\chi} & Z_{\chi\delta} & Z_{\chi\zeta} \\ Z_{\delta\alpha} & Z_{\delta\beta} & Z_{\delta\chi} & Z_{\delta\delta} & Z_{\delta\zeta} \\ Z_{\zeta\alpha} & Z_{\zeta\beta} & Z_{\zeta\chi} & Z_{\zeta\delta} & Z_{\zeta\zeta} \end{pmatrix} = \begin{pmatrix} \frac{1+c_1}{Z_{QQ}^2} & 0 & -\frac{Z_{\chi\chi}Z_{\tau W}}{Z_{QQ}} & 0 & \frac{Z_{\tau W}^2 Z_{\zeta\zeta}}{2Z_{QQ}^2} \\ 0 & \frac{1+c_2}{Z_{QQ}^2} & 0 & -\frac{Z_{\delta\delta}Z_{\tau W}}{Z_{QQ}} & \frac{Z_{\tau W}^2 Z_{\zeta\zeta}}{2Z_{QQ}^2} \\ 00 & 0 & \frac{1+c_3}{Z_{QQ}Z_{\tau\tau}} & 0 & -\frac{Z_{\tau W}Z_{\zeta\zeta}}{Z_{QQ}} 00000 \\ 0 & 0 & 0 & \frac{1+c_4}{Z_{QQ}Z_{\tau\tau}} & -\frac{Z_{\tau W}Z_{\zeta\zeta}}{Z_{QQ}} \\ 0 & 0 & 0 & 0 & \frac{1-a_3}{Z_{\tau\tau}^2} \end{pmatrix}. \tag{C37}$$

In summary, we have proven the action (C1) to be renormalizable.

APPENDIX D: LIST OF PROPAGATORS

We give here the list of propagators which can be calculated from the GZ action (2):

$$\langle \tilde{\bar{\omega}}_\mu^{ab}(k)\tilde{\omega}_\nu^{cd}(p)\rangle = \delta^{ac}\delta^{bd}\delta^{\mu\nu}\frac{-1}{p^2}\delta(p+k)(2\pi)^4, \qquad \langle \tilde{\bar{c}}^a(k)\tilde{c}^b(p)\rangle = \delta^{ab}\frac{1}{p^2}\delta(p+k)(2\pi)^4,$$

$$\langle \tilde{A}_\mu^a(p)\tilde{A}_\nu^b(k)\rangle = \frac{p^2}{p^4+\lambda^4}P_{\mu\nu}\delta^{ab}\delta(k+p)(2\pi)^4, \qquad \langle \tilde{A}_\mu^a(p)\tilde{b}^b(k)\rangle = -i\frac{p_\mu}{p^2}\delta^{ab}\delta(p+k)(2\pi)^4,$$

$$\langle b^a(p)b^b(k)\rangle = \delta^{ab}\frac{\lambda^4}{p^4}\delta(p+k)(2\pi)^4, \qquad \langle \tilde{A}_\mu^a(p)\tilde{\varphi}_\nu^{bc}(k)\rangle = \langle \tilde{A}_\mu^a(p)\tilde{\bar{\varphi}}_\nu^{bc}(k)\rangle = f^{abc}\frac{-g\gamma^2}{p^4+\lambda^4}P_{\mu\nu}(p)(2\pi)^4\delta(p+k),$$

$$\langle \tilde{b}^a(p)\tilde{\varphi}_\nu^{bc}(k)\rangle = \langle \tilde{b}^a(p)\tilde{\bar{\varphi}}_\nu^{bc}(k)\rangle = f^{abc}ip_\nu\frac{-g\gamma^2}{p^4}(2\pi)^4\delta(p+k),$$

$$\langle \tilde{\varphi}_\mu^{ab}(p)\tilde{\bar{\varphi}}_\nu^{cd}(k)\rangle = \left(f^{abr}f^{cdr}P_{\mu\nu}\frac{g^2\gamma^4}{p^2(p^4+2g^2N\gamma^4)}+\frac{-1}{p^2}\delta^{ac}\delta^{bd}\delta_{\mu\nu}\right)(2\pi)^4\delta(p+k),$$

