Relativistic theory of string breaking in QCD

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(Received 21 April 2011; published 12 September 2011)

The QCD string breaking due to quark pair creation in the vacuum confining field, possibly accompanied by vector, scalar, or Nambu-Goldstone bosons, is studied nonperturbatively. The scalar light-pair creation vertex occurs due to chiral symmetry breaking and has a confining form, which is computed explicitly together with subleading vector contributions. Dependence on light quark mass and flavor is specifically studied. The dominant scalar term is in good agreement with the $^3P_0$ model and experimental data.

I. INTRODUCTION

The topic of string breaking and effective decay Lagrangian has a long history. Among the nonperturbative (np) models, the most famous is the so-called $^3P_0$ model (see [1,2] for details, reviews and references), which can also be written in the form of the interaction Hamiltonian for the Dirac quark fields

$$H_I = g \int d^3x \, \bar{\psi}\psi, \qquad (1)$$

where $g = 2m_q\gamma$, m_q is the constituent quark mass and $\gamma \approx 0.5$ is the phenomenological parameter. In what follows, we derive effective Lagrangian in QCD, which in the relativistic invariant form can be written similarly to (1) as

$$\mathcal{L} = \int \bar{\psi}\mathcal{M}\psi d^4x, \qquad (2)$$

with $\mathcal{M}$ calculated nonperturbatively through string tension, while ψ, $\bar{\psi}$ are relativistic bispinors with current (polar) mass. The model (1) is rather successful (see [1–4] for reviews and discussion), however, not deduced from the QCD Lagrangian. In the detailed analysis of heavy quarkonia in [5], another model was suggested, where the role of $\mathcal{M}$ in (2) is played by the quark-antiquark potential $V_{Q\bar{Q}}(r)$ with confining and OGE parts of color octet vector type. An important step was done further in [4], where scalar and vector color-singlet potentials were carefully studied, and similarity of results for $^3P_0$ and scalar potential model was stressed. A successful application of the $^3P_0$ model to charmonia was done in [6] with $\gamma = 0.4$, assuming harmonic oscillator wave functions. This seemingly universal character of the model calls for the careful study of its connection with QCD.

In the present paper, we aim at the most general derivation of the string-breaking amplitude with possible accompanying radiation of γ or π, η, K, or vector and scalar mesons.

Formally, the problem of string breaking or of $q\bar{q}$ pair creation on the string connecting quark Q and antiquark $\bar{Q}$ can be reduced to the calculation of partition function

$$Z = \int DA \exp(\mathcal{L}_A)W_{Q\bar{Q}}(A)\det[m_q + \hat{D}(A)], \qquad (3)$$

where $\mathcal{L}_A$ is the standard gluonic action and $W_{Q\bar{Q}}(A)$ is the (external) Wilson loop of (heavy) quarks $Q\bar{Q}$ with fixed contour. Using world-line representation for the det term in (3) and retaining only one loop of quark q, one has

$$Z_{1\text{loop}} = \int DA \exp\mathcal{L}_A\left(-\frac{1}{2}\operatorname{tr}\int_0^\infty \frac{ds}{s}(D^4z)e^{-K}\right.$$
$$\left. \times W_{q\bar{q}}(A)W_{Q\bar{Q}}(A)\right). \qquad (4)$$

Thus, the effective Lagrangian can be obtained from the averaged product of two Wilson loops, calculated in [7],

$$\chi \equiv \langle W_{q\bar{q}}W_{Q\bar{Q}}\rangle_A \cong \frac{1}{N_c^2}\exp(-\sigma S_\Delta), \qquad (5)$$

where S_Δ is the minimal area between contours $q\bar{q}$ and $Q\bar{Q}$. For large masses m_q and m_Q, one can easily reproduce the scalar potential (sKs) model of [4] with $\mathcal{L}_{\text{nonrel}} = \int \sigma |\mathbf{x}_q - \mathbf{x}_{\bar{Q}}|dt$. However, in the small m_q limit, the transition from world-line representation of the light quark to the bispinor Lagrangian $\int \bar{\psi}\mathcal{M}\psi d^4x$ is not easy, since the appearance of the scalar operator $\mathcal{M}$ for light quark implies chiral symmetry breaking. Therefore, below we shall explicitly construct the kernel $\mathcal{M}$, which exemplifies simultaneously confinement and spontaneous Chiral Symmetry Breaking (CSB) for quark q of vanishing mass m_q in the field of external (possibly static for simplicity) quark $\bar{Q}$. When mass m_q is growing, $m_q \gg \sqrt{\sigma}$, spontaneous CSB is replaced by the explicit one, and the problem is much simpler.

To derive Effective String-Decay Lagrangian (ESDL), one can use the well-known Background Perturbation Theory (BPTh) [8], which was developed further in [9], where the confining properties of the background were taken into account. This theory has the prominent advantage, that it is infrared safe, and the Landau ghost pole and IR renormalons are absent in the total perturbation theory, where the first term is purely nonperturbative and can be

calculated via np field correlators [10]. The latter, in their turn, are calculated self-consistently through the only mass parameter of the theory, e.g., the string tension σ (see [11] for a review and references).

