Renormalization for the self-potential of a scalar charge in static space-times

Arkady A. Popov*

Department of Mathematics, Tatar State University of Humanities and Education, Tatarstan 2, Kazan 420021, Russia, and Department of Physics, Kazan State University, Kremlevskaya 18, Kazan, 420008, Russia

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A method is presented which allows for the renormalization of the self-potential for a scalar point charge at rest in static curved space-time. The method is suitable for the scalar field with arbitrary mass m and coupling to the scalar curvature. The asymptotic behavior of self-potential is obtained in the limit in which the Compton wavelength $1/m$ of the massive scalar field is much smaller than the characteristic scale of curvature of the background gravitational field. The self-force is calculated in this limit.

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I. INTRODUCTION

It is known that a charged particle interacts with the field, the source of which is this particle. In flat space-time the effect is determined by the derivative of acceleration of the charge [1]. The origin of self-interaction in curved space-times is associated with the nonlocal structure of the field. In static curved space-times and space-times with nontrivial topology the self-force can be nonzero even for the charge at rest (here and below the words ‘‘at rest’’ mean that the velocity of charge is collinear to the timelike Killing vector which always exists in a static space-time). The formal expression for the electromagnetic self-force in an arbitrary curved space-time has been derived by DeWitt and Brehme [2] and a correction was later provided by Hobbs [3]. Mino, Sasaki, and Tanaka [4] and independently Quinn and Wald [5] obtained a similar expression for the gravitational self-force on a point mass. The self-force on a charge interacting with a massless minimally coupled scalar field was considered by Quinn [6]. A discussion of the self-force in detail may be found in reviews [7–9].

Calculating the self-force one must evaluate the field that the point charge induces at the position of the charge. This field diverges and must be renormalized. There are different methods of such type of renormalization. Some of them are reviewed in [10,11]. Note also the zeta function method [12] and the ''massive field approach'' for the calculation of the self-force [13,14]. In the ultrastatic space-times the renormalization of the field of static charge can be realized by the subtraction of the first terms from DeWitt-Schwinger asymptotic expansion of a three-dimensional Euclidean Green's function [15–18]. In this paper a similar approach expands to the case of static space-times. In the framework of the suggested procedure one subtracts some terms of expansion of the corresponding Green's function of a massive scalar field with arbitrary coupling to the scalar curvature from the divergent expression obtained. The quantities of terms to be subtracted are defined by a simple rule—they no longer vanish as the field's mass goes to infinity. Such an approach is similar to renormalization introduced in the context of the quantum field theory in curved space-time [19,20]. The Bunch and Parker method [21] is used for expansion of the corresponding Green's function of a scalar field.

The organization of this paper is as follows. In Sec. II, we expand the potential of a scalar point charge as two points (in 3D space) function in powers of distance between these points and determine the procedure of renormalization. In Sec. III we calculate the renormalized self-potential of the scalar point charge in Schwarzschild space-time, as an example of the presented method. We discuss the results in Sec. IV. Our conventions are those of Misner, Thorne, and Wheeler [22]. Throughout this paper, we use units $c = G = 1$.

II. RENORMALIZATION

Let us consider an equation for the scalar massive field with source

$$\phi_{m;\mu}{}^{;\mu} - (m^2 + \xi R)\phi_m = -J = -4\pi q \int \delta^{(4)}(x - x_0(\tau)) \frac{d\tau}{\sqrt{-g^{(4)}}}, \qquad (1)$$

where ξ is a coupling of the scalar field with mass m to the scalar curvature R, $g^{(4)}$ is the determinant of the metric $g_{\mu\nu}$, q is the scalar charge, and τ is its proper time. The world line of the charge is given by $x_0^\mu(\tau)$. The metric of static space-time can be presented as follows:

$$ds^2 = -g_{tt}(x^i)dt^2 + g_{jk}(x^i)dx^j dx^k, \qquad (2)$$

where $i, j, k = 1, 2, 3$. This means that one can write the field equation in the following way:

