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Technicolor models with color-singlet technifermions and their ultraviolet extensions

Thomas A. Ryttov and Robert Shrock

C. N. Yang Institute for Theoretical Physics Stony Brook University Stony Brook, New York 11794, USA

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We study technicolor models in which all of the technifermions are color-singlets, focusing on the case in these fermions transform according to the fundamental representation of the technicolor gauge group. Our analysis includes a derivation of restrictions on the weak hypercharge assignments for the technifermions and additional color-singlet, technisinglet fermions arising from the necessity of avoiding stable bound states with exotic electric charges. Precision electroweak constraints on these models are also discussed. We determine some general properties of extended technicolor theories containing these technicolor sectors.

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I. INTRODUCTION

Electroweak symmetry breaking (EWSB) may occur dynamically, via the formation of bilinear condensates $\langle \bar{F}F \rangle$ of a set of fermions $\{F\}$ subject to a vectorial, asymptotically free gauge interaction, generically called technicolor (TC), that becomes strongly coupled at the TeV scale [1]. To communicate this symmetry breaking to the standard model (SM) fermions, which are technisinglets, one embeds technicolor in a larger, extended technicolor (ETC) gauge theory [2]. This involves gauging the generational index and combining it with the technicolor index. We denote the generational, technicolor, and ETC gauge groups as $G_{\rm gen}$, $G_{\rm TC}$, and $G_{\rm ETC}$. It follows that $G_{\rm ETC} \supset G_{\rm gen} \otimes G_{\rm TC}$. In one class of technicolor models, technifermions form a standard model family. In these models, the ETC gauge bosons are singlets under the SM gauge group, $G_{\rm SM} = {\rm SU}(3)_c \otimes G_{\rm EW}$, where $G_{\rm EW} = {\rm SU}(2)_L \otimes {\rm U}(1)_Y$, and the generators of $G_{\rm ETC}$ commute with those of $G_{\rm SM}$: $[G_{\rm ETC}, G_{\rm SM}] = \varnothing$.

However, the basic aim of technicolor, to break $G_{\rm EW}$ to electromagnetic ${\rm U}(1)_{\rm em}$ dynamically, can be realized using purely color-singlet technifermions. A minimal technicolor model of this type includes one ${\rm SU}(2)_L$ doublet of left-handed technifermions, together with the corresponding ${\rm SU}(2)_L$-singlet right-handed technifermions, all of which are color-singlets. To maintain the vectorial nature of the technicolor gauge symmetry (as is necessary in order that it does not self-break when it forms condensates), the left- and right-handed chiral components of the technifermions transform according to the same representation of $G_{\rm TC}$. We use the abbreviations 1FTC and 1DTC for one-family and one-${\rm SU}(2)_L$-technidoublet TC models, respectively [3]. Since ETC gauge bosons mediate transitions that take quarks to technifermions, it follows that in technicolor models in which the technifermions are color-singlets, some of these ETC gauge bosons transform according to the fundamental and conjugate fundamental representations of ${\rm SU}(3)_c$. Hence, in these models, commutators of the associated generators of $G_{\rm ETC}$ transform as the singlet and adjoint of ${\rm SU}(3)_c$, $[G_{\rm ETC}, G_{\rm SM}] \neq \varnothing$, and

$$G_{\rm ETC} \supset {\rm SU}(3)_c \otimes G_{\rm gen} \otimes G_{\rm TC} \quad \text{for 1DTC.} \tag{1.1}$$

In this paper, we shall construct and study technicolor models in which all of the technifermions are color-singlets and that are minimal, in the sense of being of 1DTC type. There are several motivations for this work. One is that 1DTC models can reduce technicolor corrections to W and Z propagators, since they involve fewer technifermions than 1FTC models. Another is that 1FTC models predict technivector mesons that transform as color-octets, and this prediction is in significant tension with lower bounds on the masses of such particles obtained by the ATLAS and CMS experiments at the Large Hadron Collider (LHC), as discussed further below. Yet another motivation is that 1FTC models have a very large global chiral symmetry, and when this is broken spontaneously via the formation of bilinear technifermion condensates, there may be problematically light (pseudo)-Nambu-Goldstone bosons. Recently, there has much considerable interest in 1DTC models [4–7]; reviews include Refs. [8,9]. Much of this work has made use of the group ${\rm SU}(2)_{\rm TC}$ with two technifermions in the adjoint representation (equivalently, the vector representation of an ${\rm SO}(3)_{\rm TC}$ group). Here, we will give a general discussion that focuses on 1DTC models with technifermions in the fundamental representation of the technicolor gauge group. Our model-building will focus on an ${\rm SU}(3)_{\rm TC}$ model, but, as will be seen, a number of our results, such as restrictions on hypercharge assignments, will apply rather generally to 1DTC models.

This paper is organized as follows. In Sec. II, to provide some background, we review one-family technicolor models and their ultraviolet extension to ETC theories. Section III contains preliminaries on 1DTC models, and Sec. IV contains some discussion of properties of models of this type with an ${\rm SU}(2)_{\rm TC}$ gauge group and technifermions in the fundamental and adjoint representations. In

Sec. V, we study 1DTC models with an $SU(3)_{TC}$ gauge group and technifermions in the fundamental representation. In Sec. VI, we derive some properties of ETC ultraviolet extensions of the $SU(3)_{TC}$ theory. Section VII contains some concluding remarks.

II. BACKGROUND ON ONE-FAMILY TC/ETC MODELS

To provide a contrasting background perspective for our study of TC/ETC models with color-singlet technifermions, we briefly discuss one-family TC/ETC models, which do contain some color-triplet technifermions. We take $G_{TC} = SU(N_{TC})_{TC}$ and $G_{gen} = SU(N_{gen})$, where $N_{gen} = 3$ is the observed number of SM fermion generations. In the most compact model of this type, G_{ETC} contains $G_{gen} \otimes G_{TC}$ as a maximal subgroup. This is arranged by setting

$$G_{ETC} = SU(N_{ETC})_{ETC}, \tag{2.1}$$

where

$$N_{ETC} = N_{gen} + N_{TC} = 3 + N_{TC}. \tag{2.2}$$

In accordance with this, one assumes that the technifermions form one SM family. One may assign the SM fermions and technifermions to the following representations of $G_{SM} \otimes G_{ETC}$

$$\begin{aligned} Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L &: (3, 2, N_{ETC})_{1/3,L} \\ u_R &: (3, 1, N_{ETC})_{4/3,R} \\ d_R &: (3, 1, N_{ETC})_{-2/3,R} \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} L_L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L &: (1, 2, N_{ETC})_{-1,L} \\ \nu_R &: (1, 1, N_{ETC})_{0,R} \\ \ell_R &: (1, 1, N_{ETC})_{-2,R}. \end{aligned} \tag{2.4}$$

Here, the numbers in parentheses are the dimensions of the representations of the three non-Abelian factor groups in $G_{SM} \otimes G_{ETC}$, the subscript denotes the weak hypercharge, and we suppress ETC and color indices. Indicating these indices explicitly, we have, for example, $u_R = u_R^{ai}$, where $a = 1, 2, 3$ is the color index and $i = 1, \ldots, N_{ETC}$ is the ETC index. The ETC indices i are ordered such that $i = 1, 2, 3$ are generation indices (with $u^{a1} = u^a$, $u^{a2} = c^a$, $u^{a3} = t^a$, $\ell^1 = e$, $\ell^2 = \mu$, etc.) and $4 \le i \le N_{ETC}$ are TC indices. It will be useful to distinguish between generational and technicolor indices, and so if i is in the interval $4 \le i \le N_{ETC}$, we shall usually label it as τ, standing for TC.

If one makes the minimal choice for G_{TC}, namely $G_{TC} = SU(2)_{TC}$, then, by Eq. (2.2), it follows that $G_{ETC} = SU(5)_{ETC}$. Detailed studies with reasonably ultraviolet-complete ETC models of this type were carried out in Refs. [10–16]. Refs. [13,15] also presented a critical assessment of an alternate type of TC/ETC model in which down-type quarks and charged leptons of opposite chiralities are assigned to relatively conjugate representations of $SU(5)_{ETC}$, while up-type quarks of opposite chiralities are assigned to the same representations of this group.

In order to account for the hierarchy in the three generations of SM quarks and charged leptons, the ETC gauge symmetry should break in a sequence of scales $\Lambda_{ETC,i}$, where $i = 1, 2, 3$, down to the residual exact technicolor gauge symmetry. The studies of Refs. [11–13] demonstrated how the sequential breaking of the ETC gauge symmetry can occur. A typical set of ETC breaking scales used in these studies is

$$\begin{aligned} \Lambda_{ETC,1} &\simeq 10^3 \text{ TeV}, \\ \Lambda_{ETC,2} &\simeq 10^2 \text{ TeV}, \\ \Lambda_{ETC,3} &\simeq \text{few TeV} \end{aligned} \tag{2.5}$$

for the three SM generations $i = 1, 2, 3$ [17]. Having an explicit and reasonably ultraviolet-complete ETC theory, it was possible to calculate flavor-changing neutral-current (FCNC) processes, and it was shown that in ETC theories in which the techniquarks transform in a vectorial manner under G_{ETC}, these can be adequately suppressed because of approximate residual generational symmetries [11,13,15]. A mechanism was also presented for obtaining lepton mixing and small neutrino masses [11–13]. The resultant technicolor gauge interaction confines and breaks chiral symmetry at the scale Λ_{TC}, thereby producing electroweak symmetry breaking. The W and Z pick up masses given to leading order by

$$m_W^2 = m_Z^2 \cos^2\theta_W = \frac{g^2 N_D f_{TC}^2}{4}, \tag{2.6}$$

where g is the $SU(2)_L$ gauge coupling and N_D denotes the number of $SU(2)_L$ doublets of technifermions. For one-family TC models, $N_D = 4$, so $f_{TC} \simeq 125$ GeV.

The ETC interactions lead to a mass for a fermion of the i'th generation of the generic form

$$m_{f^i} \sim \frac{\kappa \eta \Lambda_{TC}^3}{\Lambda_{ETC,i}^2}, \tag{2.7}$$

where κ is a numerical constant of order 10,

$$\eta = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_w} \frac{d\mu}{\mu}\, \gamma(\alpha_{TC}(\mu))\right] \tag{2.8}$$

is a renormalization-group factor, γ is the mass anomalous dimension of the technifermions, and $\alpha_{TC}(\mu) = g_{TC}(\mu)^2/(4\pi)$ is the TC running coupling (inherited from its UV completion in the ETC theory), depending on the Euclidean reference momentum, μ. In Eq. (2.8), Λ_w is the scale where the coupling rises to O(1). The Λ_{TC}^3 factor in

Eq. (2.7) arises from the technifermion condensate $\langle \bar{F}F \rangle$ and the $1/\Lambda^2_{\mathrm{ETC},i}$ factor arises from the propagator(s) of ETC gauge bosons mediating the transitions between SM fermions of the i'th generation and technifermions. As is evident in Eq. (2.7), the largest $\Lambda_{\mathrm{ETC},i}$ corresponds to the smallest fermion masses, namely, those of the first generation, and so forth for the other generations. It has also been shown how off-diagonal elements can be generated in the full 3×3 fermion mass matrices, whose diagonalization thus leads to quark and lepton mixing [11,13]. Related work is in Ref. [18].

Viable TC/ETC theories require a value of γ that is not too small, in order to enhance SM fermion mass generation via the renormalization-group factor η. This property can follow naturally if the theory has an approximate infrared fixed point (IRFP), i.e., IR zero of the TC beta function, so that the running TC coupling $\alpha_{\mathrm{TC}}(\mu)$ becomes large at a scale Λ_w but runs slowly (walks) as a function of the scale μ [19].

