

Reconciling anomalous measurements in $B_s - \bar{B}_s$ mixing: The role of CPT -conserving and CPT -violating new physics

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Recently observed anomalies in the $B_s \rightarrow J/\psi\phi$ decay and the like-sign dimuon asymmetry A_{sl}^b hint at possible new physics (NP) in the $B_s - \bar{B}_s$ mixing. We parameterize the NP with four model-independent quantities: the magnitudes and phases of the dispersive part M_{12} and the absorptive part Γ_{12} of the NP contribution to the effective Hamiltonian. We constrain these parameters using the four observables ΔM_s , $\Delta\Gamma_s$, the mixing phase $\beta_s^{J/\psi\phi}$, and A_{sl}^b . Our quantitative fit indicates that the NP should contribute a significant dispersive as well as absorptive part. In fact, models that do not contribute a new absorptive part are disfavored at more than 99% confidence level. We extend this formalism to include CPT violation, and show that CPT violation by itself, or even in presence of CPT -conserving new physics without an absorptive part, helps only marginally in the simultaneous resolution of these anomalies. The NP absorptive contribution to $B_s - \bar{B}_s$ mixing therefore seems to be essential, and would imply a large branching fraction for channels like $B_s \rightarrow \tau^+\tau^-$.

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I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) paradigm of quark mixing in the standard model (SM) is yet to be accurately tested in the $B_s - \bar{B}_s$ sector, and it is quite possible that the NP can affect the $B_s - \bar{B}_s$ system while keeping the $B_d - \bar{B}_d$ system untouched. Indeed, for most of the flavor-dependent NP models, the couplings relevant for the second and third generations of SM fermions are much less constrained than those for the first generation fermions, allowing the NP to play a significant role in the $B_s - \bar{B}_s$ mixing, in principle.

Over the last few years, the Tevatron experiments CDF and DØ, and to a smaller extent the B factories Belle and BABAR, have provided a lot of data on the B_s meson, most of which are consistent with the SM. There are some measurements, though, which show a significant deviation from the SM expectations, and hence point towards new physics (NP). The major ones among these are the following. (i) Measurements in the decay mode $B_s \rightarrow J/\psi\phi$ yield a large CP -violating phase $\beta_s^{J/\psi\phi}$ [1]. In addition, though the difference $\Delta\Gamma_s$ between the decay widths of the mass eigenstates measured in this decay is consistent with the SM, it allows $\Delta\Gamma_s$ values that are almost twice the SM prediction and also opposite in sign [2]. (ii) The like-sign dimuon asymmetry A_{sl}^b in the combined B data at DØ [3] is almost 4σ away from the SM expectation.

The resolutions of the above anomalies, separately or simultaneously, have been discussed in the context of

specific NP models: a scalar leptoquark model [4,5], models with an extra flavor-changing neutral gauge boson Z' or R -parity violating supersymmetry [6,7], two-Higgs doublet model [8,9], models with a fourth generation of fermions [10,11], supersymmetric grand unified models [12], supersymmetric models with split sfermion generations [13], or models with a very light spin-1 particle [14]. Possible four-fermion effective interactions that are consistent with the data have been analyzed by [15], and the results are consistent with [5]. Similar studies, based on the minimal flavor violating (MFV) models [16], and the Randall-Sundrum model [17], have been carried out.

In this paper, we try to determine, in a model-independent way, which kind of NP would be able to account for both the above anomalies simultaneously. We take a somewhat different approach than the references cited above. Rather than confining ourselves to specific models, we assume that the NP responsible for the anomalies contributes entirely through the $B_s - \bar{B}_s$ mixing, and parameterize it in a model-independent manner through the effective Hamiltonian for the $B_s - \bar{B}_s$ mixing. This effective Hamiltonian \mathcal{H} is a 2×2 matrix in the flavor basis, and the relevant NP contribution appears in its off-diagonal elements. The NP can then be parameterized by using four parameters: the magnitudes and phases of the dispersive part and the absorptive part of the NP contribution to \mathcal{H} . A “scatter-plot” analysis that constrained these four new parameters using only A_{sl}^b has been carried out in [17]. We perform a χ^2 fit to the $B_s - \bar{B}_s$ mixing observables and obtain a quantitative measure for which kind of NP is preferred by the data. This would lead us to shortlist specific NP models that have the desired properties, which can give testable predictions for other experiments. It is found that the NP needs to contribute to both the dispersive

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as well as absorptive part of the Hamiltonian in order to avoid any tension with the data.

We also extend our framework to include possible *CPT* violation in the $B_s - \bar{B}_s$ mixing, parameterized through the difference in diagonal elements of \mathcal{H} . The motivation is to check if this can obviate the need for an absorptive contribution from the NP. Such an analysis to constrain *CPT* and Lorentz-violating parameters was carried out in [18]. However they have used only A_{sl}^b and not $\beta_s^{J/\psi\phi}$ in their analysis, and their parameters are only indirectly connected to the elements of \mathcal{H} . We try to account for the two anomalies above with only *CPT* violation as the source of NP, and with a combination of *CPT* violation and the NP contribution to the off-diagonal elements of \mathcal{H} . As we will show, nothing improves the fit significantly from the SM unless there is a nonzero absorptive part in the $B_s - \bar{B}_s$ mixing amplitude.

The paper is organized as follows. In Sec. II, we introduce our formalism for the four NP parameters. In Sec. III, we summarize the experimental measurements and theoretical predictions for the observables relevant for $B_s - \bar{B}_s$ mixing. In Sec. IV, we present the results of our fits, and their implications for NP models are discussed in Sec. V. In Sec. VI, we introduce the formalism for introducing *CPT* violation, and in Sec. VII we explore the extent to which it can help resolving the anomalies. Sec. VIII summarizes our results and concludes.

II. THE EFFECTIVE HAMILTONIAN

The evolution of a $B_s - \bar{B}_s$ state can be described by the effective Hamiltonian

$$\mathcal{H} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \quad (1)$$

in the flavor basis, where M_{ij} and Γ_{ij} are its dispersive and absorptive parts, respectively. When *CPT* is conserved, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The eigenstates of this Hamiltonian are B_{sH} and B_{sL} , with masses M_{sH} and M_{sL} , respectively, and decay widths Γ_{sH} and Γ_{sL} , respectively. The difference in the masses and decay widths can be written in terms of the elements of the Hamiltonian as

$$\begin{aligned} \Delta M_s &\equiv M_{sH} - M_{sL} \approx 2|M_{12}|, \\ \Delta \Gamma_s &\equiv \Gamma_{sL} - \Gamma_{sH} \approx 2|\Gamma_{12}| \cos[\text{Arg}(-M_{12}/\Gamma_{12})]. \end{aligned} \quad (2)$$

The above expressions are valid as long as $\Delta \Gamma_s \ll M_s$, which is indeed the case here.

Since *CPT* is conserved, the effect of NP can be felt only through the off-diagonal elements of \mathcal{H} . We separate the SM and NP contributions to these terms via

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}, \quad \Gamma_{12} = \Gamma_{12}^{\text{SM}} + \Gamma_{12}^{\text{NP}}. \quad (3)$$

The NP can then be completely parameterized in terms of four real numbers: $|M_{12}^{\text{NP}}|$, $\text{Arg}(M_{12}^{\text{NP}})$, $|\Gamma_{12}^{\text{NP}}|$, and $\text{Arg}(\Gamma_{12}^{\text{NP}})$.

