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Improving relativistic modified Newtonian dynamics with Galileon *k*-mouflage

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We propose a simple field theory reproducing MOND phenomenology at galaxy scale, while predicting negligible deviations from general relativity at small scales thanks to an extended Vainshtein ("*k*-mouflage") mechanism induced by a covariant Galileon-type Lagrangian. The model passes all solar-system tests, including those of local Lorentz invariance, and its anomalous forces in binary pulsars are vanishingly small. The large-distance behavior is obtained as in Bekenstein's tensor-vector-scalar model, but with several simplifications. In particular, no fine-tuned function is needed to interpolate between the MOND and Newtonian regimes, and the vector field may be nondynamical. The field equations depend on second (and lower) derivatives, and thus avoid the generic Ostrogradski instabilities. We also underline why the proposed model is particularly efficient within the class of covariant Galileons.

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Although the hypothesis of dark matter is consistent with a wide range of observations, it might be an artifact of our Newtonian interpretation of experimental data if the gravitational $1/r^2$ law happens to not be valid at large distances. In 1983, Milgrom proposed a simple phenomenological rule, called "modified Newtonian dynamics" (MOND) [1], depending on an acceleration scale a_0: In the vicinity of a (baryonic) mass M, any test particle undergoes the standard Newtonian acceleration $a_N = GM/r^2$ if $a_N > a_0$, but a slower decreasing one $a = \sqrt{a_0 a_N} = \sqrt{GMa_0}/r$ if $a < a_0$. This law happens to fit galaxy rotation curves remarkably well, for a universal constant [2]

$$a_0 \approx 1.2 \times 10^{-10}\ \mathrm{m.s}^{-2}. \tag{1}$$

Moreover, it automatically recovers the Tully-Fisher law [3] and explains the correlation of dark matter profiles with the baryonic ones [4].

However, reproducing this phenomenological law in a consistent relativistic field theory happens to be quite difficult, as illustrated by almost three decades of abundant literature. In the present paper, we shall refine one of the best models proposed so far, in our opinion, namely, the tensor-vector-scalar (TeVeS) theory constructed by Bekenstein and Sanders [5–10]. We will show that a generalized Galileon action allows us to suppress deviations from general relativity at small distances, thanks to an extension of the Vainshtein mechanism occurring in massive gravity [11,12]. In the model we propose, the magnitude itself of the anomalous force tends towards zero at small distances, and not only its ratio to the gravitational one (as in the standard Vainshtein mechanism).

We consider a scalar-tensor theory of gravity defined by the action

$$S = \frac{c^3}{4\pi G}\int d^4x\sqrt{-g}\left(\frac{R}{4} + \mathcal{L}_{\mathrm{standard}} + \mathcal{L}_{\mathrm{MOND}} + \mathcal{L}_{\mathrm{Galileon}}\right) + S_{\mathrm{matter}}[\psi_{\mathrm{matter}}; \tilde{g}_{\mu\nu}], \tag{2}$$

where we use the sign convention of [13] and notably the mostly plus signature. The physical metric $\tilde{g}_{\mu\nu}$ will be defined in Eq. (7) below. The scalar field φ is chosen to be dimensionless, and its kinetic term is the sum of the following three contributions, where $s \equiv g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$:

$$\mathcal{L}_{\mathrm{standard}} = -\frac{\epsilon}{2}s = -\frac{\epsilon}{2}(\partial_\lambda\varphi)^2, \tag{3}$$

$$\mathcal{L}_{\mathrm{MOND}} = -\frac{c^2}{3a_0}s\sqrt{|s|}, \tag{4}$$

$$\mathcal{L}_{\mathrm{Galileon}} = -\frac{k}{3}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}R_{\gamma\delta\rho\sigma}. \tag{5}$$

Here $\varepsilon^{\alpha\beta\gamma\delta}$ denotes the Levi-Civita tensor, related to the fully antisymmetrical symbol $[\alpha\beta\gamma\delta]$ (whose values are 0 or ± 1) by $\varepsilon^{\alpha\beta\gamma\delta} \equiv (-g)^{-1/2}[\alpha\beta\gamma\delta]$. A small mass term $-\frac{1}{2}m^2\varphi^2$, with $1/m$ greater than the largest cluster sizes, might also be added to the above kinetic term.