*apopov@ksu.ru

$$\frac{1}{\sqrt{g_{tt}}\sqrt{g^{(3)}}}\frac{\partial}{\partial x^j}\left(\sqrt{g_{tt}}\sqrt{g^{(3)}}g^{jk}\frac{\partial\phi_m(x^i;x_0^i)}{\partial x^k}\right) - (m^2+\xi R(x))\phi_m(x^i;x_0^i) = -\frac{4\pi q\delta^{(3)}(x^i,x_0^i)}{\sqrt{g^{(3)}}}, \quad (3)$$

where m is the mass of the scalar field, $g^{(3)} = \det g_{ij}$, and we take into account that $d\tau/dt = \sqrt{g_{tt}}$ for the particle at rest. In the case

$$m \gg 1/L, \quad (4)$$

where L is the characteristic curvature scale of the background geometry, it is possible to construct the iterative procedure of the solution of Eq. (3) with small parameter $1/(mL)$ [19–21]. This expansion can be used in the regularization procedure of Rosenthal [13]

$$f_\mu^{\rm self}(x_0) = q\lim_{m\to\infty}\left\{\lim_{\delta\to 0}\frac{\partial(\phi(x;x_0)-\phi_m(x;x_0))}{\partial x^\mu} + \frac{qm^2 n_\mu(x_0)}{2} + \frac{qma_\mu(x_0)}{2}\right\}, \quad (5)$$

because this procedure demands the calculation of the expansion of $\phi_m(x;x_0)$ in terms of $x^\mu - x_0^\mu$ and $1/m$ accurate to order $O((x-x_0)^2) + O(1/m)$ only. In the expression (5) $\phi(x;x_0)$ is the massless field induced by scalar charge q, and x is a point near the charge's world line $x_0(\tau)$, defined as follows. At x_0 we construct a unit spatial vector n^μ, which is perpendicular to the object's world line but is otherwise arbitrary (i.e. at x_0 we have $n^\mu n_\mu = 1$, $n^\mu u_\mu = 0$). In the direction of this vector we construct a geodesic, which extends out an invariant length δ to the point $x(x_0, n^\mu, \delta)$; throughout this paper u^μ and a^μ denote the object's four-velocity and four-acceleration, at x_0, respectively.

To construct the expansion of $\phi_m(x;x_0)$, let us consider the equation for the three-dimensional Green's function $G_{\rm E}(x^i, x_0^i)$

$$\frac{1}{\sqrt{g^{(3)}}}\frac{\partial}{\partial x^j}\left(\sqrt{g^{(3)}}g^{jk}\frac{\partial G_{\rm E}(x^i,x_0^i)}{\partial x^k}\right) + \frac{g^{jk}}{2g_{tt}}\frac{\partial g_{tt}}{\partial x^j}\frac{\partial G_{\rm E}(x^i,x_0^i)}{\partial x^k} - (m^2+\xi R(x))G_{\rm E}(x^i,x_0^i) = -\frac{\delta^{(3)}(x^i,x_0^i)}{\sqrt{g^{(3)}}} \quad (6)$$

and introduce the Riemann normal coordinates y^i in 3D space with origin at the point x_0^i [23]. In these coordinates one has

$$g_{ij}(y^i) = \delta_{ij} - \frac{1}{3}\tilde{R}_{ikjl}|_{y=0}y^k y^l + O\left(\frac{y^3}{L^3}\right), \quad (7)$$

$$g^{ij}(y^i) = \delta^{ij} + \frac{1}{3}\tilde{R}^i{}_k{}^j{}_l|_{y=0}y^k y^l + O\left(\frac{y^3}{L^3}\right), \quad (8)$$

$$g^{(3)}(y^i) = 1 - \frac{1}{3}\tilde{R}_{ij}|_{y=0}y^i y^j + O\left(\frac{y^3}{L^3}\right), \quad (9)$$