We take the phases $\text{Arg}(M_{12}^{\text{NP}})$ and $\text{Arg}(\Gamma_{12}^{\text{NP}})$ to lie in the range $0 - 2\pi$.

In a large class of models, including the Minimal Flavor Violation (MFV) models, the NP contribution has no absorptive part, i.e. $\Gamma_{12} = \Gamma_{12}^{\text{SM}}$. This is true for a lot of non-MFV models too. This occurs when NP does not give rise to any new intermediate light states to which B_s or \bar{B}_s can decay. For such models, Eq. (2) implies that $\Delta \Gamma_s \leq \Delta \Gamma_s(\text{SM}) \approx 2|\Gamma_{12}^{\text{SM}}|$, i.e. the value of $\Delta \Gamma_s$ is always less than its SM prediction [19]. In such models, the NP is parameterized by only two parameters: $|M_{12}^{\text{NP}}|$ and $\text{Arg}(M_{12}^{\text{NP}})$. An analysis restricted to this class of models was performed in [20].

However there exists a complementary class of viable models where the NP contributes to Γ_{12} substantially. These include models with leptoquarks, R-parity violating supersymmetry, a light gauge boson, etc. It has been pointed out in [4] that such a nonzero absorptive part that arises naturally in these class of models can enhance $\Delta \Gamma_s$ significantly above its SM value, contrary to the popular expectations based on [19]. One notes that a new absorptive part in the mixing amplitude necessarily means new final states that can be accessed by both B_s and \bar{B}_s . The data from the direct measurements of branching ratios is extremely restrictive [15], apart from that for a few final states like $B_s \rightarrow \tau^+ \tau^-$ [5]. As we shall see later in this paper, such models are favored by the $B_s - \bar{B}_s$ mixing data. The importance of $\tau^+ \tau^-$ final states from B_d and B_s decays has also been pointed out in [21].

III. THE MEASUREMENTS

The $B_s - \bar{B}_s$ oscillation and *CP* violation therein can be quantified by four observables, viz., the mass difference ΔM_s , the decay width difference $\Delta \Gamma_s$, the *CP*-violating phase $\beta_s^{J/\psi\phi}$, and the semileptonic asymmetry a_{sl}^s .

The mass difference is measured to be

$$\Delta M_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1}, \quad (4)$$

which is consistent with the SM expectation [22]

$$\Delta M_s(\text{SM}) = (17.3 \pm 2.6) \text{ ps}^{-1}. \quad (5)$$

However measurements in the $B_s \rightarrow J/\psi\phi$ decay mode show a hint of some deviation from the SM. The *CP*-violating phase $\beta_s^{J/\psi\phi}$ in this decay is

$$\beta_s^{J/\psi\phi} = \frac{1}{2} \text{Arg} \left(- \frac{(V_{cb} V_{cs}^*)^2}{M_{12}} \right), \quad (6)$$

whose average value measured at the Tevatron experiments [1] is

$$\beta_s^{J/\psi\phi} = (0.41_{-0.15}^{+0.18}) \cup (1.16_{-0.18}^{+0.15}). \quad (7)$$

In the SM,

$$\beta_s^{J/\psi\phi}(\text{SM}) = \text{Arg}\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right) \approx 0.019 \pm 0.001. \quad (8)$$

Thus, the measured value of $\beta_s^{J/\psi\phi}$ is more than 2σ away from the SM expectation. On the other hand, the difference in the decay widths of the mass eigenstates B_H and B_L is measured to be [1]

$$\Delta\Gamma_s = \pm(0.154_{-0.070}^{+0.054}) \text{ ps}^{-1}, \quad (9)$$

while the SM expectation is [22]

$$\Delta\Gamma_s(\text{SM}) = (0.087 \pm 0.021) \text{ ps}^{-1}. \quad (10)$$

The measurement is consistent with the SM expectation to $\sim 1\sigma$; however it allows for $\Delta\Gamma_s$ values that are almost twice the SM prediction. Note that the sign of $\Delta\Gamma_s$ is undetermined experimentally and this gives us more room to play with the NP parameters.

CDF has recently announced its new results based on 5.2 fb^{-1} of data [23]:

$$|\Delta\Gamma_s| = (0.075 \pm 0.035 \pm 0.010) \text{ ps}^{-1},$$

$$\beta_s^{J/\psi\phi} = (0.02 - 0.52) \cup (1.08 - 1.55) \quad (11)$$

to 68% C. L. While we note that the results are consistent with the SM, the final Tevatron averages are still awaited. Therefore, we use the values in Eq. (9) in our analysis.

The other anomalous measurement is the like-sign dimuon asymmetry. Averaging the 9.0 fb^{-1} data of $D\bar{O}$ [3] and 1.6 fb^{-1} data of CDF [24], and adding the errors in quadrature and treating them as Gaussian, we get

$$A_{\text{sl}}^b = -(7.41 \pm 1.93) \times 10^{-3}, \quad (12)$$

which differs by more than 3σ from its SM prediction

$$A_{\text{sl}}^b(\text{SM}) = (-0.23_{-0.06}^{+0.05}) \times 10^{-3}. \quad (13)$$

Note that for A_{sl}^b , CDF has a poorer statistics than $D\bar{O}$ and therefore the average value is dominated by the $D\bar{O}$ data.

Even in the presence of new physics, the SM relationship holds:

$$A_{\text{sl}}^b = (0.506 \pm 0.043)a_{\text{sl}}^d + (0.494 \mp 0.043)a_{\text{sl}}^s, \quad (14)$$

where a_{sl}^s and a_{sl}^d are the semileptonic asymmetries for the $B_s - \bar{B}_s$ and the $B_d - \bar{B}_d$ systems, respectively. The former is related to the $B_s - \bar{B}_s$ mixing observables through

$$a_{\text{sl}}^s = \frac{\Delta\Gamma_s}{\Delta M_s} \tan\phi_s \quad (15)$$

where $\phi_s \equiv \text{Arg}(-M_{12}/\Gamma_{12})$. The latter is defined analogously. The coefficients in Eq. (14) are experimentally measured and contain information about $\Delta M_{d(s)}$, $\Delta\Gamma_{d(s)}$, and production fractions of B_d and B_s mesons. Using $a_{\text{sl}}^d = -(4.7 \pm 4.6) \times 10^{-3}$ [2], this leads to

$$a_{\text{sl}}^s = -0.010 \pm 0.006, \quad (16)$$

which is about 1.7σ away from the SM prediction

$$a_{\text{sl}}^s(\text{SM}) = (2.06 \pm 0.57) \times 10^{-5}. \quad (17)$$

The value of a_{sl}^d depends on ΔM_d , $\Delta\Gamma_d$ and ϕ_d , the parameters in the B_d sector analogous to those in Eq. (15). These parameters depend on the NP in the B_d sector, which is independent of the NP parameters in the B_s sector that we are considering. We therefore do not consider the measured values of a_{sl}^d as a direct constraint but express it in terms of ΔM_d , $\Delta\Gamma_d$, and ϕ_d , whose experimental values are taken as inputs.