Denoting as $r_{\mathrm{MOND}} \equiv \sqrt{GM/a_0}$ the scale where MOND effects start to manifest around a galaxy of baryonic mass M, and as $r_{\mathrm{V}} \equiv (8kGMa_0)^{1/4}/c < r_{\mathrm{MOND}}$ the Vainshtein radius below which scalar effects are suppressed, we will see below that $\mathcal{L}_{\mathrm{standard}}$ dominates at very large distances $r > r_{\mathrm{MOND}}/\epsilon$, $\mathcal{L}_{\mathrm{MOND}}$ at intermediate ones $r_{\mathrm{V}} < r < r_{\mathrm{MOND}}/\epsilon$, and $\mathcal{L}_{\mathrm{Galileon}}$ at small scales $r < r_{\mathrm{V}}$. As stated above, numerical fits of galaxy rotation curves [2] give the value (1) for a_0, while their flatness up to about $10 r_{\mathrm{MOND}}$ [14] imposes that the positive dimensionless constant ϵ is smaller than 0.1. The constant k entering $\mathcal{L}_{\rm Galileon}$ has dimension of $[{\rm length}]^4$, and we will show below that a numerical value

$$k \approx \left(4 \times 10^{-6} \frac{c^2}{a_0}\right)^4 \approx (100 \text{ kpc})^4 \qquad (6)$$

allows the model to pass solar-system tests while predicting MOND effects even for the lightest known dwarf galaxies.

Several ingredients of action (2) are borrowed from the MOND literature. In particular, the scalar kinetic term $\mathcal{L}_{\rm MOND}$ is well known to generate an extra acceleration $\sqrt{GMa_0}/r$ on any test mass at distance r from a source of baryonic mass M [15]. Note that we did not write it simply in terms of $|s|^{3/2}$, but that an absolute value is involved only within the square root, to ensure that the scalar field carries positive energy whatever the sign of s [16]. The standard kinetic term $\mathcal{L}_{\rm standard}$ has a negligible influence because it is multiplied by the small positive constant ϵ, but it ensures that the dynamics of the scalar field is well defined when s passes through a vanishing value [16,17].

As in Refs. [7–10], the matter action $S_{\rm matter}$ assumes that all matter fields ψ are minimally coupled to a physical metric

$$\tilde{g}_{\mu\nu} \equiv e^{-2\varphi} g_{\mu\nu} - 2\sinh(2\varphi) U_\mu U_\nu, \qquad (7)$$

where U_μ is a timelike unit vector field, i.e., $g^{\mu\nu}U_\mu U_\nu = -1$. Light deflection by galaxies and clusters would indeed be inconsistent with experiment if matter was merely coupled to the scalar field via a conformal metric $\tilde{g}_{\mu\nu} = e^{2\varphi} g_{\mu\nu}$ [5,6,16]. In a locally inertial frame where $g_{\mu\nu} = {\rm diag}(-1, 1, 1, 1)$, and choosing the observer's velocity such that $U_\mu = (1, 0, 0, 0)$ lies along his proper time direction, Eq. (7) merely means that $\tilde{g}_{00} = e^{2\varphi} g_{00}$ but $\tilde{g}_{ij} = e^{-2\varphi} g_{ij}$ (thus mimicking the behavior of pure general relativity in the presence of dark matter). Previous field theories attempting to reproduce the MOND dynamics predicted large preferred-frame effects in the solar system, inconsistent with experiment, if U_μ was assumed to be a constant vector field [7,16]. This is why it was assumed to be dynamical in the TeVeS model [8,9], with the idea that it could align with the matter's local proper time direction. However, making the vector field dynamical by adding a kinetic term $-F^2_{\mu\nu}$, or any function of it, causes this vector to be unstable [16,18,19]. In the present paper, we will see that preferred-frame effects remain negligible in the solar system even if U_μ is assumed to be constant, thanks to the Vainshtein mechanism at small distances. This mechanism also allows us to choose forms other than (7), in contrast to Refs. [7–10]. We could also choose the physical metric as $\tilde{g}_{\mu\nu} \approx e^{2\varphi} g_{\mu\nu} + B(\varphi, s)\varphi_{,\mu}\varphi_{,\nu}$, but B would need to be a fine-tuned function of both φ and its standard kinetic term, $s = (\varphi_{,\lambda})^2$, in order to be consistent with the observed light deflection by galaxies and clusters. Moreover, the conditions for consistency of the theory within matter would be quite involved [16]. The choice (7), borrowed from [7–10], is thus the most natural one in the present framework.

