Supergraph techniques for $D = 3$, $\mathcal{N} = 1$ broken supersymmetric theories

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We enlarge the usual $D = 3$ $\mathcal{N} = 1$ supergraph techniques to include the case of (explicitly or spontaneously) broken supersymmetric gauge theories. To illustrate the utility of these techniques, we calculate the two-loop effective potential of the $SQED_3$ by using the tadpole and the vacuum bubble methods. In these methods, to investigate the possibility of supersymmetry breaking, the superfields must be shifted by θ_α dependent classical superfields (vacuum expectation values), what implies in the explicit breakdown of supersymmetry in the intermediate steps of the calculation. Nevertheless, after studying the minimum of the resulting effective potential, we find that supersymmetry is conserved, while gauge symmetry is dynamically broken, with a mass generated for the gauge superfield.

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I. INTRODUCTION

Supersymmetry (susy), if it exists in nature, must be a broken symmetry since up to now mass degenerate Bose-Fermi supermultiplets have never been observed. So, every realistic model must include a mechanism of susy breakdown [1]. On the other hand, the superfield formalism developed for exactly supersymmetric theories is a powerful technique for doing calculations, and its possible extension to broken susy is welcome.

The usual way of studying supersymmetry breakdown is by treating the breaking terms in the Lagrangian (of quadratic or of higher number of fields) as interaction vertices to be incorporated as perturbations into the supersymmetry preserving theory. Still, in the works [2,3], the superfield formalism for $\mathcal{N} = 1$ was enlarged to softly broken supersymmetric models (in which no quadratic ultraviolet divergences are triggered by the breaking terms) in 3 and 4 dimensions of space-time, by treating on an equal footing all bilinear terms. The main difficulty to overcome in this extension is to calculate the inverse of the kernel of the bilinear part of the Lagrangian to obtain the superpropagators.

In this paper, we will focus on the construction of this extension for treating spontaneously broken supersymmetric gauge models in three dimensions. This situation involves bilinear breaking terms of forms different from that studied in [2], besides symmetry breaking monomials with higher number of fields. In fact, for studying the possibility of spontaneous breaking of susy, we must translate the fields by their vacuum expectation values, including θ_α coordinate dependent terms. The kernel of the resulting bilinear part of the Lagrangian is more general than that in [2], and by using their operator algebra to obtain the superpropagator of the spinorial gauge superfield, we learned that it needs a completion. In effect, in [2], the authors present an algebra of six antisymmetric plus six symmetric bi-spinor operators, as a basis on which the bi-spinor superpropagators (of the spinorial gauge potential) could be expanded. It is interesting to note that these 12 operators have a closed algebra, even if they fail as a basis for the more general form of superpropagators that we find in the example to be discussed below. We show that two other operators are required to complete a basis in the more general case.

The paper is organized as follows. In Sec. II the Supersymmetric Quantum Electrodynamics in 3D ($SQED_3$) is defined in the superfield language, the algebra of operators needed to invert the kernel of the bilinear part of the Lagrangian is developed, and the superpropagators of the shifted $SQED_3$ are derived. In Sec. III we compute the zero-, the one- and the two-loop corrections to the effective potential. The Conclusions, Sec. IV, contain some discussions of the results. Details of the calculations of the effective potential are given in the Appendices.

II. THE MODEL AND THE ALGEBRA OF OPERATORS IN 3D

In the notation of [4] (see also our Appendix A), the $\mathcal{N} = 1$ $SQED_3$ is defined by the action

$$S = \int d^5 z \left\{ \frac{1}{2} W^\alpha W_\alpha - \frac{1}{2} \overline{\nabla^\alpha \Phi} \nabla_\alpha \Phi + M \bar{\Phi} \Phi \right\}, \quad (1)$$

where $\alpha, \beta = 1, 2$ are spinorial indices. The UV finiteness of this model to all loop orders was studied in [5]. The basic elements involved in (1) are the complex (matter) scalar superfield,

$$\Phi(x, \theta) = \frac{1}{\sqrt{2}} (\Sigma + i\Pi) = \varphi(x) + \theta^\alpha \psi_\alpha(x) - \theta^2 F(x), \quad (2)$$

where Σ and Π are real superfields and the component fields φ, ψ_α and F are, respectively, a complex scalar field,

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a Dirac field and a complex scalar auxiliary field. The spinor gauge potential is given by

$$A_\alpha(x,\theta) = \chi_\alpha(x) - \theta_\alpha B(x) + i\theta^\beta V_{\beta\alpha}(x) - \theta^2[2\lambda_\alpha(x) + i\partial_{\alpha\beta}\chi^\beta(x)], \tag{3}$$

where χ_α and B are auxiliary fields, λ_α is the photino field and $V_a \equiv (\gamma_a)^{\alpha\beta}V_{\alpha\beta}$ ($a, b = 0, 1, 2$ are Lorentz indices) the 3-vector electromagnetic potential. The gauge superfield strength is defined as $W_\alpha = \frac{1}{2}D^\beta D_\alpha A_\beta$ and has, as one of its component fields, $F_{ab} = \partial_a V_b - \partial_b V_a$, the electromagnetic 3-tensor field strength.

The susy covariant spinorial derivative is given by $D_\alpha = \partial_\alpha + i\theta^\beta\partial_{\alpha\beta}$ and the susy and gauge covariant derivative by $\nabla_\alpha = D_\alpha - ieA_\alpha$. The action (1) is invariant under the following gauge transformations:

$$\Phi \to \Phi' = e^{ieK}\Phi \qquad A_\alpha \to A'_\alpha = A_\alpha + D_\alpha K, \tag{4}$$

where e is the gauge coupling constant and $K(x,\theta)$ is a real scalar superfield. Under these transformations, the electromagnetic field strength and the covariant derivative go in

$$W_\alpha \to W'_\alpha = W_\alpha \qquad \nabla_\alpha \to \nabla'_\alpha = e^{ieK}\nabla_\alpha e^{-ieK}.$$

We choose to work with the Lorentz-like gauge fixing term,

$$S_{\rm FG} = \int d^5z\left(-\frac{1}{4\alpha}\right)D^\alpha A_\alpha D^2 D^\beta A_\beta, \tag{5}$$

where α is a dimensionless parameter. With this gauge choice, the ghosts are free and can be ignored. By adding (5) to (1), writing Φ in terms of Σ and Π, and integrating by parts, the full action reads

$$S = \int d^5z\left\{\frac{1}{2}A_\alpha\left[-\frac{1}{2}D^2D^\beta D^\alpha + \frac{1}{2\alpha}D^2D^\alpha D^\beta\right]A_\beta + \frac{1}{2}\Sigma(D^2+M)\Sigma + \frac{1}{2}\Pi(D^2+M)\Pi + \frac{e}{2}(\Sigma D^\alpha\Pi A_\alpha - \Pi D^\alpha\Sigma A_\alpha) - \frac{e^2}{2}A^2(\Sigma^2+\Pi^2)\right\}. \tag{6}$$

In order to compute the effective potential of nonsusy theories, one counts on three popular methods: the Coleman and Weinberg [6], the tadpole [7] and the vacuum bubble [8] methods. In principle, all these methods can be implemented in both superfields and component fields of supersymmetric gauge theories. In four dimensions, the one-loop effective potential of the supersymmetric QED model (along with other two susy models) was evaluated by implementing the Coleman-Weinberg method in the superfield formalism [9]. However, the implementation of this method is cumbersome or even impossible beyond the one-loop order. On the other hand, the other two methods are simpler and will be used to compute the effective potential of the SQED_3 model.