In the SM, we have $\phi_s(\text{SM}) = 0.0041 \pm 0.0007$ [22]. Note that if the dominating contribution to Γ_{12s} were from a pair of intermediate c quarks, $\phi_s(\text{SM})$ would have been equal to $-2\beta_s^{J/\psi\phi}$. Since the intermediate $u - c$ and $u - u$ quark states give comparable contributions to Γ_{12s} , we have $\phi_s(\text{SM}) \neq -2\beta_s^{J/\psi\phi}(\text{SM})$ [25].

IV. THE STATISTICAL ANALYSIS

We perform a χ^2 fit to the observed quantities ΔM_s , $\Delta\Gamma_s$, $\beta_s^{J/\psi\phi}$, and a_{sl}^s , using the NP parameters $|M_{12}^{\text{NP}}|$, $\text{Arg}(M_{12}^{\text{NP}})$, $|\Gamma_{12}^{\text{NP}}|$, and $\text{Arg}(\Gamma_{12}^{\text{NP}})$. We assume all the measurements to be independent for simplicity, though the measurements of $\Delta\Gamma_s$ and $\beta_s^{J/\psi\phi}$ are somewhat correlated. The values of all the observables and their SM values are as given in Sec. III. In order to express them in terms of M_{12} , M_{12}^{SM} , Γ_{12} , and Γ_{12}^{SM} , one has to use Eq. (2) in addition. In order to take into account the errors on the SM parameters, we add the theoretical and experimental errors on our observed quantities in quadrature.

Note that since we have four observable quantities and four parameters, it is not surprising that we obtain the global minimum value of χ^2 as $\chi_{\text{min}}^2 = 0$ when all the NP parameters are allowed to vary. The questions we address here are (i) what the preferred values of the NP parameters are, and (ii) to what confidence level (C. L.) a given set of NP parameters (or SM, which is a special case of NP with $M_{12}^{\text{NP}} = \Gamma_{12}^{\text{NP}} = 0$) is allowed. The latter is obtained assuming all errors to be Gaussian. Here we give our results in terms of the goodness-of-fit contours for the joint estimations of two parameters at a time. The (1σ , 2σ , 3σ , 4σ) contours that are equivalent to p values of (0.3173, 0.0455, 0.0027, 0.0001), or confidence levels of (68.27%, 95.45%, 99.73%, 99.99%), correspond to $\chi^2 = (2.295, 6.18, 11.83, 19.35)$, respectively.

In Fig. 1, we show the 1σ , 2σ , 3σ , 4σ contours in the $|M_{12}| - \text{Arg}(M_{12})$ plane, where the other NP parameters are marginalized over. Clearly, we see a preference towards nonzero $|M_{12}^{\text{NP}}|$ as well as nonzero $\text{Arg}(M_{12}^{\text{NP}})$ values. There are two best-fit points with $\chi^2 = 0$, one at $M_{12}^{\text{NP}} \approx 6.3 \exp(2.0i) \text{ ps}^{-1}$ and the other at $M_{12}^{\text{NP}} \approx 16.2 \exp(2.8i) \text{ ps}^{-1}$, shown with crosses in Fig. 1. Actually, each of these crosses is a superimposed double, with two values of Γ_{12}^{NP} , as shown in Fig. 2. The points correspond to the constructive and destructive interference

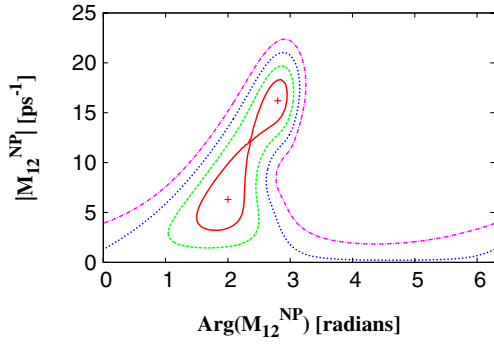


FIG. 1 (color online). The 1σ (red/solid line), 2σ (green/dashed line), 3σ (blue/dotted line), and 4σ (pink/dot-dashed line) goodness-of-fit contours in the $|M_{12}^{\text{NP}}| - \text{Arg}(M_{12}^{\text{NP}})$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses.

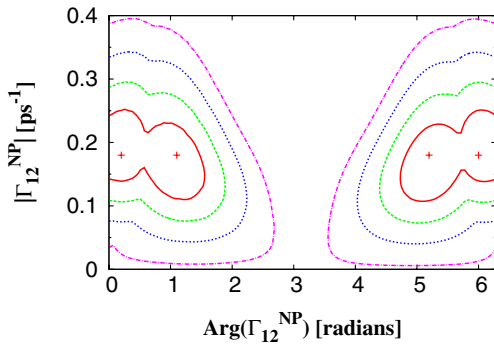


FIG. 2 (color online). The 1σ (red/solid line), 2σ (green/dashed line), 3σ (blue/dotted line), and 4σ (pink/dot-dashed line) goodness-of-fit contours in the $|\Gamma_{12}^{\text{NP}}| - \text{Arg}(\Gamma_{12}^{\text{NP}})$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses.

between the SM and NP amplitudes in order to give the measured central values of ΔM_s . The region with $M_{12}^{\text{NP}} = 0$, i.e. the x axis, is outside the 2σ region, indicating that it will be rather difficult to fit the current data without some NP contribution to the dispersive part of the $B_s - \bar{B}_s$ mixing. The contours also imply that $|M_{12}^{\text{NP}}| \lesssim 21.1 \text{ ps}^{-1}$ to 3σ .

In Fig. 2, we show the goodness-of-fit contours in the $|\Gamma_{12}^{\text{NP}}| - \text{Arg}(\Gamma_{12}^{\text{NP}})$ plane, marginalizing over other two NP parameters. As the measurements do not determine the sign of $\Delta\Gamma_s$, for any particular value of $|\Delta\Gamma_s|$, we perform the χ^2 fit for both positive and negative values and keep the minimum χ^2 of the two. This doubles the number of best-fit solutions, and the two best-fit points of Fig. 1 now split into four. For $|M_{12}^{\text{NP}}| = 6.3$, the solutions are $\Gamma_{12}^{\text{NP}} = 0.18 \exp(6.0i)$ or $0.18 \exp(5.2i)$, and for $|M_{12}^{\text{NP}}| = 16.2$, the corresponding solutions are $\Gamma_{12}^{\text{NP}} = 0.18 \exp(0.2i)$ or $0.18 \exp(1.1i)$ (both M_{12}^{NP} and Γ_{12}^{NP} are in ps^{-1} , here, and also later where not mentioned explicitly). Note that there is a reflection symmetry about $\text{Arg}(\Gamma_{12}^{\text{NP}}) = \pi$. Again, a preference for nonzero values of $|\Gamma_{12}^{\text{NP}}|$ is indicated, though

$\text{Arg}(\Gamma_{12}^{\text{NP}})$ may vanish. The region with $\Gamma_{12}^{\text{NP}} = 0$, i.e. the x axis, is outside the 4σ allowed region, indicating that NP contribution to the absorptive part of the effective Hamiltonian is highly favored. The contours also imply that $|\Gamma_{12}^{\text{NP}}| \lesssim 0.35$ at 3σ .