One-family technicolor models are subject to many constraints, such as those from precision electroweak data, neutral flavor-changing current processes, etc. In addition, 1FTC models predict color-octet technihadrons, in particular, pseudoscalar and vector technimesons. The vector technimesons are expected to have masses given by

$$\frac{m_{V,\mathrm{TC}}}{m_{\rho,\omega}} \simeq \frac{\Lambda_{\mathrm{TC}}}{\Lambda_{\mathrm{QCD}}} \simeq \frac{f_{\mathrm{TC}}}{f_\pi}\left(\frac{N_c}{N_{\mathrm{TC}}}\right)^{1/2}, \tag{2.9}$$

where $N_c = 3$, $f_\pi \simeq 93$ MeV, $\Lambda_{\mathrm{QCD}} \simeq 330$ MeV, and $f_{\mathrm{TC}} \simeq 125$ GeV, so $m_{V,\mathrm{TC}} \simeq 1.0\sqrt{3/N_{\mathrm{TC}}}$ TeV. This simple scaling behavior is approximately borne out in more sophisticated calculations of technimeson masses using solutions of the Bethe-Salpeter equation in a technicolor theory with walking behavior [20]. A similar scaling is expected to apply for the width $\Gamma_{V,\mathrm{TC}}$, so that $\Gamma_{V,\mathrm{TC}}$ would be a few hundred GeV. The ATLAS and CMS experiments at the LHC have set a lower limit of approximately 2 TeV on color-octet resonances of this type [21,22]. There is thus significant tension between these LHC data and one-family technicolor models. This tension is exacerbated by limits on the (pseudo)-Nambu-Goldstone bosons, denoted (P)NGBs, in this model [23–25]. As noted above, this provides one motivation for exploring TC/ETC models that contain only color-singlet technifermions, since these technifermions thus do not couple directly to gluons, and hence the resultant technihadrons are not subject to such severe experimental limits from current LHC (or Tevatron) data.

III. TECHNICOLOR MODELS WITH COLOR-SINGLET TECHNIFERMIONS

In this section, we begin our discussion of 1DTC models. As will be explained below, these models may, in general, also contain other technifermions that are G_{SM}-singlets. These models have the feature that all technifermions are $\mathrm{SU}(3)_c$-singlets and can thus be denoted also as color-singlet technifermion (CSTF) theories. For the models of interest here, the 1DTC property implies the CSTF property. The converse does not hold in general, since, in principle, a technicolor model could contain only color-singlet technifermions but have more than one $\mathrm{SU}(2)_L$ technidoublet. However, in practice, given the requirement of minimizing technicolor corrections to the W and Z propagators, as long as one considers CSTF models, one restricts to those of 1DTC type. Hence, in practice, one has the relation 1DTC $\Leftrightarrow$ CSTF for these properties.

The gauge symmetry that is operative at an energy scale of $\sim$1 TeV is assumed to be $G_{\mathrm{SM}} \otimes G_{\mathrm{TC}}$. We shall mainly focus on the case in which the technifermions to be in the fundamental representation of $G_{\mathrm{TC}} = \mathrm{SU}(N_{\mathrm{TC}})_{\mathrm{TC}}$ (while sometimes giving more general results), and shall consider the possible choices $G_{\mathrm{TC}} = \mathrm{SU}(2)_{\mathrm{TC}}$ and $G_{\mathrm{TC}} = \mathrm{SU}(3)_{\mathrm{TC}}$. The technicolor model is minimal in the sense that it uses the minimum content of G_{EW}-nonsinglet technifermions necessary to achieve electroweak symmetry breaking, with the left-handed and right-handed components of the technifermions transforming as

$$F^\tau_L = \begin{pmatrix} f^\tau_1 \\ f^\tau_2 \end{pmatrix}_L : (1, 2, d_{R_{\mathrm{TC}}})_{Y_{F_L}} \tag{3.1}$$

and

$$f^\tau_{1R}: (1, 1, d_{R_{\mathrm{TC}}})_{Y_{f_{1R}}}, \qquad f^\tau_{2R}: (1, 1, d_{R_{\mathrm{TC}}})_{Y_{f_{2R}}} \tag{3.2}$$

under $G_{\mathrm{SM}} \otimes G_{\mathrm{TC}}$, where $d_{R_{\mathrm{TC}}}$ denotes the dimension of the representation R_{TC}, and we again suppress the TC index τ in the notation. The electric charge operator is $Q = T_{3L} + (Y/2)$ (in units of e), so the condition that $\mathrm{U}(1)_{\mathrm{em}}$ be vectorial on these technifermions means that

$$\frac{1}{2} + \frac{Y_{F_L}}{2} = q_{f_{1L}} = q_{f_{1R}} = \frac{Y_{f_{1R}}}{2} \tag{3.3}$$

and

$$-\frac{1}{2} + \frac{Y_{F_L}}{2} = q_{f_{2L}} = q_{f_{2R}} = \frac{Y_{f_{2R}}}{2}. \tag{3.4}$$

Hence,

$$1 + Y_{f_L} = Y_{f_{1R}}, \qquad -1 + Y_{f_L} = Y_{f_{2R}}. \tag{3.5}$$

The Lagrangian mass terms $m_{f_1}\bar{f}_{1\tau L}f^\tau_{1R} + \mathrm{H.c.}$ and $m_{f_2}\bar{f}_{2\tau L}f^\tau_{2R} + \mathrm{H.c.}$ would explicitly break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, and therefore these are set to zero.

IV. $\mathrm{SU}(2)_{\mathrm{TC}}$ 1DTC MODELS

Here, we take $G_{\mathrm{TC}} = \mathrm{SU}(2)_{\mathrm{TC}}$. If $Y_{F_L} = 0$, then the theory with the fermions (3.1) and (3.2) is free of any anomalies in gauged currents. It is also free of any global $\mathrm{SU}(2)_L$ anomaly, since it contains an even number, $N_{\mathrm{gen}}(N_c + 1) + N_{\mathrm{TC}} = 14$ chiral $\mathrm{SU}(2)_L$ doublets of

fermions. (Of course, there is also no global $SU(2)_{TC}$ anomaly whether the number of technifermions transforming as fundamental representations of $SU(2)_{TC}$ is even or odd, since these are Dirac fermions, corresponding to the fact that the $SU(2)_{TC}$ theory is vectorial.) However, this model is disfavored by the fact that, as α_{TC} grows to a size of order unity and the TC interaction eventually confines and produces bilinear technifermion condensates and associated spontaneous chiral symmetry breaking at the scale Λ_{TC}, these condensates would most likely have an undesired form. The condensates form in the most attractive channel (MAC), which is $2 \times 2 \to 1$. Vacuum alignment arguments imply that the condensates should preserve the maximal possible gauge symmetry and hence would have the Majorana forms

$$\langle \epsilon_{\alpha\beta}\epsilon_{\tau\tau'} F_L^{\alpha\tau\,T} C F_L^{\beta\tau'} \rangle = 2\langle \epsilon_{\tau\tau'} f_{1L}^{\tau\,T} C f_{2L}^{\tau'} \rangle \tag{4.1}$$

and

$$\langle \epsilon_{\tau\tau'} f_{1R}^{\tau\,T} C f_{2R}^{\tau'} \rangle, \tag{4.2}$$

where here α, β are $SU(2)_L$ indices and τ, τ' are $SU(2)_{TC}$ indices. These condensates are invariant under all gauge symmetries, in particular, G_{EW}. Hence, they would not achieve the basic purpose of technicolor, which is electroweak symmetry breaking.

One might try to avoid this by assigning a nonzero weak hypercharge Y_{F_L} to F_L, which, by Eq. (3.5), would imply that at least one of the f_{jR}, $j = 1, 2$, would also have nonzero weak hypercharge. This modification would contain nonzero gauge anomalies unless one also added more fermions. A simple approach is to add an even number of (color-singlet) technicolor-singlet fermions that transform as doublets under $SU(2)_L$. This number must be even to avoid producing a global $SU(2)_L$ anomaly. The minimal such number is two, and thus we add the following color-singlet, technicolor-singlet fermions, forming two left-handed $SU(2)_L$ doublets and four corresponding right-handed $SU(2)_L$-singlets

$$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L, \qquad \psi_{jR}, \qquad j = 1, 2, \tag{4.3}$$

and

$$\psi'_L = \begin{pmatrix} \psi'_1 \\ \psi'_2 \end{pmatrix}_L, \qquad \psi'_{jR}, \qquad j = 1, 2, \tag{4.4}$$

with respective weak hypercharges Y_{ψ_L}, $Y_{\psi_{1R}}$, $Y_{\psi_{2R}}$, and $Y_{\psi'_L}$, $Y_{\psi'_{1R}}$, $Y_{\psi'_{2R}}$. It is a model-building choice whether or not one attributes a nonzero lepton number to these fermions. We do not make a definite commitment concerning this choice and hence do not refer to the ψ_i or ψ'_i as leptons, but simply as color-singlet, technicolor-singlet fermions. The hypercharges Y_{ψ_L}, $Y_{\psi_{1R}}$, $Y_{\psi_{2R}}$ satisfy relations analogous to (3.3), (3.4), and (3.5),

$$\frac{1}{2} + \frac{Y_{\psi_L}}{2} = q_{\psi_{1L}} = q_{\psi_{1R}} = \frac{Y_{\psi_{1R}}}{2} \tag{4.5}$$

and

$$-\frac{1}{2} + \frac{Y_{\psi_L}}{2} = q_{\psi_{2L}} = q_{\psi_{2R}} = \frac{Y_{\psi_{2R}}}{2} \tag{4.6}$$

so that

$$1 + Y_{\psi_L} = Y_{\psi_{1R}}, \qquad -1 + Y_{\psi_L} = Y_{\psi_{2R}}. \tag{4.7}$$

The $Y_{\psi'_L}$, $Y_{\psi'_{1R}}$, $Y_{\psi'_{2R}}$ satisfy the analogous relations with primes. Then the gauge anomalies of the form

$$\mathrm{U}(1)_Y^3, \qquad \mathrm{SU}(2)_L^2 \mathrm{U}(1)_Y, \qquad Gr^2 \mathrm{U}(1)_Y, \tag{4.8}$$

where Gr denotes graviton, are satisfied if and only if

$$d_{R_{TC}} Y_{F_L} + Y_{\psi_L} + Y_{\psi'_L} = 0. \tag{4.9}$$

Explicitly, this equation reads $2Y_F + Y_\psi + Y_{\psi'} = 0$. The solutions of this condition form a two-parameter set. In this case, it is possible that the condensation of the technifermions might proceed in the desired manner, yielding the Dirac condensates

$$\langle \bar{f}_{i\tau L} f_{iR}^{\tau} \rangle, \qquad i = 1, 2. \tag{4.10}$$

These condensates are equal for $i = 1$ and $i = 2$, up to small corrections from weak hypercharge interactions, and hence so are the associated dynamical technifermion masses for f_1 and f_2. The condensates (4.10) for each $i = 1, 2$ break G_{EW} to $\mathrm{U}(1)_{em}$, as desired. They transform as $\Delta T_{3L} = 1/2$ and $\Delta Y = 1$ operators and hence yield the tree-level mass relation $\rho = 1$, where

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \tag{4.11}$$

as is necessary to agree with experiment.

