

Shadows of colliding black holesDaisuke Nitta,¹ Takeshi Chiba,² and Naoshi Sugiyama^{1,3}¹*Department of Physics, Nagoya University, Chikusa, Nagoya 464-8602, Japan*²*Department of Physics, College of Humanities and Sciences, Nihon University, Tokyo 156-8550, Japan*³*Institute for Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8582, Japan*

(Received 21 June 2011; published 14 September 2011)

We compute the shadows of colliding black holes using the Kastor-Traschen cosmological multiblack hole solution that is an exact solution describing the collision of maximally charged black holes with a positive cosmological constant. We find that in addition to the shadow of each black hole, an eyebrowlike structure appears as the black holes come close to each other. These features can be used as probes to find the multiblack hole system at the final stage of its merger process.

DOI: 10.1103/PhysRevD.84.063008

PACS numbers: 97.60.Lf

I. INTRODUCTION

The observational evidence for the existence of black holes is mounting (see [1] for recent reviews). There are many stellar mass black holes found in the Galaxy, while one of them, Sgr A*, turns out to be a super massive black hole (SMBH) with 4.3×10^6 solar mass [2,3]. It turns out most galaxies and active galactic nuclei have at least one SMBH whose mass shows strong correlation with the mass of the spheroid component of the galaxy [4–6]. The hierarchical clustering scenario suggests such a spheroid component is formed due to a merger of smaller galaxies. Therefore it may be natural to consider the formation of a SMBH is also taking place by the merger process of smaller black holes.

Recently, the evidence for the existence of a binary black holes is provided by observing the orbital motions of stars in a galaxy by radio interferometers [7]. Moreover, from the detection of a signal periodicity in light curves, it is claimed that the binary black holes will coalesce within 500 yr [8]. However, the direct evidence of black holes is still lacking. We need unambiguous proof that this object is indeed a binary black hole. Since a black hole is defined as an object with the event horizon, we should search for phenomena associated with the existence of the event horizon.

To a distant observer, the event horizons cast shadows due to the bending of light by the black holes [9]. Observing these shadows should be compelling evidence of a coalescing black holes.

As a first step toward the study of a realistic black hole binary, we calculate the shadow of the Kastor-Traschen cosmological multiblack hole solutions [10]. The Kastor-Traschen solution is an exact solution describing the merger of maximally charged black holes with a positive cosmological constant. Although admittedly the solution is unrealistic, it is an exact and analytic solution and hence allows us to study numerically photon orbits accurately. We expect some of the features of the shadows would persist for a more realistic black hole binary

since, at least for a single black hole, the charge of black holes has little effect on the apparent shape of the shadow [11].

II. KASTOR-TRASCHEN SOLUTION

In this section, we will briefly review the Kastor-Traschen (KT) solution. The KT solution is a dynamic multiblack hole solution parameterized by n masses m_i and the positive cosmological constant Λ (see also [12]). Each black hole has a charge Q_i equals to its mass m_i (we use the geometrical units $G = c = 1$). In case of a single black hole, this solution corresponds to the Reissner-Nordstrom-de Sitter solution with charge equal to its mass. It can be reduced to the Majumdar-Papapetrou solution when $\Lambda = 0$ [13].

The metric in the cosmological coordinate is given by

$$ds^2 = -a^2 \Omega^{-2} d\tau^2 + a^2 \Omega^2 (dx^2 + dy^2 + dz^2),$$

$$a = e^{Ht} = -\frac{1}{H\tau}, \quad H = \pm \sqrt{\frac{\Lambda}{3}} \quad \Omega = 1 + \sum_i \frac{m_i}{ar_i},$$

$$r_i \equiv \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}, \quad (1)$$

where, τ and t denote conformal time and physical time, respectively. Here, $H > 0$ ($H < 0$) corresponds to expansion (contraction).

In the Majumdar-Papapetrou solution, the black holes can stay at the rest frame as if their gravity balance with electrostatic repulsions. Similarly, in the KT solution, the gravity of the black holes balance with their electrostatic repulsions, while the black holes comove with cosmic expansion.

The solution has some interesting features. Let us consider two extreme situations. First, we imagine the case that black holes are far enough from each other. If distances of black holes are at least larger than $1/|H|$, then each black hole can be treated as a single black hole. Second, we consider the case that all black holes are close enough to each other. If all black holes are located within the

black hole horizon of total mass $\sum_i m_i$, this system can be considered as a single black hole [14]. Accordingly, one can see that the KT solution describes the black hole collision because in the contracting coordinate it starts from a group of single black holes and ends up with a single black hole.