Figure 3 displays the contours in the $|M_{12}^{\text{NP}}| - |\Gamma_{12}^{\text{NP}}|$ plane, and the two NP phases are marginalized over. Not only does it show a preference for nonzero values of M_{12}^{NP} and Γ_{12}^{NP} , but the $M_{12}^{\text{NP}} = 0$ axis is outside the 2σ allowed region and the $\Gamma_{12}^{\text{NP}} = 0$ axis is outside the 4σ allowed region. The best-fit points are again superimposed doubles, whose values can be read off from the discussion above. The origin in this figure is the SM, which has $\chi_{\text{SM}}^2 = 25.85$, and lies even outside the 4σ allowed region. This dramatically quantifies the failure of the SM to accommodate the current data. The reason is evident from Eqs. (7) and (15); while $B_s \rightarrow J/\psi\phi$ prefers $\beta_s^{J/\psi\phi}$ close to $\pi/8$ or $3\pi/8$, with a probability minimum near $\beta_s^{J/\psi\phi} \approx \pi/4$, the measurement of A_{sl}^b , and hence that of a_{sl}^s , prefers large $\tan\phi_s$, forcing $\beta_s^{J/\psi\phi}$ close to $\pi/4$. This creates the tension between these two measurements.

Figure 3 also tells us that the models for which $\Gamma_{12}^{\text{NP}} = 0$, like R-parity conserving supersymmetry, universal extra dimension, and extra scalars, fermions, or gauge bosons, cannot bring the tension down even to the 4σ range, unless the data moves towards the SM expectations (and unless the new bosons are flavor-changing so as to generate a nonzero Γ_{12}^{NP}). The best-fit point with $\Gamma_{12}^{\text{NP}} = 0$ has $\chi^2 = 20.75$ and corresponds to $M_{12}^{\text{NP}} = 3.72 \exp(1.68i)$. This is further emphasized in Fig. 4, which shows the 5σ (p value of 10^{-6} , $\chi^2 = 27$) contour for those NP models where Γ_{12}^{NP} is set to vanish (within the closed contour above and under the open contour below).

One may question the optimistic SM uncertainty for $\Delta\Gamma_s$ as quoted in Eq. (10). However, this has an almost negligible effect. For example, if we increase the uncertainty by 50%, neither the best-fit points nor the confidence levels change significantly. The best-fit point in Fig. 4 has a χ^2

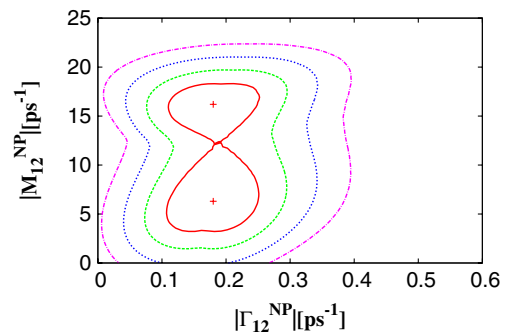


FIG. 3 (color online). The 1σ (red/solid line), 2σ (green/dashed line), 3σ (blue/dotted line), and 4σ (pink/dot-dashed line) goodness-of-fit contours in the $|M_{12}^{\text{NP}}| - |\Gamma_{12}^{\text{NP}}|$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses.

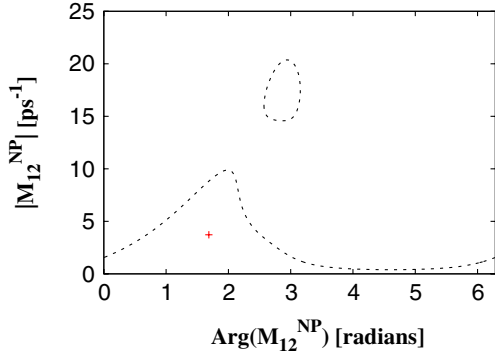


FIG. 4 (color online). The 5σ goodness-of-fit contour in the $|M_{12}^{\text{NP}}| - \text{Arg}(M_{12}^{\text{NP}})$ plane, when $\Gamma_{12}^{\text{NP}} = 0$, i.e. NP does not contribute to the absorptive part of the effective Hamiltonian. There are no points that are allowed to within 4σ . The best-fit point, with $\chi^2 = 20.75$, is denoted by a cross.

minimum of 20.58 instead of 20.75. The reason is the large deviation of a_{sl}^s from its SM value, to explain which we need a significant enhancement in $\tan\phi_s$.

V. PREFERRED NP MODELS

From the results and discussion in the previous section, it appears that

- (i) The SM by itself is strongly disfavored. Either M_{12}^{NP} or Γ_{12}^{NP} should be nonzero.
- (ii) $M_{12}^{\text{NP}} \neq 0$ but $\Gamma_{12}^{\text{NP}} = 0$ is also not allowed at 4σ , but the fit is marginally better than the SM.
- (iii) The hypothetical case where $\Gamma_{12}^{\text{NP}} \neq 0$ but $M_{12}^{\text{NP}} = 0$ is also disfavored to more than 2σ . (This is a rather natural condition, since any interaction that contributes to Γ_{12}^{NP} will necessarily contribute to M_{12}^{NP}).

Most of the NP models can contribute significantly to M_{12}^{NP} . Leading examples are the MFV models like minimal supersymmetry, universal extra dimensions, little-Higgs with T-parity, etc. Non-MFV models like a fourth chiral generation, supersymmetry with R-parity violation, two-Higgs doublet models, models with extra Z' , etc. can also contribute significantly to M_{12}^{NP} .

The NP models that can contribute significantly to Γ_{12}^{NP} , however, are rather rare. This is because the NP contribution to the absorptive part needs light particles in the final state, and there are strong limits on the decays of B_s to most of the possible light final state particles. One of the few exceptions is the mode $\tau^+\tau^-$, on which there is no available bound at this moment. Thus, the NP that contributes to Γ_{12}^{NP} has to do so via the interaction $b \rightarrow s\tau^+\tau^-$ but without affecting related decays like $b \rightarrow se^+e^-$ or $b \rightarrow s\mu^+\mu^-$ [15]. This can be achieved only in a limited subset of models, for example, those with second and third generation scalar leptoquarks, R-parity violating supersymmetry [4], or extra Z' bosons [7]. It turns out that the former can provide enough contribution to Γ_{12}^{NP} to increase $\Delta\Gamma_s$ up to

its current experimental upper bound [4,5]. The amount of NP required for this is consistent with the difference between the decay widths of B_d and B_s mesons ($\Gamma_s/\Gamma_d - 1 = (3.6 \pm 1.8)\%$ [2]) and the recent measurement of the branching ratio of $B^+ \rightarrow K^+\tau^+\tau^-$, which is less than 3.3×10^{-3} at 90% C. L. [26].

One should note here that if the $D\bar{O}$ results on the dimuon charge asymmetry survive the test of time, it will be a clear indication of the presence of a nonzero Γ_{12s}^{NP} . Such models are also favored from the CDF and $D\bar{O}$ combined result on the allowed contours for $\beta_s^{J/\psi\phi}$ and Γ_s , but we need to wait for the final Tevatron average.

