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Galileon design of slow expansion

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We show a model of the slow expansion, in which the scale invariant spectrum of curvature perturbation is adiabatically induced by its increasing mode, by applying a generalized Galileon field. In this model, initially $\epsilon \ll -1$, which then rapidly increases, and during this period the Universe is slowly expanding. There is no ghost instability, and the perturbation theory is healthy. When $\epsilon \sim -1$, the slow expansion phase ends, the available energy of field can be released and the Universe reheats. This scenario might be a viable design of the early Universe.

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I. INTRODUCTION

The observations imply that the primordial curvature perturbation is scale invariant. Thus, how to generate it is still a significant issue, especially for a single field. The curvature perturbation on a large scale consists of a constant mode and a mode dependent on time [1]. When one is dominated and scale invariant, the spectrum of curvature perturbation will be scale invariant. When the scale factor is rapidly changed while ϵ is nearly constant, the constant mode is responsible for that of inflation [2–5], while the increasing mode is for the contraction with matter [6–8], both are dual [6].

In principle, the increasing mode of metric perturbation, which is scale invariant for $\epsilon \gg 1$ [9] or $\epsilon \ll -1$ [10], might dominate the curvature perturbation. The constant mode of metric perturbation is the same as that of the constant mode of curvature perturbation. The duality of scale invariant spectrum of metric perturbation has been discussed in [11–13]. The evolution with $\epsilon \gg 1$ is slowly contracting, which is that of an ekpyrotic universe [14], while $\epsilon \ll -1$ gives the slow expansion [10], which has been applied for the island universe [15]. In a certain sense, it was first observed in Ref. [10] that the slow expansion might adiabatically generate the scale invariant spectrum of curvature perturbation, see [16] for that induced by the entropy perturbation.

When the available energy of field is released, the slow expansion phase ends and the Universe reheats. Thus, the slow expansion might be a viable scenario of the early Universe. In principle, when ϵ is constant, whether the increasing mode of the metric perturbation can be inherited by the curvature perturbation depends on the physics around the exiting [17]. However, when ϵ is rapidly changed, the thing is altered, see [18] for that of the slow contraction. During the slow expansion, the scale invariant curvature perturbation can be naturally induced by its increasing mode [19], or its constant mode [20,21].

The perturbation mode can leave the Hubble horizon during the slow expansion, if $\epsilon < 0$ [10,19], or a period after it is required to extend the perturbation mode out of the Hubble horizon [20]. Thus, in [10,19], the phantom was applied for a phenomenological study. However, there is a ghost instability. It was argued that the evolution of $\epsilon < 0$ emerges only for a period, and the phantom field might only be a simulation of a full theory without the ghost below a certain physical cutoff [22].

Recently, the cosmological application of a Galileon, [23,24], or its nontrivial generalization [25–27], has gained increased attention [28–32]. It has been found that for a generalized Galileon, $\epsilon < 0$ can be implemented stably, and there is no ghost instability. In this paper we will show a model of the slow expansion given in [19], by applying a generalized Galileon field. In this model, the perturbation theory is healthy, and the scale invariant curvature perturbation is given by itself increasing mode, which can be consistent with the observations. As will be argued, in a certain sense this validates the argument and calculations in [10,19].

The models of the early Universe, built by applying a generalized Galileon field, have been studied. In Ref. [26], the inflation model is implemented by using a generalized Galileon field. However, here we discuss an alternative to inflation. There is a slightly similar scenario in [33]. However, in [33], the adiabatic perturbation is not scale invariant; the scale invariant curvature perturbation is obtained by the conversion of the perturbations of other light scalar fields. Here, we will see how the adiabatic perturbation is naturally scale invariant.

II. AS A GENERAL RESULT

We begin with a brief review of the slowly evolving model in [19]. The quadratic action of the curvature perturbation $\mathcal{R}$ is

$$S_2 \sim \int d\eta d^3x \frac{a^2 Q}{c_s^2}(\mathcal{R}'^2 - c_s^2(\partial\mathcal{R})^2), \tag{1}$$

which is actually general for a single field, like $P(X, \varphi)$ [34], a generalized Galileon [25,26,35], and the modified gravity [36,37]. Q and c_s^2 are generally different for different models. However, $Q > 0$ and $c_s^2 > 0$ should be satisfied to avoid the ghost and gradient instabilities.

The equation of $\mathcal{R}$ is [38,39]

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right)u_k = 0, \tag{2}$$

after defining $u_k \equiv z\mathcal{R}_k$, where $'$ is the derivative for η, $z \equiv a\sqrt{2M_P^2 Q}/c_s$. Here we only conisder the case with constant c_s^2. When $k^2 \ll z''/z$, the solution of $\mathcal{R}$ given by Eq. (2) is

$$\mathcal{R} \sim C \qquad \text{is constant mode} \tag{3}$$

$$\text{or} \quad D\int \frac{d\eta}{z^2} \qquad \text{is changed mode,} \tag{4}$$

where the D mode is increasing or decaying dependent of different evolutions.