Therefore, in computing ESDL, one obtains several terms. First of all, the well-known $^3S_1$ term [12] due to gluon exchange, which in BPTh becomes the hybrid-mediated transition, discussed in [13], when confining background is important (for a small energy release). Below, we shall be interested mostly in the scalar-type terms of ESDL. As was said above, those terms occur due to CSB in the confining background, and our derivation closely follows previous papers [15–18], where CSB due to confinement in QCD was derived in great detail.

We shall specifically stress below the main mechanism, which creates for a (massless) relativistic quark simultaneously and spontaneously confinement and CSB, the mechanism which takes into account symmetry of quark spectrum of Hamiltonian with scalar interaction. For massive quarks of large mass, the same effect occurs due to explicit CSB. Integrating out quark degrees of freedom in the effective Lagrangian, one obtains chiral effective Lagrangian for the Nambu-Goldstone (NG) mesons at that point of the string, where it breaks down. In this way, one obtains ESDL with NG meson degrees of freedom.

The mechanism, described below and in [17,18] can be cast into explicitly gauge invariant form (see Appendix 1 of [17]), which is possible, since the systems under consideration ($(\bar{Q}q)$ or $(Q\bar{q})$) are white. This is in contrast to models, considering light quark pair ($q\bar{q}$) alone.

Concluding this introduction, one should stress that our approach to string breaking as a CSB light-pair creation process differs in principle from the Schwinger-type mechanism of pair creation, specific for vast electric fluxes in QED, see [19] for discussion and references.

II. CHIRAL SYMMETRY BREAKING AND SCALAR STRING BREAKING

Consider now the partition function of a light quark in the field of a static antiquark in the presence of external currents v_μ, a_μ, s, p

$$Z = \int DAD\bar{\psi}D\psi \exp[-(S_0 + S_1 + S_{\text{int}} + S_Q + S_{\bar{Q}})],$$

$$S_0 = \frac{1}{4}\int d^4x(F_{\mu\nu}^a)^2,$$

$$S_1 = -i\int d^4x\bar{\psi}^f(\hat{\partial} + m + \hat{v} + \gamma_5\hat{a} + s + i\gamma_5 p)^{fg}\psi^g,$$

$$S_{\text{int}} = -\int d^4x\bar{\psi}^f g\hat{A}^a t^a \psi^f. \tag{6}$$

Here f, g are flavor indices, S_Q and $S_{\bar{Q}}$ refer to action of external quark currents, of (possibly high mass) quark Q and antiquark $\bar{Q}$.

We shall follow derivation of [15–17], but for simplicity we shall use the simplest contour gauge, [20]so-called Balitsky gauge [21], where one can write

$$A_\mu(x) = \int_{C(x)}\alpha_\mu(u)F_{i\mu}(u)du_i, \quad \alpha_4 = 1, \quad \alpha_i = \frac{u_i}{x_i}, \tag{7}$$

and the contour $C(x)$ is going from the point $x = (\mathbf{x}, x_4)$ to the point $(\mathbf{0}, x_4)$ on the world-line of Q and then along this world-line to $x_4 = -\infty$. Note, that our final result (13) and (14) will be cast in the gauge invariant form, which is the same for all contours, connecting points x, y to the world lines of Q (or $\bar{Q}$). The independence of the resulting asymptotic expressions from the form of contours is shown in Appendix 3 of [17].

Averaging over fields A_μ, $(F_{\mu\nu})$, one can write

$$Z = \iint D\psi D\bar{\psi}\exp[-(S_1 + S_{\text{eff}})], \tag{8}$$

where S_{eff} was computed in [15–17]. Keeping only quadratic correlators and colorelectric fields for simplicity[1] one obtains (for one flavor)

$$S_{\text{eff}} = -\frac{1}{2}\int d^4x d^4y\bar{\psi}(x)\gamma_4[\psi(x)\bar{\psi}(y)]\gamma_4\psi(y)J(x,y), \tag{9}$$

where $J(x, y)$ is expressed via vacuum correlator of colorelectric fields, $[\psi\bar{\psi}]$ implies color singlet combination. Keeping only colorelectric fields, one has

$$J(x, y) \equiv \frac{g^2}{N_c}\langle A_4(x)A_4(y)\rangle = \int_0^x du_i\int_0^y dv_i D(u - v). \tag{10}$$

Here $D(w)$ is the np correlator, responsible for confinement [11],

$$\frac{g^2 tr}{N_c}\langle F_{i\mu}(u)F_{k\nu}(v)\rangle$$
$$= (\delta_{ik}\delta_{\mu\nu} - \delta_{i\nu}\delta_{\mu k})D(u - v) + O(D_1) \tag{11}$$

and we have omitted the (vector) contribution of the correlator D_1, containing gluon exchange and np corrections to it. These terms will be discussed below.