To this end, we must shift the superfield Σ in (6) by a classical superfield $\sigma(\theta)$: $\Sigma \to \Sigma + \sigma(\theta)$. As we want to study the possibility of susy breaking, this classical field $\sigma(\theta)$ must include a non zero (component) auxiliary field σ_2, that is, we must consider

$$\sigma(\theta) = \sigma_1 - \theta^2\sigma_2. \tag{7}$$

The resulting expression for the shifted action is

$$S'[\sigma_1,\sigma_2;\Sigma,\Pi,A_\alpha] \equiv \int d^5z\left\{\frac{1}{2}A_\alpha\left[-\frac{1}{2}D^2D^\beta D^\alpha + \frac{1}{2\alpha}D^2D^\alpha D^\beta + \frac{e^2}{2}\sigma^2(\theta)C^{\alpha\beta}\right]A_\beta + \frac{1}{2}\Pi(D^2+M)\Pi + \xi A^\alpha\left[\frac{e}{2}(\sigma(\theta)D_\alpha - D_\alpha\sigma(\theta))\right]\Pi + \frac{1}{2}\Sigma(D^2+M)\Sigma + \frac{e}{2}(\Sigma D^\alpha\Pi A_\alpha - \Pi D^\alpha\Sigma A_\alpha) - e^2\sigma(\theta)A^2\Sigma - \frac{e^2}{2}A^2(\Sigma^2+\Pi^2) + (D^2\sigma + M\sigma)\Sigma + \frac{1}{2}\sigma(D^2+M)\sigma\right\}. \tag{8}$$

Furthermore we introduced the parameter ξ (to be made $\xi = 1$ at the end of the calculations) in front of the mixing (A_α, Π) terms to allow to track the effects of the mixture in the intermediate steps of the calculations.

The bilinear part of the action, with external source terms added, reads

$$S_{\rm bil} = \int d^5z d^5z'\left[\frac{1}{2}A_\alpha(z)\mathcal{O}^{\alpha\beta}(z,z')A_\beta(z') + \frac{1}{2}\Pi(z)\mathcal{O}(z,z')\Pi(z') + \xi A_\alpha(z)\mathcal{O}^\alpha(z,z')\Pi(z') + \frac{1}{2}\Sigma(z)\mathcal{O}(z,z')\Sigma(z') + J(z)\Pi(z) + \eta^\alpha(z)A_\alpha(z) + G(z)\Sigma(z)\right], \tag{9}$$

where the kernel operators $\mathcal{O}$ are functions, not only of the susy covariant operators D_α and $\partial_{\alpha\beta}$ (and their square powers), but also of θ_α and θ^2:

$$\mathcal{O}_{\alpha\beta}(z, z') = \left[\frac{(1-\alpha)}{2\alpha} i\partial_{\alpha\beta}D^2 + \frac{1}{2}\left(-\frac{1+\alpha}{\alpha}\Box + e^2\sigma_1^2\right)C_{\alpha\beta} - e^2\sigma_1\sigma_2 C_{\alpha\beta}\theta^2\right]\delta^5(z-z') \tag{10a}$$

$$\mathcal{O}(z, z') = [M + D^2]\delta^5(z-z') \tag{10b}$$

$$\mathcal{O}_\alpha(z, z') = \left[-\frac{e\sigma_2}{2}\theta_\alpha - \frac{e\sigma_1}{2}D_\alpha + \frac{e\sigma_2}{2}\theta^2 D_\alpha\right]\delta^5(z-z'). \tag{10c}$$

It must be noted that the mixing between the gauge field and the matter field represented in $\mathcal{O}_\alpha$ can, in general, be avoided by using an R_ξ gauge. This is not true when $\sigma_2 \neq 0$. In this case the mixture is unavoidable and the use of the Lorentz-like gauge fixing term has the advantage of having a decoupled ghost sector.

From this action, the superpropagators can be calculated in the usual way. We start by considering the generating functional $\mathcal{Z}[J, \eta]$:

$$\mathcal{Z}[J, \eta] = \mathcal{N}\int \mathcal{D}\Sigma\mathcal{D}\Pi\mathcal{D}A_\alpha \exp(iS_{bil}), \tag{11}$$

change the superfields by

$$\begin{aligned}\Sigma(z) &\to \Sigma(z) - \int d^5z'\Delta_\Sigma(z,z')G(z'),\\ \Pi(z) &\to \Pi(z) - \int d^5z'\{\Delta(z,z')J(z') + \Delta^\alpha(z,z')\eta_\alpha(z')\},\\ A_\alpha(z) &\to A_\alpha(z) - \int d^5z'\{J(z')\Delta_\alpha(z',z) + \Delta_\alpha{}^\beta(z,z')\eta_\beta(z')\},\end{aligned} \tag{12}$$

and determine the superpropagators $\triangle$ by imposing that the terms which mix fields with currents add to zero. With these conditions, the integration in the shifted superfields can be carried out, leaving $\mathcal{Z}[J, \eta]$ as a functional of the sources:

$$\begin{aligned}\mathcal{Z}[J, \eta] = \exp\Bigg[i\iint d^5zd^5z'\Bigg\{&-\frac{1}{2}\eta^\alpha(z)\Delta_\alpha{}^\beta(z,z')\eta_\beta(z')\\ &-\frac{1}{2}J(z)\Delta(z,z')J(z') - J(z)\Delta^\alpha(z,z')\eta_\alpha(z')\\ &-\frac{1}{2}G(z)\Delta_\Sigma(z,z')G(z')\Bigg\}\Bigg].\end{aligned} \tag{13}$$

From this expression, it follows that the superpropagators are given by

$$\langle TA_\alpha(z)A_\beta(z')\rangle = i\Delta_{\alpha\beta}(z,z') = i\Theta^{-1}_{\alpha\beta}(z,z'), \tag{14a}$$

$$\begin{aligned}\langle T\Pi(z)\Pi(z')\rangle &= i\Delta(z,z')\\ &= i\mathcal{O}^{-1}(z,z') + i\xi^2\iint_{z_1,z_2}\mathcal{O}^{-1}(z,z_1)\\ &\quad\times H(z_1,z_2)\mathcal{O}^{-1}(z_2,z'),\end{aligned} \tag{14b}$$

$$\begin{aligned}\langle T\Pi(z)A_\alpha(z')\rangle &= i\Delta_\alpha(z,z')\\ &= i\xi\iint_{z_1,z_2}\mathcal{O}^{-1}(z,z_1)\mathcal{O}^\beta(z_2,z_1)\\ &\quad\times\Theta^{-1}_{\beta\alpha}(z_2,z'),\end{aligned} \tag{14c}$$

$$\langle T\Sigma(z)\Sigma(z')\rangle = i\mathcal{O}^{-1}(z,z') \tag{14d}$$

with

$$\begin{aligned}\Theta_{\alpha\beta}(z,z') &= \mathcal{O}_{\alpha\beta}(z,z') + \xi^2 Q_{\alpha\beta}(z,z'),\\ H(z,z') &= \iint_{z_1,z_2}\mathcal{O}^\alpha(z_1,z)\Theta^{-1}{}_\alpha{}^\beta(z_1,z_2)\mathcal{O}_\beta(z_2,z'),\\ Q_{\alpha\beta}(z,z') &= \iint_{z_1,z_2}\mathcal{O}_\alpha(z,z_1)\mathcal{O}^{-1}(z_1,z_2)\mathcal{O}_\beta(z',z_2).\end{aligned} \tag{15}$$

