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Conformal gravity and extensions of critical gravity

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Higher-order curvature corrections involving the conformally invariant Weyl-squared action have played a role in two recent investigations of four-dimensional gravity: in critical gravity, where they are added to the standard cosmological Einstein-Hilbert action with a coefficient tuned to make the massive ghostlike spin-2 excitations massless, and in a pure Weyl-squared action considered by Maldacena, where the massive spin-2 modes are removed by the imposition of boundary conditions. We exhibit the connections between the two approaches, and we also generalize critical gravity to a wider class of Weyl-squared modifications to cosmological Einstein gravity where one can eliminate the massive ghostlike spin-2 modes by means of boundary conditions. The cosmological constant plays a crucial role in the discussion, since there is then a "window" of negative mass-squared spin-2 modes around AdS_4 that are not tachyonic. We also construct analogous conformal and nonconformal gravities in six dimensions.

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I. INTRODUCTION

Although string theory may provide the most promising candidate for a quantum theory of gravity, there remains a tantalizing question as to whether four-dimensional gravity can be quantized in its own right. A natural approach, and one of the simplest, is to consider extending Einstein gravity by adding quadratic curvature terms, thus rendering the theory perturbatively renormalizable [1,2]. However, as is typical in a theory with more than second-order time derivatives, it contains ghostlike modes, in the form of massive spin-2 excitations. There is a way to circumvent this problem if one considers three dimensions rather than four, and so the usual massless graviton is trivial. Hence the ghostlike massive modes can become acceptable upon reversing the sign of the Einstein-Hilbert action, without creating a ghostlike physical massless graviton in the process. Examples include the well-studied topologically massive gravity [3], and the more recently discovered new massive gravity [4]. It was observed that when a cosmological constant is included, there exists some critical point [5] in the parameter space such that the massive modes disappear and are replaced by modes with logarithmic coordinate dependence [6]. The theory can be made ghost-free while retaining the standard sign for the Einstein-Hilbert action, by truncating out the log modes using standard Brown-Henneaux AdS_3 boundary conditions [7]. The theory has subsequently been generalized to a large class of three-dimensional off-shell supergravities [8–12].

Analogous critical gravities in four dimensions were subsequently proposed [13]. The Lagrangian consists of the Einstein-Hilbert term, a cosmological constant Λ, and a term constructed from the square of the Weyl tensor, with a coupling constant $\frac{1}{2}\alpha$.[1] It was shown that there is a critical relationship between α and Λ such that the massive spin-2 modes disappear by coalescing with the massless modes, resulting again in the appearance of logarithmic modes [15]. (See also [16,17].) These log modes are ghostlike in nature [18,19], but their falloff behavior at infinity is slower than the standard massless modes, and so they can be truncated out by imposing appropriate AdS_4 boundary conditions. The resulting theory appears, however, to be somewhat empty, in that the remaining massless graviton has zero on-shell energy. Furthermore, the mass and entropy of black holes in the critical theory both vanish. This critical phenomenon arises also in higher-dimensional gravities extended by adding curvature-squared terms [20], and also if certain cubic curvature terms are added [21]. (See also [22] for the $D = 3$ case.)

Recently, four-dimensional purely conformal gravity [23], where there is only a Weyl-squared term, has been revisited in [24]. It was shown that if an appropriate boundary condition is imposed, then for spherically symmetric configurations only the Schwarzschild–anti-de Sitter AdS (Schwarzschild-AdS) metric arises as

[1]Actions with a Weyl-squared term have also been considered in the context of noncommutative geometry in [14].

a black-hole solution in conformal gravity. Furthermore, its Euclidean action calculated in conformal gravity modified by a purely topological contribution from a Gauss-Bonnet term turns out to match exactly with the action of the same black hole in Einstein gravity with a cosmological constant, for an appropriate choice of the coefficient α of the Weyl-squared term in conformal gravity. The black-hole entropy calculated for the conformal gravity and for the usual Einstein gravity then precisely matches also. This leads to the possibility that the two theories at long wavelengths are in fact equivalent.

The Lagrangian for critical gravity, modulo a total derivative that does not affect the equations of motion, is given by [13]

$$\mathcal{L}^{\rm crit} = \sqrt{-g}(R - 2\Lambda + \tfrac{1}{2}\alpha C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}), \qquad (1.1)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. It turns out that the value for α required for criticality, namely, $\alpha = 3/(2\Lambda)$, is of precisely the same magnitude as that for the Weyl-squared coupling coefficient obtained in [24] for conformal gravity by imposing the Euclidean action matching condition described above. Thus the essentially vacuous nature of critical gravity is a reflection of the equivalence of the cosmological Einstein-Hilbert and the conformal theories.

In Sec. II, we review both critical gravity and the Einstein/conformal gravity duality conjecture. In conformal gravity, there exist ghostlike massive spin-2 modes in the AdS_4 background satisfying $(\Box - \frac{2}{3}\Lambda - M^2)h_{\mu\nu} = 0$, in addition to the massless spin-2 modes satisfying $(\Box - \frac{2}{3}\Lambda)h_{\mu\nu} = 0$. Spin-2 representations in AdS_4 are characterized by their lowest-energy E_0, which is given by

$$E_0 = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{3}{\Lambda}M^2}. \qquad (1.2)$$

The representation is unitary if $E_0 \geq 3$, and hence $M^2 \geq 0$ [25]. The time dependence of the modes is proportional to $e^{-iE_0 t}$, and so by analogy with the situation in Minkowski spacetime, modes may be defined to be tachyonic if E_0 becomes complex, thus leading to exponential growth in time. From (1.2), the absence of tachyons therefore requires[2]

$$M^2 \geq \tfrac{3}{4}\Lambda. \qquad (1.3)$$

Interestingly, although the massive modes in conformal gravity have $M^2 < 0$, the "mass" squared is not sufficiently negative to violate the bound (1.3), and so, although they are not unitary representations, they are not tachyonic. However, the radial falloff of these modes is slower than that for modes with $M^2 \geq 0$. In fact, they fall off more slowly than even the logarithmic modes. Thus these nonunitary modes can be truncated out by imposing appropriate boundary conditions, leaving only the massless graviton. The vanishing on-shell energy of the massless graviton in critical gravity implies that its energy in conformal gravity is exactly the same as it is in cosmological Einstein gravity.