To start one can simplify the matter and neglect possible chiral degrees of freedom, contained in the $4q$ combination in (9), replacing at large N_c

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[1]The fact, that quadratic (Gaussian) correlators yield dominant contribution is due to small vacuum correlation length $\lambda \approx 0.1$ fm, which ensures small expansion parameter $\sigma\lambda^2 \ll 1$, and is strongly supported by lattice measurements of Casimir scaling (see [11] for discussion. Keeping only colorelectric correlators is justified when angular momentum of light quark pair $q\bar{q}$ is not large. Colormagnetic correlators produce angular momentum-dependent corrections ($V_{CM} \approx \frac{3l(l+1)}{\sigma r^3}$) to the linear confinement term σr, which is due to colorelectric correlators, see [22] for details.

$$[\psi(x)\bar{\psi}(y)] \rightarrow \langle \psi(x)\bar{\psi}(y)\rangle_q \equiv S_q(x, y). \quad (12)$$

In this way one obtains

$$S_{\text{eff}} = \int d^4x d^4y \bar{\psi}(x)\tilde{\mathcal{M}}(x, y)\psi(y), \quad (13)$$

where $\mathcal{M}$ is expressed via $S_q(x, y)$

$$\tilde{\mathcal{M}}(x, y) = -iS_q(x, y)J(x, y) \quad (14)$$

and S_q is expressed via $\tilde{\mathcal{M}}$, $S_q = \frac{i}{\hat{\partial}+m+\tilde{\mathcal{M}}}$. But one can notice, that S_q contains the term in the denominator $(m + \mathcal{M})$ and even for $m = 0$ we look for a scalar $\bar{\mathcal{M}}$, which provides in S_q terms with even number of γ_μ, and scalar $\bar{\mathcal{M}}_s$ can be a self consistent solution of the nonlinear equation $\bar{\mathcal{M}}_s = \frac{1}{\hat{\partial}+m+\bar{\mathcal{M}}_s}J$. It is clear, that for $m > 0$ the situation is simplified. Below we shall find explicitly the solution of this nonlinear equation.

At this point one should stress, that $S_q(x, y)$ is the light quark Green's function in the field of antiquark. To simplify matter, we shall consider massless quark $(m = 0)$ and static antiquark $\bar{Q}$. It is clear, that without CSB S_q contains an odd number of γ matrices, hence, tr$\tilde{\mathcal{M}}(x, y)$ vanishes. To exemplify the appearance of CSB, it is convenient to consider the limit of small correlation length λ when $\tilde{M}$ factorizes, [15,16]

$$\tilde{\mathcal{M}}(x, y) = \tilde{\delta}(x_4 - y_4)M(\mathbf{x}, \mathbf{y}), \\ M(\mathbf{x}, \mathbf{y}) = J(\mathbf{x}, \mathbf{y})\beta\Lambda(\mathbf{x}, \mathbf{y}), \quad (15)$$

where $\Lambda(\mathbf{x}, \mathbf{y})$ is expanded in eigensolutions $\{\psi_k\}$ of static Dirac equation

$$\Lambda(\mathbf{x}, \mathbf{y}) = \sum_{\varepsilon_k} \psi_k(x)\text{sign}\varepsilon_k \psi_k^+(y), \qquad x \equiv |\mathbf{x}|. \quad (16)$$

$J(\mathbf{x}, \mathbf{y})$, defined in (10), behaves at large and parallel $|\mathbf{x}| \approx |\mathbf{y}|$ as

$$J(\mathbf{x}, \mathbf{y}) \approx \text{const min}(x, y). \quad (17)$$

This implies confinement, if $\Lambda(\mathbf{x}, \mathbf{y})$ ensures almost equal and parallel $\mathbf{x}, \mathbf{y}$. Hence for self-consistency, $\Lambda(\mathbf{x}, \mathbf{y})$ must tend to $\beta\delta^{(3)}(\mathbf{x} - \mathbf{y})$ at least for large $|\mathbf{x}| \sim |\mathbf{y}|$. Note that $\{\psi_k\}$ should be computed with the interaction $M(\mathbf{x}, \mathbf{x})$, which consists of the same $\{\psi_k\}$, as it happens in the mean field method. And here, the spontaneous CSB reveals itself in the fact that for scalar mean field $M(\mathbf{x}, \mathbf{x})$, the combination of $\{\psi_k\}$ entering in $\Lambda(\mathbf{x}, \mathbf{y})$ indeed has this property: in the 4×4 structure

$$\Lambda \equiv \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}$$

one obtains $\Lambda_{22} = -\Lambda_{11}$ (i.e., $\Lambda \sim \beta$), if the spectrum $\{\varepsilon_n\}$ has the symmetry property for $\varepsilon_n \rightarrow -\varepsilon_n$ of the scalar potential and in this case, the sum (16) computed in the relativistic WKB method [23] yields the smeared

$\delta^{(3)}$-function at large x, ensuring small angles between $\mathbf{x}$ are $\mathbf{y}$ [15]. The appearance of this scalar structure of $M \sim \beta\Lambda$, $\Lambda \sim \beta$ can be called the spontaneous scalar generation, which gives CSB.

