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Natural Peccei-Quinn symmetry in the 3-3-1 model with a minimal scalar sector

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In the framework of a 3-3-1 model with a minimal scalar sector we make a detailed study concerning the implementation of the Peccei-Quinn symmetry in order to solve the strong *CP* problem. For the original version of the model, with only two scalar triplets, we show that the entire Lagrangian is invariant under a Peccei-Quinn-like symmetry but no axion is produced since a $U(1)$ subgroup remains unbroken. Although in this case the strong *CP* problem can still be solved, the solution is largely disfavored since three quark states are left massless to all orders in perturbation theory. The addition of a third scalar triplet removes the massless quark states but the resulting axion is visible. In order to become realistic the model must be extended to account for massive quarks and an invisible axion. We show that the addition of a scalar singlet together with a Z_N discrete gauge symmetry can successfully accomplish these tasks and protect the axion field against quantum gravitational effects. To make sure that the protecting discrete gauge symmetry is anomaly-free we use a discrete version of the Green-Schwarz mechanism.

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I. INTRODUCTION

The standard model (SM) of the elementary particles physics successfully describes almost all of the phenomenology of the strong, electromagnetic, and weak interactions. However, from the experimental point of view, the need to go to physics beyond the standard model comes from the neutrino masses and mixing, which are required to explain the solar and atmospheric neutrino data. On the other hand, from the theoretical point of view, the SM cannot be taken as the fundamental theory since some important contemporary questions, like the number of generations of quarks and leptons, do not have an answer in its context. Unfortunately we do not know what the physics beyond the SM should be. A likely scenario is that at the TeV scale physics will be described by models which, at least, give some insight into the unanswered questions of the SM.

A way of introducing new physics is to enlarge the symmetry gauge group. For example, the gauge symmetry may be $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, instead of that of the SM. Models based on this gauge group have become known as 3-3-1 models [1–3]. Although the 3-3-1 models coincide with the SM at low energies, they explain some fundamental questions. This is the case of the number of generations cited above. In the 3-3-1 model framework, the number of generations must be three, or a multiple of three, in order to cancel anomalies. This is because the model is anomaly-free only if there is an equal number of triplets and antitriplets, including the color degrees of freedom. In this case, each generation is anomalous. The anomaly cancellation only occurs for the three, or multiple of three, generations together, and not generation by generation like in the SM. This provides, at least, a first step towards the understanding of the flavor question. Other interesting features of the 3-3-1 models concern the electric charge quantization and the vectorial character of the electromagnetic interaction [4,5]. These questions can be accommodated in the SM. However, in the 3-3-1 models these questions are related one to another and are independent of the nature of the neutrinos.

In recent literature we find studies about the most different aspects of the 3-3-1 model phenomenology. Among others, a fundamental puzzling aspect is, Why is the *CP* nonconservation in the strong interactions so small [6,7]? The last question, quantified by the $\bar{\theta}$ parameter of the effective QCD Lagrangian, is known as the strong *CP* problem. Several solutions based on different ideas have been proposed. According to the framework, they are based on unconventional dynamics [8], spontaneously broken *CP* [9–11], and an additional chiral symmetry. In the framework of introducing an additional chiral symmetry, two suggestions have been made. If this symmetry is not broken, the symmetry is realized in the Wigner-Weyl manner and the only possible way of relating this unbroken chiral symmetry with flavor conserving gluons is to have at least one massless quark [12]. This suggestion is disfavored by standard current algebra analysis [13,14]. The second possibility is that the global $U(1)$ chiral symmetry, known as $U(1)_{\rm PQ}$ [15,16], is spontaneously broken down, which implies a Nambu-Goldstone boson (NG boson), currently known as the axion [17–19].

In this paper we consider the strong *CP* problem in the framework of a version of the 3-3-1 model in which the scalar sector is minimal [20]. This model has become known as the "economical 3-3-1 model." The appealing feature of this 3-3-1 model is the natural existence of a

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Peccei-Quinn-like (PQ-like) $U(1)$ symmetry. To study the consequences of this symmetry in this model, we organize this paper as follows: in Sec. II we briefly describe the model, and in Sec. III we analyze the consequences of the natural PQ-like symmetry in the model and find that the symmetry is realized in the Wigner-Weyl manner implying three massless quarks, which disagrees with the standard current algebra analysis. Thus, we propose the introduction of two new scalar fields, η and ϕ, in order to both give a solution to the massless quarks and implement the PQ mechanism. Since this mechanism needs the $U(1)_{\mathrm{PQ}}$ to be anomalous in order to solve the strong CP problem, it does not seem *natural* to impose this symmetry on the Lagrangian. However, it could be understood as being *natural* if it is a residual symmetry of a larger one which is not anomalous and spontaneously broken. Then, we consider a Z_N discrete gauge symmetry to be a symmetry of the Lagrangian. The discrete gauge anomalies are canceled by a discrete version of the Green-Schwarz mechanism. After this, two Z_N symmetries, Z_{10} and Z_{11}, which protect the axion against quantum gravity effects, are explicitly shown. Finally, our conclusions are given in Sec. IV.