To explicitly find these superpropagators, we develop the algebra of operators used for calculating the inverse of the matrices $\mathcal{O}$. Let us begin by considering the scalar sector. Any scalar operator $\mathcal{O} = \mathcal{O}(\theta_\alpha, D_\alpha, i\partial_{\alpha\beta})$ can be expanded in terms of six scalar operators,

$$\mathcal{O} = \sum_{i=0}^{5} p_i P_i, \tag{16}$$

defined in [2] as

$$\begin{aligned}&P_0 = 1, \qquad P_1 = D^2, \qquad P_2 = \theta^2,\\ &P_3 = \theta^\alpha D_\alpha, \qquad P_4 = \theta^2 D^2, \qquad P_5 = i\partial_{\alpha\beta}\theta^\alpha D^\beta,\end{aligned} \tag{17}$$

which form a basis in this sector. The coefficients p_i are, in general, functions of the d'Alembert operator $\Box$, the parameters of the theory (masses, coupling constants, etc.) and of the components σ_1 and σ_2 of the classical superfield.

The product of the operators P_i is presented in Table I. In addition, one has the trivial results $P_0P_i = P_iP_0 = P_i$, with $i = 0, \ldots, 5$.

Working with this basis, the inversion of $\mathcal{O}$ follows immediately. Since $\mathcal{O}^{-1} = \sum_i \tilde{p}_i P_i$ in the basis $\{P_i\}$, the requirement $\mathcal{O}^{-1}\mathcal{O} = 1$ leads, after using the Table I, to a soluble system of six equations for the six unknown coefficients $\tilde{p}_i$.

TABLE I. Multiplication table in the scalar sector.

	P_1	P_2	P_3	P_4	P_5
P_1	$\Box$	$-P_0 + P_3 + P_4$	$2P_1 + P_5$	$-P_1 + \Box P_2 - P_5$	$\Box(-2P_0 + P_3)$
P_2	P_4	0	0	0	0
P_3	$-P_5$	$2P_2$	$P_3 - 2P_4$	$2P_4$	$2\Box P_2 + P_5$
P_4	$\Box P_2$	$-P_2$	$2P_4$	$-P_4$	$-2\Box P_2$
P_5	$-\Box P_3$	0	$-2\Box P_2 + P_5$	0	$\Box(P_3 + 2P_4)$

For the inversion of $\mathcal{O}_{\alpha\beta}$ we need a basis of bi-spinor operators. In [2] a ''basis'' of 12 bi-spinor operators,

$$R_i^{\alpha\beta} = i\partial^{\alpha\beta}P_i, \qquad S_i^{\alpha\beta} = C^{\alpha\beta}P_i, \tag{18}$$

was introduced.

According to the authors, any bi-spinor operator may be expanded in terms of R_i and S_i, that is,

$$\mathcal{O}_{\alpha\beta} = \sum_{i=0}^{5}(r_i R_{i,\alpha\beta} + s_i S_{i,\alpha\beta}), \tag{19}$$

where as before $r_i = r_i(\Box, c)$ and $s_i = s_i(\Box, c)$, with c labeling all the parameters of the theory. The (closed) operator algebra obeyed by R_i and S_i is reproduced in Table II. In these tables we have defined $P_{ij} \doteq P_i P_j$, where the expansion of the result of the multiplication $P_i P_j$ in terms of the six P_i must be read on Table I.

Even though (19) works for the operators $\mathcal{O}_{\alpha\beta}$ found in [2], it does not work for the inversion of the more general form of $\mathcal{O}_{\alpha\beta}$ that we have. It should be noted that any antisymmetric bi-spinor operator $S^{\alpha\beta}$ has only one independent component and can always be written as $S^{\alpha\beta} = C^{\alpha\beta}[-\frac{1}{2}S^{\gamma}_{\gamma}]$, where S^{γ}_{γ} is a scalar operator that can be expanded in terms of the six P_i. However, not all symmetric bi-spinor (which have three independent components) can be written as a product of $i\partial^{\alpha\beta} = \frac{1}{2}[D^{\alpha}D^{\beta} + D^{\beta}D^{\alpha}]$ by a scalar operator expandable in terms of the six P_i. In fact, up to two supercovariant (spinorial) derivatives, one has the independent symmetric operator

$$M^{\alpha\beta} \doteq \theta^{\alpha}D^{\beta} + \theta^{\beta}D^{\alpha}. \tag{20}$$

That $M^{\alpha\beta}$ is independent of the $R_i^{\alpha\beta}$ can be seen by explicitly applying M, or a linear combination of the six R_i operators, to an arbitrary superfield and verifying that there is no way of choosing the coefficients of the linear combination of R_i to get the same result. More easily, let us see an example of inconsistency that appears if we assume that M is a superposition of the R_i. Suppose that

$$\theta_{\alpha}D_{\beta} + \theta_{\beta}D_{\alpha} = \sum_{i=0}^{5} r_i R_{i,\alpha\beta}. \tag{21}$$

The coefficients r_i can be determined by contracting the two sides with $i\partial^{\alpha\beta}$. Using the relations $\theta_{\alpha}\theta_{\beta} = -C_{\alpha\beta}\theta^2$ and $\partial^{\alpha\beta}\partial_{\beta\gamma} = \delta^{\alpha}_{\gamma}\Box$, along with the above definitions and multiplication tables, this expression reduces to

$$\theta_{\alpha}D_{\beta} + \theta_{\beta}D_{\alpha} = -\frac{1}{\Box}R_{5,\alpha\beta}. \tag{22}$$

If we now multiply both sides of (22) on the left by θ^{α}, we get the inconsistency

$$3\theta^2 D_{\beta} = \theta^2 D_{\beta}, \tag{23}$$

showing that the assumption (21) is incorrect.

Now, by starting with $M^{\alpha\beta}$ we can define six new operators $M_i^{\alpha\beta} \doteq P_i M^{\alpha\beta}$ with at most three spinorial covariant derivatives (in the $\mathcal{N} = 1$ superfield formalism in 3 dimensions, the product of three or more covariant spinorial derivatives can be reduced to products of two or less spinorial covariant derivatives D^{α} and the (also susy covariant) space-time derivative $i\partial_{\alpha\beta}$). After a little algebraic work we can see that $M_i^{\alpha\beta} = 0$, for $i = 2, 3, 4$ and 5, and so, the only new operator, aside from $M_0^{\alpha\beta} = M^{\alpha\beta}$, is $M_1^{\alpha\beta} = D^2 M^{\alpha\beta}$. For convenience, instead of using M_1, we will work with

$$N^{\alpha\beta} \doteq i\theta^{\alpha}\partial^{\beta\gamma}D_{\gamma} + i\theta^{\beta}\partial^{\alpha\gamma}D_{\gamma} = -M_1^{\alpha\beta} + 2R_0^{\alpha\beta}. \tag{24}$$

The multiplication table of the operators M and N with the 12 (R, S) ones, that complements the Table II, is shown in the Table III.