In Sec. III, we obtain new unitary four-dimensional gravities, by generalizing the parameter choices made for critical gravity in [13]. For critical gravity, the unitarity requirement $M^2 \geq 0$ was imposed for the spin-2 modes. However, as noted above, the absence of tachyonic modes in $D = 4$ is less restrictive than this, and M^2 can be negative provided that (1.3) is still satisfied. This implies we can choose the coupling α for the Weyl-squared term in (1.1) so that the massive spin-2 modes have $3\Lambda/4 \leq M^2 < 0$. These ghostlike modes are classically stable, but can be truncated out by imposing appropriate boundary conditions, just as was done for conformal gravity in [24], leaving only the unitary massless graviton modes. Within this broader class of cosmological gravity plus Weyl-squared theories, critical gravity's specific problem of becoming vacuous after truncating the ghostlike modes is circumvented. Furthermore, since the broader class of theories has a range of allowable values for the parameter α, rather than a single critical choice, the possibility of finding a stable fixed point under the renormalization group flow becomes less demanding.

In Sec. IV, we generalize these results to six dimensions. There are three conformally invariant structures in $D = 6$. Two of these are the two independent invariants built from the cube of the Weyl tensor. The third is essentially built from second derivatives of the square of the Riemann curvature. In order to obtain a conformal equivalence to Einstein gravity, it is, in particular, necessary that Einstein metrics should also be solutions of the conformal gravity. Indeed, it was already observed that there exists a specific linear combination of the three conformal structures such that Riemann curvature-squared and cubed terms all vanish [27]. As in $D = 4$, we find that the conditions on the coefficients required for critical gravity are exactly the same as those implied by requiring Einstein/conformal gravity duality. We then observe that we can again construct a more general family of six-dimensional gravities for which the massive spin-2 modes can be eliminated by boundary conditions.

The paper ends with conclusions in Sec. V. In the Appendix, we collect some of the detailed calculations for the six-dimensional theories.

II. CRITICAL VS CONFORMAL GRAVITY IN FOUR DIMENSIONS

The Lagrangian of four-dimensional critical gravity discussed in [13],

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1, \qquad (2.1)$$

contains two parts. The first is the usual Einstein-Hilbert term with a cosmological constant,

[2]For scalar fields, the analogous requirement that E_0 be real is equivalent to the Breitenlohner-Freedman bound [26].

$$\mathcal{L}_0 = \sqrt{-g}(R - 2\Lambda). \tag{2.2}$$

The second term is quadratic in curvature, namely, the square of the Weyl tensor together with a Gauss-Bonnet term which is a total derivative:

$$\begin{aligned}\mathcal{L}_1 &= -\tfrac{1}{3}\alpha\sqrt{-g}(R^2 - 3R^{\mu\nu}R_{\mu\nu}) \\ &= \tfrac{1}{2}\alpha\sqrt{-g}(C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} - E_4),\end{aligned} \tag{2.3}$$

where

$$E_4 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \tag{2.4}$$

is the Gauss-Bonnet invariant whose integral is proportional to the Euler number. Being a total derivative in four dimensions, E_4 does not contribute to the equations of motion.

The Lagrangian $\mathcal{L}_1$ is proportional to the one for conformal gravity discussed in [23]. Defining $\mathcal{L}^{\rm conf}(\alpha) \equiv -\mathcal{L}_1$, we have

$$\mathcal{L} = \mathcal{L}_0 - \mathcal{L}^{\rm conf}(\alpha). \tag{2.5}$$

The Lagrangian admits Einstein metrics as solutions, with a cosmological constant equal to Λ. Included amongst these is the AdS_4 vacuum solution, whose curvature is given by

$$\begin{aligned}R_{\mu\nu} &= \Lambda g_{\mu\nu}, \qquad R = 4\Lambda, \\ R_{\mu\nu\rho\sigma} &= \frac{\Lambda}{3}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}).\end{aligned} \tag{2.6}$$

Writing the varied metric as $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$, and so $\delta g_{\mu\nu} = h_{\mu\nu}$, the linearized equations of motion were given in [13]. Choosing the gauge condition

$$\nabla^\mu h_{\mu\nu} = \nabla_\nu h, \tag{2.7}$$

it was shown that the trace part h vanishes by virtue of the equations of motion. The transverse and traceless spin-2 modes satisfy

$$-\alpha(\Box - \tfrac{2}{3}\Lambda)(\Box - \tfrac{2}{3}\Lambda - M^2)h_{\mu\nu} = 0, \tag{2.8}$$

where

$$M^2 = \frac{2}{3}\Lambda - \frac{1}{\alpha}. \tag{2.9}$$

The spectrum contains massless graviton modes $h^{(m)}_{\mu\nu}$ and also massive spin-2 modes $h^{(M)}_{\mu\nu}$. Their on-shell energies are given by [13]

$$E_{\rm massless} = -\frac{1}{2\kappa^2 T}\left(1 - \frac{2}{3}\alpha\Lambda\right)\int\sqrt{-g}d^4x\nabla^0 h^{\mu\nu}_{(m)}\dot{h}^{(m)}_{\mu\nu}, \tag{2.10}$$

$$E_{\rm massive} = \frac{1}{2\kappa^2 T}\left(1 - \frac{2}{3}\alpha\Lambda\right)\int\sqrt{-g}d^4x\nabla^0 h^{\mu\nu}_{(M)}\dot{h}^{(M)}_{\mu\nu}, \tag{2.11}$$

where the integration over the time coordinate is taken over an interval T, which one could take to be the natural time periodicity of AdS_4, or else just send it to infinity. Since the integrals themselves both give negative quantities, it follows that ghost modes are unavoidable, in general. In [13], the parameter α was chosen to have the critical value given by

$$\alpha = \alpha^{\rm crit} \equiv \frac{3}{2\Lambda}, \tag{2.12}$$

implying that $M^2 = 0$. In this case, because the massive modes coalesce with the massless ones, one obtains new solutions to (2.8) that are annihilated by neither second-order factor. These modes have logarithmic dependence on the AdS_4 radial coordinate, and they can be truncated out by imposing an appropriate AdS boundary condition. The resulting critical gravity is, however, rendered essentially trivial, since the energy (2.10) for the surviving massless mode vanishes. Furthermore, the mass and the entropy of the Schwarzschild black hole vanish also. The mass formulas for black holes in extended gravity can be found in [28–30].