II. A BRIEF REVIEW OF THE ECONOMICAL 3-3-1 MODEL

The different models based on a 3-3-1 gauge symmetry can be classified according to the electric charge operator

$$Q = T^3 - bT^8 + X, \tag{1}$$

where T^3 and T^8 are the diagonal Gell-Mann matrices, X refers to the quantum number of the $U(1)_X$ group, and $b = 1/\sqrt{3}$, $\sqrt{3}$. The embedding b parameter defines the model. Here, we will consider the model with both $b = 1/\sqrt{3}$ and the simplest scalar sector, which was proposed for the first time in Ref. [21]. It has become known in the literature as "economical 3-3-1 model." This model had origin in a systematic study of all possible 3-3-1 models without exotic electric charges [22].

To give a brief review of the main features of this model, let us say that it has a fermionic matter content given by

$$\begin{aligned} &\Psi_{aL} = (\nu_a, e_a, (\nu_{aR})^C)_L^T \sim (\mathbf{1}, \mathbf{3}, -1/3),\\ &e_{aR} \sim (\mathbf{1}, \mathbf{1}, -1), \qquad Q_{\alpha L} = (d_\alpha, u_\alpha, d'_\alpha)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 0),\\ &Q_{3L} = (u_3, d_3, u'_3)_L^T \sim (\mathbf{3}, \mathbf{3}, 1/3), \qquad u_{aR} \sim (\mathbf{3}, \mathbf{1}, 2/3),\\ &u'_{3R} \sim (\mathbf{3}, \mathbf{1}, 2/3), \qquad d_{aR} \sim (\mathbf{3}, \mathbf{1}, -1/3),\\ &d'_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3), \end{aligned} \tag{2}$$

where $a = 1, 2, 3$, $\alpha = 1, 2$ (from now on Latin and Greek letters always take the values 1, 2, 3 and 1, 2, respectively), and the values in the parentheses denote quantum numbers based on the $(SU(3)_C, SU(3)_L, U(1)_X)$ factor, respectively. In this model the electric charges of the exotic quarks are the same as the usual ones, i.e., $Q(d'_\alpha) = -1/3$ and $Q(u'_3) = 2/3$.

In the bosonic matter content there are only two scalar triplets, χ and ρ:

$$\begin{aligned} \chi &= (\chi^0, \chi^-, \chi_1^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3),\\ \rho &= (\rho^+, \rho^0, \rho_1^+) \sim (\mathbf{1}, \mathbf{3}, 2/3). \end{aligned} \tag{3}$$

These two scalars spontaneously break down the $SU(3)_L \otimes U(1)_X$ gauge group. The vacuum expection values (vevs) in this model satisfy the constraint

$$V_{\rho^0} \equiv \langle \mathrm{Re}\rho^0 \rangle, \qquad V_{\chi^0} \equiv \langle \mathrm{Re}\chi^0 \rangle \ll V_{\chi_1^0} \equiv \langle \mathrm{Re}\chi_1^0 \rangle.$$

With the quark, lepton, and scalar multiplets above we have the Yukawa interactions

$$\mathcal{L}_{\mathrm{Y}}^l = Y_{ab} \bar{\Psi}_{aL} e_{bR} \rho + Y'_{ab} \epsilon^{ijk} (\bar{\Psi}_{aL})_i (\Psi_{bL})_j^C (\rho^*)_k + \mathrm{H.c.}, \tag{4}$$

for leptons. Y_{ab} and Y'_{ab} are arbitrary complex matrices and Y'_{ab} is also antisymmetric. Throughout the paper we use the convention that an addition over repeated indices is implied. The lepton masses are generated by the interactions in Eq. (4). The first term gives a general tree level mass matrix for the charged leptons [20]. However, for the neutrino mass generation, the interactions in the second term are not able to provide a realistic mass spectrum at the tree level. At least 1-loop corrections must be considered in order to obtain neutrino masses compatible with the solar and atmospheric neutrino data [23].

For quarks we have

$$\begin{aligned} \mathcal{L}_{\mathrm{Y}}^q = {}& G^1 \bar{Q}_{3L} u'_{3R} \chi + G^2_{\alpha\beta} \bar{Q}_{\alpha L} d'_{\beta R} \chi^* + G^3_a \bar{Q}_{3L} d_{aR} \rho\\ &+ G^4_{\alpha a} \bar{Q}_{\alpha L} u_{aR} \rho^* + G^5_a \bar{Q}_{3L} u_{aR} \chi + G^6_{\alpha a} \bar{Q}_{\alpha L} d_{aR} \chi^*\\ &+ G^7_\alpha \bar{Q}_{3L} d'_{\alpha R} \rho + G^8_\alpha \bar{Q}_{\alpha L} u'_{3R} \rho^* + \mathrm{H.c.}, \end{aligned} \tag{5}$$

where G^i are arbitrary complex matrices. Notice that the Yukawa interactions given in Eqs. (4) and (5) are the most general allowed by the gauge symmetries. Here, we follow exactly Refs. [20,24]; i.e., no additional symmetries are imposed, contrary to what is done in Ref. [25] where a Z_2 symmetry is imposed.