The reason why a mere action $\mathcal{L}_{\rm standard} + \mathcal{L}_{\rm MOND}$, Eqs. (3) and (4) above, does not suffice to define a consistent relativistic field theory of MOND is that it would also predict an extra force $\sqrt{GM_\odot a_0}/r$ within the solar system (where $M_\odot$ denotes the mass of the Sun), in addition to the Newtonian one $GM_\odot/r^2$ and its post-Newtonian corrections. (We often use the word "force" instead of "acceleration" in the present paper; i.e., we do not write the mass of the test particle, to simplify.) This would be ruled out by tests of Kepler's third law and those of post-Newtonian dynamics. The literature thus considered "relativistic aquadraric Lagrangians" (RAQUAL), also known as "k-essence" theories in the cosmological framework, i.e., a scalar kinetic term $-\frac{1}{2}f(s)$ depending on a *function* of $s = (\varphi_{,\lambda})^2$. In order to reproduce the MOND dynamics, this kinetic term was assumed to take the form (4) for small accelerations (i.e., small values of s), while the scalar-field behavior within the solar system depended on the shape of $f(s)$ for large s. The clever choice of the literature was to impose that $f'(s)$ tends towards a constant value for large s, say f'_∞, in order to recover a Brans-Dicke behavior $\varphi = -GM_\odot/f'_\infty rc^2$ at small distances, so that the physical metric (7) reproduces the standard Schwarzschild solution up to a rescaling of the gravitational constant $G_{\rm eff} = G(1 + 1/f'_\infty)$. The parametrized post-Newtonian (PPN) parameters β and γ [20] then strictly take their general relativistic values $\beta = \gamma = 1$, and classical solar-system tests are passed. However, binary-pulsar tests are directly sensitive to the matter-scalar coupling strength $1/f'_\infty$, independently of the fact that the physical metric (7) contains a "disformal" contribution proportional to $U_\mu U_\nu$, and they impose $f'_\infty > 10^3$ [21]. As discussed in [16], such a large value is difficult to reconcile with the expression (4) needed for small s. One needs a fine-tuned interpolating function $f(s)$, of a shape similar to Fig. 3 of [16], in order to predict MOND effects while passing binary-pulsar tests. Another idea would thus be to choose a function $f(s)$ such that the scalar force at small distances is negligible with respect to the Newtonian one $GM_\odot/r^2$, instead of keeping the same radial dependence $\varphi'(r)c^2 = GM_\odot/f'_\infty r^2$. As underlined in [22], nonlinear kinetic terms (i.e., those of k-essence/RAQUAL models) can reduce scalar effects at small distances, acting as a camouflage for the scalar (hence the name "k-mouflage"). However, RAQUAL models must satisfy two conditions to have a Hamiltonian bounded by below and a well-posed Cauchy problem [16]: $f'(s) > 0$ and $2sf''(s) + f'(s) > 0$. These conditions suffice to prove that $\varphi''(r) < 0$, i.e., that $\varphi'(r)$ is a decreasing function of r, and the best we can obtain is thus an almost constant force $\varphi'(r)c^2 \approx a_0$ within the solar system, the value a_0 being imposed by the MOND regime for $r \sim r_{\rm MOND}$. But solar-system tests are precise enough to rule out a constant anomalous acceleration even numerically as tiny as (1). We must therefore look for other possible scalar kinetic terms, and this is where generalized Galileons enter our discussion.

Galileons were first introduced in the cosmological context in [23] (although they had actually already been studied in [24,25] in different frameworks). In flat spacetime, they are theories whose field equations depend only on second derivatives of a scalar field, but not on their lower (zeroth and first) nor on higher derivatives. One of their initial motivations was to generalize the key features of the decoupling limit of the DGP brane model [26], which yields an equation of motion $\propto (\Box\varphi)^2 - (\varphi_{;\mu\nu})^2$ for a scalar degree of freedom, playing a crucial role in cosmology [27,28]. References [29,30] showed how to extend Galileon models to curved spacetime without introducing higher derivatives (while now making first derivatives also enter them). The same Lagrangians can be obtained in a suitable limit of brane models including Gauss-Bonnet-Lovelock densities [31], or from dimensional reduction of such densities [32]. They can also be extended, in any dimension, to arbitrary p-forms possibly coupled to each other [33], or to general nonlinear models whose field equations depend on at most second derivatives [34]. Throughout this paper, we call Galileons this full class of models, although they go beyond the initial ones of [23].