In a new development in higher-derivative gravity, four-dimensional conformal gravity [23] was revisited in [24]. It was observed that the Euclidean action for the Schwarzschild-AdS black hole computed from $\mathcal{L}^{\rm conf}$ is identical to that calculated from the pure cosmological Einstein-Hilbert Lagrangian $\mathcal{L}_0$, provided that the parameter α is chosen to take the critical value given in (2.12). The black-hole entropy matches also. We have checked that the actions also match for the Kerr-AdS black hole. It was proposed in [24] that, subject to the imposition of appropriate boundary conditions, the Lagrangians $\mathcal{L}_0$ and $\mathcal{L}^{\rm conf}$ are equivalent at the critical point, in the long-wavelength regime. From this point of view, the "triviality" of critical gravity can be easily understood, since critical gravity (2.1) is given by

$$\mathcal{L}^{\rm crit} = \mathcal{L}_0 - \mathcal{L}^{\rm conf}(\alpha^{\rm crit}), \tag{2.13}$$

and so one is subtracting two Lagrangians that describe the same IR physics. The vanishing of the graviton energy and also the black-hole mass and entropy in critical gravity further establish the equivalence of $\mathcal{L}_0$ and $\mathcal{L}^{\rm conf}$ at the critical point.

It should be remarked that there is an issue of ghost modes in conformal gravity. The linearized equation of motion following from $\mathcal{L}^{\rm conf}$ is given by

$$\alpha(\Box - \tfrac{2}{3}\Lambda)(\Box - \tfrac{4}{3}\Lambda)h_{\mu\nu} = 0. \tag{2.14}$$

This implies that

$$M^2 = \tfrac{2}{3}\Lambda, \tag{2.15}$$

which is negative since $\Lambda < 0$ for AdS_4. The energies of the on-shell massless and massive spin-2 modes are given by

$$E^{\rm conf}_{\rm massless} = -\frac{\alpha\Lambda}{\kappa^2 T}\int \sqrt{-g}d^4x\nabla^0 h^{\mu\nu}_{(m)}\dot{h}^{(m)}_{\mu\nu}, \tag{2.16}$$

$$E^{\rm conf}_{\rm massive} = \frac{\alpha\Lambda}{\kappa^2 T}\int \sqrt{-g}d^4x\nabla^0 h^{\mu\nu}_{(M)}\dot{h}^{(M)}_{\mu\nu}. \tag{2.17}$$

Thus we see that α has to be negative for the massless graviton to have positive energy; meanwhile, the massive graviton has negative energy.

The mass squared M^2 of the massive graviton (2.15) in conformal gravity is negative, suggesting the possibility that these modes might be tachyonic. As we mentioned in the Introduction, the $SO(2,3)$ representations for massive spin-2 modes in AdS$_4$ are characterized by their lowest-energy E_0, which is given in terms of M^2 by (1.2). From now on, we shall, for convenience, take

$$\Lambda = -3, \tag{2.18}$$

so that the AdS$_4$ has "unit radius." The reality of $E_0 = \frac{3}{2} \pm \sqrt{\frac{9}{4} + M^2}$ therefore requires that

$$M^2 \geq M^2_{\rm min} \equiv -\tfrac{9}{4}. \tag{2.19}$$

As can be seen from the explicit expressions for the massive spin-2 modes obtained in [15], which have time dependence of the form $e^{-iE_0 t}$, the condition that E_0 be real ensures that the modes do not grow exponentially in time. This is essentially the statement of the absence of tachyons. The massive spin-2 modes in conformal gravity, which have $M^2 = -2$, lie within the bound (2.19), and so they are not tachyonic.

The radial dependence of the modes with $M^2_{\rm min} \leq M^2 < 0$ exhibits a slower falloff at large distances than that for modes with $M^2 \geq 0$. In fact, they fall off more slowly than even the log modes. They can therefore be truncated out by imposing an appropriate asymptotic boundary condition, as was described in [24]. This is essentially the same boundary condition that can be used to truncate out the logarithmic modes in critical gravity.

III. NEW UNITARY GRAVITIES IN FOUR DIMENSIONS

After the truncation of the massive modes, the conformal gravity described by $\mathcal{L}^{\rm conf}(\alpha_{\rm crit})$ can be viewed as being equivalent, at the classical level, to cosmological Einstein gravity $\mathcal{L}_0$ [24]. It should, however, be emphasized that conformal gravity admits Einstein metrics with an arbitrary cosmological constant as solutions, and so for a given value of α the equivalence to Einstein gravity holds only for the specific value $\Lambda = 3/(2\alpha)$ appearing in $\mathcal{L}_0$.

It is natural to consider the more restrictive case where the theory has a unique scale for the AdS vacuum determined by the cosmological constant in the theory. We then need to consider the Lagrangian (2.1). As discussed earlier, the mass of the massive spin-2 modes in this theory is given by (2.9). In [13], it was required that $M^2 \geq 0$, so that these modes will correspond to unitary representations of $SO(2,3)$. For $M^2 > 0$, they fall off faster than the massless modes, and so they could not be truncated out by imposing boundary conditions at infinity. Thus one would be stuck with having nontruncatable ghostlike modes, except in the critical case where $M = 0$, for which the resulting logarithmic modes can be truncated out on account of their slower falloff.