The most general scalar potential invariant under the gauge symmetry is

$$\begin{aligned} V_{\mathrm{H}} = {}& \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\rho^\dagger \rho)^2\\ &+ \lambda_3 (\chi^\dagger \chi)(\rho^\dagger \rho) + \lambda_4 (\chi^\dagger \rho)(\rho^\dagger \chi). \end{aligned} \tag{6}$$

One of the main features of this model is that its scalar sector is the simplest possible. In principle, this should make the scalar potential analysis easier. A study of the stability of this scalar potential is presented in Ref. [26].

III. $U(1)_{\rm PQ}$ SYMMETRY IN THE ECONOMICAL 3-3-1 MODEL

A $U(1)_{\rm PQ}$ symmetry is global and chiral [15,16]; i.e., it treats the left- and right-handed parts of a Dirac field differently. Moreover, it must be both a symmetry of the entire Lagrangian and valid only at the classical level. In renormalizable theories, the key ingredient of the $U(1)_{\rm PQ}$ is that it must be afflicted by a color anomaly; i.e., its associated current, $j_\mu^{\rm PQ}$, must obey

$$\partial^\mu j_\mu^{\rm PQ} \supset \frac{Ng^2}{16\pi^2} G\tilde{G}, \tag{7}$$

being $G\tilde{G} = \frac{1}{2}\epsilon^{\mu\nu\sigma\tau} G^b_{\mu\nu} G^b_{\sigma\tau}$, and $G^b_{\mu\nu}$ is the color field strength tensor ($b = 1, \ldots, 8$). N must not be zero.

Now, we are going to prove that the economical 3-3-1 model entire Lagrangian is naturally invariant under a $U(1)_{\rm PQ}$ symmetry transformation. To do so, we search for how many $U(1)$ symmetries the model has. First of all, we write the relations that these symmetries must obey in order to keep the entire Lagrangian invariant. From Eqs. (4)–(6) we obtain the following relations:

$$-X_{Q_3} + X_{u'_{3R}} + X_\chi = 0, \quad -X_Q + X_{d'_R} - X_\chi = 0, \tag{8}$$

$$-X_{Q_3} + X_{u_R} + X_\chi = 0, \quad -X_Q + X_{d_R} - X_\chi = 0, \tag{9}$$

$$-X_{Q_3} + X_{d_R} + X_\rho = 0, \quad -X_Q + X_{u_R} - X_\rho = 0, \tag{10}$$

$$-X_{Q_3} + X_{d'_R} + X_\rho = 0, \quad -X_Q + X_{u'_{3R}} - X_\rho = 0, \tag{11}$$

$$-X_\Psi + X_{e_R} + X_\rho = 0, \qquad -2X_\Psi - X_\rho = 0, \tag{12}$$

where the notation X_ψ above is to be understood as the $U(1)$ charge of the ψ field. Solving the equations above, we find three independent $U(1)$ symmetries. One of these is the $U(1)_X$ gauge symmetry. The other two are the usual baryon number symmetry, $U(1)_B$, and a chiral symmetry acting on the quarks, $U(1)_{\rm PQ}$. Thus, the model actually has a larger symmetry: $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_B \otimes U(1)_{\rm PQ}$. The two last symmetries are global. This is summarized in Table I. We can see that the $U(1)_{\rm PQ}$ chiral symmetry is afflicted by a color anomaly in the following way:

$$A_{\rm PQ} \propto -X_\rho - 2X_\chi = -3, \tag{13}$$

where $A_{\rm PQ}$ is the coefficient of the $[SU(3)_C]^2 U(1)_{\rm PQ}$ anomaly. Therefore, this chiral symmetry is a PQ-like symmetry. Also, notice that in this case the $U(1)_{\rm PQ}$ is an accidental symmetry; i.e., it follows from the gauge local symmetry plus renormalizability. In other words, the economical model naturally has a PQ symmetry. The naturalness of the $U(1)_{\rm PQ}$ in the economical 3-3-1 model is a key point. In our understanding, since $U(1)_{\rm PQ}$ symmetry is anomalous its imposition is not sensible in the sense that in the absence of further constraints on very high energy physics we should expect all relevant and marginally relevant operators that are forbidden only by this symmetry to appear in the effective Lagrangian with coefficient of order one, but if this symmetry follows from some other free anomaly symmetry, in our case from the gauge symmetry, all terms which violate it are then irrelevant in the renormalization group sense.