In the static limit, when one can neglect the energy transfer from the heavy quark Q to the light quark pair $q\bar{q}$ in (13), the effective mass $\mathcal{M}(x, y)$ was calculated in [15–17], using relativistic WKB approximation [23]. The result is

$$\tilde{\mathcal{M}}(x, y) = \sigma|\mathbf{x}| \cdot \tilde{\delta}^{(3)}(\mathbf{x} - \mathbf{y})\tilde{\delta}(x_4 - y_4), \quad (17)$$

where $|\mathbf{x}|$, $|\mathbf{y}|$ are distances to Q and the smeared δ-function $\tilde{\delta}^{(3)}(\mathbf{z})$ has the range $r_0 = \sigma|\frac{\mathbf{x}+\mathbf{y}}{2}|$, while the range of $\tilde{\delta}(x_4 - y_4)$ is of the order of the vacuum correlation length. The explicit form of $\tilde{\mathcal{M}}(x, y)$ in (17) is given in the Appendix. Hence for a long string, when $r_0 \gg 0.1$ fm, one can neglect the nonlocality in (13) and take into account that the equivalent contribution can be obtained, connecting the correlator $J(x, y)$ to the worldline of antiquark $\bar{Q}$, which gives the final result

$$S_{\text{eff}} = \int d^4x \bar{\psi}(x)\bar{\mathcal{M}}(x)\psi(x), \quad (18)$$

with

$$\bar{\mathcal{M}}(x) = \sigma(|\mathbf{x} - \mathbf{x}_Q| + |\mathbf{x} - \mathbf{x}_{\bar{Q}}|). \quad (19)$$

Two terms of $\bar{\mathcal{M}}(x)$ are shown in Figs. 1 and 2, which show examples of trajectories (world lines) of quarks and antiquarks with time growing in a horizontal direction. Vertical lines starting at x and y belong to contours $C(x)$, $C(y)$ used in (7). The cross-hatched region between them contributes to the kernel $\tilde{M}(x, y)$ in (17).

There are several properties of the interaction (18) and (19) which should be mentioned. First of all, the mass term $\bar{\mathcal{M}}(x)$ is scalar and hence creates the $^3P_0$ pair $\bar{q}q$.

Additional terms in $\bar{\mathcal{M}}(x)$, which are subleading at large distances, were studied in [15–17] see the Appendix below. In particular, the contribution of the term D_1 in (11) which was neglected above, yields terms proportional to β in
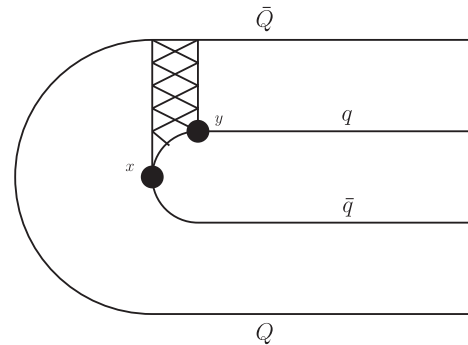

FIG. 1. The qqbar pair creating in the confining field of heavy antiquark Qbar.
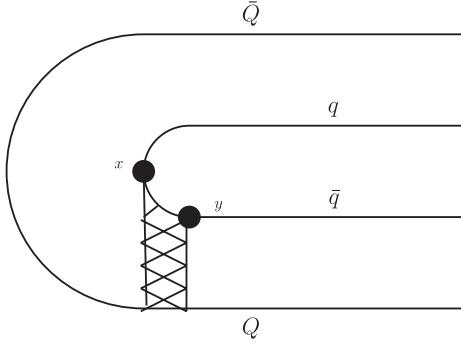
FIG. 2. The qqbar pair creating in the confining field of heavy anttiquark Q.

$\bar{\mathcal{M}}(x)$, which are of the relative order $O(\frac{\alpha_s}{\sigma r^2})$, $r = |\mathbf{x} - \mathbf{x}_Q|$, and hence can be neglected at large r.

Second, the kernel (19) does not depend on flavor, which can be checked experimentally, as discussed in the concluding section. However, the local form (19) is an approximation of the nonlocal one, given in (17) and in Appendix, Eq. (A7) derived for zero mass m_q. The effect of nonlocality is numerical reduction of $\bar{\mathcal{M}}(x)$ ([17]) by $(10 \div 15)\%$, as can be seen from Fig. 1 of [16]. For larger m_q, the range of nonlocality decreases, as shown in Appendix, Eq. (A10), and $\tilde{\mathcal{M}}(x, y)$ tends to the final form (19).