VI. CPT VIOLATION: THE FORMALISM

The analysis till now is valid only if we assume *CPT* invariance. However, the *CPT* symmetry may be violated in theories that break Lorentz invariance [27]. Indeed for local field theories, *CPT* violation requires Lorentz violation [28]. (This need not be true for nonlocal field theories as well as for theories with noncommutative space-time geometry, see [29].) In general, *CPT* violation should result in differences in masses and decay widths between particle-antiparticles pairs. However, it may be easier to identify even through oscillation experiments, which typically are sensitive to an interference between the *CPT*-conserving and *CPT*-violating interactions.

While *CPT* violation in the K system is severely constrained through the mass difference between the neutral kaons [30], the bounds on the *CPT* violating parameters in the B_d and B_s systems are rather weak. In fact, the bounds for the B_d sector are about 3 orders of magnitude weaker than those for the K sector [31]. The bounds on Lorentz-violating parameters using the data on B mesons can be found in [18] and references therein. Here we use a model-independent parameterization, like the one earlier followed in [32] and recently used by two of us [33], and determine the preferred parameter space using the data on $B_s - \bar{B}_s$ oscillations. Unlike [18], we take both A_{sl}^b and $\beta_s^{J/\psi\phi}$ data into account.

One should note that as a new physics option, *CPT* violation is not exactly at the same footing as the models mentioned before. However in the language of the effective Hamiltonian \mathcal{H} , the *CPT* violation manifests itself naturally through in a difference between the diagonal elements of \mathcal{H} . It is therefore interesting to see if the constraints on the NP coming from ΔM_s and $\Delta\Gamma_s$ can be relaxed at all with these additional degrees of freedom. *A posteriori*, we will justify the discussion on *CPT* violation by showing that if the new physics indeed turns out to be without an absorptive part, *CPT* violation might help to explain the $B_s - \bar{B}_s$ mixing data, albeit only marginally.

The *CPT* violation manifests itself in the effective Hamiltonian through the difference in the diagonal elements. We write the effective Hamiltonian in Eq. (1) as

$$\mathcal{H} = \begin{pmatrix} M_0 - \frac{i}{2}\Gamma_0 - \delta' & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_0 - \frac{i}{2}\Gamma_0 + \delta' \end{pmatrix} \quad (18)$$

and define the dimensionless *CPT*-violating complex parameter δ as

$$\delta \equiv \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}} = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}, \quad (19)$$

where $H_{ij} \equiv M_{ij} - \frac{i}{2}\Gamma_{ij}$.

The eigenvalues of \mathcal{H} are

$$\lambda = \left(M_0 - \frac{i}{2}\Gamma_0 \right) \pm \alpha y H_{12}, \quad (20)$$

where $\alpha \equiv \sqrt{H_{21}/H_{12}}$ and $y \equiv \sqrt{1 + \delta^2/4}$. The corresponding mass eigenstates are

$$|B_{sH}\rangle = p_1|B_s\rangle + q_1|\bar{B}_s\rangle, \quad |B_{sL}\rangle = p_2|B_s\rangle - q_2|\bar{B}_s\rangle, \quad (21)$$

with $|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1$, and

$$\eta_1 \equiv \frac{q_1}{p_1} = \sqrt{\frac{H_{21}}{H_{12}}} \left(\sqrt{1 + \frac{\delta^2}{4}} + \frac{\delta}{2} \right), \quad (22)$$

$$\eta_2 \equiv \frac{q_2}{p_2} = \sqrt{\frac{H_{21}}{H_{12}}} \left(\sqrt{1 + \frac{\delta^2}{4}} - \frac{\delta}{2} \right).$$

Clearly, *CPT* invariance corresponds to $\eta_1 = \eta_2$.

Let us now determine the dependence of our four observables on the *CPT*-violating parameters. The differences in masses and decay widths of the eigenstates are related to the difference in eigenvalues as

$$\lambda_1 - \lambda_2 = \Delta M + \frac{i}{2}\Delta\Gamma, \quad (23)$$

where λ_1 and λ_2 are ordered such that $\text{Re}(\lambda_1 - \lambda_2) > 0$. From Eq. (20),

$$\Delta M = M_1 - M_2 = 2\text{Re}(\alpha y H_{12}), \quad (24)$$

$$\Delta\Gamma = \Gamma_2 - \Gamma_1 = 4\text{Im}(\alpha y H_{12}). \quad (25)$$

Since $|\Gamma_{12}| \ll |M_{12}|$, we can write

$$\alpha H_{12} = |M_{12}| \left[1 - \frac{1}{4} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} - i\text{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right) \right]^{1/2}$$

$$\approx |M_{12}| \left[1 - \frac{i}{2} \text{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right) \right]. \quad (26)$$

Then Eqs. (24) and (25) yield

$$\Delta M \approx |M_{12}| \left[2\text{Re}(y) + \text{Im}(y)\text{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right) \right], \quad (27)$$

$$\Delta\Gamma \approx |M_{12}| \left[4\text{Im}(y) - 2\text{Re}(y)\text{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right) \right]. \quad (28)$$

The dependence on the *CPT*-violating parameter δ appears entirely through y .

Let us pause here for a moment and find what the above two equations tell us about the allowed parameter space. Let us first focus on the best constraint, ΔM_s , and work in the limit where Γ_{12}/M_{12} is negligible. $|M_{12}|$, and hence M_{12}^{NP} , can be arbitrarily large, as $\text{Re}(y)$ can be made arbitrarily small by an appropriate choice of δ . Similarly, $\text{Re}(y)$ can be quite large (albeit compatible with other constraints) as long as there is a near-perfect cancellation between the SM and NP mixing amplitudes, making $|M_{12}|$ small. However, the smallness of $\Delta\Gamma/\Delta M$ constrains $\text{Im}(y)/\text{Re}(y)$ to be small, thus indicating that y is almost real. Since $y = \sqrt{1 + \delta^2/4}$, this implies that δ^2 is almost real and $\text{Re}(\delta^2) \gtrsim -4$. Therefore, one would expect that δ is either almost real, or it is almost imaginary but with $|\text{Im}(\delta)| < 2$.