where the coefficients here and below are evaluated at $y^i = 0$ (i.e. at the point x_0^i), and δ_{ij} denotes the metric of a flat three-dimensional Euclidean spacetime. $\tilde{R}_{ikjl}$ and $\tilde{R}_{ij}$ denote the components of Riemann and Ricci tensors of the three-dimensional space-time with metric g_{ij}

$$R_{ij} = \tilde{R}_{ij} - \frac{g_{tt,i;j}}{2g_{tt}} + \frac{g_{tt,i}g_{tt,j}}{4{g_{tt}}^2}, \qquad R = \tilde{R} - \frac{{g_{tt,i}}^{;i}}{g_{tt}} + \frac{g_{tt,i}{g_{tt}}^{,i}}{2g_{tt}^2}, \quad (10)$$

where $g_{tt,i}$ denotes the covariant derivative of a scalar function $g_{tt}(y^j)$ with respect to y^i in 3D space with metric $g_{ij}(y^k)$ ($g_{tt,i;j}$ is the covariant derivative of a vector $g_{tt,i}$ at point $y^k = 0$ in 3D space, which coincides with the partial derivative as $\Gamma^k_{ij} = 0$ at $y^k = 0$ in the Riemann normal coordinates). All indices are raised and lowered with δ_{ij}. Defining $\bar{G}(y^i)$ by

$$\bar{G}(y^i) = \sqrt{g^{(3)}}G_{\rm E}(y^i) \quad (11)$$

and retaining in (6) only the terms with coefficients involving two derivatives of the metric or fewer one finds that $\bar{G}(y^i)$ satisfies the equation

$$\delta^{ij}\frac{\partial^2\bar{G}}{\partial y^i\partial y^j} - m^2\bar{G} + \delta^{ij}\frac{g_{tt,i}}{2g_{tt}}\frac{\partial\bar{G}}{\partial y^j} + \delta^{ij}\left(\frac{g_{tt,ik}}{2g_{tt}} - \frac{g_{tt,i}g_{tt,k}}{2g_{tt}^2}\right)y^k\frac{\partial\bar{G}}{\partial y^j} + \tilde{R}^i{}_k{}^j{}_l\frac{y^k y^l}{3}\frac{\partial^2\bar{G}}{\partial y^i\partial y^j} + \left(\frac{\tilde{R}}{3} - \xi R\right)\bar{G} = -\delta^{(3)}(y). \quad (12)$$

Let us present

$$\bar{G}(y^i) = \bar{G}_0(y^i) + \bar{G}_1(y^i) + \bar{G}_2(y^i) + \cdots, \quad (13)$$

where $\bar{G}_a(y^i)$ has a geometrical coefficient involving a derivatives of the metric at point $y^i = 0$. Then these functions satisfy the equations

$$\delta^{ij}\frac{\partial^2\bar{G}_0}{\partial y^i\partial y^j} - m^2\bar{G}_0 = -\delta^{(3)}(y), \quad (14)$$

$$\delta^{ij}\frac{\partial^2\bar{G}_1}{\partial y^i\partial y^j} - m^2\bar{G}_1 + \delta^{ij}\frac{g_{tt,i}}{2g_{tt}}\frac{\partial\bar{G}_0}{\partial y^j} = 0, \quad (15)$$

$$\delta^{ij}\frac{\partial^2\bar{G}_2}{\partial y^i\partial y^j} - m^2\bar{G}_2 + \delta^{ij}\frac{g_{tt,i}}{2g_{tt}}\frac{\partial\bar{G}_1}{\partial y^j} + \delta^{ij}\left(\frac{g_{tt,ik}}{2g_{tt}} - \frac{g_{tt,i}g_{tt,k}}{2{g_{tt}}^2}\right) \times y^k\frac{\partial\bar{G}_0}{\partial y^j} + \tilde{R}^i{}_k{}^j{}_l\frac{y^k y^l}{3}\frac{\partial^2\bar{G}_0}{\partial y^i\partial y^j} + \left(\frac{\tilde{R}}{3} - \xi R\right)\bar{G}_0 = 0. \quad (16)$$