The potential kernel (19) has similarity with the kernel, suggested by Eichten *et al.* [5], however, there was assumed its color octet and vector character, i.e., $\bar{\mathcal{M}}(x)$, was taken to be proportional to $\gamma_4 t_a$, while in our case it is scalar and color singlet. This assignment of [5] is not favored phenomenologically, see [3,4] for discussion. The kernel (19) coincides formally with the so-called sKs kernel, which was studied nonrelativistically in [6], and shown to be phenomenologically successful and close to the $^3P_0$ model.

III. EMISSION OF MESONS, ACCOMPANYING THE STRING BREAKING PROCESS

We return here to the $4q$ effecting action (9), and will follow the procedure of [18], where chiral Lagrangian was derived in conjunction with the confining kernel (17). Doing bosonization in (9) with bosonic scalar variable M_s and pseudoscalar ϕ_a, one obtains the effective quark-meson Lagrangian

$$Z = \int D\bar{\psi} D\psi DM_s D\phi_a \exp[-S_{QM}], \quad (20)$$

$$S_{QM} = -\int d^4x d^4y [\bar{\psi}_\alpha^f(x)(i(\hat{\partial} + \hat{v} + \gamma_5\hat{a} + s + i\gamma_5 p)_{\alpha\beta}^{fg}$$
$$\times \delta^{(4)}(x - y) + iM_s(x, y)\hat{U}_{\alpha\beta}^{fg}(x, y))\psi_\beta^g(y)$$
$$- 2N_f(J(x, y))^{-1}M_s^2(x, y)], \quad (21)$$

$$\hat{U}_{\alpha\beta}^{fg}(x, y) = \exp(i\gamma_5 t_n \phi_n(x, y))_{\alpha\beta}^{fg}. \quad (22)$$

Integrating out quark fields, one obtains effective chiral Lagrangian given in [17] for the field ϕ_a with external currents included.

Finally, classical equations of motion define the stationary point conditions,

$$\phi_a^{(0)}(x, y) = 0, \quad (23)$$

$$M_s^{(0)}(x, y) = \frac{-i}{4N_f} J(x, y)\text{Tr}_{f,d}(S_q(x, y)), \quad (24)$$

$$S_q(x, y) \equiv S_\phi(x, y)|_{\phi=0}. \quad (25)$$

where $\text{Tr}_{f,d}$ is the trace over flavor and bispinor indices, $S_\phi(x, y)$ is the total quark propagator with chiral and extra mesons included,

$$S_\phi(x, y) = \left\langle x \left| \frac{1}{i\hat{\Delta} + iM_s e^{i\gamma_5 t_a \phi_a}} \right| y \right\rangle \quad (26)$$

and

$$\hat{\Delta} = \hat{\partial} + m + \hat{v} + \gamma_5\hat{a} + s + i\gamma_5 p. \quad (27)$$

Note, that the Eq. (24) coincides with (14), when S-external currents are retained. Several properties of the new effective action (21) are to be noted.

First of all, chiral degrees of freedom are now contained in the new string-breaking term, which in the local approximation has the form,

$$S_{\text{str.br}}^{(\text{chiral})} = -i \int d^4x \hat{\psi}_\alpha^f(x)\hat{M}(x)Z^{(U)}\hat{U}_{\alpha\beta}^{fg}(x)\psi_\beta^g(x). \quad (28)$$

Here, f, g are flavor indices, α, β are 4-spinor indices, and color indices of $\bar{\psi}$ and ψ are suppressed, while $\bar{M}(x)\hat{U}$ are color-blind. The factor $Z^{(U)}$ in (28) takes into account both np and perturbative renormalization of the NG fields, which are nonlocal and free (without interaction) in (21) and (22), while in (28) already physical local fields are contained.

$$\hat{U} = \exp(i\gamma_5\phi^a t^a) = \exp\left(i\gamma_5\frac{\varphi_a\lambda_a}{f_\pi}\right),$$

$$\varphi_a\lambda_a \equiv \sqrt{2}\begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & K^+ \\ \pi^-, & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}_0, & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (29)$$

In a similar way the string breaking with emission or absorption of a non-NG meson is described by the first term in (21), see Fig. 2

$$S_{str.br}^{(j)} = -i \int d^4x \bar{\psi}_\alpha^f(x)\hat{j}_{\alpha\beta}^{fg}(x)\psi_\beta^g(x), \quad (30)$$

where

$$\hat{j}_{\alpha\beta}^{fg}(x) = (\hat{v} + \gamma_5\hat{a} + s + i\gamma_5 p)_{\alpha\beta}^{fg}. \quad (31)$$

At this point, one should stress that in general, when Q is a heavy quark, the decay of $Q\bar{Q}$ meson into $(Q\bar{q})(\bar{Q}q)$

mesons is possible only due to string breaking, while decay into $(Q\bar{q})(\bar{Q}q)+$ light meson is going via the string-breaking emission mechanism of Eqs. (28) and (30) and in addition via two-step process with string breaking and subsequent emission from a light quark (antiquark) in heavy-light products. When $Q(\bar{Q})$ is a light quark, there is in addition a possibility of emission of NG meson (or light vector meson etc.) directly from Q.