An alternative choice is to choose the α parameter so that M^2 lies in the range

$$-\tfrac{9}{4} \leq M^2 < 0. \tag{3.1}$$

Within this range, the massive modes are nontachyonic and classically stable in the sense that there is no exponential growth in time. Since, however, they fall off more slowly than those with $M^2 \geq 0$, one can impose boundary conditions that eliminate them from the spectrum while retaining the massless modes.[3] The condition (3.1) is satisfied by either $\alpha \geq 4$ or $\alpha < -\frac{1}{2}$. It follows from (2.10) that the choice $\alpha < -\frac{1}{2}$ implies that the massless graviton has negative energy. On the other hand, for the choice of $\alpha \geq 4$, the energy of the massless graviton remains positive. Of course, in this case, the massive modes would have negative energy. However, as we discussed, these modes can be eliminated by imposing appropriate boundary conditions, leaving just the nontrivial positive-energy massless graviton.

As we have seen, by allowing the possibility of having nontachyonic but negative-M^2 massive modes, which can then be eliminated by boundary conditions, we have now arrived at a one-parameter family ($\alpha \geq 4$) of extended gravity theories that describe just unitary massless spin-2 fields. At the quantum level, having such a family broadens the chances for finding an ultraviolet fixed point of the renormalization group flow that lands within the class of acceptable theories. This may improve the prospects for obtaining a consistent theory of quantum gravity.

IV. GENERALIZATIONS TO SIX DIMENSIONS

We now turn our attention to six dimensions. Conformal gravities in $D = 6$ have been previously studied (see, for example, [31–33]). Three independent structures can arise in the Lagrangian. Their explicit forms are (see [34], and also [35,36])

$$I_1 = C_{\mu\rho\sigma\nu}C^{\mu\alpha\beta\nu}C_{\alpha}{}^{\rho\sigma}{}_{\beta}, \quad I_2 = C_{\mu\nu\rho\sigma}C^{\rho\sigma\alpha\beta}C_{\alpha\beta}{}^{\mu\nu},$$
$$I_3 = C_{\mu\rho\sigma\lambda}(\delta^{\mu}_{\nu}\Box + 4R^{\mu}{}_{\nu} - \tfrac{6}{5}R\delta^{\mu}_{\nu})C^{\nu\rho\sigma\lambda} + \nabla_{\mu}J^{\mu}, \tag{4.1}$$

where $\nabla_\mu J^\mu$, which does not contribute to the equations of motion, can be found in [34]. In general, a Lagrangian of the form $\sqrt{-g}c_i I_i$ will give equations of motion that are not

[3] Note that the $E_0 = 0$ branch of the massless solution from (1.2) is truncated out for the same reason.

satisfied by arbitrary Einstein metrics. However, for a specific choice of the c_i (unique up to overall scaling), the equations of motion will be satisfied by any Einstein metric. This same linear combination has the feature that, modulo total derivatives, all terms of cubic and quadratic order in the Riemann tensor are absent [27]. In this form, the Lagrangian is given by

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm conf} &= \beta\Big(4I_1 + I_2 - \frac{1}{3}\tilde{I}_3 - \frac{1}{24}E_6 + \nabla_\mu \tilde{J}^\mu\Big) \\ &= \beta\Big(RR^{\mu\nu}R_{\mu\nu} - \frac{3}{25}R^3 - 2R^{\mu\nu}R^{\rho\sigma}R_{\mu\rho\nu\sigma} \\ &\quad - R^{\mu\nu}\Box R_{\mu\nu} + \frac{3}{10}R\Box R\Big), \end{aligned} \tag{4.2}$$

where $\tilde{I}_3 = I_3 - \nabla_\mu J^\mu$, and the total derivative $\nabla_\mu \tilde{J}^\mu$ can be derived from [27]. Note that E_6 is the Euler integrand, given by

$$\begin{aligned} E_6 = {}& \epsilon_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\epsilon^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3} \\ & \times R^{\mu_1\nu_1}{}_{\rho_1\sigma_1}R^{\mu_2\nu_2}{}_{\rho_2\sigma_2}R^{\mu_3\nu_3}{}_{\rho_3\sigma_3}. \end{aligned} \tag{4.3}$$

Any Einstein metric (or, in fact, any metric conformal to Einstein metric) will be a solution to the theory following from (4.2). In particular, we may consider the Schwarzschild-AdS black hole, satisfying $R_{\mu\nu} = -5g_{\mu\nu}$, with the metric

$$ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega_4^2, \qquad f = 1 + r^2 - \frac{\mu}{r^3}. \tag{4.4}$$

This is also a solution to Einstein gravity with a cosmological constant, described by the Lagrangian

$$e^{-1}\mathcal{L}_0 = R + 20. \tag{4.5}$$

Note that we have chosen the cosmological constant so that the AdS_6 vacuum is of unit radius. The thermodynamic quantities for the black hole (4.4) are given by

$$T = \frac{3 + 5r_+^2}{4\pi r_+}, \qquad S = \frac{2}{3}\pi^2 r_+^4, \qquad M = \frac{2}{3}\pi r_+^3(1 + r_+^2), \tag{4.6}$$

where r_+ is the horizon radius. The Euclidean action is given by

$$I_6^{\rm Ein} = \frac{2\pi^2 r_+^4(1 - r_+^2)}{3(3 + 5r_+^2)}. \tag{4.7}$$

Substituting the Euclideanized solution (4.4) into the action $I_6^{\rm conf} = \int d^6x\mathcal{L}^{\rm conf}$, we find that the contribution from $(4I_1 + I_2 - \frac{1}{3}\tilde{I}_3)$ converges. The contribution from the $\nabla_\mu \tilde{J}^\mu$ term vanishes. The integral of E_6 itself diverges, but if, following the same strategy as in [24], one adds in the associated boundary term that arises in the definition of the Euler number for manifolds with boundary, it contributes a pure topological number. The Euclidean action $I_6^{\rm conf}$ then turns out to be proportional to $I_6^{\rm Ein}$. To be specific, we have

$$I_6^{\rm Ein} = I_6^{\rm conf}|_{\beta=-(1/24)}. \tag{4.8}$$

We have also checked this equality for the Kerr-AdS black hole [37,38], using also results from [39], in the case that the two angular momenta are equal. It is straightforward to check, using the Wald formula [40], that the Schwarzschild-AdS black-hole entropy matches exactly also. This suggests, therefore, that as in $D = 4$, Einstein gravity emerges from conformal gravity.