The scale invariance of $\mathcal{R}$ requires $\frac{z''}{z} \sim \frac{2}{(\eta_* - \eta)^2}$, which implies

$$z \sim \frac{a\sqrt{Q}}{c_s} \sim \frac{1}{\eta_* - \eta} \qquad \text{for constant mode} \tag{5}$$

$$\text{or} \quad (\eta_* - \eta)^2 \qquad \text{for increasing mode} \tag{6}$$

has to be satisfied, where initially $\eta \ll -1$. In a certain sense, both evolutions are dual [6]. The results will be different if c_s^2 is changed, however, which we will not be involve here. In principle, both a and Q can be changed, and together contribute to the change of z. However, only one among them is changed, while another is hardly changed could be interesting, e.g. the inflation, given by (5), or the contraction dominated by the matter, given by (6), in which a is rapidly changed, while Q is hardly changed.

However, the case can also be the inverse. When Q is rapidly changed while a is hardly changed, the scale invariant spectrum of curvature perturbation can also be induced by either by its constant mode [18–20], given by (5), or its increasing mode [19], given by (6). Though both cases give the scale invariant spectrum, both pictures are distinct. In general, for the picture in [19], initially $|\epsilon| \gg 1$, which is then rapidly decreasing, the slow evolution of the scale factor ends when $|\epsilon| \sim 1$. While for that in [18,20,21], initially $|\epsilon| \lesssim 1$, which then is rapidly increasing. In addition, for [18,20,21], during the slow evolution, the perturbation mode is actually still inside the Hubble horizon. Thus, a period after it is required to extend the perturbation mode out of the Hubble horizon, while in [19], the perturbation mode can naturally leave the Hubble horizon during the slow evolution. There is also not the problem pointed out in [40].

Here, we will discuss that in [19]. We have generally $Q = \epsilon$ for a single field action $P(X, \varphi)$ [34], while the case is slightly complex for a generalized Galileon [25,26]. However, as will be shown in the following section, we actually have $Q \sim |\epsilon|$.

Thus, $Q = |\epsilon|$ will be set for general discussion in the following. In principle, $|\epsilon|$ is dependent on a. However, it can be observed that a is nearly constant for $|\epsilon| \gg 1$. Thus, for (6), we have

$$Q = |\epsilon| \sim \Lambda_*^4 (t_* - t)^4, \tag{7}$$

since $\eta \sim t$, where Λ_* is the $1/t_*$ dimension. The Hubble parameter is given by

$$H \sim \frac{1}{\Lambda_*^4 (t_* - t)^5}. \tag{8}$$

Thus, a is given by

$$\left|\ln\left(\frac{a}{a_*}\right)\right| \sim \frac{1}{\Lambda_*^4 (t_* - t)^4} \sim \frac{1}{|\epsilon|}. \tag{9}$$

When initially $\Lambda_*(t_* - t) \gg 1$, i.e.$|\epsilon| \gg 1$, the evolution corresponds to the slow expansion for $\epsilon \ll -1$, or the slow contraction for $\epsilon \gg 1$, since $a/a_* \simeq 1$. The slow evolution ends when $\Lambda_*(t_* - t) \simeq 1$, at which $|\epsilon| \sim 1$.

When $k^2 \simeq z''/z$, the perturbation mode is leaving the horizon, and hereafter it freezes out. This horizon could be referred to as the $\mathcal{R}$ horizon

$$1/\mathcal{H}_{\text{freeze}} = \sqrt{\left|\frac{z}{z''}\right|} \simeq \eta_* - \eta. \tag{10}$$

Thus, the physical $\mathcal{R}$ horizon is $a/\mathcal{H}_{\text{freeze}} \simeq t_* - t$, while the Hubble horizon is $1/H$ given by Eq. (8). Here, the evolutions of the $\mathcal{R}$ horizon and the Hubble horizon are different. While when a is rapidly changed and $|\epsilon|$ is unchanged, e.g. inflation, both evolutions are mostly the same. The reason is that for inflation, $z''/z \sim a''/a$, thus

$$1/\mathcal{H}_{\text{freeze}} \simeq \sqrt{\left|\frac{z}{z''}\right|} \simeq \sqrt{\left|\frac{a}{a''}\right|} \sim 1/\mathcal{H}, \tag{11}$$

while here a is constant and $|\epsilon|$ is rapidly changed, we do not have $z''/z \sim a''/a$.

When $k^2 \gg z''/z$, i.e. the perturbation is deep inside the $\mathcal{R}$ horizon, u_k oscillates with a constant amplitude. The quantization of u_k is well defined for $Q \sim |\epsilon| > 0$, which gives its initial value. The evolutions of a, $1/H$, and $a/\mathcal{H}_{\text{freeze}}$ are plotted in Fig. 1 for the slow expansion. It can be found that the perturbation mode first leaves the $\mathcal{R}$ horizon, after which it is freezed out, but it is still inside the Hubble horizon. However, since the Hubble horizon is decreasing, after awhile the perturbation mode will inevitably be extended outside it, and become the primordial perturbation on the super Hubble scale.