III. SHADOW OF A SINGLE BLACK HOLE

We begin by computing the shadow of a single black hole to understand some asymptotic behaviors. The KT solution in case of a single black hole is equivalent to the extreme Reissner-Nordstrom-de Sitter solution. This solution can be rewritten in the static coordinate. The metric is then given by

$$ds^2 = -VdT^2 + V^{-1}dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$V(R) = \left(1 - \frac{M}{R}\right)^2 - H^2R^2. \quad (2)$$

Taking $V = 0$, we obtain event horizons which are given by

$$R_{\pm} = \frac{1}{2|H|} (1 \pm \sqrt{1 - 4M|H|}), \quad (3)$$

where R_+ and R_- denote cosmological and black hole horizons, respectively. The cosmological horizon has a similar feature of the horizon in de Sitter space-time since it becomes a future (past) horizon if the Universe is contracting (expanding). On the other hand, the black hole horizon is a usual black hole horizon in the RDdS solution.

We define a momentum of a photon using the affine parameter λ as $P^\mu \equiv dx^\mu/d\lambda$. Here, P_T and P_ϕ are constants corresponding to the time shift and the rotational symmetry. Using the null condition, we obtain a geodesic equation at $\theta = \pi/2$

$$\left(\frac{dR}{d\lambda}\right)^2 + VR^{-2}b^2 = 1, \quad (4)$$

where, $b \equiv P_\phi/P_T$ denotes the impact parameter. The ‘‘effective potential’’ b^2V/R^2 has a local maximum at $R = 2M$. A sphere with this radius of $R = 2M$ is known as the ‘‘photon sphere’’ inside of which the photon orbits become unstable. Note that the orbits of these photons are unstable. One can hence find that the critical value of the impact parameter b_c is given by

$$b_c = \frac{4M}{\sqrt{1 - 16M^2H^2}}. \quad (5)$$

If $b \leq b_c$, the photons are captured by the black hole.

Next, we transform to the cosmological coordinate for computing a shadow of the black hole seen from an observer. The transformation between the static and cosmological coordinates is given by [10] as $ar = R - M$, $t = T + h(R)$, and $dh(R)/dR = -HR^2/[(R - M)V(R)]$. So, from Eq. (3), the event horizons in the cosmological coordinate are given by

$$ar_{\pm} = \frac{1}{2|H|} (1 \pm \sqrt{1 - 4M|H|}) - M, \quad (6)$$

where r_+ and r_- correspond to the cosmological and the black hole horizons, respectively.

The shape of the shadow is different according to whether the observer is in the expanding or contracting coordinate. In the expanding coordinate, the observer who is in the asymptotic de Sitter space-time can see the shadow, since the photons can travel from inside the cosmological horizon to infinite distance. While in the contracting coordinate, the observer can never see the shadow in the asymptotically de Sitter space-time but one can see only inside the cosmological horizon.

Since the colliding black holes must be considered in the contracting coordinate, let us consider the situation where an observer is near inside the cosmological horizon ($r_{\text{obs}} \rightarrow r_+$) in the contracting coordinate. We define the following parameters, which form the celestial coordinate system, as

$$\alpha \equiv -\frac{ar_{\text{obs}}P^{(\phi)}}{P^{(\tau)}}, \quad \beta \equiv \frac{ar_{\text{obs}}P^{(\theta)}}{P^{(\tau)}}, \quad (7)$$

where $P^{(\mu)}$ are the momenta in the local inertial frame and ar_{obs} is the physical distance between the observer and the center of the coordinate. Because the shadow’s shape of a single nonrotating black hole is a circle in the α - β plane due to the rotational symmetry, we only compute when $\beta = 0$. Using the transformation from the static to the cosmological coordinate and the critical value of the impact parameter (5), we obtain the critical value of α as

$$\alpha_c = \frac{4M\epsilon}{\sqrt{1 + 4M|H|}}, \quad \epsilon \equiv a|H|(r_+ - r_{\text{obs}}). \quad (8)$$

Note that $\epsilon \ll 1$ since we locate an observer near the cosmological horizon.