Now let us consider the *CP*-violating observables $\beta_s^{J/\psi\phi}$ and a_{sl}^s . The effective value of the former may be obtained in the presence of *CPT* violation by considering the decay rates of B_s and \bar{B}_s to a final *CP* eigenstate f_{CP} as [33]

$$\Gamma(B_s(t) \rightarrow f_{CP}) = |A_f|^2 [|f_+(t)|^2 + |\xi_{f_1}|^2 |f_-(t)|^2 + 2\text{Re}(\xi_{f_1} f_-(t) f_+^*(t))], \quad (29)$$

$$\Gamma(\bar{B}_s(t) \rightarrow f_{CP}) = \left| \frac{A_f}{\eta_2} \right|^2 [|f_-(t)|^2 + |\xi_{f_2}|^2 |\bar{f}_+(t)|^2 + 2\text{Re}(\xi_{f_2} \bar{f}_+(t) f_-^*(t))], \quad (30)$$

with

$$\xi_{f_1} \equiv \eta_1 \frac{\bar{A}_f}{A_f}, \quad \xi_{f_2} \equiv \eta_2 \frac{\bar{A}_f}{A_f}, \quad \omega \equiv \frac{\eta_1}{\eta_2}. \quad (31)$$

Here A_f and \bar{A}_f are the amplitudes for the processes $B_s \rightarrow f_{CP}$ and $\bar{B}_s \rightarrow f_{CP}$, respectively. The time evolutions are given by

$$f_-(t) = \frac{1}{1 + \omega} (e^{-i\lambda_1 t} - e^{-i\lambda_2 t}),$$

$$f_+(t) = \frac{1}{1 + \omega} (e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}), \quad (32)$$

$$\bar{f}_+(t) = \frac{1}{1 + \omega} (\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}).$$

The final state in $B_s \rightarrow J/\psi\phi$ is not a *CP* eigenstate but a combination of *CP*-even and *CP*-odd final states, which may be separated using angular distributions. With the transversity angle distribution [34], the time-dependent decay rate to the *CP*-even state is given by the coefficient of $(1 + \cos^2\theta)$, while the time-dependent decay rate to the *CP*-odd state is given by the coefficient of $\sin^2\theta$.

The value of effective $\beta_s^{J/\psi\phi}$ in this process is determined by writing the time evolutions (29) and (30) in the form

$$\Gamma(B_s(t) \rightarrow f_{CP}) = c_1 \cosh(\Delta\Gamma_s t/2) + c_2 \sinh(\Delta\Gamma_s t/2) + c_3 \cos(\Delta M_s t) + c_4 \sin(\Delta M_s t), \quad (33)$$

$$\Gamma(\bar{B}_s(t) \rightarrow f_{CP}) = \bar{c}_1 \cosh(\Delta\Gamma_s t/2) + \bar{c}_2 \sinh(\Delta\Gamma_s t/2) + \bar{c}_3 \cos(\Delta M_s t) + \bar{c}_4 \sin(\Delta M_s t). \quad (34)$$

The direct CP violation in $B_s \rightarrow J/\psi\phi$ is negligible; i.e. $|\bar{A}_f/A_f| \approx 1$. Also, $|\Gamma_{12}/M_{12}| \ll 1$, so that in the absence of CPT violation, $|\eta_1| = |\eta_2| = 1$. Then in terms of $\xi_f \equiv \xi_{f_1} = \xi_{f_2} = \alpha \bar{A}_f/A_f$, one can write

$$\sin(2\beta_s^{J/\psi\phi}) = -\eta_{CP} \frac{2[-\text{Im}(\omega) - \text{Re}(\xi_{f_1})\text{Im}(\omega) + \text{Im}(\xi_{f_1}) + \text{Im}(\xi_{f_1})\text{Re}(\omega)]}{[1 + |\omega|^2 + 2|\xi_{f_1}|^2 + 2\text{Re}(\xi_{f_1}) - 2\text{Re}(\xi_{f_1})\text{Re}(\omega) - 2\text{Im}(\xi_{f_1})\text{Im}(\omega)]}, \quad (36)$$

$$\sin(2\bar{\beta}_s^{J/\psi\phi}) = -\eta_{CP} \frac{2[-|\xi_{f_2}|^2\text{Im}(\omega) + \text{Re}(\xi_{f_2})\text{Im}(\omega) + \text{Im}(\xi_{f_2}) + \text{Im}(\xi_{f_2})\text{Re}(\omega)]}{[2 + |\xi_{f_2}|^2(1 + |\omega|^2) - 2\text{Re}(\xi_{f_2}) + 2\text{Re}(\xi_{f_2})\text{Re}(\omega) - 2\text{Im}(\xi_{f_2})\text{Im}(\omega)]}. \quad (37)$$

Though the analysis of the B_s and \bar{B}_s modes needs to be performed separately, here we assume identical detection and tagging efficiencies for both and use the average of Eq. (36) and Eq. (37) for our fit.

The semileptonic CP asymmetry a_{sl}^s is measured through the ‘‘wrong-sign’’ lepton signal:

$$a_{\text{sl}}^s = \frac{\Gamma(\bar{B}_s(t) \rightarrow \mu^+ X) - \Gamma(B_s(t) \rightarrow \mu^- X)}{\Gamma(\bar{B}_s(t) \rightarrow \mu^+ X) + \Gamma(B_s(t) \rightarrow \mu^- X)}. \quad (38)$$

Here,

$$\Gamma(B_s(t) \rightarrow \mu^- X) = |\eta_1 f_- A(B_s \rightarrow \mu^+ X)|^2, \quad (39)$$

$$\Gamma(\bar{B}_s(t) \rightarrow \mu^+ X) = |(f_-/\eta_2) A(\bar{B}_s \rightarrow \mu^+ X)|^2, \quad (40)$$

and since $|A(\bar{B}_s \rightarrow \mu^+ X)| = |A(B_s \rightarrow \mu^+ X)|$,

$$a_{\text{sl}}^s = \frac{\frac{1}{|\eta_2|^2} - |\eta_1|^2}{\frac{1}{|\eta_2|^2} + |\eta_1|^2} = \frac{1 - |\alpha|^4}{1 + |\alpha|^4}, \quad (41)$$

which is independent of the CPT -violating parameter δ . That the semileptonic asymmetry does not contain a CPT violating term in the leading order was also noted earlier [35].

VII. CPT VIOLATION: THE STATISTICAL ANALYSIS

In this section, we perform a χ^2 fit to the observables ΔM_s , $\Delta\Gamma_s$, the effective phase $\beta_s^{J/\psi\phi}$, and a_{sl}^s . Let us first assume that there is no CPT -conserving NP contribution coming from M_{12}^{NP} and Γ_{12}^{NP} , so that the only relevant

$$\frac{c_4}{c_1} = -\frac{\bar{c}_4}{\bar{c}_1} = \frac{2\text{Im}(\xi_f)}{1 + |\xi_f|^2} \approx -\eta_{CP} \sin(2\beta_s^{J/\psi\phi}), \quad (35)$$

where η_{CP} is the CP eigenvalue of f_{CP} .

When CPT is violated, the effective phases $\beta_s^{J/\psi\phi}$ and $\bar{\beta}_s^{J/\psi\phi}$, measured through $B_s(t)$ and $\bar{B}_s(t)$ decays, respectively, will turn out to be different. Indeed, the difference between these effective phases will be a clean signal of CPT violation.

NP contribution is CPT violating and is parameterized by $\text{Re}(\delta)$ and $\text{Im}(\delta)$. The allowed parameter space is shown in Fig. 5. It turns out that in this case, the value of χ_{min}^2 is ≈ 16.4 (at $\delta = 0.008 + 0.958i$ and $\delta = -0.024 + 0.958i$), marginally better than the one obtained in the ($\Gamma_{12}^{\text{NP}} = 0$, $M_{12}^{\text{NP}} \neq 0$) case discussed above in Fig. 4. There are some, albeit small, regions in the parameter space that are allowed to 4σ . However a fit good to 3σ or better is still not possible.