$$\langle \tilde{\varphi}_\mu^{ab}(p)\tilde{\varphi}_\nu^{cd}(k)\rangle = \langle \tilde{\bar{\varphi}}_\mu^{ab}(p)\tilde{\bar{\varphi}}_\nu^{cd}(k)\rangle = f^{abr}f^{cdr}P_{\mu\nu}\frac{g^2\gamma^4}{p^2(p^4+2g^2N\gamma^4)}(2\pi)^4\delta(p+k). \tag{D1}$$

with

$$P_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \tag{D2}$$

APPENDIX E: DETAILS OF THE CALCULATION OF THE EFFECTIVE ACTION FOR THE FURTHER REFINED GZ ACTION

1. Determination of the LCO parameters $\delta\zeta$, $\delta\alpha$, $\delta\chi$, and $\delta\rho$

We shall start from expression (18), determine the quadratic part, and integrate out all the fields. The quadratic action is given by

$$\begin{aligned}\Sigma_{\rm CGZ}^{\rm quad} = &\int d^dx\Big[A_\mu^a\delta^{ab}\Big(-\delta_{\mu\nu}\partial^2+\Big(1-\frac{1}{\alpha}\Big)\partial_\mu\partial_\nu\Big)A_\nu^b \\ &+\bar{\varphi}\partial^2\varphi-\gamma^2 gf_{abc}A_\mu^a(\varphi_\mu^{bc}+\bar{\varphi}_\mu^{bc})\Big] \\ &+\int d^4x\Big[Q\bar{\varphi}_i^a\varphi_i^a+\frac{1}{2}\tau A_\mu^aA_\mu^a-\frac{1}{2}\zeta\tau^2-\alpha QQ \\ &-\chi Q\tau\Big]+\int d^4x\Big[\frac{1}{2}\bar{G}\bar{\varphi}_i^a\bar{\varphi}_i^a+\frac{1}{2}G\varphi_i^a\varphi_i^a+\rho G\bar{G}\Big],\end{aligned}$$

where we have immediately integrated out the ghost fields, c, $\bar{c}$, ω, $\bar{\omega}$, as they only appear trivially. We have also already integrated out the b field, where α is formally equal to zero.

As a first step, we integrate out the φ and $\bar{\varphi}$ fields. For this, we shall split φ, $\bar{\varphi}$, G, and $\bar{G}$ into real and imaginary components,

$$\begin{aligned}&\bar{\varphi}_i^a = U_i^a + iV_i^a, \qquad \varphi_i^a = U_i^a - iV_i^a, \\ &\bar{G} = X + iY, \qquad G = X - iY,\end{aligned} \tag{E1}$$

so that the part depending on φ and $\bar{\varphi}$ in expression (E1) becomes

$$\begin{aligned}&\int d^dx(U_i^a\partial^2U_i^a+V_i^a\partial^2V_i^a-2\gamma^2gf_{abc}A_\mu^aU_\mu^{bc}+QU^2 \\ &\quad+QV^2+XU^2-XV^2-2YU_i^aV_i^a+\rho X^2+\rho Y^2) \\ &\quad=\int d^dx\Big(\frac{1}{2}\begin{bmatrix}U_\mu^{ab} & V_\mu^{ab}\end{bmatrix} \\ &\quad\times\begin{bmatrix}2(\partial^2+Q+X) & -2Y \\ -2Y & 2(\partial^2+Q-X)\end{bmatrix} \\ &\quad\times\begin{bmatrix}U_\mu^{ab} \\ V_\mu^{ab}\end{bmatrix}-2\gamma^2gf_{abc}A_\mu^aU_\mu^{bc}\Big).\end{aligned}$$