Let us now consider the linearization of conformal gravity around the AdS_6 background. For Einstein gravity (4.5), the spin-2 graviton is massless, satisfying

$$-(\Box + 2)h_{\mu\nu}^{(m)} = 0. \tag{4.9}$$

(Recall that we have set $\Lambda = -5$.) For conformal gravity (4.2), the full set of equations of motion and linearization around the AdS_6 vacuum are given in the Appendix. It turns out the spin-2 modes satisfy

$$\beta(\Box + 2)(\Box + 6)(\Box + 8)h_{\mu\nu} = 0. \tag{4.10}$$

Thus in the six-dimensional conformal gravity, there are two massive spin-2 modes, with negative mass squared, in addition to the massless graviton. The masses are given by

$$M_1^2 = -4 \quad \text{and} \quad M_2^2 = -6. \tag{4.11}$$

The condition for the absence of tachyon modes is that the lowest-energy E_0 of the $SO(2, 5)$ representations should be real, where E_0 is given by

$$E_0(E_0 - 5) = M^2. \tag{4.12}$$

This implies that

$$M^2 \geq -\frac{25}{4}. \tag{4.13}$$

Thus both M_1 and M_2 satisfy this bound, even though both these massive modes violate the bound $E_0 \geq 5$ for unitary representations. Since they have $M^2 < 0$, their falloff at large distances is slower than the modes with $M^2 \geq 0$, and hence they can be truncated out by imposing an appropriate AdS boundary condition while the massless graviton is retained.

Using standard Ostrogradsky or Noether techniques, we find that the on-shell energy of the massless graviton in the conformal gravity is given by

$$E = \frac{1}{4\kappa^2}(24\beta)\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int\sqrt{-g}d^5x\dot{h}_{\mu\nu}\nabla^0 h^{\mu\nu}. \tag{4.14}$$

For $\beta = -1/24$, this is precisely the on-shell energy of the Einstein gravity (4.5), further establishing the equivalence of Einstein gravity and conformal gravity at the classical level.

We may also interpret the above discussion from the point of view of critical gravity, whose Lagrangian is given by

$$\mathcal{L}_6 = \mathcal{L}_6^0 - \mathcal{L}_6^{\rm conf}. \tag{4.15}$$

It is clear that the theory admits a unique AdS_6 vacuum. Furthermore, any Einstein metrics with $\Lambda = -5$, including the Schwarzschild black hole (4.4), are also solutions. Linearizing the theory around the AdS_6 vacuum, it is easy to verify that the trace mode is trivial and the remaining spin-2 modes satisfy the equation

$$-(\Box + 2)(1 + \beta(\Box + 6)(\Box + 8))h_{\mu\nu} = 0. \tag{4.16}$$

Thus in addition to the massless graviton, there are two massive modes with

$$M_\pm^2 = -5 \pm \sqrt{1 - \frac{1}{\beta}}. \tag{4.17}$$

The on-shell energy for the massless graviton is given by

$$E = -\frac{1}{4\kappa^2}(1 + 24\beta)\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int\sqrt{-g}d^5x\dot{h}_{\mu\nu}\nabla^0 h^{\mu\nu}. \tag{4.18}$$

The criticality condition is $\beta = -1/24$, for which the massless graviton therefore has zero energy. Furthermore, one of the two massive gravitons becomes massless. The remaining massive mode has $M^2 = -10$, which violates the no-tachyon bound (4.13).

As in the case of $D = 4$, we can consider alternative parameter choices such that the massive spin-2 modes both satisfy the bound

$$-\frac{25}{4} \le M_\pm^2 < 0. \tag{4.19}$$

These modes, with nonunitary representations, can nevertheless be truncated out by imposing appropriate AdS_6 boundary conditions. Furthermore, we would like the remaining massless graviton to have positive energy, as given by (4.18). These requirements can all be met by choosing

$$\beta \ge 1. \tag{4.20}$$

Note that $\beta = 1$ corresponds to another critical point, where the two massive spin-2 modes have the same mass, $M_\pm^2 = -5$. Restoring the general cosmological constant Λ, defined by $R_{\mu\nu} = \Lambda g_{\mu\nu}$, the condition (4.20) becomes $\beta(-\Lambda) \ge 5$.

Finally, we remark that at the $\beta = -1/24$ critical point in the $D = 6$ theory, there are still surviving massive spin-2 modes, since we have only the one parameter β to adjust. We may also add a Weyl-squared term $\frac{1}{2}\alpha\sqrt{-g}C^2$ to the Lagrangian. The linearized equation for the spin-2 modes is now given by

$$-(\Box + 2)(1 - \tfrac{1}{3}\alpha(\Box + 6) + \beta(\Box + 6)(\Box + 8))h_{\mu\nu} = 0. \tag{4.21}$$

A tricritical point is then achieved with $\alpha = 15/8$ and $\beta = 1/16$, at which the linearized equation becomes

$$-(\Box + 2)^3 h_{\mu\nu} = 0. \tag{4.22}$$

V. CONCLUSIONS

In this paper we have developed some new ideas for constructing higher-derivative theories of gravity that avoid the difficulties with massive spin-2 ghost modes that typically plague such theories. In four dimensions, it was observed in [13] that if a term proportional to the square of the Weyl tensor is added to the usual Einstein-Hilbert Lagrangian with a cosmological constant, then although generically one finds that the fluctuations around the AdS_4 background describe the usual massless spin-2 graviton and also ghostlike massive spin-2 modes, it is possible to tune the coefficient of the Weyl-squared term so as to make the massive modes massless also. In fact, the energies of the massless modes then vanish in this critical theory. There are, however, now also modes with a logarithmic coordinate dependence, which can have negative energies. Since their falloff at infinity is slower than that of the massless modes, they can be removed by imposing appropriate boundary conditions. However, since the massless modes that remain have zero energy, the resulting theory is somewhat trivial.