The function $\bar{G}_0(y^i)$ satisfies the condition

$$\tilde{R}^i{}_k{}^j{}_l y^k y^l\frac{\partial^2\bar{G}_0}{\partial y^i\partial y^j} - \tilde{R}^i_j y^j\frac{\partial\bar{G}_0}{\partial y^i} = 0, \quad (17)$$

since $\bar{G}_0(y^i)$ can be the function only of $\delta_{ij}y^iy^j$. Therefore Eq. (16) may be rewritten

$$\delta^{ij}\frac{\partial^2\bar{G}_2(y^i)}{\partial y^i\partial y^j} - m^2\bar{G}_2(y^i) + \delta^{ij}\frac{g_{tt,i}}{2g_{tt}}\frac{\partial\bar{G}_1}{\partial y^j} + \left[\frac{1}{3}\tilde{R}^i_k + \delta^{ij}\left(\frac{g_{tt,jk}}{2g_{tt}} - \frac{g_{tt,j}g_{tt,k}}{2g_{tt}{}^2}\right)\right]y^k\frac{\partial\bar{G}_0}{\partial y^i} + \left(\frac{\tilde{R}}{3} - \xi R\right)\bar{G}_0 = 0. \tag{18}$$

Let us introduce the local momentum space associated with the point $y^i = 0$ by making the 3-dimensional Fourier transformation

$$\bar{G}_a(y^i) = \iint_{-\infty}^{+\infty}\int\frac{dk_1dk_2dk_3}{(2\pi)^3}\exp(ik_iy^i)\bar{G}_a(k^i). \tag{19}$$

It is not difficult to see that

$$\bar{G}_0(k^i) = \frac{1}{k^2 + m^2}, \tag{20}$$

$$\bar{G}_1(k^i) = i\frac{\delta^{ij}g_{tt,i}k_j}{2g_{tt}(k^2 + m^2)^2}, \tag{21}$$

where $k^2 = \delta^{ij}k_ik_j$. In momentum space Eq. (18) gives

$$-(k^2 + m^2)\bar{G}_2(k^i) + i\frac{\delta^{ij}g_{tt,i}k_j}{2g_{tt}}\bar{G}_1(k^i) + \left[ik_i\left(\frac{\tilde{R}^i_k}{3} + \delta^{ij}\frac{g_{tt,jk}}{2g_{tt}} - \delta^{ij}\frac{g_{tt,j}g_{tt,k}}{2g_{tt}{}^2}\right)y^k + \left(\frac{\tilde{R}}{3} - \xi R\right)\right]\bar{G}_0(k^i) = 0. \tag{22}$$

Hence

$$\bar{G}_2(k^i) = \frac{-\frac{\delta^{ij}g_{tt,ij}}{2g_{tt}} + \frac{\delta^{ij}g_{tt,i}g_{tt,j}}{2g_{tt}{}^2} - \xi R}{(k^2 + m^2)^2} + \frac{k_ik_j\delta^{ik}\delta^{jl}(\frac{2}{3}\tilde{R}_{jl} + \frac{g_{tt,jl}}{g_{tt}} - \frac{5g_{tt,j}g_{tt,l}}{4g_{tt}{}^2})}{(k^2 + m^2)^3}. \tag{23}$$