IV. MATRIX ELEMENTS OF STRING DECAY AND HADRON EMISSION

In this section, we shall study physical matrix elements based on effective actions (18), (28), and (30).

The basic quantity, which can be calculated from the effective Lagrangians, derived above, is the two-step amplitude, which incorporates string-breaking transition to intermediate two-body states and back to the $Q\bar{Q}$ state.

In this way, the transition amplitude from the state $(Q\bar{Q})_n$ to $(Q\bar{Q})_m$ via string breaking to the intermediate states $(Q\bar{q})_{n_2}(\bar{Q}q)_{n_3}$ with energy $E_{n_2 n_3}$ can be written as (see [24–26] for details, a similar amplitude was introduced in [5])

$$w_{nm}(E) = \frac{1}{N_c} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{n_2, n_3} \frac{\tilde{J}_{nn_2n_3}\tilde{J}^+_{mn_2n_3}}{E - E_{n_2n_3}(\mathbf{p})}. \quad (32)$$

Here, $\tilde{J}_{nn_2n_3}(\mathbf{p})$ is the overlap matrix element with $\bar{M}(\mathbf{x}) \to M(\mathbf{q})$

$$\tilde{J}_{nn_2n_3}(\mathbf{p}) = \int \bar{y}_{123}\frac{d^3\mathbf{q}}{(2\pi)^3}\Psi_n^+(\mathbf{p} + \mathbf{q})\bar{M}(\mathbf{q})\psi_{n_2}(\mathbf{q})\psi_{n_3}(\mathbf{q}), \quad (33)$$

and $\bar{y}_{123}$ is the trace of normalized spin-tensors corresponding to spin-angular parts of meson states, while Ψ_n, ψ_n are the radial parts, tables of $\bar{y}_{123}$ are given in [24,26], e. g., for the 1^{--} state n, one has $\bar{y}_{123} = \frac{i}{\omega}(q_i - \frac{p_i\omega}{2(\omega+\Omega)})$, where ω, Ω are average kinetic energies of light and heavy quark, respectively, tables of ω, Ω are given in Appendix 1 of [24].

For the string-breaking operator (19), the overlap integral (33) has the form

$$\tilde{J}_{nn_2n_3}(\mathbf{p}) = \sigma \int \bar{y}_{123}\frac{d^3\mathbf{q}}{(2\pi)^3}\Psi_n^+(\mathbf{p} + \mathbf{q})$$
$$\times \left(\left| \frac{d\psi_{n_2}(\mathbf{q})}{d\mathbf{q}} \right| \psi_{n_3}(\mathbf{q}) + \left| \frac{d\psi_{n_3}(\mathbf{q})}{d\mathbf{q}} \right| \psi_{n_2}(\mathbf{q}) \right). \quad (34)$$

Since heavy-light meson wave functions are well reproduced by a single oscillator function [24–26],

$$\psi_{n_2}(\mathbf{q}) = \left(\frac{2\sqrt{\pi}}{\beta_2} \right)^{3/2} e^{-(\mathbf{q}^2/2\beta_2^2)},$$

the resulting effective vertex $\bar{M}(\mathbf{q})$ can be written as

$$\bar{M}_{\text{eff}}(\mathbf{q}) = 2\sigma\frac{\langle q_{\text{eff}} \rangle}{\beta_2^2} \approx \frac{2\sigma}{\beta_2} \equiv M_\omega \quad (35)$$

This constant parameter M_ω was systematically used in [24–26] to calculate decays of bottomonium. In this case, $\beta_2(B) = 0.48$ GeV and $M_\omega \approx 0.8$ GeV; this value was exactly used in [24–26], and one can see that it agrees with the vertex mass operator $\bar{M}(x)$.

Using w_{nm}, one can calculate production and decay of all states involved.

V. COMPARISON TO $^3P_0$ AND jKj MODELS

Our derivation of the effective Lagrangian ([17]) implies, that the basic interaction behind the string breaking is the scalar confining term acting between the light and heavy (anti)quark, (with subdominant terms of vector color-singlet OGE type, etc.) This is in exact correspondence with the so-called sKs model, studied in detail in [6] and compared to the $3P_0$ model, where it was shown that both are successful and give similar results. In this way, our derivation of $\bar{M}(|\mathbf{x}|)$ in (19) gives additional theoretical status for both sKs and $^3P_0$ models as candidates for the string-breaking kernel. The difference between our approach and sKs model in [6] lies in the treatment of quark motion in the decay matrix elements, which was taken nonrelativistic in [6], with constituent quark mass m_q, while in our approach we are using relativistic formalism for light quarks with current (zero) mass. In particular, our formalism for calculation of the factor $\bar{y}_{123}$ via vertex Z_i factors in Appendixes 1 and 2 of [26] and the use of relativistic string Hamiltonian for $(Q\bar{Q})_n$ and $(Q\bar{q})_n$ states, takes into account relativistic kinematics. With all that difference, results of our analysis are qualitatively and even quantitatively similar to those of sKs and $^3P_0$ models.