Recently, purely conformal gravity where there is only a Weyl-squared term was revisited [24]. In this case there are again massless and massive spin-2 modes around an AdS_4 background, and again the massive modes are ghostlike. However, their mass squared is actually negative, although not sufficiently negative to be tachyonic. This means that they fall off more slowly at infinity than do the massless modes, and so they can be eliminated by imposing appropriate boundary conditions. In [24], it was shown that by tuning the coefficient of the Weyl-squared action appropriately, it could be matched for Euclideanized solutions with the Euclidean cosmological Einstein action for the same configuration. It was argued that conformal gravity is then really equivalent to cosmological Einstein gravity. In fact, the value needed for the Weyl-squared coefficient is exactly the same as the one required in [13] for critical gravity. This provides a new insight into the trivial nature of critical gravity once the logarithmic modes are eliminated, in that its action is essentially just the difference between two actions that provide equivalent descriptions of long-wavelength physics.

The main purpose of this paper was to construct a new one-parameter family of higher-derivative gravities for which the ghostlike massive spin-2 modes can be eliminated. We did this by relaxing the assumption that was made in [13] that the mass squared of the massive spin-2 modes should be non-negative. This condition is needed if one wants the massive modes to carry unitary representations under $SO(2,3)$, but since they are in any case ghostlike and need to be truncated, this is not really a crucial requirement. The key point is that because the background is AdS_4 rather than Minkowski spacetime, there is a window of allowed *negative* values of mass squared for which the modes are nontachyonic, and thus classically stable.

Furthermore, precisely because the mass squared is negative, the falloff of these modes is slower than the falloff of the massless modes. Thus one can impose boundary conditions to eliminate the undesired massive modes while retaining the massless modes. The massive modes lie in the desired negative mass-squared range if the parameter α in (1.1) satisfies

$$\alpha(-\Lambda) > 12, \tag{5.1}$$

i.e. $\alpha > 4$ if we normalize the cosmological constant of AdS_4 canonically to $\Lambda = -3$. (Although our primary concern in this paper is for a negative cosmological constant, we expect the above inequality to hold for a positive cosmological constant also.)

We then extended our discussion to consider gravities in six dimensions. By taking a suitable linear combination of the three possible conformally invariant terms, one can construct a conformal gravity in six dimensions that admits all Einstein metrics as solutions. One can again tune the overall coefficient so that the action of Euclideanized AdS black holes matches with that calculated for the cosmological Einstein-Hilbert action. There are now two sets of massive spin-2 modes in addition to the massless ones, and both have mass-squared values that are negative but not tachyonic. Thus, as in the four-dimensional case studied in [24], one can eliminate the ghostlike massive modes by imposing appropriate boundary conditions, suggesting the equivalence of Einstein and conformal gravity in six dimensions too.

An essential idea underlying the proposal for conformal gravity in [24] is that one may be able to "have one's cake and eat it too" by reaping the renormalizability benefits of the higher-derivative theory in the ultraviolet regime while still having an equivalence to conventional Einstein gravity in the infrared. One motivation for seeking families of potentially acceptable theories of gravity as we have done in this paper, rather than isolated examples, comes from quantum considerations. If one does have a renormalizable theory, then the question arises as to how it behaves in the high-energy limit under the renormalization group flow. One possibility is that the family of theories we have considered ($\alpha \geq 4$ in four dimensions; $\beta \geq 1$ in six dimensions) might start from a finite α or β and flow to a fixed point at the conformally invariant limit ($\alpha = \infty$ or $\beta = \infty$), as possibly suggested by the results in [41]. An advantage of starting from a finite α or β at lower energies, rather than just using the conformally invariant theory at all energy scales, would be that one would, in general, have the more tightly restricted solution space of Einstein gravity plus quadratic corrections, flowing to the less restrictive scale invariance of conformal gravity only in the high-energy limit. Thus the extensions of critical gravity we have considered here may be of relevance for a quantum theory of gravity.

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APPENDIX: EQUATIONS AND LINEARIZATION OF $D = 6$ CONFORMAL GRAVITY

Equations of motion

The Lagrangian for the $D = 6$ conformal gravity we study in this paper is given by (4.2). There are five terms. The contributions $E^{(i)}_{\mu\nu}$ to Einstein equations of motion from each term are summarized as follows:

1): $RR^{\mu\nu}R_{\mu\nu} \Rightarrow$

$$E^{(1)}_{\mu\nu} = \Big(\Box(R_{\lambda\sigma}R^{\lambda\sigma}) + \nabla_\lambda\nabla_\sigma(RR^{\lambda\sigma}) - \frac{1}{2}RR_{\lambda\sigma}R^{\lambda\sigma}\Big)g_{\mu\nu} + R_{\lambda\sigma}R^{\lambda\sigma}R_{\mu\nu} + 2RR_{\lambda\mu}R^\lambda{}_\nu$$
$$+ \Box(RR_{\mu\nu}) - \nabla_\mu\nabla_\nu(R_{\lambda\sigma}R^{\lambda\sigma}) - \nabla_\lambda\nabla_\mu(RR^\lambda{}_\nu) - \nabla_\lambda\nabla_\nu(RR^\lambda{}_\mu),$$