When $k^2 \ll z''/z$, the amplitude of the perturbation spectrum is $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \sqrt{k^3}|\frac{u_k}{z}|$. Thus,

$$\mathcal{P}_{\mathcal{R}} \simeq \left|\frac{1}{a\sqrt{c_s|\epsilon|}(\eta_* - \eta)M_P}\right|^2 \simeq \frac{|\epsilon|}{c_s M_P^2} H^2, \tag{12}$$

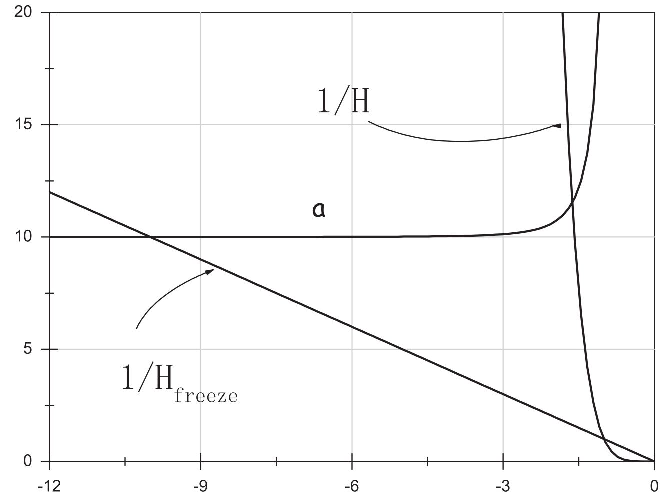

FIG. 1. The evolutions of a, the Hubble horizon, and the $\mathcal{R}$ horizon during the slow expansion given by Eq. (7). $a_* = 10$ is set. During this phase, due to the rapid change of H and $H_{\rm freeze}$, the perturbation mode is initially inside both horizons, i.e. $\lambda \sim a \ll 1/H_{\rm freeze} \ll 1/H$ will naturally leave the $\mathcal{R}$ horizon, i.e.$\lambda \sim a > 1/H_{\rm freeze}$, and then the Hubble horizon, i.e.$\lambda \sim a > 1/H$.

where $Q \sim |\epsilon|$ is applied. The perturbation is given by the increasing mode (4), because a is hardly changed and $|\epsilon|$ is decreasing. When $|\epsilon| \sim 1$, the change of a begins to become non-negligible. Though $|\epsilon|$ is still decreasing, a is increased exponentially. Thus, this mode will become the decaying mode at a certain time $t_f \sim \mathcal{O}(t_*)$ shortly after $|\epsilon| \sim 1$. In principle, the spectrum of $\mathcal{R}$ should be calculated around t_f. Thus,

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \sqrt{\frac{|\epsilon_f|}{c_s M_P^2}} H_f. \tag{13}$$

The Universe reheats around or after t_f, and hereafter the perturbation is dominated by its constant mode, until it enters into the Hubble horizon during radiation or matter domination. We assume that $|\epsilon_f| \sim 1$. Thus, $\Lambda_*^4 (t_* - t_f)^4 \sim 1$. Eq. (13) becomes

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \frac{\Lambda_*}{M_P \sqrt{c_s}}, \tag{14}$$

which is a general result of the slow evolution in [19], i.e. the evolution of $|\epsilon|$ follows Eq. (7) and c_s^2 is constant.

III. A GALILEON DESIGN OF SLOW EXPANSION

Here, we will show a detailed model of the slow expansion given in [10,19]. While the scenario of the slow contraction given in [19] is slightly alike with that in [18], which might be studied in detail elsewhere.

A. The background

We consider a generalized Galileon as

$$\mathcal{L} \sim -e^{4\varphi/\mathcal{M}} X + \frac{1}{\mathcal{M}^8} X^3 - \frac{1}{\mathcal{M}^7} X^2 \Box \varphi, \tag{15}$$

where $\mathcal{M}$ is the energy scale. Here, the sign before $e^{4\varphi/\mathcal{M}} X$ is negative. However, as will be shown this model does not have ghost and gradient instabilities, since $Q > 0$ and $c_s^2 > 0$. The evolution of background is determined by

$$\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8} X^2 + \frac{24}{\mathcal{M}^7} H \dot{\varphi} X\right) \ddot{\varphi} + 3\left(-e^{4\varphi/\mathcal{M}} + \frac{3}{\mathcal{M}^8} X^2\right) H \dot{\varphi} + \left(-\frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} + \frac{6\dot{H}\dot{\varphi}^2}{\mathcal{M}^7} + \frac{18 H^2 \dot{\varphi}^2}{\mathcal{M}^7}\right) X = 0, \tag{16}$$

$$3H^2 M_P^2 = -e^{4\varphi/\mathcal{M}} X + \frac{5}{\mathcal{M}^8} X^3 + \frac{6}{\mathcal{M}^7} X \dot{\varphi}^3 H. \tag{17}$$