However, this desired pattern of condensation is not guaranteed. It is also possible that, even with nonzero weak hypercharge assignments, the theory would still prefer to form the Majorana condensates (4.1) and (4.2) instead of the Dirac condensates (4.10). If this happened, then these Majorana condensates (4.1) and (4.2) would break only the $\mathrm{U}(1)_Y$ part of G_{EW}, while preserving the $SU(2)_L$ part. Both of the condensates (4.1) and (4.2) transform as $\Delta Y = 2Y_{F_L}$ operators. Indeed, this latter type of condensation could actually be favored by a vacuum alignment argument on the grounds that it only breaks one of the four generators of G_{EW}, while the Dirac condensates (4.10) break all of the generators, leaving one linear combination (the electric charge, Q) invariant. The use of the $SU(2)_{TC}$ gauge group in the one-family technicolor models does not encounter this problem because in that case the Majorana-type condensates would break $SU(3)_c$ and $\mathrm{U}(1)_{em}$ and hence are excluded by a vacuum alignment argument [26].

One way of avoiding this problem with undesired condensates that has been investigated is to assign the

technifermions to an adjoint representation of $SU(2)_{TC}$ [4–9]. This is also useful in reducing technicolor contributions to W and Z propagator corrections. This model often denoted the minimal walking technicolor (MTW) model, since, first, it includes a minimal content of G_{EW}-nonsinglet technifermions, and, secondly, it can achieve walking behavior with a minimal set of technifermions. For our general discussion here, it will be useful to remark on some properties of this model, especially concerning vacuum alignment. Since the adjoint representation of SU(2) is equivalent to the vector representation of SO(3), we may denote the left-handed technifermions as

$$\vec{F}_L = \begin{pmatrix} \vec{f}_1 \\ \vec{f}_2 \end{pmatrix}_L, \tag{4.12}$$

and the right-handed technifermions as $\vec{f}_{1R}$, and $\vec{f}_{2R}$. The number of $SU(2)_L$ chiral doublets is odd (equal to $N_{gen}(N_c + 1) + 3 = 15$), and hence one is led to introduce one such $SU(2)_L$, namely, the color-singlet, technisinglet fermions of Eq. (4.3). The condition of zero gauge anomalies is satisfied if and only if $3Y_{F_L} + Y_{\psi_L} = 0$.

A basic question is whether the $SU(2)_{TC}$ theory with two (massless) Dirac fermions in the adjoint representation evolves into the infrared in the desired manner, with confinement and spontaneous chiral symmetry breaking via the formation of bilinear fermion condensates that break G_{EW} to $U(1)_{em}$, or whether, instead, it evolves to an infrared conformal phase with no such SχSB. Lattice studies suggest that this theory evolves into the infrared toward an exactir fixed point (IRFP) which is at a sufficiently small value of α_{TC} that there is no SχSB or formation of any fermion condensates. Since one needs such condensates for electroweak symmetry breaking, one would have to add requisite four-fermion terms to allow SχSB at a smaller value of α_{TC} [7]. Presuming such a term is generated by an ultraviolet completion, one then may examine the condensates that form.

The most attractive bilinear fermion condensation channel is $3 \times 3 \to 1$, and there are several condensates that, *a priori*, could form in this channel. Since the scalar product of two vectors of SO(3) is symmetric under interchange of the vectors, while the weak $SU(2)_L$ contraction with $\epsilon_{\alpha\beta}$ is antisymmetric, the resultant Majorana condensate vanishes identically:

$$\langle \epsilon_{\alpha\beta} \vec{F}_L^{\alpha T} \cdot C \vec{F}_L^{\beta} \rangle = 0. \tag{4.13}$$

Hence, the $\vec{f}_{1L}$ and $\vec{f}_{2L}$ must condense via Dirac condensates with the corresponding right-handed technifermions. A vacuum alignment argument leads to the conclusion that these condensates are

$$\langle \bar{\vec{f}}_{1L} \cdot \vec{f}_{1R} \rangle, \qquad \langle \bar{\vec{f}}_{2L} \cdot \vec{f}_{2R} \rangle. \tag{4.14}$$

These transform as $\Delta T_{3L} = 1/2$, $\Delta Y = 1$ operators, breaking G_{EW} in the desired manner to $U(1)_{em}$. The same vacuum alignment argument implies that none of the following condensates form: (i) the Dirac condensates $\langle \bar{\vec{f}}_{1L} \cdot \vec{f}_{2R} \rangle$ and $\langle \bar{\vec{f}}_{2L} \cdot \vec{f}_{1R} \rangle$, which transform as $\Delta T_{3L} = 1/2$ but violate charge, and (ii) the Majorana condensates $\langle \vec{f}_{1R}^T \cdot C \vec{f}_{1R} \rangle$ and $\langle \vec{f}_{2R}^T \cdot C \vec{f}_{2R} \rangle$, which preserve $SU(2)_L$ but violate charge. For the case $Y_{F_L} \neq 0$, this argument also implies that there is no formation of the Majorana condensate $\langle \vec{f}_{1R}^T \cdot C \vec{f}_{2R} \rangle$, which would preserve $SU(2)_L$ and transform as a $\Delta Y = \Delta Q = 2Y_{F_L}$ operator. If $Y_{F_L} = 0$, this condensate could form, but we shall demonstrate below that the assignment $Y_L = 0$ generically leads to a problem with unobserved exotically charged matter. We next proceed to investigate a different class of technicolor models.

V. $SU(3)_{TC}$ 1DTC MODELS

A. General construction

Here, we construct and study models with the technicolor gauge group $SU(3)_{TC}$ and technifermions transforming according to the fundamental representation of this group. These technifermions thus comprise the requisite special case of Eqs. (3.1) and (3.2). Because the number of $SU(2)_L$ chiral doublets is then odd, namely $N_{gen}(N_c + 1) + N_{TC} = 12 + 3 = 15$, it is necessary to add an odd number of additional $SU(2)_L$ doublets to avoid a global $SU(2)_L$ anomaly. We choose to add the minimal number, viz., one, with the color-singlet, technicolor-singlet fermions of Eq. (4.3). The resultant theory is free of all anomalies in gauged currents if and only if

$$d_{R_{TC}} Y_{F_L} + Y_{\psi_L} = 0, \qquad i.e., \qquad 3Y_{F_L} + Y_{\psi_L} = 0. \tag{5.1}$$

The solutions of this condition form a one-parameter set [27]. It will be useful to give a general classification of the types of solutions in this set. First, there are three discrete special cases. We denote these with the abbreviations ZY, SMY and RSMY, standing for "zero Y_{F_L} and Y_{ψ_L}", "SM-type Y" and "reversed-sign SM-type Y" assignments:

$$\text{ZY: } Y_{F_L} = Y_{\psi_L} = 0 \Rightarrow \quad q_{f_1} = q_{f_2} + 1 = q_{\psi_1} = q_{\psi_2} + 1 = \frac{1}{2} \tag{5.2}$$

and, with $N_{TC} = 3$,

$$\text{SMY: } Y_{F_L} = \frac{1}{3}, \qquad Y_{\psi_L} = -1 \Rightarrow \quad q_{f_1} = q_{f_2} + 1 = \frac{2}{3}, \quad q_{\psi_1} = q_{\psi_2} + 1 = 0 \tag{5.3}$$

or

$$\text{RSMY: } Y_{F_L} = -\frac{1}{3}, \qquad Y_{\psi_L} = 1 \Rightarrow$$
$$q_{f_1} = q_{f_2} + 1 = \frac{1}{3},$$
$$q_{\psi_1} = q_{\psi_2} + 1 = 0. \tag{5.4}$$

Indicating the charges explicitly, we have, for the $SU(2)_L$ doublets,

$$\text{ZY}: F_L = \begin{pmatrix} f_1^\tau(1/2) \\ f_2^\tau(-1/2) \end{pmatrix}_L, \qquad \psi_L = \begin{pmatrix} \psi_1(1/2) \\ \psi_2(-1/2) \end{pmatrix}_L, \tag{5.5}$$

$$\text{SMY}: F_L = \begin{pmatrix} f_1^\tau(2/3) \\ f_2^\tau(-1/3) \end{pmatrix}_L, \qquad \psi_L = \begin{pmatrix} \psi_1(0) \\ \psi_2(-1) \end{pmatrix}_L, \tag{5.6}$$

and

$$\text{RSMY}: F_L = \begin{pmatrix} f_1^\tau(1/3) \\ f_2^\tau(-2/3) \end{pmatrix}_L, \qquad \psi_L = \begin{pmatrix} \psi_1(1) \\ \psi_2(0) \end{pmatrix}_L, \tag{5.7}$$

with corresponding charge assignments for the f_{jR}^τ and ψ_{jR}. Note that in the SMY case, even though the technifermions f_1 and f_2 have the same electric charges as the up-type and down-type quarks, respectively, they cannot mix with these quarks, since this would violate the exact $SU(3)_c$ color symmetry (as well as the exact $SU(3)_{\rm TC}$ technicolor symmetry). The same statement applies for the RSMY case, where f_2^c and f_1^c have the same electric charges as the up-type and down-type quarks, respectively. The SMY assignments coincide with the usual ones in the standard model, and the RSMY assignments are obtained by reversing the signs of the hypercharges in the SMY case. Only for the SMY and RSMY cases is one of the ψ_j neutral; for the SMY and RSMY choices, respectively, this is ψ_1 and ψ_2, as indicated in the equations above [28]. Note that one can equivalently describe the RSMY case in terms of charge-conjugated fermions with SMY hypercharge assignments, viz., $F_{\tau,R}^c$, with $Y = 1/3$, L_R^c with $Y = -1$, etc. In this form, the technifermions would transform as conjugate fundamental, rather than fundamental, representations of $SU(3)_{\rm TC}$. However, without loss of generality, we will keep the forms as in Eq. (5.4).

The others in the continuous one-parameter set of solutions of the anomaly cancellation condition (5.1) form four different classes:

$$\text{I: } Y_{\psi_L} > 1, \quad \text{so} \quad q_{\psi_i} > 0 \quad \text{for both } i = 1 \quad \text{and} \quad i = 2 \tag{5.8}$$

$$\text{II: } Y_{\psi_L} < -1, \quad \text{so} \quad q_{\psi_i} < 0 \quad \text{for both } i = 1 \quad \text{and} \quad i = 2 \tag{5.9}$$

$$\text{III: } -1 < Y_{\psi_L} < 0, \quad \text{so} \quad 0 < q_{\psi_1} < \frac{1}{2} \quad \text{and} \quad -1 < q_{\psi_2} < -\frac{1}{2} \tag{5.10}$$

and

$$\text{IV: } 0 < Y_{\psi_L} < 1, \quad \text{so} \quad \frac{1}{2} < q_{\psi_1} < 1 \quad \text{and} \quad -\frac{1}{2} < q_{\psi_2} < 0. \tag{5.11}$$

Restrictions on these hypercharge assignments will be given below.