2): $R^3 \Rightarrow$

$$E^{(2)}_{\mu\nu} = \Big(3\Box R^2 - \frac{1}{2}R^3\Big)g_{\mu\nu} + 3R^2R_{\mu\nu} - 3\nabla_\mu\nabla_\nu R^2,$$

3): $R^{\mu\nu}R^{\lambda\rho}R_{\mu\lambda\nu\rho} \Rightarrow$

$$E^{(3)}_{\mu\nu} = -\frac{1}{2}R^{\sigma\delta}R^{\lambda\rho}R_{\sigma\lambda\delta\rho}g_{\mu\nu} + \frac{3}{2}R^{\rho\sigma}R_{\rho\mu\sigma\lambda}R^\lambda{}_\nu + \frac{3}{2}R^{\rho\sigma}R_{\rho\nu\sigma\lambda}R^\lambda{}_\mu$$
$$+ \Box(R^{\rho\sigma}R_{\rho\mu\sigma\nu}) + \nabla^\sigma\nabla^\delta(R^{\lambda\rho}R_{\lambda\sigma\rho\delta})g_{\mu\nu} - \nabla^\lambda\nabla_\mu(R^{\rho\sigma}R_{\rho\lambda\sigma\nu})$$
$$- \nabla^\lambda\nabla_\nu(R^{\rho\sigma}R_{\rho\lambda\sigma\mu}) - \nabla_{(\sigma}\nabla_{\lambda)}(R_\mu{}^\sigma R_\nu{}^\lambda) + \nabla_\sigma\nabla_\lambda(R_{\mu\nu}R^{\sigma\lambda})$$

$$4): R^{\mu\nu}\Box R_{\mu\nu} = -g^{\mu\nu}\nabla_\mu R^{\lambda\rho}\nabla_\nu R_{\lambda\rho} \Rightarrow$$

$$E^{(4)}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}(g^{\sigma\delta}\nabla_\sigma R^{\lambda\rho}\nabla_\delta R_{\lambda\rho}) - (2\nabla^\sigma R_{\mu\lambda}\nabla_\sigma R_\nu{}^\lambda + \nabla_\mu R_{\sigma\lambda}\nabla_\nu R^{\sigma\lambda}) + 2\nabla_\lambda(R_{\sigma(\mu}\nabla_{\nu)}R^{\lambda\sigma}) + 2\nabla_\lambda(\nabla^\lambda R^\sigma{}_{(\nu}R_{\mu)\sigma}) - 2\nabla_\sigma(\nabla_{(\mu}R_{\nu)\lambda}R^{\sigma\lambda}) + \Box^2 R_{\mu\nu} + \nabla_\sigma\nabla_\lambda\Box R^{\sigma\lambda}g_{\mu\nu} - \nabla_\lambda\nabla_\nu(\Box R^\lambda{}_\mu) - \nabla_\lambda\nabla_\mu(\Box R^\lambda{}_\nu)$$

$$5): R\Box R = -g^{\mu\nu}\nabla_\mu R\nabla_\nu R \Rightarrow$$

$$E^{(5)}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}(g^{\sigma\lambda}\nabla_\sigma R\nabla_\lambda R) - \nabla_\mu R\nabla_\nu R + 2(\Box R)R_{\mu\nu} + 2(\Box^2 R)g_{\mu\nu} - 2\nabla_\mu\nabla_\nu\Box R. \qquad \text{(A1)}$$

The complete equation of motion following from (4.2) is then

$$E^{(1)}_{\mu\nu} - \frac{3}{25}E^{(2)}_{\mu\nu} - 2E^{(3)}_{\mu\nu} - E^{(4)}_{\mu\nu} + \frac{3}{10}E^{(5)}_{\mu\nu} = 0. \qquad \text{(A2)}$$

Linearization

The theory admits Einstein metrics with $R_{\mu\nu} = \Lambda g_{\mu\nu}$, with arbitrary Λ. We consider the linearization around the AdS_6 background, namely, $R_{\mu\nu\rho\sigma} = \frac{1}{5}\Lambda(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$. Writing the varied metric as $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$, and so $\delta g_{\mu\nu} = h_{\mu\nu}$, the linearized Einstein tensor is given by

$$\mathcal{G}^L_{\mu\nu} = R^L_{\mu\nu} - \tfrac{1}{2}R^L g_{\mu\nu} - \Lambda h_{\mu\nu}, \qquad \mathcal{G}^L \equiv g^{\mu\nu}\mathcal{G}^L_{\mu\nu}, \qquad \text{(A3)}$$

$$R^L_{\mu\nu} = \nabla^\lambda\nabla_{(\mu}h_{\nu)\lambda} - \tfrac{1}{2}\Box h_{\mu\nu} - \tfrac{1}{2}\nabla_\mu\nabla_\nu h, \qquad \text{(A4)}$$

$$R^L = \nabla^\mu\nabla^\nu h_{\mu\nu} - \Box h - \Lambda h. \qquad \text{(A5)}$$

(We have also defined $R^L_{\mu\nu}$, the linearization of $R_{\mu\nu}$, and introduced $h = g^{\mu\nu}h_{\mu\nu}$.) With these preliminaries, we find that the five linearized contributions of the equations of motion listed above are given by

$$1): (2\Lambda\Box\mathcal{G}^L + 6\Lambda\nabla^\lambda\nabla^\sigma\mathcal{G}^L_{\lambda\sigma} - 4\Lambda^2\mathcal{G}^L)g_{\mu\nu} + 30\Lambda^2\mathcal{G}^L_{\mu\nu} + 6\Lambda\Box\mathcal{G}^L_{\mu\nu} - 2\Lambda\nabla_\mu\nabla_\nu\mathcal{G}^L - 6\Lambda\nabla^\sigma\nabla_\mu\mathcal{G}^L_{\sigma\nu} - 6\Lambda\nabla^\sigma\nabla_\nu\mathcal{G}^L_{\sigma\mu} + 14\Lambda g_{\mu\nu}\Box R^L + 2\Lambda^2 g_{\mu\nu}R^L - 14\Lambda\nabla_\mu\nabla_\nu R^L,$$