We require that initially $\epsilon \ll -1$, and behaves as Eq. (7). This can be found by requiring $e^{4\varphi/\mathcal{M}} X \simeq \frac{5X^3}{\mathcal{M}^8}$ in Eq. (17). This gives

$$e^{\varphi/\mathcal{M}} = \left(\frac{5}{4}\right)^{1/4} \frac{1}{\mathcal{M}(t_* - t)}. \tag{18}$$

Thus,

$$\dot{\varphi} = \frac{\mathcal{M}}{(t_* - t)}. \tag{19}$$

Thus,

$$H \simeq \frac{\dot{\varphi}^5}{\mathcal{M}^7} \simeq \frac{1}{\mathcal{M}^2 M_P^2 (t_* - t)^5} \tag{20}$$

is induced. Thus, for $\mathcal{M} M_P \sim \Lambda_*^2$, Eq. (8) is obtained. This gives Eq. (7), which is just the required evolution.

Equations (16) and (17) are numerically solved in Figs. 2 and 3. We can see that Eqs. (19) and (20) can be highly consistent with accurate solutions for a long range of time. A significant deviation only occurs around $t_f \sim \mathcal{O}(t_*)$. We might think that the slow expanding phase ends when the significant deviation appears, and the reheating begins. However, it might be possible that the reheating of the Universe begins some time after the significant deviation occurs, since the perturbation generated during this period only are the perturbation on a small scale, which does not have to be scale invariant.

Equations (19) and (20) imply $H\dot{\varphi}\mathcal{M} \ll X$, $H\dot{\varphi}/\mathcal{M}^3 \ll e^{2\varphi/\mathcal{M}}$, and $H\dot{\varphi} \ll \ddot{\varphi}$, since

$$H \sim \frac{1}{(t_* - t)^5} \ll \frac{1}{(t_* - t)} \tag{21}$$

for $|\epsilon| \gg 1$, i.e.$\sqrt{\mathcal{M} M_P}(t_* - t) \gg 1$. Thus, Eq. (16) is approximately

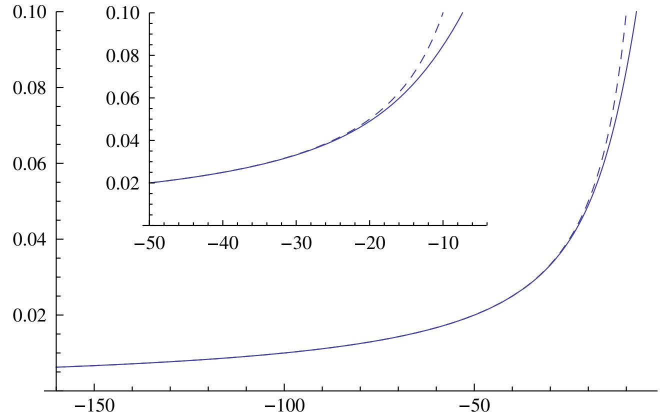

FIG. 2 (color online). The evolution of $\dot{\varphi}$ with respect to the time. The initial values of φ and $\dot{\varphi}$ are required to satisfy Eqs. (18) and (19), respectively. The parameter $\mathcal{M} = 0.01M_P$. The dashed line is that of Eq. (19). The inset is that around $t_f \sim \mathcal{O}(t_*)$.

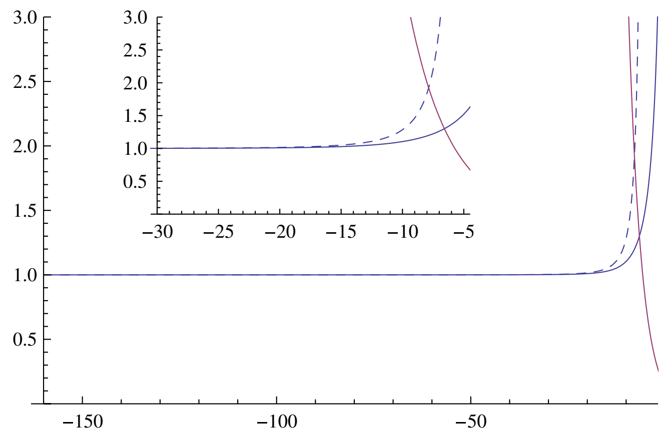

FIG. 3 (color online). The evolutions of a and H with respect to the time. The red line is that of H. The black line is that of a, while the black dashed line is that of Eq. (9). The inset is that around $t_f \sim \mathcal{O}(t_*)$.