Therefore, applying Gaussian integration, we find for the integration over φ and $\bar{\varphi}$

$$\int[d\varphi][d\bar{\varphi}]\exp[-\Sigma_{\rm CGZ}^{\rm quad}] = \exp\left[\frac{1}{2}\lambda^4 A_\mu^k\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)A_\mu^k + \cdots\right](\det P_{\mu\nu}^{ab,cd})^{-1/2}, \quad \text{(E2)}$$

where we recall that λ is defined as $\lambda^4 = 2\gamma^4 g^2 N$. $P_{\mu\nu}^{ab,cd}$ is given by

$$P_{\mu\nu}^{ab,cd} = \delta_{\mu\nu}\delta^{ab}\delta^{cd}\begin{bmatrix} 2(\partial^2+Q+X) & -2Y \\ -2Y & 2(\partial^2+Q-X)\end{bmatrix}, \quad \text{(E3)}$$

and the $\cdots$ stands for the other terms in $\Sigma_{\rm CGZ}^{\rm quad}$ [see (E1)], i.e. terms purely in A and the vacuum terms. The second step is to integrate out the gluon field A_μ^a. Combining the expression (E2) with the terms purely in A from the quadratic action, we obtain

$$\int[dA]e^{[-(1/2)A_\mu^a\delta^{ab}(-\delta_{\mu\nu}\partial^2+(1-(1/\alpha))\partial_\mu\partial_\nu-\lambda^4(\partial^2+Q-X/\partial^4+2Q\partial^2+Q^2-X^2-Y^2)+\tau\delta_{\mu\nu})A_\nu^b]} = \left[\det\left(-\delta_{\mu\nu}\partial^2+\left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu-\lambda^4\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\delta_{\mu\nu}\right)\right]^{-1/2}. \quad \text{(E4)}$$

Therefore, the total effective action at one loop is given by

$$e^{-W(Q,\tau,G,\bar{G})} = (\det P_{\mu\nu}^{ab,cd})^{-1/2}\left[\det\left(-\delta_{\mu\nu}\partial^2+\left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu-\lambda^4\delta_{\mu\nu}\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\delta_{\mu\nu}\right)\right]^{-1/2} \times e^{[-\int d^4x[-(1/2)\zeta\tau^2-\alpha QQ-\chi Q\tau(1/2)+\rho G\bar{G}]]}. \quad \text{(E5)}$$

In order to find $\delta\zeta$, $\delta\alpha$, $\delta\chi$, and $\delta\rho$ at one loop, we need to find the first order infinities of the previous expression. These shall be present in the two determinants which we need to evaluate.

Let us start with the first determinant of $P_{\mu\nu}^{ab,cd}$. In general, we can write

$$(\det P_{\mu\nu}^{ab,cd})^{-1/2} = e^{-(1/2)\,{\rm Tr}\ln P_{\mu\nu}^{ab,cd}} = e^{-(1/2)d(N^2-1)^2\,{\rm Tr}\ln P}. \quad \text{(E6)}$$

As we are taking the trace, we know that ${\rm Tr}\ln P = {\rm Tr}\ln P'$, with P' the diagonalization of P. Therefore, after diagonalization, we find

$$(\det P_{\mu\nu}^{ab,cd})^{-1/2} = \exp\left[-\frac{1}{2}d(N^2-1)2\,{\rm Tr}\left(\ln\left(-\partial^2-Q+\sqrt{X^2+Y^2}\right) + \ln\left(-\partial^2-Q-\sqrt{X^2+Y^2}\right)\right)\right]. \quad \text{(E7)}$$

Employing the standard formula [38]

$$\text{Tr}\ \ln(-\partial^2+M^2) = -\frac{\Gamma(-d/2)}{(4\pi)^{d/2}}\frac{1}{(M^2)^{-d/2}}, \quad \text{(E8)}$$

we obtain the following infinity:

$$(\det P)^{-1/2} = \exp\left[\frac{1}{\epsilon}\frac{(N^2-1)^2}{4\pi^2}[Q^2+X^2+Y^2]+c_1\right], \quad \text{(E9)}$$

where c_1 is a constant term.