Unfortunately, when χ and ρ acquire vevs different from zero, a subgroup of $U(1)_X \otimes U(1)_{\rm PQ}$ remains unbroken; i.e., the symmetry-breaking pattern is

$$SU(3)_L \otimes U(1)_X \otimes U(1)_{\rm PQ} \stackrel{\langle\chi\rangle}{\to} SU(2)_L \otimes U(1)_Y \otimes U(1)'_{\rm PQ} \stackrel{\langle\rho\rangle}{\to} U(1)_Q \otimes U(1)''_{\rm PQ}, \tag{14}$$

where $U(1)_Q$ is the electromagnetic symmetry. The $SU(3)_C$ and $U(1)_B$ groups have been omitted in the expression above because these are both unbroken and irrelevant to the current analysis. An explicit expression of the $U(1)'_{\rm PQ}$ symmetry can be easily written as

$$U(1)'_{\rm PQ} \equiv U(1)_{\rm PQ} + 3U(1)_X. \tag{15}$$

Also, note that $U(1)'_{\rm PQ}$ and $U(1)''_{\rm PQ}$ are PQ-like symmetries because these are chiral and afflicted by a color anomaly.

As a consequence of the unbroken $U(1)''_{\rm PQ}$ chiral symmetry [i.e., $U(1)''_{\rm PQ}$ is realized in the Wigner-Weyl manner], no axion appears in the scalar mass spectrum. Instead of that, some quarks remain massless after the spontaneous symmetry breaking, and these will remain massless to all orders of perturbation theory.

To illustrate the preceding, we explicitly calculate the mass spectra of scalars and quarks. First, we calculate the scalar mass spectrum

$$m^2_{H_1,H_2} = \lambda_1 V^2_{\rho^0} + (V^2_{\chi^0} + V^2_{\chi^0_1})\lambda_2 \pm \sqrt{(V^2_{\rho^0}\lambda_1 - (V^2_{\chi^0} + V^2_{\chi^0_1})\lambda_2)^2 + (V^2_{\chi^0} + V^2_{\chi^0_1})V^2_{\rho^0}\lambda_3^2}, \tag{16}$$

TABLE I. Assignment of quantum charges in the economical 3-3-1 model.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	χ
$U(1)_X$	0	1/3	2/3	−1/3	−1/3	−1	2/3	−1/3
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0
$U(1)_{\rm PQ}$	−1	1	0	0	−1/2	−3/2	1	1

$$m^2_{H_3^\pm} = \frac{1}{2}(V^2_{\rho^0} + V^2_{\chi^0} + V^2_{\chi_1^0})\lambda_4, \tag{17}$$

where V_{ρ^0}, V_{χ^0}, $V_{\chi_1^0}$ are the vevs of ρ^0, χ^0, χ_1^0, respectively. For simplicity, all the vevs have been assumed to be real. Additionally, there are exactly 8 NG bosons that will become the longitudinal components of the 8 gauge bosons [21]. The absence of one physical massless state (or axion) in the scalar spectrum shows that the $U(1)''_{\rm PQ}$ symmetry remains unbroken after the spontaneous symmetry breaking.

On the other hand, in the quark spectra, there are three massless states, one in the up-quark sector and two in the down-quark sector. First, consider the up-quark mass matrix at the tree level which is written as

$$\bar{\mathbf{u}}_L M_u^{(0)} \mathbf{u}_R \equiv \frac{1}{\sqrt{2}} \bar{\mathbf{u}}_L \begin{bmatrix} G^4_{11}V_{\rho^0} & G^4_{12}V_{\rho^0} & G^4_{13}V_{\rho^0} & G^8_1 V_{\rho^0} \\ G^4_{21}V_{\rho^0} & G^4_{22}V_{\rho^0} & G^4_{23}V_{\rho^0} & G^8_2 V_{\rho^0} \\ G^5_1 V_{\chi^0} & G^5_2 V_{\chi^0} & G^5_3 V_{\chi^0} & G^1 V_{\chi^0} \\ G^5_1 V_{\chi_1^0} & G^5_2 V_{\chi_1^0} & G^5_3 V_{\chi_1^0} & G^1 V_{\chi_1^0} \end{bmatrix} \mathbf{u}_R, \tag{18}$$

where $\bar{\mathbf{u}}_L \equiv (\bar{u}_{1L}, \bar{u}_{2L}, \bar{u}_{3L}, \bar{u}'_{3L})$ and $\mathbf{u}_R \equiv (u_{1R}, u_{2R}, u_{3R}, u'_{3R})^T$. The third and fourth rows of the $M_u^{(0)}$ matrix are proportional; thus there is a massless up quark (we refer to this massless up quark simply as u) at the tree level. An analytical expression for this massless state can be given but it is useless for our analysis. Later we give arguments that the u quark remain massless to all orders of perturbation theory [27]. Similarly, the down-quark mass matrix at the tree level, $M_d^{(0)}$, defined as $\frac{1}{\sqrt{2}}\bar{\mathbf{d}}_L M_d^{(0)} \mathbf{d}_R$, reads