The new ingredient of the present paper is the third scalar kinetic term of action (2), $\mathcal{L}_{\rm Galileon}$, which strongly suppresses all scalar-field effects at small distances, as we will show below. This action (up to the factor $-8k/3$) was obtained in Eq. (24) of Ref. [32] by dimensional reduction of the Gauss-Bonnet-Lovelock density. It is written here in the compact form of [30], and is also in the general classes considered in [24,33,34]. It is easy to check that all field equations involve at most second derivatives, because of the antisymmetry of the Levi-Civita tensors entering (5).

Although it is straightforward to write the full field equations deriving from action (2), for both φ and the metric, it will suffice in the present paper to obtain the perturbative solution for a static and spherically symmetric φ in a Schwarzschild background $ds^2 = -(1 - r_s/r)c^2dt^2 + dr^2/(1 - r_s/r) + r^2 d\Omega^2$, where $r_s \equiv 2GM/c^2$ denotes the Schwarzschild radius. It can be checked *a posteriori* that the backreaction of the scalar field on the metric has a negligible effect as compared to present experimental bounds. We will even assume that $r \gg r_s$ to simplify the expressions. More details will be provided in a forthcoming publication [35]. Denoting as before $\varphi' \equiv d\varphi/dr$, and integrating the field equation for φ once, we find

$$4k\frac{r_s}{r^2}\varphi'^2 + \frac{r^2c^2}{a_0}\varphi'^2 + \epsilon r^2\varphi' \approx \frac{r_s}{2}, \tag{8}$$

where the origin of the different terms is obvious from (3)–(5). The large number of field derivatives involved in $\mathcal{L}_{\rm Galileon}$, Eq. (5), is responsible for the negative power of r in the first term of (8). This is the central idea of the Vainshtein mechanism, since it makes this first term dominate at small distances. Imposing now that $\varphi' \to 0$ for $r \to \infty$ (otherwise φ would diverge at infinity), we can immediately write the unique solution of (8), which is a mere second-order polynomial equation for φ':

$$\varphi' = \left(\sqrt{\frac{8k}{r^2} + \frac{r^2c^4}{GMa_0} + \left(\frac{\epsilon r^2}{r_s}\right)^2} + \frac{\epsilon r^2}{r_s}\right)^{-1}. \tag{9}$$

We thus easily recover the asymptotic Brans-Dicke behavior $\varphi' \approx GM/\epsilon r^2c^2$ for very large distances r, the MOND regime $\varphi' \approx \sqrt{GMa_0}/rc^2$ at intermediate scales, but a small derivative $\varphi' \approx r/\sqrt{8k}$ at small distances. Note that this small-distance behavior of φ' does not depend at all on the mass M, which not only generates the right-hand side (source) of Eq. (8), but also the background Schwarzschild geometry entering the first term of (8) via the Riemann tensor of (5). This universal small-distance behavior means that, paradoxically, any body strictly generates the same scalar force $\varphi' c^2$ in its vicinity. Figure 1 illustrates the three regimes of solution (9). Neglecting its ϵ contribution, the maximum value $\varphi'_{\rm max} = (GMa_0/2k)^{1/4}/2c$ is reached at $r_{\rm V} = (8kGMa_0)^{1/4}/c$, which thus defines a transition radius.

In order to predict the MOND phenomenology in a galaxy of baryonic mass M, we need $r_{\rm V} < r_{\rm MOND}$, i.e., $k < GMc^4/8a_0^3$. The lightest dwarf galaxies for which we have evidence for dark matter or MOND effects thus give an upper bound for the constant k. Draco-like dwarfs correspond to a baryonic mass M between 10^5 and 10^6 solar masses, but even tiny clusters of only $10^3 M_\odot$ seem to be dominated by dark matter [36]. Let us thus be conservative and impose $k \approx G(10^3 M_\odot)c^4/8a_0^3$. This is the numerical value given in Eq. (6) above.