B. Option of augmenting the model for walking behavior

As so far constructed, the $SU(3)_{\rm TC}$ theory has only $N_{tf} = 2$ (Dirac) technifermions, which is well below the range of values of N_{tf} where continuum and lattice studies indicate that walking behavior occurs. Since this walking property is desirable to enhance SM fermion masses (provided that the associated mass anomalous dimension is, in fact, not too small), one thus looks for ways to augment the fermion content of the theory so as to produce walking. In order for this theory to have walking, one would choose the content of technifermions to be such as to yield an approximate IR fixed point at a value $\alpha_{\rm TC} = \alpha_{IR}$ that is slightly greater than the minimum value for spontaneous chiral symmetry breaking, denoted α_{cr}. The key fact that one can make use of is that although some technifermions must be nonsinglets under $G_{\rm EW}$ with representations as given in Eqs. (3.1) and (3.2), other technifermins may be $G_{\rm EW}$-singlets and, indeed, fully $G_{\rm SM}$-singlets. Since technifermions transforming like those in Eqs. (3.1) and (3.2) contribute to W and Z propagator corrections, and since one would like to minimize these contributions, one is thus naturally led to choose any additional technifermions to be $G_{\rm EW}$-singlets. These must also be color-singlets, since otherwise some technivector mesons would transform as color-octets, and one would encounter the same problem with the LHC lower limits on the masses of such particles that one does with one-family technicolor models. So the additional technifermions that would be added for walking behavior should be $G_{\rm SM}$-singlets. This type of device has been used before to get walking, e.g., in [5]. Although it is not mandatory to take these $G_{\rm SM}$-singlet technifermions to transform according to the fundamental representation, we shall do so here because this makes possible a simpler embedding of the technicolor model in the eveloping extended technicolor theory. As we shall show in a later section, the structure of the ETC ultraviolet extension of the model is strongly affected by whether one includes or does not include these additional $G_{\rm SM}$-singlet, technisinglet fermions.

In contrast to 1FTC models, in which all technifermions are $G_{\rm SM}$-nonsinglets, in the present type of model some technifermions are not just color-singlets, but also

electroweak singlets. This fact, together with the fact that the technicolor gauge interaction is vectorial, means that, at the technicolor level, no gauge symmetry forbids these $G_{\rm SM}$-singlet technifermions from having nonzero Lagrangian masses in the effective Lagrangian that is operative slightly above TeV scale. Consequently, there are, in principle, two parameters that we may choose in determining the structure of the $G_{\rm SM}$-singlet technifermion sector of the augmented model, namely, the overall number of such fermions, and their masses. We begin with a discussion of the case in which all of the technifermions have zero Lagrangian masses and then proceed to remark on the more general case in which some $G_{\rm SM}$-singlet technifermions have nonzero Lagrangian masses. Throughout this discussion, it should be recalled that all technifermions gain dynamical masses of order $\Lambda_{\rm TC}$ from the confinement and formation of chiral-symmetry-breaking bilinear condensates that form in the technicolor theory.

Analyses of the ladder approximation to the Dyson-Schwinger equation for the technifermion propagator suggest that $\alpha_{cr} = \pi/(3C_f)$, where C_f is the quadratic Casimir invariant for the technifermion representation R of SU($N_{\rm TC}$) [19]. Setting this equal to the two-loop value of $\alpha_{\rm IR}$ yields an estimate for $N_{tf,cr}$, defined as the value of N_{tf} such that, for $N_{tf} < N_{tf,cr}$ and $N_{tf} > N_{tf,cr}$, the theory evolves into the IR with SχSB and without SχSB, respectively. In the former case, where SχSB occurs, the IRFP is approximate, since when the technifermions condense and gain dynamical masses, one integrates them out in the effective low-energy field theory applicable below the confinement and condensation scale $\Lambda_{\rm TC}$, so that the beta function changes to a pure gauge beta function, which has no perturbative IRFP [29]. In contrast, in the latter case of no SχSB, the IRFP is exact, and the theory is conformal in the IR. For $N_{\rm TC} = 3$, this method yields the estimate $N_{tf,cr} \simeq 12$ [19]. Although the standard analysis of the Dyson-Schwinger equation neglects instantons, which enhance spontaneous chiral symmetry breaking, it may still achieve reasonable accuracy because it also neglects an effect (having to do with a reduction in the integration interval over virtual Euclidean momenta due to confinement of the technifermions) that goes in the opposite direction, reducing the tendency to SχSB [30]. There have been a number of recent lattice studies of this theory [31]. Higher-order calculations, up to four-loop order, of the IR zero of the beta function, i.e., the value of the approximate or exact IRFP, and of the technifermion mass anomalous dimension γ evaluated at this zero, have also been given [32,33]. Since the model, as constructed so far, has $N_{wk} = 2$ Dirac technifermions in Eqs. (3.1) and (3.2), one would envision adding $N_{tf,cr} - 2$ additional massless Dirac technifermions. One would choose these additional technifermions to be singlets under $G_{\rm EW}$ (as well as SU(3)$_c$) to ensure that they do not contribute to modifications of the W and Z propagators. The number of SU(2)$_L$-doublet technifermions is denoted $N_{tf,ewd}$ and the additional, $G_{\rm EW}$-singlet (ews), technifermions is denoted as $N_{tf,ews}$, so that the total number of technifermions in the theory is $N_{tf} = N_{tf,ewd} + N_{tf,ews} = 2 + N_{tf,ews}$.

More generally, one could allow the possibility that some of the $G_{\rm SM}$-singlet technifermions may have nonzero Lagrangian masses. A constraint on these masses is that they should be small enough, relative to the scale Λ_w where $\alpha_{\rm TC}$ grows to O(1), so that the technifermions still contribute enough to the beta function coefficients to give rise to the approximate IRFP that, in turn, yields walking behavior. For if this condition were not met, i.e., if some technifermion masses were larger than Λ_w, then they would have been integrated out of the low-energy effective theory applicable at scales below Λ_w, and thus would not contribute to the beta function in this theory. In the absence of all Lagrangian masses for the technifermions, if one turned off all other gauge interactions, this theory would have a large (nonanomalous) global chiral symmetry group SU(N_{tf})$_L \otimes$ SU(N_{tf})$_R \otimes$ U(1)$_V$, which would be spontaneously broken by the various condensates, giving rise to various $G_{\rm SM}$-singlet Nambu-Goldstone bosons. Since NGBs have derivative couplings, their interaction amplitudes are attenuated at low energies $\sqrt{s}$ by factors of $\sqrt{s}/f_{\rm TC} \propto \sqrt{s}/\Lambda_{\rm TC}$. The presence of nonnegligible and, in general, nondegenerate, Lagrangian masses for these electroweak-singlet technifermions would reduce the formal global chiral symmetry group and increase the masses of the (P)NGBs.

C. Instanton breaking of number symmetries

We next discuss the SU(2)$_L$ instanton-induced breaking of certain global number symmetries. By an extension of the analysis carried out in Ref. [34], we see that, in addition to the breaking of quark number N_q and baryon number, $B = N_c N_q = 3N_q$ by SU(2)$_L$ instantons, these also break the number symmetry associated with the ψ_L fields and the SU(2)$_L$-doublet technifermions, and hence also total technibaryon number, $B_{\rm TC}$, even though a subset of the fermions contributing to this are SU(2)$_L$-singlets. At temperatures low compared with the electroweak scale, these SU(2)$_L$ instantons are exponentially suppressed, but at temperatures higher than this scale, they are not suppressed [35].

D. Phenomenology of and constraints on ψ fermions

The color-singlet, technisinglet ψ fermions are a notable feature of this type of TC/ETC model, because if one retains the normal property of the SM, that quarks and leptons come in families for each generation, then one cannot assign a generation index to these ψ fermions, since there is no corresponding fourth generation of quarks. Note that, even before dealing with phenomenological constraints on a fourth generation of quarks, one would

not want to add them to this model, since this reinstates the problem of an odd number of $SU(2)_L$ doublets. Thus, with the ψ fermions, one has a qualitatively new kind of (technisinglet) fermion, namely, one with no usual generational index. Alternately, if one were to consider the ψ fermions as a fourth generation of leptons, then this would, *ipso facto*, redefine the meaning of the term "generation", which hitherto had meant a family of SM quarks and leptons. This has important implications for an ETC theory, since it requires one to postulate a new kind of ETC gauge-mediated transition between the ψ fermions and technifermions that does not involve the usual generational index. This transition is necessary in order for ETC interactions to give masses to these ψ fermions. Indeed, the masses of ψ_1 and ψ_2 must be quite large in order not to conflict with current lower mass limits from LEP and hadron colliders. (Actually, ψ_1 and ψ_2 are group eigenstates and not, in general, mass eigenstates; here and below, when we refer to the masses of ψ_j, $j = 1, 2$, we mean the primary mass eigenstates in these group eigenstates.)

The phenomenology of the ψ fermions depends on the hypercharge assignment that is made to define the model. We proceed to derive constraints on this assignment. For the hypercharge assignments ZY and I–IV, neither ψ_1 nor ψ_2 is electrically neutral. In these cases, no mixing can occur between ψ_1 or ψ_2 (or their conjugates) and the SM leptons, and, as a consequence, the lighter member of the set $\{\psi_1, \psi_2\}$ is stable. We denote this lighter (ℓ) member as ψ_ℓ and the heavier (h) member as ψ_h. In the early universe, as the temperature T decreases below the scale of the mass m_{ψ_ℓ}, there will generically be residual ψ_ℓ's or their charge conjugates, ψ_ℓ^c's, depending on initial ψ-number asymmetries and physics in the UV completion of the theory that could give rise to such asymmetries. First, let us consider the case in which there is a residual population of ψ_ℓ fermions. There are then two subcases to analyze. With hypercharge assignments for which the ψ_ℓ is negatively charged, as the temperature cools sufficiently, this fermion will form Coulombic bound states with protons, $(p\psi_\ell)$. With hypercharge assignments for which the ψ_ℓ is positively charged, this fermion will form Coulombic bound states with electrons, $(e\psi_\ell)$. We treat these subcases in sequence. For the subcase with $q_{\psi_\ell} < 0$, which leads to a $(p\psi_\ell)$ bound state, the binding energy in the ground state is, to lowest order in $\alpha_{\rm em}$, [36],

$$E_C[(p\psi_\ell)] = \frac{q_{\psi_\ell}^2 \alpha_{\rm em}^2 \mu_{p\psi_\ell}}{2} \simeq \frac{q_{\psi_\ell}^2 \alpha_{\rm em}^2 m_p}{2}, \qquad (5.12)$$

where μ_{ij} is the reduced mass

$$\mu_{ij} = \frac{m_i m_j}{m_i + m_j}. \qquad (5.13)$$

To infer the last equality of Eq. (5.12), we have used the fact that $\mu_{p\psi_\ell} \simeq m_p$, since $m_{\psi_\ell} \gg m_p$ as required by current data. Hence, numerically,

$$E_C[(p\psi_\ell)] = (25.0\ {\rm keV}) q_{\psi_\ell}^2. \qquad (5.14)$$

Similarly, for the subcase with $q_{\psi_\ell} > 0$, which leads to a $(e\psi_\ell)$ bound state, the binding energy in the ground state is

$$E_C[(e\psi_\ell)] = \frac{q_{\psi_\ell}^2 \alpha_{\rm em}^2 \mu_{e\psi_\ell}}{2} \simeq \frac{q_{\psi_\ell}^2 \alpha_{\rm em}^2 m_e}{2}, \qquad (5.15)$$

where here $\mu_{e\psi_\ell} \simeq m_e$, since $m_{\psi_\ell} \gg m_e$. Hence, numerically, for this subcase,

$$E_C[e\psi_\ell)] = (13.6\ {\rm eV}) q_{\psi_\ell}^2. \qquad (5.16)$$

These Coulombic bound states would thus form as $k_B T$ decreases below the respective E_C values in Eq. (5.14) or (5.16). They would be stable heavy states with masses in excess of 100 GeV and nonintegral electric charges. There could also be Coulombic bound states involving multiple ψ_ℓ's with higher-Z nuclei in the case where $q_{\psi_\ell} < 0$. Extensive searches for massive states with exotic, nonintegral charges have been carried out in matter (often as part of free quark searches), reaching very stringent upper limits on their concentration, measured in terms of the number fraction $N_{\rm ec}/N_{\rm nuc}$, where ec denotes exotic-charge, and $N_{\rm ec}$ and $N_{\rm nuc}$ are the respective numbers of exotic-charge particles and nucleons in a given sample [25,37–39]. Let us denote $Q_{\rm ec}$ as the charge of the exotically charged (ec) particle (in units of e, as usual). These 95% CL upper bounds include [38]

$$\frac{N_{\rm ec}}{N_{\rm nucl}} < 1.17 \times 10^{-22} \quad {\rm for}\ 0.18 < |q_{\rm ec}| < 0.82, \qquad (5.17)$$

and [37]

$$\frac{N_{\rm ec}}{N_{\rm nucl}} < 4.71 \times 10^{-22} \quad {\rm for}\ |q_{\rm ec}| > 0.16. \qquad (5.18)$$

Many experiments looking for particles with exotic electric charges have been motivated by the search for free quarks, and hence have focused on the values $|q| = 1/3$ and $|q| = 2/3$ (in units of e). Some of these have reported considerably more stringent upper limits on $N_{\rm ec}/N_{\rm nucl}$, extending down to $\sim 10^{-26}$ [40]. A remark is in order here concerning the possibility that the hypercharge assignments are such that $|q_{\psi_\ell}| \ll 1$. There have been a number of searches for such electrically charged particles with charges whose magnitude is much smaller than 1 (in units of e), often called "milli-charged" particles [41]. These have again set very stringent upper limits on such particles. Recent reviews of searches for fractionally charged particles include Refs. [25,39].