Therefore the shape of the shadow is a circle with this radius of $4M\epsilon/\sqrt{1 + 4M|H|}$ in α - β space. One can see that this radius becomes smaller when the observer approaches the cosmological horizon, $r_{\text{obs}} \rightarrow r_+$, due to the geometry on this space-time.

It is necessary to take into account a black hole that is not in the center of the coordinate as a more general case. Then the black hole moves toward the center of the coordinate in the contracting coordinate. We find that its shape is the same as a centered black hole.

IV. SHADOWS OF COLLIDING BLACK HOLES

Let us consider a two black hole system as an example of colliding black holes. It is convenient to adopt the cylindrical coordinate (r, z, ϕ) , because the system has the axial symmetry in this case. Then the locations of the black holes are given by $(x_i, y_i, z_i) = (0, 0, d_i)$, where $i = 1, 2$ and we set $d_1 = -d_2$. We set an observer at a fixed point inside a cosmological horizon in the physical coordinate.

For simplicity, we take $\phi_{\text{obs}} = 0$ and $\theta_{\text{obs}} = \pi/2$ in terms of the polar coordinate.

Now let us consider ray tracing. We have to shoot photons from all the directions to the observer in the contracting coordinate to see the shadows of colliding black holes. Instead, technically it is easier to consider the time reversal of this system. Namely, we shoot photons from the observer to all directions in the expanding coordinate.

Using the parameters α and β that have been defined by Eqs. (7), the initial momenta for photons at the observer are given by

$$P^r = -\frac{P_\tau}{a^2} \sqrt{1 - (\alpha^2 + \beta^2)/(ar_{\text{obs}})^2}, \quad P^z = \frac{P_\tau \beta}{a^3 r_{\text{obs}}},$$

$$P^\phi = \frac{P_\tau \alpha}{a^3 r_{\text{obs}}^2}, \quad \text{for } \theta_{\text{obs}} = \frac{\pi}{2}. \quad (9)$$

The above equations show that the shadows must lie inside the circle in the celestial coordinates α and β with a radius of ar_{obs} ($\leq ar_+$) due to the condition $1 - (\alpha^2 + \beta^2)/(ar_{\text{obs}})^2 \geq 0$. We can easily extend the above initial conditions for arbitrary observers at $\theta_{\text{obs}} \neq \pi/2$ by rotating the two-dimensional vector (P^r, P^z) in the z - y plane.

We then numerically calculate the photon's geodesic equations from the observer in the expanding coordinate. The photons that eventually fall into the black hole horizons are regarded as shadows.

Figure 1 shows that the shadows of two black holes with same masses $m_1 = m_2$ at each physical time t seen by observers at $z_{\text{obs}} = 0$ ($\theta_{\text{obs}} = \pi/2$) and $\phi_{\text{obs}} = 0$ with $\epsilon = 0.01$. We take $M = m_1 + m_2 = 0.1/|H|$. The initial positions of two black holes are $d_1 = -d_2 = 4.5 \times 10^{-8}/|H|$.

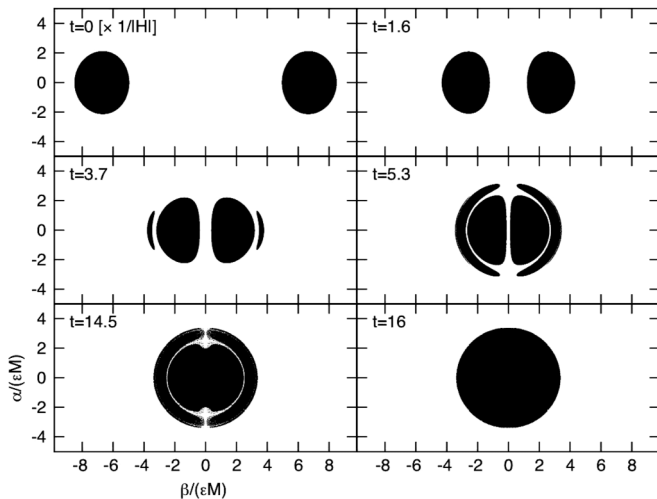


FIG. 1. Black hole shadows for the two black hole system plotted in α - β space normalized by ϵM with each physical time $t/|H|^{-1} = 0, 1.6, 3.7, 5.3, 14.5, 16$. Here we have used the following parameters: $\theta = \pi/2$, $m_1 = m_2 = M/2$, $H = -0.1/M$, and $\epsilon = 0.01$.