Substituting (19)–(21) and (23) in (13) and integrating leads to

$$\bar{G}_0(y^i) + \bar{G}_1(y^i) + \bar{G}_2(y^i) = \frac{\exp(-my)}{8\pi}\left[\frac{2}{y} - \frac{g_{tt,i}}{2g_{tt}}\frac{y^i}{y} + \frac{1}{m}\left(-\frac{\delta^{ij}g_{tt,ij}}{4g_{tt}} + \frac{3\delta^{ij}g_{tt,i}g_{tt,j}}{16g_{tt}{}^2} - \xi R + \frac{\tilde{R}}{6}\right) + \left(-\frac{g_{tt,ij}}{4g_{tt}} + \frac{5g_{tt,i}g_{tt,j}}{16g_{tt}{}^2} - \frac{\tilde{R}_{ij}}{6}\right)\frac{y^iy^j}{y}\right], \tag{24}$$

where

$$y = \sqrt{\delta_{ij}y^iy^j}. \tag{25}$$

Using the definition of $\bar{G}(y^i)$ (11), expansion (9), and expressions (10) one finds

$$\begin{aligned}G_{\mathrm{E}}(y^i) &= \left[1 + \frac{1}{6}R_{ij}y^iy^j + O\left(\frac{y^3}{L^3}\right)\right]\bar{G}(y^i)\\ &= \frac{\exp(-my)}{8\pi}\left[\frac{2}{y} - \frac{g_{tt,i}}{2g_{tt}}\frac{y^i}{y} + \frac{1}{m}\left(-\frac{\delta^{ij}g_{tt,ij}}{4g_{tt}} + \frac{3\delta^{ij}g_{tt,i}g_{tt,j}}{16g_{tt}{}^2} - \xi R + \frac{\tilde{R}}{6}\right) + \left(-\frac{g_{tt,ij}}{4g_{tt}} + \frac{5g_{tt,i}g_{tt,j}}{16g_{tt}{}^2} + \frac{\tilde{R}_{ij}}{6}\right)\frac{y^iy^j}{y}\right.\\ &\quad\left. + O\left(\frac{1}{m^2L^3}\right) + O\left(\frac{y}{mL^3}\right) + O\left(\frac{y^2}{L^3}\right)\right]\\ &= \frac{1}{8\pi}\left\{\frac{2}{y} - \frac{g_{tt,i}}{2g_{tt}}\frac{y^i}{y} - 2m + \frac{1}{m}\left[-\frac{\delta^{ij}g_{tt,ij}}{12g_{tt}} + \frac{5\delta^{ij}g_{tt,i}g_{tt,j}}{48g_{tt}{}^2} - \left(\xi - \frac{1}{6}\right)R\right] + m\frac{g_{tt,i}y^i}{2g_{tt}} + m^2y\right.\\ &\quad + \left[\frac{\delta^{ij}g_{tt,ij}}{12g_{tt}} - \frac{5\delta^{ij}g_{tt,i}g_{tt,j}}{48g_{tt}{}^2} + \left(\xi - \frac{1}{6}\right)R\right]y + \left(-\frac{g_{tt,ij}}{6g_{tt}} + \frac{13g_{tt,i}g_{tt,j}}{48g_{tt}{}^2} + \frac{R_{ij}}{6}\right)\frac{y^iy^j}{y} + O\left(\frac{1}{m^2L^3}\right)\\ &\quad\left. + O\left(\frac{y}{mL^3}\right) + O\left(\frac{y^2}{L^3}\right) + O\left(\frac{my^3}{L^3}\right) + O(m^3y^2)\right\}.\end{aligned} \tag{26}$$

In the arbitrary coordinates of 3D space

$$y^i \to u^i(x_0)\Delta s \equiv -\sigma^i, \tag{27}$$

where $u^i(x_0)$ is the unit tangent vector to the shortest geodesic connecting points x_0 and x which is calculated at points x_0 and directed from x_0 to x. Δs is the distance between these points along the considered geodesic. Therefore,