$$\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8}X^2\right)\ddot{\varphi} - \frac{4}{\mathcal{M}}e^{4\varphi/\mathcal{M}}X \simeq 0 \tag{22}$$

for $\sqrt{\mathcal{M}M_P}(t_* - t) \gg 1$. It can be found that Eq. (22) is consistent with Eqs. (18) and (19). Thus, the equation of the perturbation $\delta\varphi$ of φ is

$$\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{4\mathcal{M}^8}\dot{\varphi}^4\right)\delta\ddot{\varphi} - \frac{4}{\mathcal{M}}e^{4\varphi/\mathcal{M}}\dot{\varphi}\delta\dot{\varphi} + \frac{15}{\mathcal{M}^8}\dot{\varphi}^3\ddot{\varphi}\delta\dot{\varphi} - \left(\frac{4}{\mathcal{M}}\ddot{\varphi} + \frac{8}{\mathcal{M}^2}\dot{\varphi}^2\right)e^{4\varphi/\mathcal{M}}\delta\varphi \simeq 0.$$

When Eqs. (18) and (19) are considered, the solution is

$$\delta\varphi \sim (t_* - t)^6, \qquad \text{is decaying mode} \tag{23}$$

$$\text{or} \quad 1/(t_* - t), \qquad \text{is increasing mode.} \tag{24}$$

The decaying mode is negligible. The increasing mode is dominated. Thus, $\delta\varphi \sim \frac{\dot{\varphi}}{\mathcal{M}}$. Thus, for $\mathcal{M}\Delta t \gg 1$, $\delta\varphi \ll \Delta\varphi$. Thus, if initially $\delta\varphi \ll \varphi$ is satisfied, it will be valid all along. When the time arrives around t_f, Eq. (21) will be not right. Thus, Eq. (22) cannot be found. This explains why there will be significant deviation for Eq. (19) around t_f.

There might be other fluids, However, their energies generally do not increase, since the expansion is slow. Thus for $|\epsilon| \gg 1$, i.e.$\sqrt{\mathcal{M}M_P}(t_* - t) \gg 1$, the evolution of background, given by Eqs. (19) and (20), is stable.

B. The curvature perturbation

$\mathcal{R}$ satisfies Eq. (2). We follow the definitions and calculations of Refs. [26,35] Here, the generalized Galileon action is (15). Thus, it is found that

$$\mathcal{F} = -e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} + \frac{8X}{\mathcal{M}^7}(\ddot{\varphi} + H\dot{\varphi}) - \frac{8X^4}{\mathcal{M}^{14}M_P^2} \simeq \frac{7}{2\mathcal{M}^4(t_* - t)^4} \tag{25}$$

$$\mathcal{G} = -e^{4\varphi/\mathcal{M}} + \frac{15X^2}{\mathcal{M}^8} + \frac{12H\dot{\varphi}^3}{\mathcal{M}^7} + \frac{12X^4}{\mathcal{M}^{14}M_P^2} \simeq \frac{5}{2\mathcal{M}^4(t_* - t)^4} \tag{26}$$

for $\mathcal{M}(t_* - t) \gg 1$. In [26], the results are applied to that of inflation, however, which are actually general for arbitrary evolution. Thus, Q is given by

$$Q = \frac{\mathcal{F}X}{M_P^2(H - \frac{2\dot{\varphi}X^2}{\mathcal{M}^7M_P^2})^2} \sim \frac{M^{14}M_P^2\mathcal{F}}{\dot{\varphi}^8} \simeq \mathcal{M}^2M_P^2(t_* - t)^4, \tag{27}$$

where Eqs. (19) and (20) are applied. Thus, $Q \sim |\epsilon| > 0$, which is just required here, satisfies Eq. (7). There is not the ghost instability. Here, the importance of $X^2\Box\varphi$ is obvious, because if it disappears in (15), $\mathcal{F}$ is given by

$$\mathcal{F} = -e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} \simeq -\frac{1}{2\mathcal{M}^4(t_* - t)^4} < 0, \tag{28}$$

$Q > 0$ will hardly be obtained, which is consistent with $Q = \epsilon < 0$ for this case. This indicates that it is $X^2\Box\varphi$ that alters the sign of Q, and leads $Q \sim |\epsilon| > 0$. The c_s^2 is given by

$$c_s^2 = \frac{\mathcal{F}}{\mathcal{G}} \sim 1.4. \tag{29}$$

Thus, $c_s^2 > 0$ is constant, which is also just required. The sign of c_s^2 is determined by the signs of $\mathcal{F}$ and $\mathcal{G}$, which are both are positive. Here, obviously $\mathcal{F} > 0$ is also required to assure $c_s^2 > 0$. Thus, there are no ghost and gradient instabilities, and the effective theory is healthy.