To complete our discussion, we consider the other case, where the ψ-number asymmetry is such that there is a residual abundance of ψ_ℓ^c rather than ψ_ℓ fermions. Then a similar argument applies. With hypercharge assignments for which the ψ_ℓ^c is negatively charged, as the temperature cools sufficiently, this fermion will form Coulombic bound states with protons, $(p\psi_\ell^c)$, and for hypercharge assignments for which the ψ_ℓ^c is positively charged, this fermion

will form Coulombic bound states with electrons, $(e\psi_\ell^c)$. As before, these are ruled out down to extremely low number densities by experimental searches. These limits disfavor hypercharge assignments for which neither ψ_1 nor ψ_2 is electrically neutral.

Combining these results, we infer that the hypercharge assignments ZY and I–IV are generically disfavored by upper limits on exotic-charged particles in matter. This leaves the discrete hypercharge assignments (5.3) and (5.4) as being generically allowed. We note some caveats concerning this exclusion result. First, it is, in principle, possible that, in contrast with normal matter, there was a negligibly small ψ_ℓ-particle number asymmetry in the early universe and the ψ_ℓ and ψ_ℓ^c particles annihilated to very high precision (before forming Coulombic bound states), leaving an undetectably small residual population of these fermions or antifermions. A second type of exception would hold for values of hypercharge such that the magnitude of the charge $|q_{\psi_\ell}|$ is extremely close to 1 and m_{ψ_ℓ} happens to be such that the Coulombic bound states could be experimentally indistinguishable from neutral atoms of a usual heavy nucleus. However, we also note that the type of reasoning that we have used to disfavor various hypercharge assignments is evidently more general than the particular case of $N_{\rm TC} = 3$ and technifermions in the fundamental representation, and can also be applied to other TC/ETC models [42]. A comment is in order concerning possible Coulombic bound states of the ψ fermions with technibaryons. As will be discussed below, the lightest technibaryon is likely to be electrically neutral. Hence, it is unlikely that such bound states would form.

We discuss some further phenomenology pertaining to the two allowed hypercharge cases denoted SMY and RSMY. For the SMY case, the ψ_1 is electrically neutral and would contribute to the invisible decay width of the Z unless its mass is greater than $m_Z/2$, and similarly for the ψ_2 in the RSMY case. The measurement of the invisible width of the Z by LEP I and its consistency with three light $SU(2)_L$-doublet neutrinos thus implies that in these two respective cases, $m_{\psi_\ell} > m_Z/2$, so that these decays are kinematically forbidden. Even stronger lower bounds have been obtained from analyses of LEP II data, which imply that such a fourth $SU(2)_L$-doublet lepton-like fermion, either neutral or charged, must have a mass greater than about 90–110 GeV, where the range reflects model-dependent details of how these mix with, and couple to, the known leptons [25,43].

To give the ψ_i masses that are large enough to satisfy these experimental constraints poses a challenge for this type of model. With such large masses, one must also be careful that the model yields a value of the ratio m_{ψ_1}/m_{ψ_2} that is sufficiently close to unity to avoid an excessively large contribution to the ρ parameter measuring the violation of custodial symmetry. We discuss this further below.

For the SMY case, ψ_1 and ψ_2 can mix with the usual three generations of neutrinos and charged leptons, respectively. These mixings affect the weak charged-current decays of the ψ_j, $j = 1, 2$. Moreover, there will also be decays of the ψ_j, $j = 1, 2$, that are mediated by weak neutral currents. To see why these occur, we recall that the necessary and sufficient conditions for the diagonality of the leptonic neutral weak current are that leptons of a given electric charge and chirality must have the same weak T and T_3 (equivalently, weak T and Y) [44]. In general, the presence of electroweak-singlet neutrinos renders the leptonic neutral weak current nondiagonal, and hence this is true for the present model, both because of the ν_R^i in the standard model augmented to include neutrino masses and because of the ψ_{1R} and ψ_{2R}. This follows because we can write a ν_R as ν_L^c for any generation, and ψ_{jR} as ψ_{jL}^c. Consequently, in addition to charged-current decays, there are also has neutral-current decays of these leptons. A similar discussion applies for the RSMY case, where charge conservation allows the ψ_2 to mix with neutrinos and the ψ_1 to mix with charged leptons.

E. Some phenomenology of the technihadrons

Another topic of interest is the technihadrons in the model. We begin with the technibaryons. There are several possibilities here. As noted above, the f_i have zero Lagrangian masses, since these would break the electroweak gauge symmetry. Because of the formation of the technifermion condensates (4.10) at $\Lambda_{\rm TC}$, these technifermions f_1 and f_2 gain dynamical masses that are, up to small hypercharge corrections, equal to each other. There are several resultant spin-$1/2$ and spin-$3/2$ technibaryons for this $N_{\rm TC} = 3$ case. The classification of these is similar to the classification of the usual baryons composed of u and d quarks. The lightest technibaryons would be the spin-$1/2$ techninucleons (using lower indices),

$$p_{\rm TC} = (f_1 f_1 f_2), \qquad n_{\rm TC} = (f_2 f_2 f_1). \tag{5.19}$$

These have electric charges

$$\begin{aligned} q_{p_{\rm TC}} &= 1, \qquad q_{n_{\rm TC}} = 0 \quad \text{for SMY case} \\ q_{p_{\rm TC}} &= 0, \qquad q_{n_{\rm TC}} = -1 \quad \text{for RSMY case} \end{aligned} \tag{5.20}$$

Since the technifermions have zero Lagrangian masses, and gain dynamical masses that are equal (of order $\Lambda_{\rm TC}$), up to small electromagnetic corrections, the techniproton and technineutron are almost degenerate, with masses given, to leading order, by

$$\frac{m_{p_{\rm TC}, n_{\rm TC}}}{m_{p,n}} \simeq \frac{\Lambda_{\rm TC}}{\Lambda_{\rm QCD}} \simeq \frac{f_{\rm TC}}{f_\pi}\left(\frac{N_c}{N_{\rm TC}}\right)^{1/2}. \tag{5.21}$$

Hence, with $f_{\rm TC} \simeq 125$ GeV, and $N_{\rm TC} = 3$, it follows that

$$m_{p_{\rm TC}, n_{\rm TC}} \simeq 1.25 \text{ TeV}. \tag{5.22}$$

There would be an electromagnetic mass splitting between these techninucleons (TCNs) of order

$$|m_{p_{\rm TC}} - m_{n_{\rm TC}}| \sim \frac{\alpha_{\rm em}}{R_{\rm TCN}} \sim \alpha_{\rm em}\Lambda_{\rm TC} \sim \text{few GeV}, \quad (5.23)$$

where $R_{\rm TCN}$ is the spatial size of a techninucleon. The techninucleon that is charged (viz., $p_{\rm TC}$ for the SMY case and $n_{\rm TC}$ for the RSMY case) is heavier, because of its Coulombic self-energy [45]. For the SMY case, the $p_{\rm TC}$ would thus decay via a weak charged-current transition to the $n_{\rm TC}$ via the channels

$$\begin{aligned} \text{SMY: } & p_{\rm TC} \to n_{\rm TC} + e^+ + \nu_e, \\ & p_{\rm TC} \to n_{\rm TC} + \mu^+ + \nu_\mu, \\ & p_{\rm TC} \to n_{\rm TC} + \{\text{hadrons}\}^+, \end{aligned} \quad (5.24)$$

where in the last line, $\{\text{hadrons}\}^+$ refers to the possible hadronic final states that can be produced with a few GeV of energy, including π^+, $\pi^+\pi^0$, ρ^+, and states with higher pion multiplicity. If the mass splitting between $p_{\rm TC}$ and $n_{\rm TC}$ is large enough, the decay $p_{\rm TC} \to n_{\rm TC} + \tau^+ + \nu_\tau$ might also occur, although it would, in any case, be suppressed by the small phase-space available. Since both the masses and the magnitude of the mass splitting for these techninucleons are larger than those of the actual nucleons by the factor $f_{\rm TC}/f_\pi$, a rough estimate of the decay rate $\Gamma(p_{\rm TC} \to n_{\rm TC} + \ell^+ + \nu_\ell)$ for $\ell = e$ or $\ell = \mu$ could be obtained by simple scaling as

$$\Gamma(p_{\rm TC} \to n_{\rm TC} + \ell^+ + \nu_\ell) \simeq \Gamma(n \to p + e^- + \bar{\nu}_e)\left(\frac{\Lambda_{\rm TC}}{\Lambda_{\rm QCD}}\right)^5, \quad (5.25)$$

where $\Gamma(n \to p + e^- + \bar{\nu}_e) = 1/\tau_n$, with $\tau_n = 0.886 \times 10^3$ sec. and $\Lambda_{\rm TC}/\Lambda_{\rm QCD} \simeq (f_{\rm TC}/f_\pi)\sqrt{N_c/N_{\rm TC}} = f_{\rm TC}/f_\pi$ in the present model with $N_{\rm TC} = 3$. For the inclusive weak decay rate, neglecting phase-space suppressed modes, we can estimate $\Gamma(p_{\rm TC}) = (2 + N_c)\Gamma(p_{\rm TC} \to n_{\rm TC} + \ell^+ + \nu_\ell)$. Combining this with Eq. (5.25), we obtain the estimate for the lifetime

$$\tau_{p_{\rm TC}} \sim \frac{1}{5}\left(\frac{f_\pi}{f_{\rm TC}}\right)^5 \tau_n \simeq 10^{-15} \text{ sec.} \quad (5.26)$$

The time t in the early universe by which the temperature T has decreased to $T \sim m_{p_{\rm TC}} \sim 1$ TeV is $t \sim 10^{-12}$ sec. After this time, within a few e-foldings of the lifetime $\tau_{p_{\rm TC}}$, most of the $p_{\rm TC}$ techninucleons would have decayed to $n_{\rm TC}$s's.