$$G_{\rm E}(x^i;x_0^i) = \frac{1}{8\pi}\left\{\frac{2}{\sqrt{2\sigma}} + \frac{g_{tt,i}\sigma^i}{2g_{tt}\sqrt{2\sigma}} - 2m + \frac{1}{m}\left[-\frac{g_{tt,i}{}^{;i}}{12g_{tt}} + \frac{5g_{tt,i}g_{tt}{}^{,i}}{48g_{tt}^2} - \left(\xi - \frac{1}{6}\right)R(x_0)\right] - m\frac{g_{tt,i}\sigma^i}{2g_{tt}} + m^2\sqrt{2\sigma}\right.$$
$$+\left[\frac{g_{tt,i}^{;i}}{12g_{tt}} - \frac{5g_{tt,i}g_{tt}^{,i}}{48g_{tt}^2} + \left(\xi - \frac{1}{6}\right)R(x_0)\right]\sqrt{2\sigma} + \left(-\frac{g_{tt,i;j}}{6g_{tt}} + \frac{13g_{tt,i}g_{tt,j}}{48{g_{tt}}^2} + \frac{R_{ij}(x_0)}{6}\right)\frac{\sigma^i\sigma^j}{\sqrt{2\sigma}}$$
$$\left. + O\left(\frac{1}{m^2L^3}\right) + O\left(\frac{\sqrt{\sigma}}{mL^3}\right) + O\left(\frac{\sigma}{L^3}\right) + O\left(\frac{m\sigma^{3/2}}{L^3}\right) + O(m^2\sigma)\right\}, \tag{28}$$

where $g_{tt,i}$ denotes the covariant derivative of a scalar function $g_{tt}(x_0)$ with respect to x_0^i in 3D space with metric $g_{ij}(x_0)$ ($g_{tt,i;j}$ is the covariant derivative of a vector $g_{tt,i}$ at point x_0 in 3D space),

$$\sigma = \frac{g_{ij}(x_0)}{2}\sigma^i\sigma^j \tag{29}$$

is one-half the square of the distance between the points x_0^i and x^i along the shortest geodesic connecting them, and (see, e.g., [24,25])

$$\sigma^i = -(x^i - x_0^i) - \frac{1}{2}\Gamma^i_{jk}(x^j - x_0^j)(x^k - x_0^k)$$
$$- \frac{1}{6}\left(\Gamma^i_{jm}\Gamma^m_{kl} + \frac{\partial\Gamma^i_{jk}}{\partial x_0^l}\right)(x^j - x_0^j)(x^k - x_0^k)(x^l - x_0^l)$$
$$+ O((x - x_0)^4), \tag{30}$$

where the Christoffel symbols Γ^i_{jk} are calculated at the point x_0.

Now we can use the expansion of

$$\phi_m(x^i;x_0^i) = 4\pi q G_{\rm E}(x^i;x_0^i) \tag{31}$$

in the regularization procedure (5). But if we take the limits before the partial differentiation in (5), then the last two terms do not appear in the expression for $f_\mu^{\rm self}(x_0)$. And in the considered case of a charge at rest in a static spacetime we can renormalize the self-potential as

$$\phi_{\rm ren}(x) = \lim_{x_0\to x}(\phi(x;x_0) - \phi_{\rm DS}(x;x_0)), \tag{32}$$

where

$$\phi_{\rm DS}(x^i;x_0^i) = q\left(\frac{1}{\sqrt{2\sigma}} + \frac{\partial g_{tt}(x_0)}{\partial x_0^i}\frac{\sigma^i}{4g_{tt}(x_0)\sqrt{2\sigma}} - m\right), \tag{33}$$

and $\phi(x;x_0)$ is the solution of (1) in the case of arbitrary mass m (even $m = 0$). Finally the self-force acting on a static scalar charge is

$$f_\mu^{\rm self}(x) = -\frac{q}{2}\frac{\partial\phi_{\rm ren}(x)}{\partial x^\mu}. \tag{34}$$

III. THE SCHWARZSCHILD SPACE-TIME

Let us verify the above scheme for the well-known case of a black hole space-time [26,27]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$
$$f(r) = \left(1 - \frac{2M}{r}\right). \tag{35}$$