$$2): 108\Lambda^2\mathcal{G}^L_{\mu\nu} + 36\Lambda g_{\mu\nu}\Box R^L + 36\Lambda^2 g_{\mu\nu}R^L - 36\nabla_\mu\nabla_\nu R^L,$$

$$3): 3\Lambda R_\mu{}^\lambda{}_\nu{}^\sigma\mathcal{G}^L_{\lambda\sigma} + 6\Lambda^2\mathcal{G}^L_{\mu\nu} + \Lambda\Box\mathcal{G}^L_{\mu\nu} + \Box(R_\mu{}^\lambda{}_\nu{}^\sigma\mathcal{G}^L_{\lambda\sigma}) + g_{\mu\nu}\nabla^\sigma\nabla^\delta(R_\sigma{}^\lambda{}_\delta{}^\rho\mathcal{G}^L_{\lambda\rho}) - \nabla^\lambda\nabla_\mu(R_\lambda{}^\sigma{}_\nu{}^\delta\mathcal{G}^L_{\sigma\delta}) - \nabla^\lambda\nabla_\nu(R_\lambda{}^\sigma{}_\mu{}^\delta\mathcal{G}^L_{\sigma\delta}) - \frac{3}{2}\Lambda\nabla^\sigma\nabla_\mu\mathcal{G}^L_{\sigma\nu} - \frac{3}{2}\Lambda\nabla^\sigma\nabla_\nu\mathcal{G}^L_{\sigma\mu} + \Lambda\Box\mathcal{G}^L_{\mu\nu} + 3\Lambda^2 g_{\mu\nu}R^L + 3\Lambda g_{\mu\nu}\Box R^L - 3\Lambda\nabla_\mu\nabla_\nu R^L,$$

$$4): 2\Lambda\Box\mathcal{G}^L_{\mu\nu} + \Box^2\mathcal{G}^L_{\mu\nu} + g_{\mu\nu}\nabla^\lambda\nabla^\sigma\Box\mathcal{G}^L_{\lambda\sigma} - \nabla^\lambda\nabla_\mu\Box\mathcal{G}^L_{\lambda\nu} - \nabla^\lambda\nabla_\nu\Box\mathcal{G}^L_{\lambda\mu} + g_{\mu\nu}\Lambda\Box R^L + g_{\mu\nu}\Box^2 R^L - \nabla_\mu\nabla_\nu\Box R^L,$$

$$5): 2(\Lambda g_{\mu\nu}\Box R^L + g_{\mu\nu}\Box^2 R^L - \nabla_\mu\nabla_\nu\Box R^L). \qquad \text{(A6)}$$

Thus for the traceless and transverse spin-2 modes $h_{\mu\nu}$, the linearized Einstein tensor is $\mathcal{G}^L_{\mu\nu} = -\frac{1}{2}(\Box + 2)h_{\mu\nu}$ and the Ricci scalar is $R^L = 0$. It follows that the linearized equation of motion is given by (4.10).

Hamiltonian

The quadratic fluctuations S_2 for the following action S are given by

$$S = \frac{1}{2\kappa^2}\int\sqrt{-g}d^6x\Big[R - \beta\Big(RR^{\mu\nu}R_{\mu\nu} - \frac{3}{25}R^3 - 2R^{\mu\nu}R^{\lambda\rho}R_{\mu\lambda\nu\rho} - R^{\mu\nu}\Box R_{\mu\nu} + \frac{3}{10}R\Box R\Big)\Big], \qquad \text{(A7)}$$

$$S_2 = -\frac{1}{8\kappa^2}\int\sqrt{-g}d^6x[\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} - 2h_{\mu\nu}h^{\mu\nu} + \beta(\nabla_\lambda\Box h_{\mu\nu}\nabla^\lambda\Box h^{\mu\nu} - 16\Box h^{\mu\nu}\Box h_{\mu\nu} + 76\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} - 96h^{\mu\nu}h_{\mu\nu})]. \qquad \text{(A8)}$$

The Hamiltonian is

$$H = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int d^5x(\dot{h}_{\mu\nu}\Pi^{(1)\mu\nu} + \partial_t(\nabla_0 h_{\mu\nu})\Pi^{(2)\mu\nu} + \partial_t(\Box h_{\mu\nu})\Pi^{(3)\mu\nu} - L), \qquad \text{(A9)}$$

where

$$
\begin{aligned}
\Pi^{(1)\mu\nu} &= -\frac{\sqrt{-g}}{4\kappa^2}[\nabla^0 h^{\mu\nu} + \beta(76\nabla^0 h^{\mu\nu} + 16\nabla^0\Box h^{\mu\nu} + \nabla^0\Box^2 h^{\mu\nu})],\\
\Pi^{(2)\mu\nu} &= -\frac{\beta\sqrt{-g}}{4\kappa^2}[-16g^{00}\Box h^{\mu\nu} - g^{00}\Box^2 h^{\mu\nu}],\\
\Pi^{(3)\mu\nu} &= -\frac{\beta\sqrt{-g}}{4\kappa^2}[\nabla^0\Box h^{\mu\nu}].
\end{aligned} \tag{A10}
$$

Then one can obtain the energy of massless gravitons as

$$
E = -\frac{1}{4\kappa^2}(1+24\beta)\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int\sqrt{-g}d^5x\dot{h}_{\mu\nu}\nabla^0 h^{\mu\nu}. \tag{A11}
$$

In our case, the Wald formula is

$$
S = -\frac{1}{8G}\int_H \epsilon_{ab}\epsilon_{cd}\left(\frac{\partial L}{\partial R_{abcd}} + \nabla_{(mn)}\frac{\partial L}{\partial\nabla_{(mn)}R_{abcd}}\right)d\Sigma, \tag{A12}
$$

where ϵ_{ab} is the bi-normal vector of the horizon normalized to satisfy $\epsilon_{ab}\epsilon^{ab} = -2$.

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