An analogous discussion, with obvious changes, applies for the case of RSMY hypercharge assignments. Thus, here,

$$\begin{aligned} \text{RSMY: } & n_{TC} \to p_{\rm TC} + e + \bar{\nu}_e, \\ & n_{\rm TC} \to p_{\rm TC} + \mu + \bar{\nu}_\mu, \\ & n_{\rm TC} \to p_{\rm TC} + \{\text{hadrons}\}^-. \end{aligned} \quad (5.27)$$

Similarly, the $n_{\rm TC}$ lifetime, $\tau_{n_{\rm TC}}$, for the RSMY case, would be essentially equal to $\tau_{p_{\rm TC}}$ for the SMY case. In each of these two respective cases, the lightest technibaryon would be stable against weak decay. Although this technibaryon would be electrically neutral, it would not be a weakly interacting massive particle (WIMP), for several reasons. First, it is composed of electrically charged technifermions, and these could interact with a photon via magnetic and electric form factors, just as is the case with the actual neutron. Second, it would have residual strong interactions via exchange of technipions (the longitudinal components of W and Z bosons), on the length scale $\sim 1/m_{W,Z}$ and further strong residual interactions via exchange of technivector mesons, on the length scale of $\sim 1/(1\ \text{TeV})$. Some related work on technibaryons as possible sources of dark matter is in Refs. [6,46,47]. We shall not pursue this topic here, but it merits further study.

There would also be heavier, spin-3/2 technibaryons, split in mass from these techninucleons by the technigluon hyperfine interaction,

$$(f_1 f_1 f_1), \quad (f_1, f_1, f_2), \quad (f_1, f_2, f_2), \quad (f_2, f_2, f_2) \quad (5.28)$$

with respective charges 2, 1, 0, −1 and 1, 0, −1, −2 for the SMY and RSMY cases. The spectrum of the technicolor theory would also contain mesons with various $J^{\rm PC}$ values and technigluballs. The three true Goldstone bosons are absorbed by the $W^\pm$ and Z, but there would be some (pseudo) Nambu-Goldstone bosons (PNGBs) involving the additional $G_{\rm SM}$-singlet technifermions included for walking behavior. The masses of these PNGBs would depend on the bare masses that we assigned to these additional $G_{\rm SM}$-singlet technifermions. If these masses were small compared with $\Lambda_{\rm TC}$, so that the PNGBs were close to being true NGBs, they would be characterized by derivative couplings and hence would tend to decouple at energies low compared with $\Lambda_{\rm TC}$.

F. TC corrections to W and Z propagators

Although perturbation theory cannot be used to estimate the technicolor contribution to W and Z propagator corrections, since the technicolor interaction is strongly coupled at the mass scale m_W and m_Z, one nevertheless often refers to the perturbative estimate as a crude guide. The most important of these W and Z propagator corrections are embodied in the S and T parameters [48]. As background, we recall that the fermions in Eq. (3.1) and (3.2) have zero Lagrangian masses and gain dynamical masses from the confinement and spontaneous chiral symmetry breaking at the scale $\Lambda_{\rm TC}$, due to their technicolor

gauge interactions. Since $G_{\rm EW}$ interactions are quite weak at the scale $\Lambda_{\rm TC}$, these dynamical masses of f_1 and f_2 are equal, up to small corrections. In QCD, the constituent (dynamical) quark masses are roughly 330 MeV, while $f_\pi = 93$ MeV. If one takes the ratio of the dynamical technifermion mass divided by f_π to be roughly similar to the ratio (330 MeV)/(93 MeV) = 3.5 in QCD, then the technifermion dynamical mass $\Sigma_{\rm TC} \simeq 850$ GeV. Hence, $m_Z^2/\Sigma_{\rm TC}^2 \simeq 1 \times 10^{-2}$.

We recall that for the case with one doublet of fermions, as in Eq. (3.1) and (3.2) with masses that are approximately degenerate and are large compared with m_Z, the perturbative contribution to the S parameter is $\Delta S_{\rm pert} = 1/(6\pi)$. If the technicolor theory has N_D $\mathrm{SU}(2)_L$ doublets of technifermions and these transform as the representations R of $G_{\rm TC}$, then one has

$$(\Delta S)_{\rm pert} = \frac{N_D d_{R_{\rm TC}}}{6\pi}. \tag{5.29}$$

For the $\mathrm{SU}(3)_{\rm TC}$ theory, $d_{R_{\rm TC}} = 3$, so that the perturbative estimate of the contributions of the technifermions to S is $(\Delta S)_{\rm pert} = 1/(2\pi) \simeq 0.16$.

We also need to analyze the effects of the ψ fields. Because these are technisinglets, they are weakly interacting at the scale m_Z, so that their contributions to loop corrections to the W and Z propagators can be reliably calculated perturbatively. For the same reason, higher-loop corrections due to these ψ fields are expected to be reasonably small compared to the one-loop correction. We use the exact one-loop expression for the contribution to S, which is (e.g., [49])

$$\begin{aligned}(\Delta S)_\psi = \frac{1}{6\pi}[&2(3+2Y_{\psi_L})r_1 + 2(3-2Y_{\psi_L})r_2 \\ &- Y_{\psi_L}\ln(r_1/r_2) + \frac{1}{2}[(3+2Y_{\psi_L})r_1 + Y_{\psi_L}]G(r_1) \\ &+ \frac{1}{2}[(3-2Y_{\psi_L})r_2 - Y_{\psi_L}]G(r_2)],\end{aligned} \tag{5.30}$$

where

$$r_i = \left(\frac{m_{\psi_i}}{m_Z}\right)^2 \tag{5.31}$$

and

$$G(r) = -4\sqrt{4r-1}\,\arctan\left[\frac{1}{\sqrt{4r-1}}\right]. \tag{5.32}$$

Note that

$$\begin{aligned}&(\Delta S)_\psi \text{ is invariant under the interchange } m_{\psi_1} \leftrightarrow m_{\psi_2} \\ &\quad \text{with} \quad Y_{\psi_L} \to -Y_{\psi_L}.\end{aligned} \tag{5.33}$$

It follows that $(\Delta S)_\psi$ does not depend on Y_{ψ_L} if $m_{\psi_1} = m_{\psi_2}$. For the experimentally allowed range of m_{ψ_j}, $j = 1, 2$, lying about 100 GeV, the exact expression is well approximated by

$$(\Delta S)_{\rm pert} = \frac{1}{6\pi}\left[1 - 2Y_{\psi_L}\ln\left(\frac{m_{\psi_1}}{m_{\psi_2}}\right) + \frac{(1+4Y_{\psi_L})}{20}\left(\frac{m_Z}{m_{\psi_1}}\right)^2 + \frac{(1-4Y_{\psi_L})}{20}\left(\frac{m_Z}{m_{\psi_2}}\right)^2 + O\left(\frac{m_Z^4}{m_{\psi_i}^4}\right)\right]. \tag{5.34}$$

If ψ_1 and ψ_2 are nearly degenerate, they simply contribute an additional amount $1/(6\pi)$ to S, so that, combining this with the rough perturbative estimate of the technifermion contributions, one would obtain, as the estimate of the total new addition to S, the result $(\Delta S)_{\rm pert} = (3+1)/(6\pi) = 2/(3\pi) = 0.21$. Mixing of the W and Z with charged and neutral technivector mesons, respectively, also affects these corrections. The property of walking might reduce this contribution to S somewhat [50], but one already knows that in QCD-like theories, the full nonperturbative contribution to S is larger than the perturbative estimate by approximately a factor of 2 [48], so it could be challenging to try to reduce this sufficiently, even in a walking TC theory. The value $S \simeq 0.2$ is larger than the region of S values (forming a tilted elliptical region in an S-T plot [51]) favored by experiment.

Therefore, to minimize the ψ contributions to S, one would want to have the following mass orderings:

$$m_{\psi_1} < m_{\psi_2} \quad \text{for SMY case} \tag{5.35}$$

and

$$m_{\psi_1} > m_{\psi_2} \quad \text{for RSMY case.} \tag{5.36}$$

In both cases, the second term, $-2Y_{\psi_L}\ln(m_{\psi_1}/m_{\psi_2})$, in the square brackets in Eq. (5.34) is negative and helps to reduce the contribution to S from the first term. Note that in both of these cases, the mass orderings that minimize the contribution to S are such that the neutral member of the ψ doublet is lighter than the charged member. It will be useful to consider two illustrative sets of mass values,

$$\text{SMY}: \; m_{\psi_1} = 120 \text{ GeV}, \qquad m_{\psi_2} = 160 \text{ GeV} \tag{5.37}$$

and

$$\text{RSMY}: \; m_{\psi_1} = 160 \text{ GeV}, \qquad m_{\psi_2} = 120 \text{ GeV} \tag{5.38}$$

as well as a continuous variation of the heavier ψ in each case, with the lighter ψ fixed at the value of 120 GeV. The amount by which m_{ψ_2}/m_{ψ_1} can exceed unity in the SMY case, or m_{ψ_1}/m_{ψ_2} can exceed unity in the RSMY case, is constrained in at least two ways. First, it is a challenge in this model for the ETC interaction to produce such large masses for both ψ_1 and ψ_2, and this challenge is especially severe for the heavier of these, in the two respective cases. Given the experimental lower limit on the lighter of the two, m_{ψ_ℓ}, the more one tries to increase the ratio m_{ψ_h}/m_{ψ_ℓ}, the more of a problem it is to achieve this with credible ETC interactions. Second, the larger the ratio of, and the splitting between, the masses of the heavier and lighter of the ψs, the greater is the violation of custodial symmetry

and the larger is the contribution to the ρ parameter. This is given by

$$\Delta\rho = \frac{G_F}{8\pi^2\sqrt{2}} f(m_{\psi_1}^2, m_{\psi_2}^2) = \frac{f(m_{\psi_1}^2, m_{\psi_2}^2)}{16\pi^2 v^2}, \qquad (5.39)$$

where $v = 2m_W/g = 246$ GeV and

$$f(x,y) = x + y - \frac{2xy}{x-y}\ln\left(\frac{x}{y}\right). \qquad (5.40)$$

As is evident from Eq. (5.40), $f(x,y) = f(y,x)$ and $f(x,x) = 0$. The corresponding contribution to T is $\Delta T = \alpha_{\rm em}(m_Z)^{-1}\Delta\rho$. By convention, T is defined with the SM contribution from the t quark removed. Any new contribution is restricted experimentally to lie within the tilted elliptical region in the S, T plane [51]. With the illustrative mass values (5.37) and ((5.38) for the SMY and RSMY hypercharge assignments, respectively, we find that the additional contribution from the ψ fermions to S is 0.022. For comparison, if ψ_1 and ψ_2 had degenerate masses equal to the smaller value, $m_{\psi_1} = m_{\psi_2} = 120$ GeV, then this contribution would be 0.056, while if they had degenerate masses equal to the larger value, $m_{\psi_1} = m_{\psi_2} = 160$ GeV, then this contribution would be 0.055, so there is a significant reduction in S due to the nondegeneracy in masses of ψ_1 and ψ_2. Summing this with the technifermion contribution, we have, for the given hypercharge assignment and these illustrative sets of ψ_j values for the SMY and RSMY cases, the total perturbative estimate that the new $G_{\rm EW}$-nonsinglet fermions in this model contribute $(\Delta S)_{\rm pert} \simeq 0.18$. This is on the high side of the values preferred by current global precision electroweak fits, but appears to be admissible. For the same ψ_j masses, we find that the new contribution to T is 0.03, which is easily small enough to be allowed by experimental constraints. Thus, the main restriction on how large the ratio m_{ψ_h}/m_{ψ_ℓ} can be comes more from the difficulty of producing such a heavy ψ_h from reasonable ETC interactions than from the ρ (T) parameter.