Therefore, the consistent expansion of $\mathcal{O}_{\alpha\beta}$ that replaces (19) is given by

TABLE II. Partial multiplication table in the gauge sector, $(XY)^{\alpha\beta} = X^{\alpha\gamma}Y_{\gamma}^{\beta}$.

	R_j	S_j
R_i	$\Box S_{ij}$	R_{ij}
S_i	R_{ij}	S_{ij}

TABLE III. Partial multiplication table in the gauge sector ($X \doteq M - S_3$ and $Y \doteq N - R_3 + S_5$).

	N	M
S_0	N	M
S_1	$-\Box M + 2R_1$	$-N + 2R_0$
S_2	0	0
S_3	$N - 2R_4$	$M - 2R_2$
S_4	$2R_4$	$2R_2$
S_5	$-\Box(-M + 2R_2) + 2R_5$	$N - 2R_3 - 2R_4$
R_0	$\Box(M + S_3) + R_5$	$N - R_3 - S_5$
R_1	$-\Box(N - R_3 - 2S_1 - S_5)$	$-\Box(M - 2S_0 + S_3) - R_5$
R_2	0	0
R_3	$-\Box(-M - S_3 + 2S_4) + R_5$	$N - R_3 - 2\Box S_2 - S_5$
R_4	$2\Box S_4$	$2\Box S_2$
R_5	$-\Box(-Y + 2\Box S_2)$	$-\Box(-X + 2S_4) + R_5$
N	$-\Box(-4R_2 + S_3 + 6S_4) - 2R_5$	$2R_3 + 4R_4 - 6\Box S_2 + S_5$
M	$-2N - 4R_4 + 6\Box S_2 - 3S_5$	$-2M - 4R_2 + 3S_3 + 6S_4$

	S_0	S_1	S_2	S_3	S_4	S_5
N	N	$\Box M$	$2R_2$	$N - 2R_4$	$2R_4$	$-\Box(M - 2R_2)$
M	M	N	0	$M + 2R_2$	0	$-N - 2R_4$

	R_0	R_1	R_2	R_3	R_4	R_5
N	$-\Box X - R_5$	$-\Box Y$	$2\Box S_2$	$-\Box(X + 2S_4) - R_5$	$2\Box S_4$	$\Box(Y + 2\Box S_2)$
M	$-Y$	$-\Box X - R_5$	0	$-Y + 2\Box S_2$	0	$-\Box(-X + 2S_4) + R_5$

$$\mathcal{O}_{\alpha\beta} = \sum_{i=0}^{5}(r_i R_{i,\alpha\beta} + s_i S_{i,\alpha\beta}) + mM_{\alpha\beta} + nN_{\alpha\beta}, \quad (25)$$

where the set of 14 operators $\{R_i, S_i, M, N\}$ forms a basis in the gauge sector. The inverse operator $\mathcal{O}^{-1}$ is obtained from its definition $\mathcal{O}^{-1,\alpha\beta}\mathcal{O}_{\beta\gamma} = \delta^{\alpha}_{\gamma}$ and the fact that $\mathcal{O}^{-1}$ must have a expansion similar to that of $\mathcal{O}$, (25), with coefficients $\{\tilde{r}_i, \tilde{s}_i, \tilde{m}, \tilde{n}\}$ to be determined.

The bilinear mixing terms in (8) give rise to a spinorial mixing superpropagator $\langle T\Pi(z)A_\alpha(z')\rangle$. As mentioned earlier, this is a consequence of the translation of the scalar

TABLE IV. Multiplication table in the mixing sector.

	T^1_β	T^2_β	T^3_β	T^4_β
T^1_α	$-S_2$	$-R_2$	$-S_4$	$-R_4$
T^2_α	R_2	$\Box S_2$	R_4	$\Box S_4$
T^3_α	$\frac{1}{2}(M - S_3) - S_4$	$-R_4 - \frac{1}{2}S_5 - \frac{1}{2}N$	$-\Box S_2 + \frac{1}{2}S_5 + \frac{1}{2}N$	$-\frac{1}{2}\Box(M - S_3 + 2R_2)$
T^4_α	$R_3 + \frac{1}{2}S_5 - \frac{1}{2}N + R_4$	$\frac{1}{2}\Box(M + S_3 + 2S_4) + R_5$	$-\frac{1}{2}\Box(M + S_3 - 2R_2) - R_5$	$\Box(\Box S_2 - R_3 - \frac{1}{2}S_5 + \frac{1}{2}N)$
T^5_α	$-\frac{1}{2}(M + S_3) + S_0$	$R_0 - R_3 + \frac{1}{2}S_5 + \frac{1}{2}N$	$-\frac{1}{2}N + S_1 + \frac{1}{2}S_5$	$\frac{1}{2}\Box(M - S_3) + R_1 + R_5$
T^6_α	$-R_0 - \frac{1}{2}S_5 + \frac{1}{2}N$	$-\frac{1}{2}\Box(M - S_3 + 2S_0) - R_5$	$\frac{1}{2}\Box(M + S_3) - R_1$	$\Box(R_3 - \frac{1}{2}S_5 - \frac{1}{2}N - S_1)$
T^7_α	S_2	R_2	S_4	R_4
T^8_α	$-R_2$	$-\Box S_2$	$-R_4$	$-\Box S_4$

	T^5_β	T^6_β	T^7_β	T^8_β
T^1_α	$\frac{1}{2}(M - S_3)$	$-\frac{1}{2}(N + S_5)$	0	0
T^2_α	$R_3 + \frac{1}{2}S_5 - \frac{1}{2}N$	$\frac{1}{2}\Box(M + S_3) + R_5$	0	0
T^3_α	$-\frac{1}{2}N - \frac{1}{2}S_5$	$\frac{1}{2}\Box(M - S_3)$	$-R_2 + \frac{1}{2}S_3 + S_4 - \frac{1}{2}M$	$R_4 - \Box S_2 + \frac{1}{2}S_5 + \frac{1}{2}N$
T^4_α	$\frac{1}{2}\Box(M + S_3) + R_5$	$\Box(R_3 + \frac{1}{2}S_5 - \frac{1}{2}N)$	$-R_3 - \frac{1}{2}S_5 + \frac{1}{2}N - R_4 + \Box S_2$	$\Box(R_2 - \frac{1}{2}S_3 - S_4 - \frac{1}{2}M) - R_5$
T^5_α	$R_0 - S_1$	$\Box S_0 - R_1$	$R_2 - \frac{1}{2}S_3 - S_4 + \frac{1}{2}M$	$-R_4 + \Box S_2 - \frac{1}{2}S_5 - \frac{1}{2}N$
T^6_α	$R_1 - \Box S_0$	$\Box(S_1 - R_0)$	$+R_3 + R_4 - \Box S_2 + \frac{1}{2}S_5 - \frac{1}{2}N$	$\Box(-R_2 + \frac{1}{2}S_3 + S_4 + \frac{1}{2}M) + R_5$
T^7_α	$R_2 - S_4$	$\Box S_2 - R_4$	0	0
T^8_α	$R_4 - \Box S_2$	$\Box(S_4 - R_2)$	0	0

superfield by its vacuum expectation value. So, it is also convenient to define, for the expansion of $\mathcal{O}_\alpha$, the basis of eight spinorial operators

$$\begin{aligned} T^1_\alpha &= \theta_\alpha \qquad T^2_\alpha = i\partial_{\alpha\beta}\theta^\beta \qquad T^3_\alpha = \theta_\alpha D^2 \\ T^4_\alpha &= i\partial_{\alpha\beta}\theta^\beta D^2 \qquad T^5_\alpha = D_\alpha \qquad T^6_\alpha = i\partial_{\alpha\beta}D^\beta \\ T^7_\alpha &= \theta^2 D_\alpha \qquad T^8_\alpha = i\partial_{\alpha\beta}\theta^2 D^\beta \end{aligned} \tag{26}$$

The results of their multiplications are presented in Table IV.