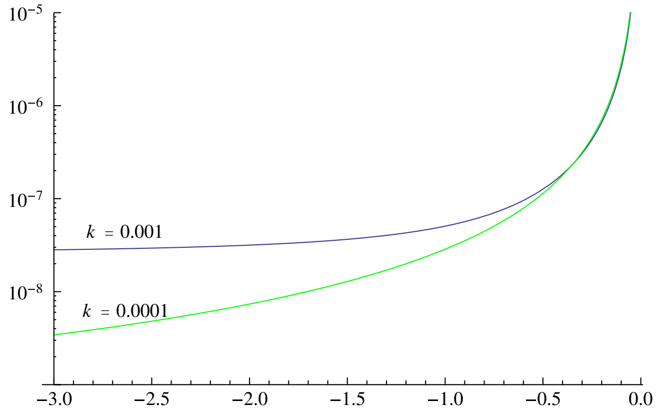

FIG. 4 (color online). The evolutions of the amplitude of curvature perturbation for different k with respect to the time. The green and black lines are that with different k. Here, the time axis is rescaled as $\mathcal{M}t$ for the convenience of numerical calculation, and t is that in Figs. 2 and 3, $\mathcal{M} = 0.01$.

We plot the evolution of the amplitude of the curvature perturbation in Fig. 4, and the spectrum of perturbation in Fig. 5. We can see that the perturbation initially does not increase, since it is inside the $\mathcal{R}$ horizon. The increase begins after the perturbation mode leaves the $\mathcal{R}$ horizon. The longer the wavelength of the perturbation, the earlier the perturbation leaves the $\mathcal{R}$ horizon, and the earlier it begins to increase. However, since the shorter the wavelength of perturbation is, the larger its initial amplitude is, all perturbation modes will eventually have the same amplitude.

There is a cutoff $k_{\rm cutoff}$ in Fig. 5, which is given by

$$k_{\rm cutoff} \sim \mathcal{H}_{\rm inifr}, \tag{30}$$

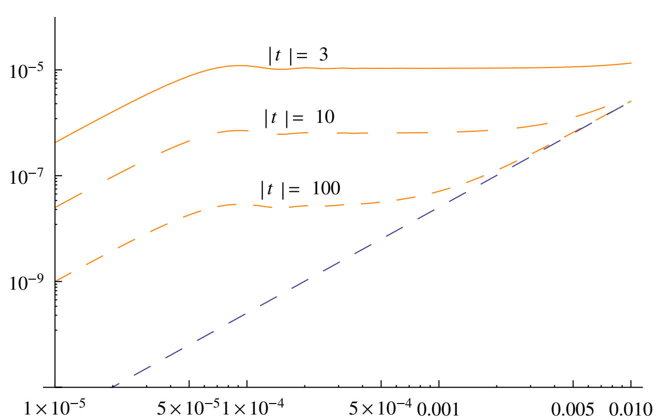

FIG. 5 (color online). The spectrum of curvature perturbation at different times with respect to k. The black dashed line is the initial spectrum. The short dashed, long dashed, and solid orange lines are the spectra at different times, respectively. There is a cutoff $k_{\rm cutoff} \sim 5 \times 10^{-5}$, below which the spectrum is not scale invariant, which is explained in the text.

where $\mathcal{H}_{\rm inifr}$ is $\mathcal{H}_{\rm freeze}$ at the initial time, and can be changed with the difference of the initial parameters in the numerical calculation. The spectrum is scale invariant for $k > k_{\rm cutoff}$. However, for $k < k_{\rm cutoff}$, since the corresponding perturbation modes are outside the $\mathcal{R}$ horizon all along, only their amplitudes are increasing but the shape of the spectrum is not altered [41,42].

The spectrum of $\mathcal{R}$ is scale invariant. The amplitude of spectrum is given by Eq. (14)

$$\mathcal{P}_{\mathcal{R}}^{1/2} \sim \sqrt{\frac{\mathcal{M}}{c_s M_P}}, \tag{31}$$

where $\Lambda_* \sim \sqrt{\mathcal{M} M_P}$ is applied. $\mathcal{P}_{\mathcal{R}}^{1/2} \sim 10^{-5}$ requires $\mathcal{M} \sim 10^{-10} c_s M_P$. Thus, $\mathcal{M} \sim 10^9$ GeV for $c_s \simeq 1$. The only adjusted parameter in this model is fixed by the observation. There is no other fine-tuning.

C. The reheating

When the slowly expanding phase ends, the energy of a Galileon field is required to be released into the radiation, and the Universe reheats. Hereafter, the evolution of the hot "big bang" cosmology begins. We can notice that before this, the perturbation mode has left the Hubble horizon.

Here, in a certain sense, the reheating is like that of inflation. The preheating theory after inflation has been developed in [43,44]. In general, during the preheating phase after inflation the energy of inflaton will be rapidly released by the parametric resonance effects, due to the coupling of inflaton with other fields. This issue has been extensively studied, see [45–47] for reviews.