The second determinant requires a bit more effort to be evaluated. Let us call the corresponding matrix K. We thus calculate

$$(\det K_{\mu\nu}^{ab})^{-1/2} = e^{-(1/2)(N^2-1){\rm Tr}\ln K_{\mu\nu}}. \quad \text{(E10)}$$

Therefore, we need to determine

$${\rm Tr}\ln K_{\mu\nu} = {\rm Tr}\ln\left(\delta_{\mu\nu}\left(-\partial^2-\lambda^4\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\right)\right) + {\rm Tr}\ln\left(\delta_{\mu\nu}+\frac{1}{(-\partial^2-\lambda^4(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2})+\tau)}\left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right). \quad \text{(E11)}$$

For the first term, we can easily take the trace over the Lorentz indices, while for the second term, we need to use $\ln(1+x) = x-\frac{x^2}{2}+\cdots$, then take the trace of the diagonal elements of the second term, and again employ $x-\frac{x^2}{2}+\cdots = \ln(1+x)$. After these operations, we obtain

$$\mathrm{Tr}\ln K_{\mu\nu} = d\,\mathrm{Tr}\ln\left(\left(-\partial^2 - \lambda^4\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\right)\right) + \mathrm{Tr}\ln\left(1+\frac{1}{(-\partial^2-\lambda^4(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2})+\tau)}\left(1-\frac{1}{\alpha}\right)\partial^2\right),$$

which can be written as

$$\mathrm{Tr}\ln K_{\mu\nu} = (d-1)\mathrm{Tr}\ln\left(\left(-\partial^2 - \lambda^4\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\right)\right) + \mathrm{Tr}\ln\left(\left(-\partial^2 - \lambda^4\left(\frac{\partial^2+Q-X}{\partial^4+2Q\partial^2+Q^2-X^2-Y^2}\right)+\tau\right)+\left(1-\frac{1}{\alpha}\right)\partial^2\right).$$

The first term of this expression can be written as[7]

$$(d-1)[\mathrm{Tr}\ln(p^6+(\tau-2Q)p^4+(\lambda^4+Q^2-X^2-Y^2-2Q\tau)p^2-Q\lambda^4+X\lambda^4+Q^2\tau-X^2\tau-Y^2\tau) - \mathrm{Tr}\ln(p^4-2Qp^2+Q^2-X^2-Y^2)] = (d-1)(\mathrm{Tr}\ln(p^2-x_1)+\mathrm{Tr}\ln(p^2-x_2) + \mathrm{Tr}\ln(p^2-x_3)-\mathrm{Tr}\ln(p^2-x_4)-\mathrm{Tr}\ln(p^2-x_5)), \tag{E12}$$

where x_1, x_2, and x_3 are the solutions of the equation

$$x^3+(\tau-2Q)x^2+(\lambda^4+Q^2-X^2-Y^2-2Q\tau)x - Q\lambda^4+X\lambda^4+Q^2\tau-X^2\tau-Y^2\tau=0$$

and x_4 and x_5 of the equation $x^2-2Qx+Q^2-X^2-Y^2=0$. After determining $x_1,\ldots,x_5$, we can apply the standard formula (E8) again, so we ultimately find for the first term

$$-\frac{3}{16\pi^2}\frac{1}{\epsilon}(\tau^2-2\lambda^4)+c_2, \tag{E13}$$

with c_2 a constant, which is not of our current interest. For the second term of (E11), we can perform an analogous analysis, whereby we find that this term is proportional to α and therefore does not contribute to the determinant as $\alpha\to 0$. Therefore, the second determinant ultimately gives

$$(\det K^{ab}_{\mu\nu})^{-1/2} = \exp\left[(N^2-1)\frac{3}{32\pi^2}\frac{1}{\epsilon}(\tau^2-2\lambda^4)+c_2\right]. \tag{E14}$$