In particular, the width and shift of the state $(Q\bar{Q})_n$ is given by

$$(\Delta E_n, \Gamma_n) = (\text{Re}\,w_{nn}, -\text{Im}\,w_{nn}), \quad (36)$$

and for the decay $\Upsilon(4S) \to B\bar{B}$ one obtains

$$\Gamma_{4S}(B\bar{B}) = \left(\frac{M_\omega}{2\omega} \right)^2 0.0033|J_{B\bar{B}}(p)|^2 (\text{GeV}). \quad (37)$$

One can now estimate M_ω from the width of the decay $\Upsilon(4S) \to B\bar{B}$, as it was done in [25]. The wave functions of all participants have been found with good accuracy from the relativistic string Hamiltonian [27] and parameterized by 15 oscillator functions. Comparing experimental value $\Gamma_{\text{exp}}(\Upsilon(4S) \to B\bar{B}) = (20.5 \pm 2.5)$ MeV [28] with (34), one obtains $M_\omega \cong 0.8$ GeV. Another check of our kernel (19) and its average value M_ω is the decay of $\psi(3770)$ into $D\bar{D}$ with the width $\Gamma_{\text{exp}} \cong 25.4$ MeV [28]. Following [27], one takes this state as 1^3D_1 and approximates it with 5 oscillator functions. Calculating (33) with $\bar{M}(q) \to M_\omega = 0.8$ GeV, one obtains $\Gamma_{th} = 22$ MeV,

which agrees with $\Gamma_{\exp}$ within experimental ($\sim 10\%$) accuracy.[2]

One can now compare this value with (19) and take into account, that the r.m.s. radii for $\Upsilon(4S)$ and B are 0.9 fm and 0.5 fm [27], respectively. Therefore, one obtains $\sigma 2 r_B \approx 0.9$ GeV, and $\sigma r_\Upsilon \approx 0.81$ GeV, in good agreement with (35).

In a similar way, the decay of $\Upsilon(5S)$ into six channels of $B_i \bar{B}_k$, where $B_i(B_k) = B, B^*, B_s, B_s^*$ were considered in [29]. The total width computed in [29] is $\Gamma_{\text{tot}} = 116$ MeV M_ω^2, where M_ω is in GeV. Comparing to $\Gamma_{\text{tot}}(\exp) = (110 \pm 13)$ MeV [28], one has $M_\omega = (0.91 \div 1.03)$ GeV in reasonable agreement with (35). Also, the experimental ratio of decay into beauty-strange mesons is well reproduced in [29], which supports the flavor-blind kernel $\bar{M}$ in (14).

We now turn to the string-breaking emission process. It is best studied in the transitions of the type $(Q\bar{Q})_n \rightarrow (Q\bar{Q})_{n'}\pi\pi$ [24,25,29] and in $(Q\bar{Q})_n \rightarrow (Q\bar{Q})_n \eta$ in [30].

In [24,25], the subthreshold string breaking was considered for the decay of $\Upsilon(4S)$, while in [29], the decay of $\Upsilon(5S)$ was analyzed, In all cases of this double-pion or single eta (n, n') transitions, the effective $Z^{(U)}$ factor appears to be small, $Z^{(\pi)} \approx Z^{(\eta)} \approx O(\frac{f_\pi}{M_\omega})$.

In conclusion, we have derived the relativistic string-breaking kernel, which is the scalar color-singlet confining interaction, which is flavor blind and nonlocal for zero mass $q\bar{q}$ pair, tending to the confining σr potential for a long breaking string. This form is close to $^3P_0$ and sKs models in the nonrelativistic formalism.

ACKNOWLEDGMENTS

APPENDIX

1. Properties of the mass kernel, Eq. (17)

To simplify calculations, one can choose the correlator $D(z)$ in the Gaussian form, $D(z) = D(0)\exp(-\frac{z^2}{4\lambda^2})$, where λ is the vacuum correlation length, $\lambda \approx (0.1 \div 0.2)$ fm, then $\sigma = \frac{1}{2}\int D(z)d^2z = 2\pi D(0)\lambda^2$. In the final expressions for the asymptotics of $\mathcal{M}(\mathbf{x}, \mathbf{y})$, only σ enters, therefore the value of λ and the form of $D(z)$ influences only the small x, y region (the same is true for the dependence on the contours of the contour gauge, see Appendix 3 of [17]). Now the mass operator of Eq. (15) has the form

$$M(p_4 = 0, \mathbf{x}, \mathbf{y}) \equiv M(\mathbf{x}, \mathbf{y}) = \sigma(\mathbf{xy})f(\mathbf{x}, \mathbf{y})\beta\Lambda(\mathbf{x}, \mathbf{y}), \quad \text{(A1)}$$