We therefore need to add the CPT -conserving NP to the CPT -violating contribution. However we have already seen in the preceding section that M_{12}^{NP} and Γ_{12}^{NP} together are capable of explaining the data by themselves. Therefore the fit using δ , M_{12}^{NP} as well as Γ_{12}^{NP} is redundant. With six independent parameters and only four observables, not only is $\chi_{\text{min}}^2 = 0$ guaranteed, but no effective

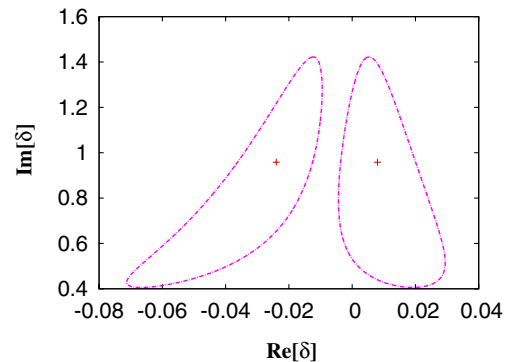


FIG. 5 (color online). The 4σ goodness-of-fit contours in the $\text{Re}(\delta) - \text{Im}(\delta)$ plane, when the only relevant NP contribution is CPT violating, parameterized entirely by δ . There are no points that are allowed to within 3σ . The crosses show the best-fit points, with $\chi^2 = 16.4$.

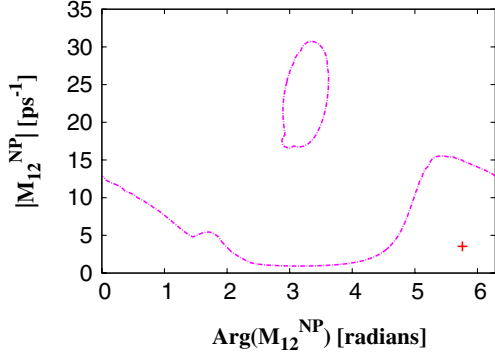


FIG. 6 (color online). The 4σ goodness-of-fit contours in the $|M_{12}^{\text{NP}}| - \text{Arg}(M_{12}^{\text{NP}})$ plane, when $\Gamma_{12}^{\text{NP}} = 0$, i.e. NP does not contribute to the absorptive part of the effective Hamiltonian. The CPT -violating complex parameter δ has been marginalized over. There are no points that are allowed to within 3σ . The cross shows the best-fit point, with $\chi^2 = 14.3$.

limits on CPT -conserving and CPT -violating parameters are generated.

We, therefore, go directly to the possibility where there is CPT -conserving NP but without an absorptive part: $\Gamma_{12}^{\text{NP}} = 0$. We have already observed (Fig. 4) that the entire region in the $|M_{12}^{\text{NP}}| - \text{Arg}(M_{12}^{\text{NP}})$ is outside the 4σ region in such a scenario. We would now ask what happens if we enhance the two-parameter NP with two more CPT violating parameters, *viz.*, $\text{Re}(\delta)$, and $\text{Im}(\delta)$. This scenario is interesting because, as we have seen before, only very specific kind of NP can contribute to Γ_{12}^{NP} , which would be tested severely in near future. In case no evidence for the relevant NP is found (e.g. the branching ratio of $B_s \rightarrow \tau^+ \tau^-$ is observed to be the same as its SM prediction), the next step would be to check if CPT violation, along with the NP contribution through M_{12}^{NP} , would be able to account for the anomalies. For example, one may want to determine β_s and $\bar{\beta}_s$ of Eqs. (36) and (37) separately and see whether they are different.

Figure 6 shows the situation in the $|M_{12}^{\text{NP}}| - \text{Arg}(M_{12}^{\text{NP}})$ plane. As compared to Fig. 4, one can see that once we marginalize over δ , we now have some regions allowed to within 4σ (within the closed contour above and below the open contour) but none within 3σ . Indeed, $\chi_{\text{min}}^2 = 14.3$ at $M_{12}^{\text{NP}} = 3.54 \exp(5.76i)$. This clearly does not improve the goodness-of-fit substantially, indicating that there is no good alternative for Γ_{12}^{NP} .

Figure 7 shows the situation in the complex δ plane, when M_{12}^{NP} has been marginalized over. The best-fit point corresponds to $\delta = -0.01 + 1.40i$, which gives $\chi_{\text{min}}^2 = 14.3$ as mentioned earlier. The CPT conserving point ($\delta = 0$) lies outside the 4σ region. As expected from the discussion in Sec. VI, the allowed values of δ are close to the $\text{Re}(\delta)$ or $\text{Im}(\delta)$ axis, with $|\text{Im}(\delta)|$ restricted to 2. One observes that the current data allows rather large (~ 1) positive values of $\text{Im}(\delta)$ at 4σ .

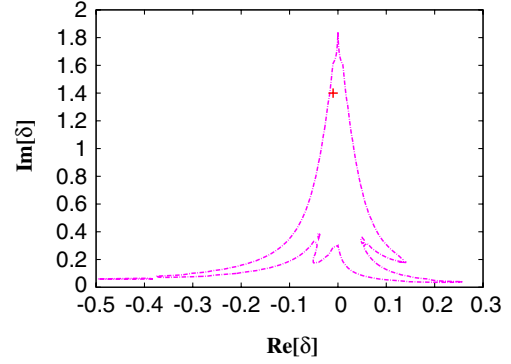


FIG. 7 (color online). The 4σ goodness-of-fit contours in the $\text{Re}(\delta) - \text{Im}(\delta)$ plane, when the complex NP parameter M_{12}^{NP} is marginalized over, while Γ_{12}^{NP} has been constrained to vanish. There are no points that are allowed to within 3σ . The cross shows the best-fit point, with $\chi^2 = 14.3$.

VIII. CONCLUSION

Any flavor-dependent new physics model can in general affect both mass and width differences in the $B_s - \bar{B}_s$ system. It can also affect the CP -violating phase, as well as the dimuon asymmetry, which was found by the $D0$ collaboration to have an anomalously large value. With these four observables, one can constrain the free parameters of the new physics model. We have used the model-independent approach where we consider the effective $B_s - \bar{B}_s$ mixing Hamiltonian \mathcal{H} and parameterize the NP through its contribution to \mathcal{H} . We quantify the goodness-of-fit for the SM and NP parameter values by performing a combined χ^2 fit to all the four measurements. The tension of the data with the SM is clear by the high value of χ^2 at the SM. Moreover, it is observed that we need NP to contribute to the dispersive as well as absorptive part of the off-diagonal elements of \mathcal{H} in order for the current data to be explained. The absorptive contribution, in particular, can be obtained from a very limited set of models, which will be severely tested in near future.

We also introduce the possibility of CPT violation by adding unequal NP contributions to the diagonal elements of \mathcal{H} . We explicitly show how CPT violation might affect the observables, especially dwelling on the effect on $\beta_s^{J/\psi\phi}$. Taken alone, the CPT violation cannot affect the dimuon asymmetry, and it can make the fit to the $B_s - \bar{B}_s$ mixing data only marginally better. In combination with a CPT conserving NP, it can enhance the allowed parameter space for that NP; however it does not seem to be able to obviate the need of an absorptive contribution from NP.