In the momentum space ($i\partial_{\alpha\beta} \to k_{\alpha\beta}$) and after an extensive use of these multiplication tables, the superpropagators (14a)–(14d) can be written as

$$\langle TA_\alpha(k,\theta)A_\beta(-k,\theta')\rangle = i\bigg\{\sum_{i=0}^{5}(r_i R_{i,\alpha\beta} + s_i S_{i,\alpha\beta}) + mM_{\alpha\beta} + nN_{\alpha\beta}\bigg\}\delta^2(\theta-\theta'), \tag{27a}$$

$$\langle T\Pi(k,\theta)\Pi(-k,\theta')\rangle = i\bigg(\sum_{i=0}^{5} a_i P_i\bigg)\delta^2(\theta-\theta'), \tag{27b}$$

$$\langle T\Pi(k,\theta)A_\alpha(-k,\theta')\rangle = i\bigg(\sum_{i=1}^{8} b_i T^i_\alpha\bigg)\delta^2(\theta-\theta'), \tag{27c}$$

$$\langle T\Sigma(k,\theta)\Sigma(-k,\theta')\rangle = i[c_0 P_0 + c_1 P_1]\delta^2(\theta-\theta'). \tag{27d}$$

The coefficients $r_i \cdots c_1$ are listed in the Appendix B. In the rest of the paper we shall study the symmetry properties of the vacuum of the $SQED_3$ model, by calculating the effective potential up to 2-loops in the perturbation theory. For the 1-loop calculation, we use the tadpole method [7], while for the 2-loop one, we use the vacuum bubble method [8]. As we will see, susy remains unbroken up to 2-loops, while the internal $U(1)$ gauge symmetry is broken.

III. THE EFFECTIVE POTENTIAL UP TO TWO-LOOPS

A. THE CLASSICAL POTENTIAL

The classical effective action can be read from (8). The terms depend only on the classical field σ are

$$\Gamma_{\rm cl} = \int d^5z \frac{1}{2}\sigma(D^2 + M)\sigma \equiv -\int d^3x U_{\rm cl}(\sigma_1,\sigma_2),$$

where the second equality defines the Classical Potential. After integrating in the θ variables we get $U_{\rm cl}(\sigma_1,\sigma_2) = -\frac{1}{2}\sigma_2^2 - M\sigma_1\sigma_2$. The classical potential can also be obtained by integrating the tree-level Σ supertadpole (8):

$$\Gamma^{(\Sigma)}_{\rm cl} = \int d^5z \Sigma(D^2\sigma + M\sigma) \tag{28}$$

where, in component fields, $\Sigma \doteq \Sigma_1(x) + \theta^\alpha\Psi_\alpha(x) - \Sigma_2(x)\theta^2$. Starting from this tadpole we have two alternatives for computing the classical potential. We can work in the superfield approach and adopt the superfield Miller's recipe [10,11] or we can jump to the component approach. We choose the last option because it is simpler in the calculations at 1-loop level (next section). Substituting Σ in terms of its component fields in (28) and integrating in θ, we obtain

$$\Gamma^{(\Sigma)}_{\rm cl} = \int d^3x[M\sigma_2\Sigma_1(x) + (M\sigma_1 + \sigma_2)\Sigma_2(x)].$$

From this expression we can easily recognize the tree-level $\Sigma_1(\Sigma_2)$ tadpoles and set up the tadpole equations:

$$\frac{\partial U_{\rm cl}}{\partial\sigma_1} = -M\sigma_2 \tag{29}$$

$$\frac{\partial U_{\rm cl}}{\partial\sigma_2} = -(M\sigma_1 + \sigma_2). \tag{30}$$

By integrating these equations, we get, as before,

$$U_{\rm cl}(\sigma_1,\sigma_2) = -\frac{1}{2}\sigma_2^2 - M\sigma_1\sigma_2. \tag{31}$$

B. ONE- AND TWO-LOOPS CONTRIBUTIONS TO THE EFFECTIVE POTENTIAL

Having determined the explicit form of the shifted superpropagators, we are ready to compute the one- and the two-loops contributions to the effective potential. Since the coefficients of the superpropagators are merely functions of k^2 (and of the parameters of the shifted theory) we do hide their intricate structures in the intermediate stages of the computations. This is possible because the Grassmann calculus needed to reduce the θ integrations to a single θ integration involves only ($\theta_\alpha, D_\alpha, k_{\alpha\beta}$) manipulations.

1. One-loop contribution

At the one-loop order, we use the tadpole method. Figure 1 shows the two contributions to the tadpoles. Their corresponding integrals are

$$\Gamma_1 = \int d^2\theta \int \frac{d^3k}{(2\pi)^3}\bigg[e\langle D^\alpha\Pi(k,\theta)A_\alpha(-k,\theta)\rangle + \frac{e}{2}\langle\Pi(k,\theta)D^\alpha A_\alpha(-k,\theta)\rangle - \frac{e^2}{2}\sigma(\theta)\langle A^\alpha(k,\theta)A_\alpha(-k,\theta)\rangle\bigg]\int d^3x\Sigma(x,\theta) \tag{32}$$

As discussed in [12] and reproduced in our previous paper [13], to study the possibility of susy breaking it is enough to calculate the radiative corrections to the effective potential up to linear dependence in σ_2. So, in the following we will restrict our calculations to this approximation. Inserting the superpropagators (27) and integrating by parts, one obtains

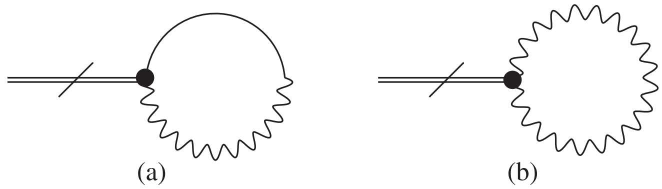

FIG. 1. One-loop contributions to the Σ tadpole of the shifted $SQED_3$. Double-solid line represents the Σ scalar superpropagator, solid-wavy line the $\langle \Pi A \rangle$ mixed superpropagator, and wavy line the gauge superpropagator.