We can now estimate the order of magnitude of scalar effects within the solar system. Since we chose a physical metric of the disformal form (7), like in TeVeS, one could naively conclude that the PPN parameters β and γ keep their general relativistic values $\beta = \gamma = 1$. However, such parameters do not have any meaning here, because the PPN formalism assumes that no length scale enters the theory, and it needs the gravitational potential to be $\propto 1/r$ at the Newtonian order. Therefore, the consequences of our anomalous potential $\varphi \propto r^2 + \text{const}$ at small distances cannot be analyzed in the standard way. Let us thus merely compare the anomalous force $\varphi' c^2 \approx rc^2/\sqrt{8k}$ it generates on a test particle, with the post-Newtonian forces $\sim(GM_\odot/r^2) \times (r_s/r)$ which have been precisely tested in the solar system. Since their ratio is $(r^2/r_s)^2/\sqrt{2k} \approx (r/22\ \text{AU})^4$ for the chosen value (6) of the constant k, the scalar effects are thus smaller than 10^{-5} times post-Newtonian ones at the Earth's orbit, i.e., negligible with

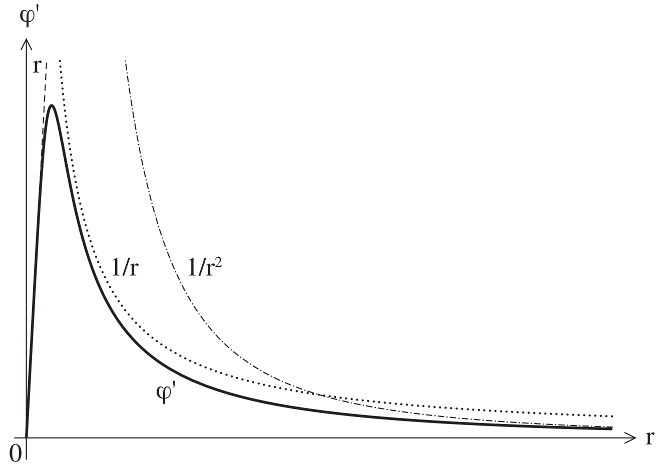

FIG. 1. Behavior of the scalar force $\varphi' c^2$ in the three regimes of the theory: $\varphi' \propto r$ at small distances (k-mouflage mechanism), $\varphi' \propto 1/r$ in the intermediate range (MOND regime), and $\varphi' \propto 1/r^2$ at large distances (asymptotic Brans-Dicke theory).

respect to the best experimental constraints. This ratio becomes even smaller for inner planets, and is $<10^{-7}$ for Mercury's orbit. On the other hand, scalar effects grow for outer planets, but they remain 2×10^{-5} smaller than post-Newtonian forces at the orbit of Mars, and 3×10^{-3} at Jupiter's. This is consistent with the most precise planetary data. When considering the Moon's orbit, the contribution of the Earth to the Riemann tensor entering (5) dominates over that of the Sun, and φ is almost spherically symmetric with respect to the Earth's center. Denoting now as r the distance to this center, we get again the universal behavior $\varphi' c^2 \approx rc^2/\sqrt{8k}$, which must be compared to the Newtonian accelerations caused by both the Earth and the Sun and their post-Newtonian corrections. We find that scalar effects on the Moon's motion are 10^{-8} smaller than post-Newtonian ones, i.e., 4 orders of magnitude smaller than the tightest experimental constraints derived from lunar laser ranging [20].

Preferred-frame effects can be estimated in a similar way. Assuming that the solar system is moving with a velocity $\boldsymbol{w}$ with respect to the preferred frame where $U_\mu = (1, 0, 0, 0)$, we compute the contributions proportional to $\boldsymbol{w}^2$ in $\tilde{g}_{00}$ and to w^i in $\tilde{g}_{0i}$, and their radial derivatives give us the magnitude of the anomalous scalar forces. Comparing them to those generated by the α_1 term in the PPN formalism [20] (while the terms corresponding to α_2 and α_3 vanish in the present model [7]), we find that their ratio is $\lesssim 8r^3/(r_s \alpha_1 \sqrt{2k}) \approx (r/10^4 \text{ AU})^3/\alpha_1$, thus giving scalar effects similar to those of an $\alpha_1 \approx 2 \times 10^{-12}$ at the orbit of Mars. As above, when considering the Moon's orbit around the Earth, the local value of $\varphi' \approx r/\sqrt{8k}$ must be used (r now denoting the Earth-Moon distance), and we find that preferred-frame effects caused by the scalar field are similar to those of an $\alpha_1 \approx 10^{-15}$. Since the tightest constraint $\alpha_1 < 10^{-4}$ comes from lunar laser ranging, we conclude that the model (2)–(7) does not predict any detectable violation of local Lorentz invariance in the solar system.