Generalizing this analysis, in Fig. 1, we show the total estimate for ΔS from the technifermions and the ψ fermions, as a function of $m_{\psi_2} > m_{\psi_1}$ for the SMY case, with m_{ψ_1} fixed at the value of 120 GeV. In Fig. 2 we plot the corresponding contribution from the nondegenerate ψ_1 and ψ_2 fermions to T. The same figures apply for the RSMY case with m_{ψ_1} and m_{ψ_2} interchanged. If we restrict ΔT to be less than, say, 0.1, then this restricts the heavier of the ψ_js, i.e., m_{ψ_2} for SMY and m_{ψ_1} for RSMY, to be less than approximately 195 GeV.

With the mass ordering (5.35) for the SMY hypercharge case, the $(\psi_2)^-$ will decay to $(\psi_1)^0$ via a charged-current weak transition, via a virtual W^- which could produce $\ell^-\bar{\nu}_\ell$ with $\ell = e$, μ, τ, as well as $d\bar{u}$ and $s\bar{c}$. Similarly, with the mass ordering (5.36) for the RSMY hypercharge case, the $(\psi_1)^+$ will decay to $(\psi_2)^0$, producing $\ell^+\nu_\ell$, $u\bar{d}$, and $c\bar{s}$ final states. Treating these cases together, it follows

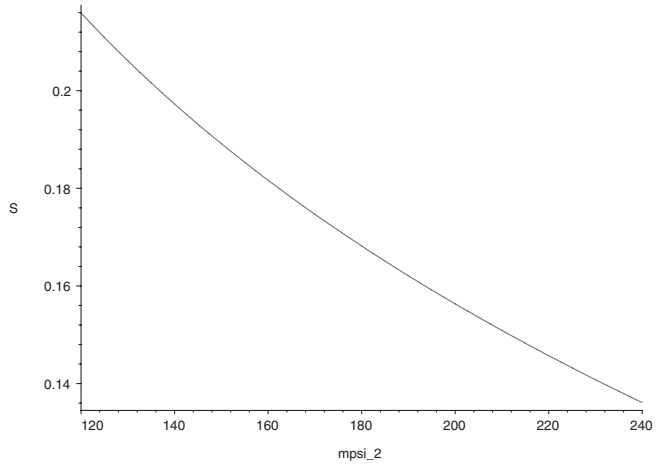

FIG. 1. Contribution to S from technifermions and the ψ fermions, as a function of m_{ψ_2} in units of GeV, with m_{ψ_1} fixed at the illustrative value $m_{\psi_1} = 120$ GeV, for the SMY hypercharge assignments of Eq. (5.3). The same figure applies to the RSMY hypercharge assignments with m_{ψ_1} and m_{ψ_2} interchanged.

that the inclusive weak decay rate of the ψ_h due to these charged-current decays would be $\Gamma(\psi_h) = (3 + 2N_c)\Gamma_\mu$, up to phase-space factors reflecting the substantial mass of ψ_ℓ relative to ψ_h. If we neglect mixing effects, then for the illustrative values (5.37), using a calculation of the phase-space suppression factor, of 3×10^{-3}, from Ref. [52], we obtain the rough estimate

$$\tau_{\psi_h} \simeq \frac{1}{9} \times (3 \times 10^2)\left(\frac{m_\mu}{m_{\psi_2}}\right)^5 \tau_\mu$$
$$\simeq (10^{-20}\ \text{sec})\left(\frac{160\ \text{GeV}}{m_{\psi_h}}\right)^5. \qquad (5.41)$$

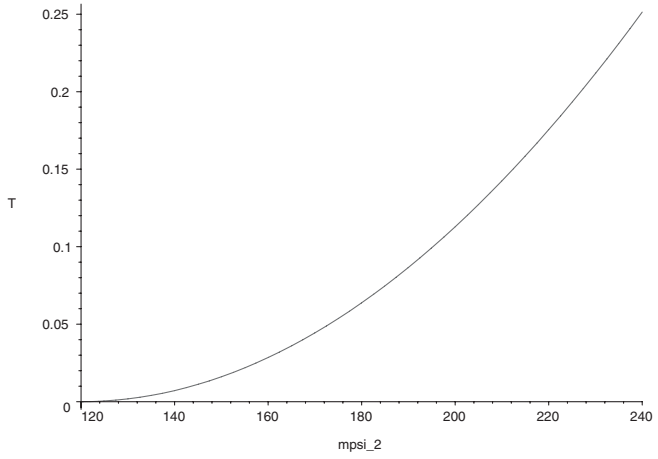

FIG. 2. Contribution to T from the ψ fermions, as a function of m_{ψ_2} in units of GeV, with m_{ψ_1} fixed at the illustrative value $m_{\psi_1} = 120$ GeV. The contribution to T is invariant under the interchange of m_{ψ_1} and m_{ψ_2}.

There will also be mixing effects that would allow ψ_h to undergo charged-current decays without such phase-space suppression, but this is already a very short lifetime. We note that, owing to the fact that the leptonic weak neutral-current contains nondiagonal terms [44], the ψ_h could also decay via neutral-current reactions, which would reduce its lifetime.

VI. EXTENDED TECHNICOLOR THEORIES CONTAINING 1DTC SECTORS

A. General

So far we have studied technicolor models which have a minimal, single $SU(2)_L$ doublet of left-handed technifermions, together with corresponding right-handed technifermions (all of which are color-singlets), as in Eqs. (3.1) and (3.2), focusing on the case of $G_{TC} = SU(3)_{TC}$ with technifermions in the fundamental representation. We have analyzed some phenomenological restrictions on this model and have noted the optional addition of a set of G_{SM}-singlet technifermions to get walking behavior. We now proceed to analyze properties of extended technicolor theories that contain these technicolor sectors. We use the term ''ultraviolet extension'' to refer to these, rather than the more ambitious term ''ultraviolet completion'', because additional ingredients would be needed to account fully for the precise values of the fermion masses and mixings, etc.

The basic purpose of extended technicolor is to communicate the electroweak symmetry breaking in the technifermion sector to the SM fermions, which are technisinglets, and thereby to give them masses. As was noted in the introduction, because some ETC gauge bosons transform as fundamental representations of $SU(3)_c$, it follows that commutators of the corresponding generators of G_{ETC} with their hermitian conjugates yield generators that transform according to the singlet and adjoint representation of $SU(3)_c$, which implies the structural property (1.1). With $G_{gen} = SU(3)_{gen}$ and $G_{TC} = SU(3)_{TC}$, Eq. (1.1) takes the form, for non-Abelian factor groups,

$$G_{ETC} \supset SU(3)_c \otimes SU(3)_{gen} \otimes SU(3)_{TC}. \tag{6.1}$$

In 1DTC models the ETC gauge bosons also carry weak hypercharge, Y. The structure of the ETC theory depends on whether or not one includes the additional G_{SM}-singlet fermions. To have a compact notation to refer to these two types of 1DTC theories, we introduce the abbreviations 1DTCM and 1DTCA for the minimal 1DTC model and the 1DTC model augmented with the additional G_{SM}-singlet technifermions. Although the 1DTCM model, without additional ingredients, does not exhibit walking behavior, it serves as a useful contrast to the 1DTCA model as regards respective ETC theories. The structural formula, Eq. (6.1) holds for both 1DTCM and 1DTCA models; however, as we will show, G_{ETC} also includes $SU(2)_L$ as a subgroup in the case of the 1DTCA model.

B. ETC Ultraviolet extension of a 1DTCM model

Although the exchanges of ETC gauge bosons that produce the masses of the SM fermions and of the ψ fermions involve strong coupling and nonperturbative physics, the quantum numbers carried by the ETC gauge bosons can be determined by an analysis of the basic perturbative vertices. For the 1DTCM model, one can group the various types of fermions of a given chirality $\chi = L, R$ in the two sets

$$[\{u^{ai}\}, \{\nu^i\}, f_1^\tau, \psi_1]_\chi \tag{6.2}$$

and

$$[\{d^{ai}\}, \{\ell^i\}, f_2^\tau, \psi_2]_\chi, \tag{6.3}$$

where a, i, and τ are color, generation, and technicolor gauge indices, and $\{u^{ai}\}$ and $\{d^{ai}\}$ denote the respective sets of $Q = 2/3$ and $Q = -1/3$ quarks, each with $N_{gen}N_c = 9$ members, $\{\nu^i\}$ and $\{\ell^i\}$ denote the corresponding sets of $N_{gen} = 3$ neutrinos and leptons, and the other fermions were given in Eqs. (3.1), (3.2), and (4.3). Some ETC-mediated transitions operate within each set. Among these are, first, vectorial transitions of the form $q^{ai} \to q^{bi}$, where $q = u$ or $q = d$, mediated by the color gluons in the $SU(3)_c$ subgroup of Eq. (1.1). Secondly, there are vectorial technicolor transitions $f_j^\tau \to f_j^{\tau'}$, $j = 1, 2$, mediated by the technigluons of the G_{TC} subgroup in Eq. (1.1). Third, there are transitions involving generational indices, $q^{ai} \to q^{aj}$, where $q = u$ or $q = d$, $\nu^i \to \nu^j$, and $\ell^i \to \ell^j$, involving ETC gauge bosons in the $SU(3)_{gen}$ subgroup in Eq. (1.1). Then there are the $\binom{4}{2} = 6$ types of ETC-mediated transitions between each group of fermions in the set (6.2) and, separately, in the set (6.3). Of these six types of transitions, three enable the SM fermions and the ψ fermions to make transitions to technifermions and hence pick up masses. The other three involve transitions between the subsets of fermions $\{u^{ai}\}$, $\{\nu^j\}$, and ψ_1 on the one hand, and, separately, between $\{d^{ai}\}$, $\{\ell^j\}$, and ψ_2. Moreover, various commutators of the nondiagonal ETC generators corresponding to these gauge bosons and their hermitian conjugates produce diagonal generators in Cartan subalgebras of G_{ETC}.

An important property of the 1DTCM model is that there is a 1–1 correspondence between the fermions in the set (6.2) and in the set (6.3). This reflects a kind of left-right extension of the basic $SU(2)_L$ symmetry according to which the upper member of an $SU(2)_L$ doublet can be transformed into the lower member of the same doublet. In particular, this means that all ETC-mediated transitions occur between chiral fermions that transform in the same way under $SU(2)_L$ (as doublets for all left-handed fermions, and singlets for all right-handed fermions). Therefore,

in the 1DTCM model, all ETC gauge bosons are $SU(2)_L$ singlets, and

$$[G_{\rm ETC}, {\rm SU}(2)_L] = \varnothing \quad \text{for 1DTCM model.} \tag{6.4}$$

This commutativity does not hold in 1DTCA models, as will be discussed below.