$$\Gamma_1 = i\int d^2\theta \int \frac{d^3k}{(2\pi)^3}[(eb_5(k) - 2eb_3(k) + e^2\sigma_1 s_1(k)) + (eb_7(k) - e^2\sigma_2 s_1(k) + e^2\sigma_1 s_4(k))\theta^2]\int d^3x\Sigma(x,\theta), \tag{33}$$

which, after integrating in the θ variables, gives

$$\Gamma_1 = i\int \frac{d^3k}{(2\pi)^3}\bigg[(eb_5(k) - 2eb_3(k) + e^2\sigma_1 s_1(k)) \times \int d^3x\Sigma_2(x) - (eb_7(k) - e^2\sigma_2 s_1(k) + e^2\sigma_1 s_4(k)) \times \int d^3x\Sigma_1(x)\bigg]. \tag{34}$$

From this expression we can directly read the tadpole equations for the components Σ_1 and Σ_2:

$$\frac{\partial U_1}{\partial \sigma_1} = i\int \frac{d^3k}{(2\pi)^3}[eb_7(k) - e^2\sigma_2 s_1(k) + e^2\sigma_1 s_4(k)] \tag{35}$$

$$\frac{\partial U_1}{\partial \sigma_2} = i\int \frac{d^3k}{(2\pi)^3}[-eb_5(k) + 2eb_3(k) - e^2\sigma_1 s_1(k)]. \tag{36}$$

The coefficients b_i and s_i, which are functions of σ_1 and σ_2, are given up to a linear dependence in σ_2 and in α (this last restriction is for simplicity of calculation) in the Appendix B. Solving this pair of equations (in the α- and σ_2-linear approximation), we get

$$U_1(\sigma_1, \sigma_2) = \frac{\xi\alpha e^2 M\sigma_1\sigma_2}{2} i^2 \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2(k^2 + M^2)} = -\frac{\alpha\xi}{8\pi}e^2\sigma_1\sigma_2, \tag{37}$$

a result that depends on the gauge parameter α and is zero in the Landau gauge ($\alpha = 0$). From (37) we see that neither susy nor gauge symmetry are dynamically broken at one-loop order. This outcome has already been obtained long ago in [12].

We should emphasize that the gauge dependence of the effective potential is a well known fact [14–17]. In spite of this fact, the Nielsen Identities show that the value of the potential at its minimum and the values of the generated masses are independent of the gauge parameter α. So, the conclusions about breakdown of symmetries, got from the analysis of the minimum of the effective potential, are in fact gauge independent. Let us now extend our study to the two-loop level.

2. Two-loop contributions

At this order we use the vacuum bubble method [8]. The seven supergraphs that contribute to the effective potential are depicted in Fig. 2. Nevertheless, once the integration over the θ variables have been carried out [18], only the diagrams 2(a), 2(c) and 2(d) survive in our approach (linear contribution in α and σ_2). The corresponding integrals are shown in the Appendix C.

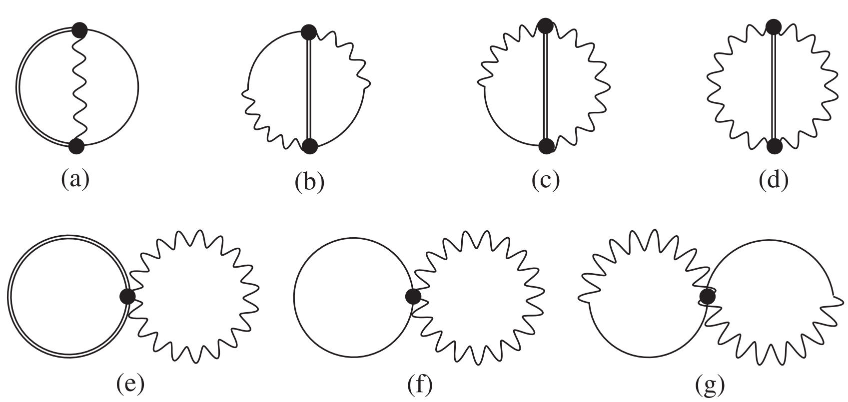

FIG. 2. Two-loop vacuum bubbles for the shifted $SQED_3$. Solid lines stand for Π scalar superpropagators.

The integrations over the internal momenta were done (with dimensional regularization) using the results in [19,20]. The result exhibits the following structure

$$U_2(\sigma_1, \sigma_2) = \sigma_1\sigma_2[F(m_1^2, M^2) + \alpha G(m_1^2, M^2)]e^4, \quad (38)$$

where $m_1^2 = e^2\sigma_1^2/2$. However, for convenience of the analysis of the minimum of the effective potential, we rewrite this result in the form

$$U_2(\sigma_1, \sigma_2) = \sigma_2\left[f\left(\frac{e\sigma_1}{M}\right) + \alpha g\left(\frac{e\sigma_1}{M}\right)\right]\frac{e^3}{64\pi^2}, \quad (39)$$

where the finite functions $f(x)$ and $g(x)$ are given by

$$f(x) = \frac{1}{x} + \frac{1}{\sqrt{2}}\frac{1}{1 + x/(2\sqrt{2})} - \frac{\sqrt{2}}{1 + \sqrt{2}x} + \frac{2}{x^3}\ln\left[\frac{1 + \sqrt{2}x}{(1 + \frac{x}{\sqrt{2}})^2}\right] \quad (40)$$

and

$$g(x) = -\frac{2x}{(2 + \sqrt{2}x)^2} + \xi\left(-\frac{2\sqrt{2}x^2}{x^2 + 3\sqrt{2}x + 4} + 4x\ln\left[\frac{2\sqrt{2} + x}{\sqrt{2} + x}\right]\right) + \xi^2\left(-\frac{4x(6 + \sqrt{2}x)}{(4 + \sqrt{2}x)^2} + x\ln\left[\frac{2\sqrt{2} + x}{\sqrt{2} + x}\right]\right), \quad (41)$$

At this point a remark is in order. Even if susy or gauge symmetry is broken, a phase (rotational) symmetry is preserved [6] in the effective potential. In our variables Σ and Π (real and imaginary components of the superfield Φ), the rotational gauge symmetry can be recovered by the substitutions $\sigma_1^2 \to \sigma_1^2 + \pi_1^2$, $\sigma_2^2 \to \sigma_2^2 + \pi_2^2$ and $\sigma_1\sigma_2 \to \sigma_1\sigma_2 + \pi_1\pi_2$ in the results (31), (37), and (38). Here π_1 and π_2 are the components of the translation $\pi = \pi_1 - \theta^2\pi_2$ in the field Π (which for simplicity we did not considered). This symmetry is not evident when the two-loop contribution to the effective potential is written in the form (39).

By collecting the results of zero-, one- and two-loops, and by choosing, for simplicity, the Landau gauge ($\alpha = 0$), the effective potential turns out

$$U_{\text{eff}}(\sigma_1, \sigma_2) = -\frac{1 + \delta z}{2}\sigma_2^2 - (M + \delta M)\sigma_1\sigma_2 + \frac{e^3}{64\pi^2}f\left(\frac{e\sigma_1}{M}\right)\sigma_2. \quad (42)$$

In this result we have introduced two counterterms: the matter field wave function renormalization counterterm δz and the mass renormalization counterterm δM. They must be fixed by the renormalization prescriptions on the effective potential. It must be noted that both one- and two-loop contributions (in dimensional regularization) are finite. Despite its appearance, $f(x)$ is a finite monotonically decreasing function,

$$f(x) = \frac{1}{\sqrt{2}} - \frac{7}{8\sqrt{2}}x^2 + \frac{133}{96}x^3 - \frac{223}{64\sqrt{2}}x^4 + \mathcal{O}(x^5) \quad \text{for } x \ll 1, \quad (43)$$

running from $f(x = 0) = 1/\sqrt{2}$ to $f(x = \infty) = 0$.