In the case $x^i - x_0^i = \delta^i_r(r - r_0)$

$$\sigma^r = -(r - r_0) + \frac{1}{4f(r_0)}\frac{df(r_0)}{dr_0}(r - r_0)^2$$
$$- \frac{1}{6}\left(\frac{3}{4f(r_0)^2}\left(\frac{df(r_0)}{dr_0}\right)^2 - \frac{d^2f(r_0)}{2f(r_0)dr_0^2}\right)(r - r_0)^3$$
$$+ O\left(\frac{(r - r_0)^4}{L^3}\right), \tag{36}$$

$$\sigma = \frac{(r - r_0)^2}{2f(r_0)}\left[1 - \frac{df(r_0)}{2f(r_0)dr_0}(r - r_0)\right.$$
$$\left. + \left(\frac{5}{16f(r_0)^2}\left(\frac{df(r_0)}{dr_0}\right)^2 - \frac{d^2f(r_0)}{6f(r_0)dr_0^2}\right)(r - r_0)^2\right]$$
$$+ O\left(\frac{(r - r_0)^5}{L^3}\right), \tag{37}$$

and the expansion of $\phi(r;r_0)$ for the massive field ($mL \gg 1$) is

$$\phi_m(r;r_0) = 4\pi q G_{\rm E}(r;r_0)$$
$$= \frac{q\sqrt{1 - \frac{2M}{r_0}}}{|r - r_0|} - qm + \frac{q}{2}\left(-\frac{M}{3r_0^3\sqrt{1 - \frac{2M}{r_0}}} + \frac{mM}{r_0^2(1 - \frac{2M}{r_0})} + \frac{m^2}{\sqrt{1 - \frac{2M}{r_0}}}\right)(r - r_0) + O\left(\frac{q}{mL^2}\right) + O\left(\frac{q(r - r_0)}{mL^3}\right)$$
$$+ O\left(\frac{q(r - r_0)^2}{L^3}\right) + O(qm^2(r - r_0)^2) + O\left(\frac{qm(r - r_0)^3}{L^3}\right). \tag{38}$$

Therefore the renormalized counterterm for the massless field is

$$\phi_{\rm DS}(r;r_0) = \frac{q\sqrt{f(r_0)}}{|r-r_0|}. \qquad (39)$$

This expression coincides with the unrenormalized potential $\phi(x;x')$ of a scalar point charge at rest in Schwarzschild space-time in the case $\xi = 0$, $m = 0$ and $t = t'$, $\theta = \theta'$, $\varphi = \varphi'$ [26]. Consequently the renormalized expression for the self-potential is

$$\phi_{\rm ren}(r) = 0, \quad \text{and} \quad f_i^{\rm self}(r) = 0. \qquad (40)$$

Note the derivative of (38) $\partial\phi(r;r_0)/\partial r$ differs from the corresponding expression in [14] by the term $-qM/(6r_0^3\sqrt{1-2M/r_0})$.

IV. CONCLUSION

The considered approach gives the possibility to renormalize (32) the self-potential of scalar point charge q at rest in static space-time (2) and to calculate the self-force (34) acting on this charge. Note that in the case in which the Compton wavelength $1/m$ of the massive scalar field is much smaller than the characteristic scale L of curvature of the background gravitational field at the considered point x, we can obtain the approximated expression for the renormalized self-potential

$$\begin{aligned}\phi_{\rm ren}(x) &= \lim_{x_0\to x}(\phi_m(x;x_0) - \phi_{\rm DS}(x;x_0)) \\ &= \frac{q}{2m}\left[-\frac{g_{tt,i}{}^{;i}}{12g_{tt}} + \frac{5g_{tt,i}g_{tt}{}^{,i}}{48{g_{tt}}^2} - \left(\xi - \frac{1}{6}\right)R\right] \\ &\quad + O\left(\frac{q}{m^2L^3}\right). \end{aligned} \qquad (41)$$

Of course the order of this expression in $1/(mL)$ is less than the correspondent order of $\phi_{\rm ren}$ for the massless field (or field with mass $m \lesssim 1/L$). However, the expression (41) can be used for the verification of asymptotic behavior of $\phi_{\rm ren}$ in the limit $m \to \infty$.

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