We can now combine both results (E9) and (E14) to find

$$W(Q,\tau,G,\bar{G}) = -\frac{(N^2-1)}{4\pi^2}\frac{1}{\epsilon}\left(\frac{3}{8}\tau^2+(N^2-1)\times(Q^2+G\bar{G})-\frac{3}{4}\lambda^4\right)+c, \tag{E15}$$

with c a constant term. Therefore, at one loop we obtain

$$\delta\zeta = -\frac{1}{\epsilon}\frac{3}{16\pi^2}(N^2-1), \qquad \delta\alpha = -\frac{1}{\epsilon}\frac{1}{4\pi^2}(N^2-1)^2,$$
$$\delta\chi = 0, \qquad \delta\rho = \frac{1}{\epsilon}\frac{1}{4\pi^2}(N^2-1)^2. \tag{E16}$$

2. Calculation of the effective action

We can now proceed in a very similar fashion as in Sec. E 1. We can split the one-loop effective potential in a few parts. A first part, $\Gamma^{(1)}_a$, is the equivalent of $(\det P)^{-1/2}$ in expression (E5)

$$\Gamma^{(1)}_a = (N^2-1)^2\left[-\frac{1}{\epsilon}\frac{1}{4\pi^2}(M^4+\rho\rho^\dagger) + \frac{1}{16\pi^2}\left(\left(M^2-\sqrt{\rho\rho^\dagger}\right)^2\ln\frac{M^2-\sqrt{\rho\rho^\dagger}}{\bar{\mu}^2} + \left(M^2+\sqrt{\rho\rho^\dagger}\right)^2\ln\frac{M^2+\sqrt{\rho\rho^\dagger}}{\bar{\mu}^2}-2(M^2+\rho\rho^\dagger)\right)\right]. \tag{E17}$$

The second part, the equivalent of $(\det K)^{-1/2}$, is given by

$$\Gamma^{(1)}_b = \frac{3(N^2-1)}{64\pi^2}\left[-\frac{2}{\epsilon}(m^4-2\lambda^4)-\frac{5}{6}(m^4-2\lambda^4) + y_1^2\ln\frac{(-y_1)}{\bar{\mu}}+y_2^2\ln\frac{(-y_2)}{\bar{\mu}}+y_3^2\ln\frac{(-y_3)}{\bar{\mu}} - y_4^2\ln\frac{(-y_4)}{\bar{\mu}}-y_5^2\ln\frac{(-y_5)}{\bar{\mu}}\right], \tag{E18}$$

where y_1, y_2, and y_3 are the solutions of the equation

[7]We shall replace $-\partial^2$ by p^2 from now on and work in momentum space.

$$y^3 + (m^2 + 2M^2)y^2 + (\lambda^4 + M^4 - \rho\rho^\dagger + 2M^2m^2)y + M^2\lambda^4 + 1/2(\rho + \rho^\dagger)\lambda^4 + M^4m^2 - m^2\rho\rho^\dagger = 0$$

and y_4 and y_5 of the equation $y^2 + 2M^2y + M^4 - \rho\rho^\dagger = 0$.

The third part is the constant term of the GZ action,

$$\Gamma_c^{(1)} = -d\gamma_0^4(N^2 - 1). \tag{E19}$$

From Eq. (A42), we can calculate that[8]

$$\gamma_0^4 = Z_{\gamma^2}^2\gamma^4, \quad \text{with } Z_{\gamma^2}^2 = 1 + \frac{3}{2}\frac{g^2N}{16\pi^2}\frac{1}{\epsilon}, \tag{E20}$$

so we find

$$\begin{aligned}\Gamma_c^{(1)} &= -d(N^2-1)\gamma_0^4 = -4(N^2-1)\gamma^4 - 4\frac{3}{2}(N^2-1)\frac{g^2N}{16\pi^2}\frac{1}{\epsilon}\gamma^4 + \frac{3}{2}\frac{g^2N}{16\pi^2}\gamma^4(N^2-1) \\ &= -2(N^2-1)\frac{\lambda^4}{Ng^2} - 6(N^2-1)\frac{\lambda^4}{32\pi^2}\frac{1}{\epsilon} + \frac{3}{2}\frac{\lambda^4}{32\pi^2}(N^2-1).\end{aligned} \tag{E21}$$