As the radiative corrections are finite, their effects in the redefinition of the mass and the wave function normalization are finite and we can adopt for convenience a "minimal subtraction renormalization prescription": $\delta z = \delta M = 0$, resulting that, up to two loops, the renormalized effective potential is given by

$$U_{\text{eff}}(\sigma_1, \sigma_2) = -\frac{1}{2}\sigma_2^2 - \sigma_2\left[x - \left(\frac{e^2}{8\pi M}\right)^2 f(x)\right]\frac{M^2}{e}. \quad (44)$$

where $x = e\sigma_1/M$.

From the Euler Lagrange equation for σ_2, that is, $\partial U_{\text{eff}}(\sigma_1, \sigma_2)/\partial\sigma_2 = 0$, we get

$$\sigma_2 = \left[\left(\frac{e^2}{8\pi M}\right)^2 f\left(\frac{e\sigma_1}{M}\right) - x\right]\frac{M^2}{e}. \quad (45)$$

After inserting this result into the expression for $U_{\text{eff}}(\sigma_1, \sigma_2)$, one obtains

$$U_{\text{eff}} = \frac{M^4}{2e^2}\left[x - \left(\frac{e^2}{8\pi M}\right)^2 f(x)\right]^2, \quad (46)$$

which satisfies $U_{\text{eff}} \geq 0$. Its minimum ($U_{\text{eff}} = 0$) occurs for

$$\frac{e\sigma_1}{M} = \left[\frac{e^2}{8\pi M}\right]^2 f\left(\frac{e\sigma_1}{M}\right), \quad (47)$$

which also implies $\sigma_2 = 0$. In perturbation expansion, by hypothesis $e^2/8\pi M \ll 1$, and the Eq. (47) has the solution $\sigma_1 \cong e^3/\sqrt{2}64\pi^2 M \neq 0$. In short, the minimum of the effective potential is $U_{\text{eff}} = 0$ and occurs at $\sigma_2 = 0$ and $\sigma_1 \neq 0$. This result means [12,13] that supersymmetry is preserved, but the gauge symmetry is dynamically broken, with a mass $m_1 = (\frac{e^2}{8\pi M})^2\frac{M}{2} \neq 0$ generated for the gauge superfield. By only making the shift $\sigma = \sigma_1$, in paper [21], the breakdown of the gauge symmetry was studied with a similar conclusion. As a result of these corrections the gauge superpropagator (B1) is given by

$$\langle TA_\alpha(k, \theta)A_\beta(-k, \theta')\rangle = \frac{i}{2(k^2 + m_1^2)}\left[-C_{\alpha\beta} + \frac{k_{\alpha\beta}D^2}{k^2}\right]\delta^2(\theta - \theta').$$

By the component decomposition of A_α, presented in the Appendix A, we obtain for the component field propagators

$$\langle T\chi_\alpha(k)\chi_\beta(-k)\rangle = \frac{ik_{\alpha\beta}}{2k^2(k^2+m_1^2)}$$
$$\langle T\lambda_\alpha(k)\lambda_\beta(-k)\rangle = -\frac{ik_{\alpha\beta}}{2(k^2+m_1^2)}$$
$$\langle T\chi_\alpha(k)\lambda_\beta(-k)\rangle = -\frac{iC_{\alpha\beta}}{2(k^2+m_1^2)}$$
$$\langle TV_a(k)V_b(-k)\rangle = -\frac{2i}{k^2+m_1^2}\left(\eta_{ab} - \frac{k_a k_b}{k^2}\right)$$
$$a, b = 0, 1, 2$$
$$\langle TB(k)B(-k)\rangle = 0.$$

where $V_a \equiv (\gamma_a)^{\alpha\beta}V_{\alpha\beta}$ is the 3-vector electromagnetic potential.

IV. CONCLUDING REMARKS

In this paper, we developed the algebra of spinorial operators involved in the calculation of the superpropagators for gauge and matter field models in 3D. This algebra is useful in the presence of shifts of the superfields by θ spinorial dependent expectation values, as needed to calculate the effective potential to study the possibility of dynamical supersymmetry breakdown. As an example, this algebra is applied in the calculation of the superpropagators of the supersymmetric quantum electrodynamics $SQED_3$. The shift of superfields with such a θ dependent part implies in bilinear mixing of the gauge and matter fields that cannot be eliminated by using an R_ξ gauge fixing term. The inversion of the quadratic part of the Lagrangian results very arduous in component or in the superfield formalism. The use of this algebra systematizes the calculation of the superpropagators, and it is helpful in the calculation of the superpropagators of any $\mathcal{N} = 1$ supersymmetric model in 3D. The effective potential for $SQED_3$ is calculated up to two loops, with the conclusion that supersymmetry is preserved, but gauge symmetry is dynamically broken with the generation of mass for the gauge superfield.

ACKNOWLEDGMENTS

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APPENDIX A: THE SUPERFIELD EXPANSIONS

In component fields the matter and gauge superfields can be written as [4]:

$$\Sigma(x,\theta) = \Sigma_1(x) + \theta^\alpha\Psi_\alpha - \theta^2\Sigma_2(x), \quad \alpha,\beta = 1,2 \tag{A1a}$$
$$\Pi(x,\theta) = \Pi_1(x) + \theta^\alpha\Xi_\alpha - \theta^2\Pi_2(x) \tag{A1b}$$
$$A_\alpha(x,\theta) = \chi_\alpha(x) - \theta_\alpha B(x) + i\theta^\beta V_{\beta\alpha}(x) - \theta^2[2\lambda_\alpha(x) + i\partial_{\alpha\beta}\chi^\beta(x)] \tag{A1c}$$
$$W_\alpha(x,\theta) = \lambda_\alpha(x) + \theta^\beta f_{\beta\alpha}(x) + \theta^2 i\partial_{\alpha\beta}\lambda^\beta(x) \tag{A1d}$$

with $f_{\alpha\beta}(x) = -\frac{1}{2}(\partial^\gamma_\alpha V_{\gamma\beta} + \partial^\gamma_\beta V_{\gamma\alpha})$. In addition, the usual (vector) gauge potential is given by $v^a \equiv (\gamma^a)_{\alpha\beta}V^{\alpha\beta}$, and the (tensor) gauge field strength by $F_{ab} \equiv \partial_a v_b - \partial_b v_a = \frac{i}{2}\in_{abc}(\gamma^c)^{\alpha\beta}f_{\alpha\beta}$.

APPENDIX B: THE SUPERPROPAGATOR COEFFICIENTS

In order to calculate the superpropagators (27) we start with the matrices (10a)–(10c) and go through all the operations indicated in formulae (14a)–(14d). These manipulations involve a lot of algebraic calculation using the operator algebra presented in Sec. II. The complete result is very cumbersome. In the results shown below, we only kept the terms up to linear dependence in the field component σ_2, which are enough to discuss the possibility of susy breakdown [12,13]. We will also limit our calculations of the effective potential to the Landau gauge ($\alpha = 0$) and so, for simplicity, in the calculation of the superpropagators we restrict ourselves to linear terms in α.