$$\begin{bmatrix} G^6_{11}V_{\chi^0} & G^6_{12}V_{\chi^0} & G^6_{13}V_{\chi^0} & G^2_{11}V_{\chi^0} & G^2_{12}V_{\chi^0} \\ G^6_{21}V_{\chi^0} & G^6_{22}V_{\chi^0} & G^6_{23}V_{\chi^0} & G^2_{21}V_{\chi^0} & G^2_{22}V_{\chi^0} \\ G^3_1 V_{\rho^0} & G^3_2 V_{\rho^0} & G^3_3 V_{\rho^0} & G^7_1 V_{\rho^0} & G^7_2 V_{\rho^0} \\ G^6_{11}V_{\chi_1^0} & G^6_{12}V_{\chi_1^0} & G^6_{13}V_{\chi_1^0} & G^2_{11}V_{\chi_1^0} & G^2_{12}V_{\chi_1^0} \\ G^6_{21}V_{\chi_1^0} & G^6_{22}V_{\chi_1^0} & G^6_{23}V_{\chi_1^0} & G^2_{21}V_{\chi_1^0} & G^2_{22}V_{\chi_1^0} \end{bmatrix}, \tag{19}$$

where $\bar{\mathbf{d}}_L \equiv (\bar{d}_{1L}, \bar{d}_{2L}, \bar{d}_{3L}, \bar{d}'_{1L}, \bar{d}'_{2L})$ and $\mathbf{d}_R \equiv (d_{1R}, d_{2R}, d_{3R}, d'_{1R}, d'_{2R})^T$. Since the first and fourth rows, and the second and fifth rows, are proportional to each other, the $M_d^{(0)}$ matrix has two eigenvalues equal to zero (we refer to these massless down quarks as d and s). Thus, the economical model has three massless quark states: one in the up-quark sector and two in the down-quark sector. In other words, the economical 3-3-1 model has a remaining unbroken chiral symmetry, $U(1)''_{\rm PQ}$, which allows us to transform $u_L \to e^{i\alpha}u_L$, $d_L \to e^{i\alpha}d_L$, $s_L \to e^{i\alpha}s_L$, leaving the Lagrangian invariant. This symmetry will protect these massless quarks to acquire mass at any level of perturbation theory [27]. At this point it is important to say that, since the $U(1)''_{\rm PQ}$ symmetry is anomalous, these quarks will acquire mass only through QCD nonperturbative effects (for example, by instanton effects [28]). Although the quarks could acquire some mass through these nonperturbative processes, this is in conflict with both chiral QCD and lattice calculation where the ratio m_u/m_d is 0.410 ± 0.036 [13,14,29].

Before considering a possible solution to the problem mentioned above, for the sake of completeness, we find it important to say that in Ref. [20] one-loop contributions to the up-quark mass matrix were calculated, even though a subtle flaw makes these contributions not right. To demonstrate that, we exactly follow the same lines as in Ref. [20]. There, in Sec. 4, the authors consider, for simplicity, one-loop contributions to the submatrix

$$M^{(0)}_{u_3 u'_3} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} G^5_3 V_{\chi^0} & G^1 V_{\chi^0} \\ G^5_3 V_{\chi_1^0} & G^1 V_{\chi_1^0} \end{bmatrix}, \tag{20}$$

where $M^{(0)}_{u_3 u'_3}$ is written in the base (u_3, u'_3). The other two massive quark states, u_1 and u_2, which acquire mass at tree level [$m_1 = G^4_{11}V_{\rho^0}/\sqrt{2}$, $m_2 = G^4_{22}V_{\rho^0}/\sqrt{2}$, see Eq. (27) in Ref. [20]] are not important in the analysis. The matrix Eq. (20) mixes together the states u_3 and u'_3. A combination of them will be a massless quark and the orthogonal combination acquires a mass $\sim V_{\chi_1^0}$.

Now, the idea is to calculate the one-loop contributions coming from the Feynman diagrams in Fig. 1 to the up-quark mass submatrix defined in Eq. (20). Following Ref. [20], we get

$$\begin{aligned} \Delta_{u_{3L}, u'_{3R}} &= -2iV_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u'_3}(G^1)^2 \\ &\quad \times \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2 - M^2_{u'_3})^2(p^2 - M^2_{\chi^0})(p^2 - M^2_{\chi_1^0})} \\ &\equiv 2V_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u'_3}(G^1)^2 I(M^2_{u'_3}, M^2_{\chi^0}, M^2_{\chi_1^0}), \end{aligned} \tag{21}$$