The data on all the observables considered in this paper is still relatively preliminary; the deviations from the SM are only at about $2 - 3\sigma$ level, and future data may either confirm these deviations or expose them as statistical fluctuations. If the errors and uncertainties shrink keeping the central values more or less intact, this will mean

- (i) The SM is strongly disfavored. Moreover, the relevant NP should be flavor-dependent, as we do not see much deviation in the $B_d - \bar{B}_d$ sector.
- (ii) The NP models that do not contribute to the absorptive amplitude of the $B_s - \bar{B}_s$ mixing are also strongly disfavored if CPT is conserved. The best bets are those NP models that provide both dispersive and absorptive amplitudes in the $B_s - \bar{B}_s$ mixing. This also gives rise to new decay channels for B_s . For example, one might find the branching ratio of $B_s \rightarrow \tau^+ \tau^-$ enhanced significantly from its SM expectation.
- (iii) Without any CPT -conserving NP, only CPT violation is only of marginal help, as it cannot enhance the semileptonic asymmetry. Even in combination with the CPT -conserving dispersive NP, it cannot allow regions in the parameter space to better than 3σ .

To summarize, the NP models that contribute an absorptive part to $B_s - \bar{B}_s$ mixing seem to be essential if one wants to explain the data on $\beta_s^{J\psi\phi}$ and A_{sl}^b simultaneously. There is only a limited set of such models, and they will be severely tested in near future. In the scenario that such an absorptive NP contribution is ruled out, one may have to resort to CPT violation in order to explain the data. A prominent signature of such a CPT violation would be a difference in $\beta_s^{J\psi/\phi}$ and $\bar{\beta}_s^{J\psi/\phi}$ as shown in Eqs. (36) and (37).

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- [1] T. Aaltonen *et al.* (CDF Collaboration), CDF Report No. CDF/PHYS/BOTTOM/CDFR/9787, 2009; V.M. Abazov *et al.* (D0 Collaboration), D0 Report No. 5928-CONF, 2009.
 - [2] D. Asner *et al.* (Heavy Flavor Averaging Group), [arXiv:1010.1589](https://arxiv.org/abs/1010.1589).
 - [3] V.M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. D* **82**, 032001 (2010); *Phys. Rev. Lett.* **105**, 081801 (2010); for the 9.0 fb^{-1} data, see V.M. Abazov *et al.* (D0 Collaboration), [arXiv:1106.6308](https://arxiv.org/abs/1106.6308).
 - [4] A. Dighe, A. Kundu, and S. Nandi, *Phys. Rev. D* **76**, 054005 (2007).
 - [5] A. Dighe, A. Kundu, and S. Nandi, *Phys. Rev. D* **82**, 031502 (2010).
 - [6] N. G. Deshpande, X. G. He, and G. Valencia, *Phys. Rev. D* **82**, 056013 (2010); R.-M. Wang, Y.-G. Xu, M.-L. Liu, and B.-Z. Li, *J. High Energy Phys.* **12** (2010) 034.
 - [7] A. K. Alok, S. Baek, and D. London, *J. High Energy Phys.* **07** (2011) 111.
 - [8] B. A. Dobrescu, P. J. Fox, and A. Martin, *Phys. Rev. Lett.* **105**, 041801 (2010).
 - [9] B. Dutta, S. Khalil, Y. Mimura, and Q. Shafi, [arXiv:1104.5209](https://arxiv.org/abs/1104.5209).
 - [10] D. Choudhury and D. K. Ghosh, *J. High Energy Phys.* **02** (2011) 033.
 - [11] S. Nandi and A. Soni, *Phys. Rev. D* **83**, 114510 (2011).
 - [12] B. Dutta, Y. Mimura, and Y. Santoso, *Phys. Rev. D* **82**, 055017 (2010); J. K. Parry, *Phys. Lett. B* **694**, 363 (2011).
 - [13] M. Endo, S. Shirai, and T. T. Yanagida, *Prog. Theor. Phys.* **125**, 921 (2011).
 - [14] S. Oh and J. Tandean, *Phys. Lett. B* **697**, 41 (2011).
 - [15] C. W. Bauer and N. D. Dunn, *Phys. Lett. B* **696**, 362 (2011).
 - [16] K. Blum, Y. Hochberg, and Y. Nir, *J. High Energy Phys.* **09** (2010) 035.
 - [17] A. Datta, M. Duraisamy, and S. Khalil, *Phys. Rev. D* **83**, 094501 (2011).
 - [18] A. Kostelecky and R. Van Kooten, *Phys. Rev. D* **82**, 101702 (2010).
 - [19] Y. Grossman, *Phys. Lett. B* **380**, 99 (1996).
 - [20] Z. Ligeti, M. Papucci, G. Perez, and J. Zupan, *Phys. Rev. Lett.* **105**, 131601 (2010).
 - [21] Y. Grossman, Z. Ligeti, and E. Nardi, *Phys. Rev. D* **55**, 2768 (1997).
 - [22] A. Lenz and U. Nierste, [arXiv:1102.4274](https://arxiv.org/abs/1102.4274); *J. High Energy Phys.* **06** (2007) 072.
 - [23] G. Giurgiu (CDF Collaboration) Proc. Sci., ICHEP2010 (2010) 236. Also see G. Giurgiu, *ICHEP, Paris, 2010*.
 - [24] T. Aaltonen *et al.* (CDF Collaboration), CDF Report No. 9015, 2007.
 - [25] A. Lenz, *Nucl. Phys. B, Proc. Suppl.* **177**, 81 (2008).
 - [26] K. Trabelsi (Belle Collaboration), in *SEL11, TIFR, Mumbai*, and available at <http://www.tifr.res.in/~sel11>.
 - [27] S. Coleman and S. Glashow, *Phys. Rev. D* **59**, 116008 (1999).
 - [28] O. W. Greenberg, *Phys. Rev. Lett.* **89**, 231602 (2002).
 - [29] M. Chaichian, A. D. Dolgov, V. A. Novikov, and A. Tureanu, *Phys. Lett. B* **699**, 177 (2011).
 - [30] K. Nakamura *et al.* (Particle Data Group Collaboration), *J. Phys. G* **37**, 075021 (2010).
 - [31] V. A. Kostelecky and N. Russell, *Rev. Mod. Phys.* **83**, 11 (2011).
 - [32] A. Datta, E. A. Paschos, and L. P. Singh, *Phys. Lett. B* **548**, 146 (2002); K. R. S. Balaji, W. Horn, and E. A. Paschos, *Phys. Rev. D* **68**, 076004 (2003).
 - [33] A. Kundu, S. Nandi, and S. K. Patra, *Phys. Rev. D* **81**, 076010 (2010).
 - [34] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, *Phys. Lett. B* **369**, 144 (1996); A. S. Dighe, I. Dunietz, and R. Fleischer, *Eur. Phys. J. C* **6**, 647 (1999).
 - [35] A. Pais and S. B. Treiman, *Phys. Rev. D* **12**, 2744 (1975); **16**, 2390(E) (1977). S. Bar-Shalom, G. Eilam, M. Gronau, and J. L. Rosner, *Phys. Lett. B* **694**, 374 (2011).