The gauge superpropagator $\langle AA\rangle$ is given by

$$\langle TA_\alpha(k,\theta)A_\beta(-k,\theta')\rangle = i\sum_{i=0}^{5}(r_iR_{i,\alpha\beta} + s_iS_{i,\alpha\beta} + mM_{\alpha\beta} + nN_{\alpha\beta})\delta^2(\theta-\theta'), \tag{B1}$$

with

$$r_0 = -r_3 = -\frac{1}{2}r_4 = \frac{1}{2k^2}s_2 = -\frac{\sigma_1\sigma_2e^2}{4k^2(k^2+m_1^2)^2},$$
$$r_2 = s_3 = s_4 = 0,$$
$$r_5 = -\frac{\xi^2\alpha\sigma_1\sigma_2e^2M}{2k^4(k^2+m_1^2)(k^2+M^2)},$$
$$s_0 = -\frac{\alpha}{2k^2} - \frac{1}{2(k^2+m_1^2)},$$
$$s_1 = s_5 = \frac{\alpha\sigma_1\sigma_2e^2(k^2(1-\xi^2)+M^2)}{2k^4(k^2+m_1^2)(k^2+M^2)} - \frac{\sigma_1\sigma_2e^2}{4k^2(k^2+m_1^2)^2}.$$
$$r_1 = \frac{1}{2k^2(k^2+m_1^2)} - \frac{\alpha[(M^2+m_1^2)k^2 + (k^2)^2 + M(Mm_1^2 + e^2\xi^2\sigma_1\sigma_2)]}{2(k^2)^2(k^2+M^2)(k^2+m_1^2)}$$
$$n = 0 \qquad m \sim O(\alpha^2),$$

where $m_1^2 = e^2\sigma_1^2/2$.

The scalar superpropagator $\langle \Pi\Pi \rangle$ exhibits the following structure

$$\langle T\Pi(k,\theta)\Pi(-k,\theta')\rangle = i\left(\sum_{i=0}^{5} a_i P_i\right)\delta^2(\theta-\theta'), \tag{B2}$$

where

$$a_0 = \frac{M}{k^2+M^2} + \frac{\xi^2\alpha\sigma_1 e^2(k^2(2M\sigma_1+\sigma_2)-M^2\sigma_2)}{2k^2(k^2+M^2)^2}, \qquad a_1 = -\frac{1}{k^2+M^2} + \frac{\xi^2\alpha\sigma_1 e^2(M(M\sigma_1+2\sigma_2)-k^2\sigma_1)}{2k^2(k^2+M^2)^2},$$

$$a_2 = 2k^2 a_5 = -\frac{2\xi^2\alpha\sigma_1\sigma_2 e^2 M}{(k^2+M^2)^2}, \qquad a_3 = \frac{1}{2}a_4 = \frac{\xi^2\alpha\sigma_1\sigma_2 e^2(k^2-M^2)}{2k^2(k^2+M^2)^2}.$$

The superpropagator $\langle \Pi A \rangle$ takes the form

$$\langle T\Pi(k,\theta)A_\alpha(-k,\theta')\rangle = i\sum_{i=1}^{8} b_i T^i_\alpha \delta^2(\theta-\theta'), \tag{B3}$$

where

$$b_1 = \frac{\xi e\sigma_2 M[(\alpha+1)(k^2+M^2)+2m_1^2\alpha\xi^2]}{2(k^2+m_1^2)(k^2+M^2)^2},$$

$$b_3 = b_2 = -\frac{\xi e\sigma_2[2k^4+2k^2[\alpha(\xi^2-1)m_1^2+M^2]-2\alpha(\xi^2+1)m_1^2M^2]}{4k^2(k^2+m_1^2)(k^2+M^2)^2},$$

$$b_4 = \frac{\xi e\sigma_2 M[(\alpha-1)k^2(k^2+M^2)-2\alpha m_1^2[(\xi^2-1)k^2-M^2]]}{2k^4(k^2+m_1^2)(k^2+M^2)^2}$$

$$b_5 = -\frac{\xi\alpha e(M\sigma_1+\sigma_2)}{2k^2(k^2+M^2)}, \quad b_7 = -Mb_8 = \frac{M\sigma_2}{\sigma_1}b_6 = \frac{eM\alpha\xi\sigma_2}{2k^2(k^2+M^2)}.$$

These coefficients are not exact, they only exhibit the contributions up to linear terms in α and in σ_2, what is enough for our purposes. The exact results are rather cumbersome, even if their calculation do not present any technical difficulty.

Finally, the superpropagator $\langle \Sigma\Sigma \rangle$ is given by

$$\langle T\Sigma(k,\theta)\Sigma(-k,\theta')\rangle = i[c_0P_0 + c_1P_1]\delta^2(\theta-\theta'), \tag{B4}$$

where

$$c_0 = \frac{M}{k^2+M^2}, \qquad c_1 = -\frac{1}{k^2+M^2}.$$

APPENDIX C: TWO-LOOP CALCULATIONS

The supergraphs contributing to the effective potential at two-loop order, in the vacuum bubble method [8], are shown in Fig. 2. To study the possibility of supersymmetry breaking it is enough to calculate the effective potential up to linear order in the component σ_2 of the classical value of the matter superfield (7). Only diagrams (a), (c) and (d) in Fig. 2 have contributions starting linearly in σ_2 and also, independent or linear in the gauge parameter α. Using the expressions for the superpropagators and performing the D-algebra with the help of the SusyMath package [18], we get the UV finite results:

$$U_{2(a)} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\left[\frac{\alpha\xi^2\sigma_1\sigma_2 e^4 M[M^2-(k+q)^2]k^2}{(k^2+m_1^2)^2(k+q)^2(M^2+q^2)[(k+q)^2+M^2]^2} + \frac{\sigma_1\sigma_2 e^4 M[\alpha\xi^2 k.q+(k+q)^2]}{(k^2+m_1^2)^2(k+q)^2(M^2+q^2)[(k+q)^2+M^2]}\right] \tag{C1}$$

$$U_{2(c)} = \alpha\xi\sigma_1\sigma_2 M e^4\int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{(k+q)^2}{k^2(k^2+M^2)(M^2+q^2)[(k+q)^2+m_1^2]^2} \tag{C2}$$

$$U_{2(d)} = \frac{1}{4}\int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{1}{(k^2+m_1^2)^2(q^2+M^2)[(k+q)^2+m_1^2]^2} \times\left[\frac{\sigma_1\sigma_2 Me^4(k^2+k\cdot q)[2(1-2\alpha)k^4-4\alpha m_1^2k^2-2\alpha m_1^4]}{2k^4} - \frac{\alpha\sigma_1\sigma_2 Mm_1^4e^4(k^2+k\cdot q)}{(k+q)^4} - \frac{\sigma_1\sigma_2 Mm_1^2e^4(2\alpha k^2+m_1^2)(k^2+k\cdot q)}{k^2(k+q)^2}\right] \tag{C3}$$

The other supergraph contributions are of order α^2 or σ_2^2, that is, $U_{2(b)} = \mathcal{O}(\alpha^2, \sigma_2)$, $U_{2(e)} = \mathcal{O}(\alpha^2, \sigma_2^2)$, $U_{2(f)} = U_{2(g)} = \mathcal{O}(\alpha, \sigma_2^2)$.

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