where $I(M^2_{u'_3}, M^2_{\chi^0}, M^2_{\chi_1^0})$ is defined as

$$\begin{aligned} I(M^2_{u'_3}, M^2_{\chi^0}, M^2_{\chi_1^0}) \\ \equiv -i \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2 - M^2_{u'_3})^2(p^2 - M^2_{\chi^0})(p^2 - M^2_{\chi_1^0})}, \end{aligned} \tag{22}$$

and $\Delta_{u_{3L}, u'_{3R}}$ is the one-loop contribution to the element $(M^{(0)}_{u_3 u'_3})_{12}$ given by the Feynman diagram in Fig. 1(a). The value of the integral in Eq. (22) is not relevant in our analysis and thus it is not calculated. Now, $\Delta_{u_{3L}, u_{3R}}$ is found in a similar way from the diagram in Fig. 1(b),

$$\begin{aligned} \Delta_{u_{3L}, u_{3R}} &= -2iV_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u'_3} G^5_3 G^1 \\ &\quad \times \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2 - M^2_{u'_3})^2(p^2 - M^2_{\chi^0})(p^2 - M^2_{\chi_1^0})} \\ &= \frac{G^5_3}{G^1}\Delta_{u_{3L}, u'_{3R}}. \end{aligned} \tag{23}$$

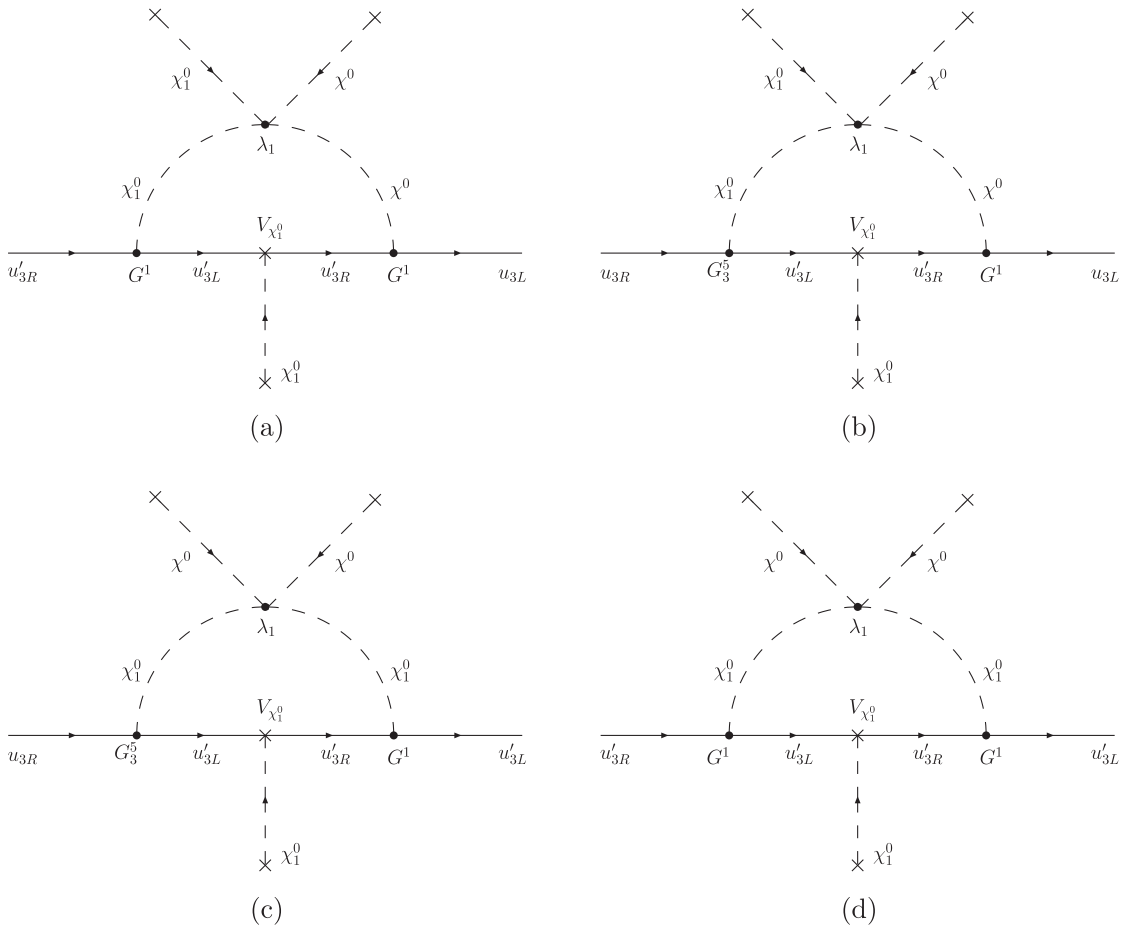

FIG. 1. One-loop contributions to the up-quark mass matrix.