Binary-pulsar tests are much more subtle to compute. A precise analysis would need either to study the (time-dependent) dynamics of a binary system at least up to order $\mathcal{O}(1/c^3)$, or to be able to relate scalar multipoles at infinity to their local matter sources in spite of the nonlinearities of the Vainshtein mechanism. In the present paper, we shall only estimate the rough order of magnitude of scalar radiation by comparing the local scalar forces to those of the precisely studied Brans-Dicke–like theories. Let us first note that the monopolar radiation, naively of order $\mathcal{O}(1/c)$, is actually reduced to order $\mathcal{O}(1/c^5)$ because the local scalar solution generated by any body does not depend on time; this is similar to the case of standard scalar-tensor theories [37]. On the other hand, the dipolar radiation starts at order $\mathcal{O}(1/c^3)$ in spite of the fact that the local scalar solution is the same around any body, independently of its mass. Indeed, if the two bodies of a binary system do not have the same mass (say, $m_A \neq m_B$), they do not move on the same orbit around their common center of mass, and the global scalar field they generate defines a preferred (oriented) direction in space. The dominant scalar radiation is thus a dipole, and we can estimate its order of magnitude by multiplying the one predicted in standard scalar-tensor theories [see, e.g., Eq. (6.52b) of Ref. [37]] by the square of the ratio of the present scalar force between the two bodies $(rc^2/\sqrt{8k})$ and the standard one. We get that the scalar-field contribution to the time derivative of the orbital period is of order $\dot{P} \sim -GcP^3(m_A - m_B)^2(m_A + m_B)/(32\pi^2 k m_A m_B)$, and numerically at least 10^{-32} smaller than the tightest experimental uncertainties. Although this estimate might be erroneous by some large numerical coefficient, we can conclude anyway that the present model should easily pass all binary-pulsar tests.

Although the Galileon field equations involve at most second derivatives, and thus avoid the generic instability related to higher derivatives, this does not suffice to prove that these models are stable. One should carefully analyze both the boundedness from below of their Hamiltonian density and the well-posedness of their Cauchy problem. The Hamiltonian of flat-space Galileons is straightforward to derive [35,38], but the situation is much more complex in curved spacetime, because all field equations for φ and $g_{\mu\nu}$ involve second derivatives of both of them. Around a given background, one should thus diagonalize the kinetic terms in order to test the stability of perturbations and the hyperbolicity of their field equations. For instance, it would have no meaning to freeze $g_{\mu\nu}$ and study the perturbations of φ only in the scalar field equation (similarly to Brans-Dicke theory, which seems to contain a ghost scalar field for $-3/2 < \omega_{\text{BD}} < 0$ if one freezes the Jordan metric, whereas studying simultaneously the dynamics of $g_{\mu\nu}$ and φ shows that the theory is stable even for such a slightly negative ω_{BD}). This problem of the consistency of Galileon field theories thus goes beyond the scope of this paper, and we postpone it for a forthcoming publication [35].

Let us finally comment on our choice of Lagrangian (5) to obtain a k-mouflage mechanism reducing scalar effects at small distances. It happens to be the most efficient one amongst all those that we have analyzed. The highest-order Galileon Lagrangian which is nontrivial in four-dimensional flat space [23,29,30], namely, $\mathcal{L}_5 = -k_5 \varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}[\varphi_{;\gamma\rho}\varphi_{;\delta\sigma} - \frac{3}{4}(\varphi_{,\lambda})^2 R_{\gamma\delta\rho\sigma}]$, also generates a small $\varphi'(r) \approx \sqrt{r}/(84k_5)^{1/4}$ at small distances, but its slower radial dependence makes scalar effects still marginally detectable in the solar system (while giving again fully negligible preferred-frame effects). Imposing as above that the MOND phenomenology should occur in the lightest known dwarf galaxies, we get that scalar effects in the solar system are $\sim(r/6\ \mathrm{AU})^{7/2}$ times post-Newtonian forces, i.e., about the order of magnitude of the most precise bounds. Similarly, scalar effects on the Moon's motion are 10^{-4} smaller than post-Newtonian ones, i.e., of the order of current limits. A full fit of planetary data taking into account the possible presence of such a small scalar force would thus be necessary to test whether it is already excluded or not. If not, this opens the exciting possibility to detect them in future higher-precision solar-system observations. All the other known covariant Galileon actions are either total derivatives in four dimensions, or they yield a vanishing scalar field equation around a Schwarzschild background [like Eq. (23) of Ref. [32]], or predict a negative $\varphi''(r)$ and therefore too-large scalar forces in the solar system. On the other hand, generalized Galileon actions [34] involving nondifferentiated fields φ and/or negative powers of $s \equiv (\varphi_{,\lambda})^2$ can provide alternative models [35], but less natural than (5).

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