An analysis of the basic vertices and associated transitions determines the quantum numbers of the ETC gauge bosons. In addition to the gauge bosons in the $SU(3)_c$, $SU(3)_{\rm gen}$, and $SU(3)_{\rm TC}$ subgroups of $G_{\rm ETC}$, we have, for both 1DTCM and 1DTCA models, the following transitions involving fermions, where, as before, a, i, and τ are color, generation, and technicolor gauge indices and $\chi = L, R$:

$$u^{ai}_\chi \to f^\tau_{1\chi} + V^{ai}_\tau, \qquad d^{ai}_\chi \to f^\tau_{2\chi} + V^{ai}_\tau, \tag{6.5}$$

$$\nu^i_\chi \to f^\tau_{1\chi} + V^i_\tau, \qquad \ell^i_\chi \to f^\tau_{2\chi} + V^i_\tau, \tag{6.6}$$

and

$$\psi_{1\chi} \to f^\tau_{1\chi} + V_\tau, \qquad \psi_{2\chi} \to f^\tau_{2\chi} + V_\tau. \tag{6.7}$$

Three other types of transitions, with their associated ETC gauge bosons, are

$$u^{ai}_\chi \to \nu^j_\chi + V^{ai}_j, \qquad d^{ai}_\chi \to \ell^j_\chi + V^{ai}_j, \tag{6.8}$$

$$u^{ai}_\chi \to \psi_{1\chi} + V^{ai}, \qquad d^{ai}_\chi \to \psi_{2\chi} + V^{ai}, \tag{6.9}$$

and

$$\nu^i_\chi \to \psi_{1\chi} + V^i, \qquad \ell^i_\chi \to \psi_{2\chi} + V^i. \tag{6.10}$$

One can read off from these transitions the representation content of the associated ETC gauge bosons under the product group (6.1). We list these in Table I. Hermitian conjugates of ETC gauge bosons corresponding to nondiagonal generators are understood; for example, $V^\tau_{ai} = (V^{ai}_\tau)^\dagger$, etc. ETC gauge bosons corresponding to diagonal, Cartan generators occur in a block-diagonal manner in accordance with the subgroup structure of Eq. (6.1). Since the ETC gauge bosons are $SU(2)_L$-singlets for a 1DTCM model, it follows that their electric charges are given by $Q_V = Y_V/2$.

As for one-family TC/ETC models, in order for ETC interactions to account for the generational hierarchy in SM fermion masses, the $SU(3)_{\rm gen}$ part of the ETC gauge symmetry should break sequentially at scales Λ_i, $i = 1, 2, 3$, where i is a generation index. Typical values of these scales for the one-family TC/ETC models of Refs. [11–15] were listed in Eq. (2.5), and roughly similar values would apply here. At each stage of this sequential generational ETC symmetry breaking, the ETC gauge bosons corresponding to generators in the coset space gain masses of order the respective breaking scale. Thus, the ETC gauge bosons containing a generational index i gain masses of order Λ_i, and the ETC gauge bosons that contain two generational indices, such as V^i_j, gain masses of order Λ_k, where $k = \min(i, j)$. Thus, for example, V^1_2 and V^1_3 would gain masses $\sim\Lambda_1$, etc. Because the ψ fermions must gain masses greater than about 100 GeV, the V_τ ETC gauge bosons involved in the transitions connecting these ψ's with the technifermions must gain masses of order the lowest ETC symmetry-breaking scale, Λ_3. These properties are indicated in Table I. As with fermions, the actual mass eigenstates of the vector bosons resulting from ETC symmetry breaking would involve linear combinations of the ETC group eigenstates, in accordance with the symmetries that are operative at the given mass scale. (This is the ETC analogue of the mixing of the electroweak gauge bosons of $SU(2)_L$ and $U(1)_Y$ to form the physical vector boson mass eigenstates W and Z in the process of EWSB.)

We come next to the choice of a possible ETC group, $G_{\rm ETC}$, for this 1DTCM model. Here the ETC group is considerably more complicated than was the case for the one-family TC/ETC models, where one had the simplifying commutativity property $[G_{\rm ETC}, G_{\rm SM}] = \varnothing$ and the

TABLE I. Properties of ETC gauge bosons (g.b.) in 1DTCM models. Here $\{a, b\}$, $\{i, j\}$; and $\{\tau, \tau'\}$ are $SU(3)_c$ color, $SU(3)_{\rm gen}$, and $SU(3)_{\rm TC}$ gauge indices, respectively, and A denotes adjoint representation. In the comments column, seq. bk. refers to the fact that these gauge bosons have masses corresponding to the sequential breaking of the $SU(3)_{\rm gen}$ gauge symmetry at the scales Λ_i, $i = 1, 2, 3$. The Y_{F_L} values for the SMY and RSMY cases are given in Eqs. (5.11) and (5.18).

ETC g.b.	$SU(3)_c$	$SU(3)_{\rm gen}$	$SU(3)_{\rm TC}$	$U(1)_Y$	comments
V^a_b	A	1	1	0	exact color sym.
V^i_j	1	A	1	0	seq. bk.
$V^\tau_{\tau'}$	1	1	A	0	exact TC sym.
V^{ai}_τ	$\square$	$\square$	$\bar{\square}$	$1/3 - Y_{F_L}$	seq. bk.
V^i_τ	1	$\square$	$\bar{\square}$	$-1 - Y_{F_L}$	seq. bk.
V_τ	1	1	$\bar{\square}$	$-4Y_{F_L}$	bk., Λ_3
V^{ai}_j	$\square$	A	1	4/3	seq. bk.
V^{ai}	$\square$	$\square$	1	$1/3 + 3Y_{F_L}$	seq. bk.
V^i	1	$\square$	1	$-1 + 3Y_{F_L}$	seq. bk.

relation (2.2). One way to embed the $SU(3)_{gen}$ and $SU(3)_{TC}$ groups in an ETC group is to choose the latter to be $SU(N_{ETC})_{ETC}$ with

$$N_{ETC} = N_{gen}(N_c + 1) + N_{TC} + 1 = 12 + 3 + 1 = 16, \tag{6.11}$$

with the left-handed and right-handed chiral fermions assigned to the vectorlike representations of $SU(16)_{ETC}$,

$$\mathcal{F}_\chi = \begin{pmatrix} \{u^{ai}\} & \{\nu^i\} & f_1^\tau & \psi_1 \\ \{d^{ai}\} & \{\ell^i\} & f_2^\tau & \psi_2 \end{pmatrix}_\chi, \qquad \chi = L, R. \tag{6.12}$$

This construction is somewhat analogous to the $SU(14)_{ETC}$ model of Ref. [53], with the difference that here we use an $SU(3)_{TC}$ group rather than an $SU(2)_{TC}$ group for the reasons discussed above, which also necessitated the inclusion of the ψ fermions [54]. We also note the toy ETC model in [54] that focuses on the third generation and can account for the t quark mass.

Clearly, this $SU(16)_{ETC}$ group is a much more complicated ETC group than the $SU(5)_{ETC}$ group for the one-family TC/ETC theory analyzed in detail in Refs. [10–15]. To proceed, one would choose an appropriate set of additional ETC-nonsinglet fermions that would render the full ETC theory a chiral gauge theory. A necessary property of this set of additional ETC fermions would be that the ETC theory would be asymptotically free and, as the reference energy scale μ decreased from large values, some of them would form bilinear condensates in such a manner as to produce a sequential breaking of the ETC gauge symmetry down to the residual exact $SU(3)_{TC}$ subgroup. To account for the mass hierarchy of the three SM fermion generations, the ETC symmetry would break in a sequence of three scales, $\Lambda_{ETC,i}$, $i = 1, 2, 3$. Presumably, the breaking scales would be roughly comparable to those of Eq. (2.5). Since a formula similar to Eq. (2.7) also applies to the mass generation for the ψ_1 and ψ_2 fermions, and since these have to have quite large masses, it would be necessary that the V_τ ETC gauge bosons gain masses at approximately the $\Lambda_{ETC,3}$ scale, as indicated in Table I. Some ingredients for the requisite ETC gauge symmetry breaking could be adopted from the previous studies of one-family models, as well as generalizations thereof [55]. As in the models analyzed in Ref. [12], it would be necessary to break the left-right symmetry of the representations in Eq. (6.12) so as to avoid conflict with experimental upper limits on right-handed charged currents. As was demonstrated in [12], this can also produce the chirally nonsymmetric weak hypercharge assignments for SM fermions. It would also be necessary to address all of the usual issues with ETC models, including producing large enough SM fermion masses while respecting constraints from flavor-changing neutral-current processes, generating the large mass splitting between the t and b quarks, producing very small nonzero neutrino masses, designing the additional ETC-nonsinglet fermion sector in such a manner that the desired sequential breaking pattern and associated condensate formation is plausible, within the context of the most attractive channel formalism, etc.

C. ETC Ultraviolet extension of a 1DTCA model

In a 1DTCA model, the chiral fermions can be grouped into the sets (6.2) and (6.3) together with the set of G_{SM}-singlet technifermions, which we shall label $s^\tau_{p,\chi}$, where $\chi = L, R$ and p is a copy (flavor) index. Hence, in a 1DTCA model, the ETC-mediated transitions between a fermion in the set (6.2) with $\chi = L$ and $s^\tau_{p,L}$ or between a fermion in the set (6.3) with $\chi = L$ and $s^\tau_{p,L}$ involve the emission of an ETC gauge boson that transforms as the fundamental (doublet) representation of $SU(2)_L$. The commutators of the generators corresponding to these $SU(2)_L$-doublet ETC gauge bosons and their hermitian conjugates produce the singlet and adjoint representation of $SU(2)_L$. Hence, the gauge group of the ETC ultraviolet extension of a 1DTCA model is larger than that for a 1DTCM model. In particular, this means that Eq. (1.1) is expanded to

$$G_{ETC} \supset SU(3)_c \otimes SU(2)_L \otimes G_{gen} \otimes G_{TC} \quad \text{for a 1DTCA model,} \tag{6.13}$$

and, in contrast to the commutativity property (6.4) for a 1DTCM model, we have

$$[G_{ETC}, SU(2)_L] \neq \varnothing \quad \text{for a 1DTCA model.} \tag{6.14}$$

The electric charge of an ETC gauge boson in a 1DTCA model is given by the full formula $Q_V = T_{3,V} + (Y_V/2)$.

The quantum numbers of the ETC gauge bosons in a 1DTCA model can be worked out in a manner similar to those of a 1DTCM model. They include the gauge bosons in the 1DTCM model, as summarized in Table I, together with others, generically denoted as X-type gauge bosons, that are involved in transitions of fermions in the sets (6.2) and (6.3) to the $s^\tau_{p,\chi}$ fermions. For example, the transition

$$Q_L^{ai\alpha} \to s^\tau_{p,L} + X^{ai\alpha}_\tau, \tag{6.15}$$

where α is an $SU(2)_L$ gauge index, involves the emission of an ETC gauge boson $X^{ai\alpha}$ that transforms as the fundamental representation, $\square$, under $SU(3)_c$, $\square$ under $SU(3)_{gen}$, $\bar{\square}$ under $SU(3)_{TC}$, and $\square$ under $SU(2)_L$, with $Y = 1/3$. The quantum numbers of the other X-type ETC gauge bosons in a 1DTCA model can be worked out in a similar manner. Evidently, the ETC ultraviolet extension of a 1DTCA model is more complicated than the $SU(16)_{ETC}$ extension of the 1DTCM model.

VII. CONCLUSIONS

In this paper, we have investigated some TC/ETC models with color-singlet technifermions and a single $SU(2)_L$ doublet of technifermions. We have considered two types

of models, with and without additional G_{SM}-singlet technifermions. We have analyzed a number of constraints on these models, including constraints on hypercharge assignments for the technifermions and for the associated color-singlet, technisinglet fermions ψ arising from the necessity to avoid exotically charged Coulombic bound states, on which there are very stringent experimental upper limits. We have also determined some properties of ETC ultraviolet extensions of these technicolor models. The results are of use for further studies of theories with dynamical electroweak symmetry breaking. Data that are forthcoming from the LHC will soon elucidate whether electroweak symmetry breaking is, indeed, dynamical.

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