One-loop contributions to $(M^{(0)}_{u_3u'_3})_{21}$ and $(M^{(0)}_{u_3u'_3})_{22}$, found from the Feynman diagrams in 1(c) and 1(d), respectively, are also proportional to each other, i.e.,

$$\Delta_{u'_{3L},u_{3R}} = \frac{G_3^5}{G^1}\Delta_{u'_{3L},u'_{3R}}. \tag{24}$$

Therefore, when considering simultaneously all the one-loop contributions above, the $M^{(0)}_{u_3u'_3}$ becomes

$$\frac{1}{\sqrt{2}}\begin{bmatrix} G_3^5\left(V_{\chi^0} + \frac{\Delta_{u_{3L},u'_{3R}}}{G^1}\right) & G^1\left(V_{\chi^0} + \frac{\Delta_{u_{3L},u'_{3R}}}{G^1}\right) \\ G_3^5\left(V_{\chi_1^0} + \frac{\Delta_{u'_{3L},u'_{3R}}}{G^1}\right) & G^1\left(V_{\chi_1^0} + \frac{\Delta_{u'_{3L},u'_{3R}}}{G^1}\right)\end{bmatrix}. \tag{25}$$

This matrix still has a determinant equal to zero. In other words, we have shown that one combination of the up quarks still remains massless, as it should be. In the down-quark sector a similar analysis can be easily made. Thus, what makes the contributions to the up-quark and down-quark masses made in Ref. [20] not right is that those contributions were not considered simultaneously.

To conclude, the 3-3-1 economical model has three massless quarks (one up quark and two down quarks) to all order of perturbation theory, which is in conflict with both chiral QCD and lattice calculation where the ratio m_u/m_d is 0.410 ± 0.036 [14]. Therefore, the economical model is not realistic and it must be modified to overcome that difficulty. One manner of doing that is introducing a new scalar triplet, η:

$$\eta = (\eta^0, \eta^-, \eta_1^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3). \tag{26}$$

When the scalar triplet, η, is introduced into the model, the Yukawa Lagrangian given in Eq. (5) has the following extra terms:

$$\mathcal{L}^q_{\rm Y,extra} = G_a^9\bar{Q}_{3L}u_{aR}\eta + G_{\alpha a}^{10}\bar{Q}_{\alpha L}d_{aR}\eta^* + G^{11}\bar{Q}_{3L}u'_{3R}\eta + G_{\alpha\beta}^{12}\bar{Q}_{\alpha L}d'_{\beta R}\eta^* + \text{H.c.} \tag{27}$$

As can be seen from Eq. (5) and Eq. (27), the quark fields interact with different neutral scalar fields simultaneously. Hence, flavor-changing neutral currents (FCNCs) are, in general, induced. This characteristic is shared by most of multi-Higgs models [30]. In order to suppress the FCNC effects we must use some model dependent strategies, for instance, choosing an appropriate direction in the vev space, resorting to heavy scalars and/or small mixing angles in the quark and the scalar sectors, and considering adequate Yukawa coupling matrix textures [3,30–32]. In particular, in this model the exotic quarks have the same electric charge as the ordinary ones. This means that they can mix with the later ones and hence also induce FCNC. However, this kind of FCNC is suppressed when the vev which controls the exotic quark masses is taken much larger than the electroweak mass scale [3,32]. FCNC also occurs in models which have an extra neutral vector boson. They can be handled in a similar way. See, for example, [33]. Finally, from Eq. (4) we see that the lepton sector of the model is not afflicted by FCNC.

The most general scalar potential invariant under the gauge symmetry, $V = V_{\rm H} + V_{\rm NH}$, has now the following extra terms:

$$V_{\rm H,extra} = \mu_\eta^2 \eta^\dagger \eta + \lambda_5 (\eta^\dagger \eta)^2 + \eta^\dagger \eta [\lambda_6 (\rho^\dagger \rho) + \lambda_7 (\chi^\dagger \chi)] + \lambda_8 (\rho^\dagger \eta)(\eta^\dagger \rho) + \lambda_9 (\chi^\dagger \eta)(\eta^\dagger \chi), \tag{28}$$

and

$$V_{\rm NH} = \mu_4^2 \chi^\dagger \eta + f \epsilon^{ijk} \eta_i \rho_j \chi_k + \lambda_{10} (\chi^\dagger \eta)^2 + \lambda_{11} (\chi^\dagger \rho)(\rho^\dagger \eta) + \lambda_{12} (\chi^\dagger \eta)(\eta^\dagger \eta) + \lambda_{13} (\chi^\dagger \eta)(\rho^\dagger \rho) + \lambda_{14} (\chi^\dagger \eta)(\chi^\dagger \chi) + {\rm H.c.} \tag{29}$$

Now, when the scalar triplets acquire vevs, it is straightforward to see that the quark mass matrices do not have determinant equal to zero; thus all the quarks are massive. Additionally, as we will show below, there will be no accidental anomalous PQ-like symmetry.