We will apply the instant preheating mechanism [48] for the case given here. We consider the straight coupling of φ with the χ particle as

$$\mathcal{L} \sim g^2(\varphi - \varphi_{\rm reh})^2\chi^2, \tag{32}$$

where g is the coupling constant. The effective mass of the χ particle is $M_{\chi{\rm eff}}^2 \sim g^2(\varphi - \varphi_{\rm reh})^2$. When the φ field arrives at the region around $\varphi_{\rm reh}$, $M_{\chi{\rm eff}}^2 \lesssim \dot{M}_{\chi{\rm eff}}$, the adiabatic condition is broken, and the production of χ particles will be inevitable. This generally occurs in a region around $\varphi_{\rm reh}$, $\Delta\varphi \lesssim \dot{\varphi}_{\rm reh}/g$, in which $\dot{\varphi}_{\rm reh}$ is the velocity of φ through $\varphi_{\rm reh}$. Thus, the production of χ particles is instantaneous, $\Delta t_{\rm reh} \sim 1/\sqrt{g\dot{\varphi}_{\rm reh}}$.

The number density n_χ of the χ particle is

$$n_\chi = \frac{1}{2\pi^2}\int n_k k^2 dk \simeq \frac{g^{3/2}\dot{\varphi}_{\rm reh}^{3/2}}{8\pi^3}, \tag{33}$$

where n_k is the occupation number of the χ particle. Thus, $\rho_\chi = n_\chi M_\chi \sim g^2\dot{\varphi}_{\rm reh}^2$, since $M_{\chi{\rm eff}} \sim g(\varphi - \varphi_{\rm reh}) \sim g\dot{\varphi}_{\rm reh}\Delta t_{\rm reh}$. The energy drained by the production of χ particle is

$$\frac{\rho_\chi}{\rho_{\varphi \mathrm{reh}}} \sim \frac{g^2}{8\pi^3} \mathcal{M}^6 M_P^2 (t_* - t_{\mathrm{reh}})^8, \tag{34}$$

where Eqs. (19) and (20) are applied, and $\rho_{\varphi \mathrm{reh}}$ is the energy density of φ around t_{reh}. We assume $t_f \sim t_{\mathrm{reh}}$ for simplicity, i.e. the reheating occurs at the time when the slow expansion ends. Thus, $\mathcal{M}^2 M_P^2 (t_* - t_{\mathrm{reh}})^4 \sim 1$. This implies

$$\frac{\rho_\chi}{\rho_{\varphi \mathrm{reh}}} \sim \frac{g^2 \mathcal{M}^2}{8\pi^3 M_P^2}. \tag{35}$$

We generally require $\mathcal{M} \ll 1$ and $g < 1$. Thus, $\rho_\chi/\rho_{\varphi \mathrm{reh}} \ll 1$, which indicates that for such a single preheating, the energy of φ can hardly be released completely, and the Universe is still dominated by ρ_φ, which will continue all along, since the energy density of φ is increasing with the expansion of the Universe, while that of the χ particle is decreasing.

However, there might be $\mathcal{N}$ couplings, one of which is like that in (32). We can find, after doing similar calculations, that when

$$\mathcal{N} > \frac{M_P^2}{g^2 \mathcal{M}^2}, \tag{36}$$

the release of the energy of φ will be complete. The sketch of this reheating course is plotted in the upper panel in Fig. 6. We assume that the χ particle produced is rapidly transferred into the radiation. In this case, the reheating temperature T_r is approximately determined by $\rho_{\varphi \mathrm{reh}} \sim T_r^4$. Thus, we have

$$T_r \sim \left(\frac{\dot{\varphi}^{10}}{\mathcal{M}^{14} M_P^2}\right)^{1/4} \sim \mathcal{M}^{1/4} M_P^{3/4}, \tag{37}$$

where $\mathcal{M}^2 M_P^2 (t_* - t_{\mathrm{reh}})^4 \sim 1$ is applied again. Thus, if $\mathcal{M} \sim 10^{-10} M_P$, we have $T_r \sim 10^{15}$ GeV.

Here, $\mathcal{N} \gg 1$ is feasible; however, it might be uncomfortable. $\mathcal{N} \gg 1$ is required because the energy of φ has

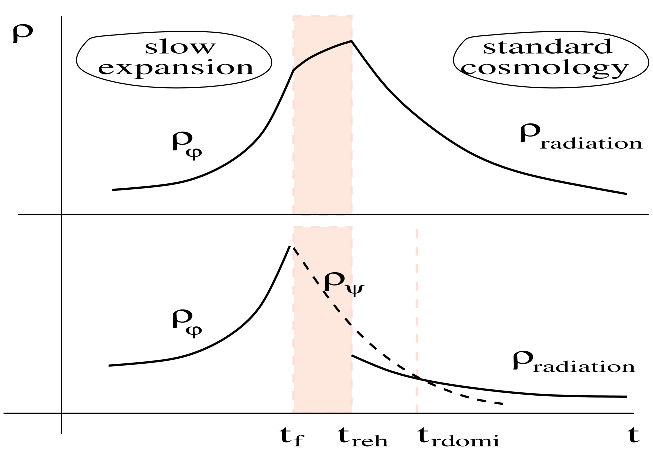

FIG. 6 (color online). The sketch of the evolution of the energy density ρ for different reheating courses discussed here.