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where

$$f(\mathbf{x}, \mathbf{y}) = \frac{1}{2\sqrt{\pi}\lambda} \int_0^1 ds \int_0^1 dt \exp[-(\hat{x}s - \hat{y}t)^2],$$

$$\hat{x}, \hat{y} = \frac{1}{2\lambda}(x, y), \quad \text{(A2)}$$

with asymptotics $f(\mathbf{x}, \mathbf{x}) = \frac{1}{|\mathbf{x}|}$, $|\mathbf{x}| \rightarrow \infty$, and

$$\Lambda(\mathbf{x}, \mathbf{y}) = \sum_n \psi_n(\mathbf{x})\text{sign}\varepsilon_n \psi_n^+(\mathbf{y}), \quad \text{(A3)}$$

while $\psi_n(\mathbf{z})$ satisfy equations

$$\left(\frac{\boldsymbol{\alpha}}{i}\frac{\partial}{\partial \mathbf{z}} + \beta m\right) + \beta \int \mathcal{M}(\mathbf{z}, \mathbf{w})\psi_n(\mathbf{w})d\mathbf{w} = \varepsilon_n \psi_n(\mathbf{z}). \quad \text{(A4)}$$

The mean-field-type Eqs. (A1) and (A4) have been solved in [15–17], using the relativistic WKB method [23], and below are listed expressions for the asymptotics of $M(\mathbf{x}, \mathbf{y})$, extracted from [17], $M(\mathbf{x}, \mathbf{y})$ is a 4×4 matrix, which can be represented in the 2×2 form, with entries expressed via Pauli matrices σ_i,

$$\mathcal{M}_{ik}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}, \quad \mathcal{M}_{ik} = a_{ik}\hat{1} + \mathbf{b}_{ik}\boldsymbol{\sigma}. \quad \text{(A5)}$$

In the region where $\cos\theta$ between $\mathbf{x}, \mathbf{y}$ is close to 1, and $|\mathbf{x}| \sim |\mathbf{y}|$, one obtains

$$a_{11} = \sigma|\mathbf{x}|\Delta_1(x, y, \theta), \qquad \mathbf{b}_{11} = -\frac{\mathbf{L}\sigma^2}{4\pi}\Delta_1'(x, y, \theta)$$

$$a_{12} = \frac{\sigma^2}{2\pi}\Delta_{12}, \qquad \mathbf{b}_{12} = \frac{\sigma^2}{2\pi}(\mathbf{n}\Delta_{12}' + (\mathbf{n} \times \mathbf{L})\Delta_{12}''. \quad \text{(A6)}$$

The symmetry of $\mathcal{M}_{ik}$ is $\mathcal{M}_{22} = \mathcal{M}_{11}$, $\mathcal{M}_{21} = -\mathcal{M}_{12}$. Here, $(|\mathbf{x}| \equiv x, |\mathbf{y}| \equiv y)$

$$\Delta_1(x, y, \theta) = \frac{\sigma^2 x^2 K_1(\sigma x\sqrt{(x-y)^2 + \theta^2 x^2})}{2\pi^2\sqrt{(x-y)^2 + \theta^2 x^2}}, \quad \text{(A7)}$$

$$\Delta_1'(x, y, \theta) = K_0\left(\sigma xy\sqrt{\theta^2 + \frac{(x-y)^2}{xy}}\right). \quad \text{(A8)}$$

Note that Δ_1 plays the role of smeared normalized function $\tilde{\delta}^{(3)}(\mathbf{x} - \mathbf{y})$:

$$\int \Delta_1(x, y, \theta)d^3y = 1, \quad \text{(A9)}$$

where one is using the relation $\int \delta(1 - \cos\theta)d\cos\theta = \frac{1}{2}$. All functions $\Delta_{12}, \Delta_{12}', \Delta_{12}''$ are antisymmetric in x, y and therefore are zero in the local limit $x = y$, therefore they are not given here, see [17].

Expressions (A7) and (A8) are calculated for the case $m_q = 0$, however the derivation in [15,17] is valid also in the case $m \neq 0$, where one can approximately at large x replace σx in (A7) by $\sigma x + m_q$, so that Δ_1 becomes

$$\Delta_1(x, y, \theta) \to \frac{(\sigma x + m_q)^2 K_1((\sigma x + m_q)\sqrt{(x-y)^2 + \theta^2 x^2})}{2\pi^2 \sqrt{(x-y)^2 + \theta^2 x^2}}.$$

(A10)

One can see in (A10), that the range of nonlocality, $|x - y|_{\text{eff}}$, in the limit of large m_q tends to zero, and one can replace Δ_1 by $\delta^{(3)}(\mathbf{x} - \mathbf{y})$. This leads to a moderate increase of effective confinement with growing m_q. The same sort of mass dependence of confining interaction between quark and antiquark was observed recently on the lattice [31].

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