The fourth part requires some calculation. We first find

$$\frac{1}{4Z_\rho Z_G^2\rho}\left(\frac{\sigma_3^2}{g^2} + \frac{\sigma_4^2}{g^2}\right) = \frac{1}{2}\frac{48(N^2-1)^2}{53N}\left(1 - \frac{53}{6}\frac{1}{\epsilon}\frac{Ng^2}{16\pi^2} - Ng^2\frac{53}{24}\frac{\rho_1}{(N^2-1)^2}\right)\frac{\rho\rho^\dagger}{g^2}, \tag{E22}$$

and second

$$\begin{aligned}&\frac{\alpha'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_1^2}{g^2} + \frac{\zeta'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_2^2}{g^2} - \frac{\chi'}{4\alpha'\zeta' - \chi'^2}\frac{\sigma_1\sigma_2}{g^2} = \frac{\zeta_0 m^4}{2g^2} + \frac{\alpha_0 M^4}{g^2} \\ &\quad + \frac{1}{\epsilon}\left(\frac{13N\zeta_0 m^4}{96\pi^2} + \frac{M^4(N^2-1)^2}{4\pi^2}\right) - \frac{\zeta_1 m^4}{2} - M^4\alpha_1 + M^2m^2\chi_1,\end{aligned} \tag{E23}$$

so that

$$\begin{aligned}\Gamma_d^{(1)} &= \frac{1}{2}\frac{48(N^2-1)^2}{53N}\left(1 - \frac{53}{6}\frac{1}{\epsilon}\frac{Ng^2}{16\pi^2} - Ng^2\frac{53}{24}\frac{\rho_1}{(N^2-1)^2}\right)\frac{\rho\rho^\dagger}{g^2} + \frac{\zeta_0 m^4}{2g^2} + \frac{\alpha_0 M^4}{g^2} + \frac{1}{\epsilon}\left(\frac{13N\zeta_0 m^4}{96\pi^2} + \frac{M^4(N^2-1)^2}{4\pi^2}\right) \\ &\quad - \frac{\zeta_1 m^4}{2} - M^4\alpha_1 + M^2m^2\chi_1.\end{aligned} \tag{E24}$$

As a check on our results, we see that all the infinities cancel, so we find

$$\begin{aligned}\Gamma^{(1)} &= \frac{(N^2-1)^2}{16\pi^2}\left[\left(M^2 - \sqrt{\rho\rho^\dagger}\right)^2\ln\frac{M^2 - \sqrt{\rho\rho^\dagger}}{\bar{\mu}^2} + \left(M^2 + \sqrt{\rho\rho^\dagger}\right)^2\ln\frac{M^2 + \sqrt{\rho\rho^\dagger}}{\bar{\mu}^2} - 2(M^2 + \rho\rho^\dagger)\right] \\ &\quad + \frac{3(N^2-1)}{64\pi^2}\left[-\frac{5}{6}(m^4 - 2\lambda^4) + y_1^2\ln\frac{(-y_1)}{\bar{\mu}} + y_2^2\ln\frac{(-y_2)}{\bar{\mu}} + y_3^2\ln\frac{(-y_3)}{\bar{\mu}} - y_4^2\ln\frac{(-y_4)}{\bar{\mu}} - y_5^2\ln\frac{(-y_5)}{\bar{\mu}}\right] \\ &\quad - 2(N^2-1)\frac{\lambda^4}{Ng^2} + \frac{3}{2}\frac{\lambda^4}{32\pi^2}(N^2-1) + \frac{1}{2}\frac{48(N^2-1)^2}{53N}\left(1 - Ng^2\frac{53}{24}\frac{\rho_1}{(N^2-1)^2}\right)\frac{\rho\rho^\dagger}{g^2} + \frac{9}{13}\frac{N^2-1}{N}\frac{m^4}{2g^2} \\ &\quad - \frac{24}{35}\frac{(N^2-1)^2}{N}\frac{M^4}{g^2} - \frac{161}{52}\frac{N^2-1}{16\pi^2}\frac{m^4}{2} - M^4\alpha_1 + M^2m^2\chi_1.\end{aligned} \tag{E25}$$

[8]For the explicit loop calculations of the Z factors, we refer to [35].

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