Here, the celestial coordinates α and β are normalized by ϵM in order to keep the shape of the shadows independent of a location of the observer.

At $t = 0$ and $t = 1.6/|H|$, the black holes are still far away from each other. However, one can find that their shapes are a little bit elongated in the α direction and squeezed in the β direction from the circles with a radius of $4m_i\epsilon/\sqrt{1 + 4m_i|H|} \sim 1.82\epsilon M$ when they are considered as single black holes in α - β space. This deformation is caused by the existence of the other black hole in the opposite side.

At $t = 3.7/|H|$, an eyebrowlike structure around each black hole appears. This kind of structure is quite unique to the multiblack hole system. The reason why these structures appear is the following. Let us consider the winding orbit of a photon around the black hole [15]. These orbits form the photon sphere as we have mentioned in Sec. III. If the impact parameter of the photon is slightly smaller than the radius of the photon sphere, this photon will eventually fall into a black hole horizon. On the other hand, for a slightly larger impact parameter, the winding photon will gradually increase the distance to the black hole and eventually go away from the black hole, or fall into the horizon of the other black hole. The latter case creates the eyebrowlike shadow along the main shadow. The situation is similar to the particle motion in the Malumdar-Papapetrou solution [16]

At $t = 5.3/|H|$, the eyebrowlike structures grow and the main shadows come close to each other. One can find there still remains a region that photons can go through between the main shadows. The reason why such a region remains is the following. In a single black hole system, a black hole horizon is enclosed with the photon sphere. On the other hand, in a two black hole system, two photon spheres intersect at the x - z plane where the photons cannot fall into either one of black holes. Accordingly photons can go through around this plane, which corresponds to $\beta = 0$ in the celestial coordinate until two black holes merge and form a horizon. Actually, this interaction between two photon spheres cause the deformation of black hole shadows at $t = 0$, $t = 1.6/|H|$, and $t = 3.7/|H|$.

At $t = 14.5/|H|$, two main shadows have merged and there no longer exists a region at $\beta = 0$ where photons can go through. This implies that the merger process of two black hole horizons took place before the photons reach the center of the coordinate. Eventually a shadow of a single black hole appears at $t = 16/|H|$. The shape of the shadow is a circle with a radius of $3.38\epsilon M$ in the celestial coordinate, which corresponds to the photon sphere of a single black hole with M as described by Eq. (8). The duration of these merger processes may be estimated as $\approx 15/|H| \approx 20\text{hr}(M/10^8 M_\odot)$.

Finally, let us mention a situation when one observes from arbitrary directions. We have calculated shadows for several different values of angle θ_{obs} at $t = 3.7$. As we

decrease θ_{obs} from $\pi/2$, the left main shadow of Fig. 1 becomes elongated, and eventually merges with the eyebrowlike structure of the right side and forms a ring structure surrounding the right main shadow [17].

V. SUMMARY AND CONCLUSIONS

In this paper, we have calculated photon paths during the collision two black holes and drawn their shadows seen by a distant observer. In a realistic case, the merger process is dynamical and has to be solved by utilizing numerical relativity. Although this must be an ultimate goal, instead, we employ the KT solution, which is the exact solution of the multiblack hole system in the contracting or expanding coordinate, as a first step. While we admit the KT solution is far from the reality [the charge is $Q = 10^{46} \text{esu}(M/10^8 M_{\odot})$], this exact solution enables us to handle evolution of black hole horizons. Moreover, it is rather easy to calculate the photon paths in this space-time accurately.

We expect that the following two features of black hole shadows obtained here are general and appear in the

realistic situation. The first is the eyebrowlike structure that shows up during the merger process. The second is the region on the plane perpendicular to the merger direction that photons can go through until the last epoch of the merger. These features in the shadows can be used as probes to find the multiblack hole system at the final stage of its merger process.

ACKNOWLEDGMENTS

D.N. would like to thank Shuichiro Yokoyama for useful discussions. This work was supported in part by a Grant-in-Aid for Scientific Research from JSPS [No. 20540280 (T.C.) and No. 22340056(N.S.)] and in part by Nihon University (T.C.). This work was also supported in part by the Grant-in-Aid for Scientific Research on Priority Areas No. 467 ‘‘Probing the Dark Energy through an Extremely Wide and Deep Survey with Subaru Telescope.’’ This research has also been supported in part by World Premier International Research Center Initiative, MEXT, Japan.

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