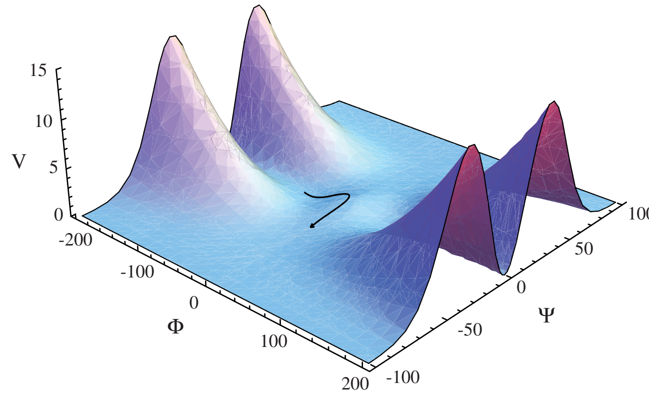

FIG. 7 (color online). The figure of the effective potential for the exiting from the slow expansion. The black solid line is the motive trajectory of the field in (φ, ψ) space.

to be released completely one time, or since the energy density of φ is increasing, the Universe will dominated by φ all along. However, we also could consider another channel of the reheating, like that in phantom inflation [59]. The energy of φ is first shifted to the kinetic energy of a normal field, e.g.ψ, and then the energy of ψ is released by the instant preheating. The sketch of this reheating course is plotted in the lower panel in Fig. 6. Here, the energy of ψ is not required to be released completely, since $\rho_\psi \sim 1/a^6$ is decreasing faster than that of the radiation, the Universe will be dominated by that of early or late radiation.

We can implement it by considering the potential of φ, illustrated in Fig. 7. We require that it is only significant around or after $|\epsilon| \sim 1$, and is negligible $|\epsilon| \gg 1$. Then we introduce a waterfall field ψ, coupled to φ. The effective mass of ψ is initially positive and becomes negative around $|\epsilon| \sim 1$. Thus, ψ will roll down along its potential. Almost all the energy of φ will be shifted to $\rho_\psi \sim \dot{\psi}^2$. This energy will be expected to be released by the instant reheating. Thus, there could be a suitable reheating after the slow expansion ends, after which the evolution of hot "big bang" cosmology begins.

IV. DISCUSSION

When initially $\epsilon \ll -1$ and is rapidly increasing, the Universe is slowly expanding. The spectrum of curvature perturbation generated during such a phase of slow expansion can be scale invariant. This provides a mechanism by which an alternative scenario of the early Universe can be imagined. Here, we show a model of such a scenario by applying an effective action of a generalized Galileon.

In principle, $\epsilon < 0$ implies ghost instability. However, in this model, because of the introduction of a Galileon field, there is no ghost instability, and the perturbation theory is healthy. In Refs. [10,19], a phantom was applied for implementing slow expansion. In the calculations of perturbation, for consistency, $|\epsilon|$ is used, though the initial value of perturbation is still pathologically defined. However, in the model given here, it can be found that actually $Q \simeq |\epsilon|$. This in a certain sense validates the argument and calculations used in [10,19], i.e. the phantom field might be a simple simulation of a full theory without the ghost below a certain physical cutoff, which can give the same results as that of a full theory, when the replacement of ϵ with $|\epsilon|$ is done.

When $\epsilon \sim -1$, the slow expansion ends. The exiting to a hot Universe is only a simple reheating, since the Universe expands all along. Thus, there is no problem with how the bouncing is implemented in bouncing cosmologies [14,49,50]. We have discussed possible implementations of reheating, and found that the available energy of a Galileon field can completely released, and the Universe can reheat to a suitable temperature. Thus, the model of the slow expansion given here might be a viable design of the early Universe.

The material compares of model with the observations is certainly interesting, which will place rigid constraints on the model. The results obtained will be expected to either improve or rule out this model. We will investigate it elsewhere. However, it should be pointed that we only bring one of all possible implementations of the slow expansion. In principle, there might be other effective actions of a generalized Galileon, or modified gravity, which could give the same evolution of background. Thus, for the slow expansion, it might also be significant to find alternative implementations to the model given here, which will help to improve the flexibility of the slow expansion to the observations.

Here, the scale factor is asymptotic to a constant value in the infinite past, and there is no singularity point. Thus, in a certain sense, the slow expansion scenario offers a solution to the cosmological singularity problem. However, it can also be imagined that after the available energy of the field is released, it might be placed again in the bottom of its effective potential, and after the Universe undergoes the radiation and matter periods, the field might dominate again and roll again with increasing energy. This models an eternally expanding cyclic universe [51–53], i.e. H oscillates periodically, while a expands all along. The implementation of this cyclic universe might be interesting for refining the model given here.

Here, the constant c_s is set. However, its change will obviously enlarge the space of solutions of the scale invariance of curvature perturbation [54–58]. In a certain sense all possibilities of the changes of a, Q, and c_s^2 might be interesting to explore further.

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