Returning to the question of the PQ symmetry, we note that due to these new terms in the Lagrangian the charges of the $U(1)$ symmetries must obey the following relations,

$$-X_{Q_3} + X_{u_R} + X_\eta = 0, \quad -X_{Q_3} + X_{u'_R} + X_\eta = 0, \tag{30}$$

$$-X_Q + X_{d'_R} - X_\eta = 0, \quad -X_Q + X_{d_R} - X_\eta = 0, \tag{31}$$

$$X_\rho + X_\eta + X_\chi = 0, \qquad -X_\chi + X_\eta = 0, \tag{32}$$

besides the ones given in Eqs. (8)–(12). Solving Eqs. (8)–(12) and Eqs. (30)–(32) simultaneously, we find that there are only two $U(1)$ symmetries, $U(1)_X$ and $U(1)_B$. The assignment of quantum charges for these two $U(1)$ symmetries when η is included is shown in Table II. Thus, in this case, in contrast to the previous one, the $U(1)_{\rm PQ}$ is not allowed by the gauge symmetry. But, if the Lagrangian is slightly modified by imposing a Z_2 symmetry such that $\chi \to -\chi$, $u'_{3R} \to -u'_{3R}$, $d'_{\beta R} \to -d'_{\beta R}$ and all the other fields being even under Z_2, the trilinear term of the scalar potential, $f\epsilon^{ijk}\eta_i\rho_j\chi_k$, is eliminated. Consequently, the $U(1)_{\rm PQ}$ symmetry is automatically introduced. This can be seen by solving Eqs. (8)–(12) and Eqs. (30)–(32) without the equation

$$X_\rho + X_\eta + X_\chi = 0. \tag{33}$$

Note that, in addition to the assignment of quantum charges given in Table I, the charge $U(1)_{\rm PQ}$ of the η triplet scalar is 1. Unfortunately, the axion that appears when the neutral components of the scalar triplets acquire vev is visible. This is easy to see as follows. In this model the χ field is responsible for breaking the symmetry from $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Thus, to obtain an invisible axion, $V_{\chi_1^0}$ that breaks the PQ symmetry must be greater than 10^9 GeV. But, when χ acquires a vev the combination $U(1)'_{\rm PQ} = U(1)_{\rm PQ} + 3U(1)_X$ is not broken. Therefore, the new PQ symmetry is truly broken when the ρ field acquires a vev. As $V_{\rho^0} \lesssim 246$ GeV, the axion induced is visible. A visible axion was long ago ruled out by experiments [34].

One usual way to resolve that problem is to introduce an electroweak scalar singlet, ϕ [17,18]. Its role is to break the PQ symmetry at a scale much larger than the electroweak scale. This field does not couple directly to quarks and leptons; however, it acquires a PQ charge by coupling to the scalar triplets. With the PQ charges given in Table I, the ϕ scalar acquires a PQ charge by coupling to the η, ρ, χ scalar triplets through the interaction term

$$\lambda_{\rm PQ} \epsilon^{ijk} \eta_i \rho_j \chi_k \phi. \tag{34}$$

From this coupling, the ϕ field obtains a PQ charge of -3. Also, notice that this term is permitted provided the ϕ field is odd under the Z_2 symmetry, i.e., $Z_2(\phi) = -\phi$. However, the Z_2 and gauge symmetries do not prohibit some terms in the scalar potential violating the PQ symmetry, such as ϕ^2, ϕ^4, $\rho^\dagger\rho\phi^2$, $\eta^\dagger\eta\phi^2$, $\chi^\dagger\chi\phi^2$, from appearing. Thus, the PQ symmetry should be imposed. Since the PQ symmetry is anomalous, it is somewhat awkward to do so. However, there is a way to overcome this difficulty. Consider that the entire Lagrangian is invariant under a Z_N discrete gauge symmetry [35], with $N \geq 5$, instead of a Z_2 symmetry. The Z_N charge assignment that allows the scalar potential to be naturally free of awkward terms violating the PQ symmetry must satisfy the following minimal conditions:

TABLE II. Assignment of quantum charges when η is included.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	(χ, η)
$U(1)_X$	0	1/3	2/3	−1/3	−1/3	−1	2/3	−1/3
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0

$$Z_N(\phi) \neq (0, N/2, N/3, N/4), \tag{35}$$

$$Z_N(\eta) + Z_N(\rho) + Z_N(\chi) \neq pN, \tag{36}$$

$$-Z_N(\chi) + Z_N(\eta) = rN, \qquad p, r \in \mathbb{Z}, \tag{37}$$

and, obviously, the other ones that leave the rest of the Lagrangian invariant under Z_N. The $-Z_N(\chi) + Z_N(\eta) = rN$ condition, with $r \in \mathbb{Z}$, is necessary to allow the terms in the scalar potential given in Eq. (29), except the trilinear $f\epsilon^{ijk}\eta_i\rho_j\chi_k$ term and, thus, avoid the appearance of an additional dangerous massless scalar in the physical spectrum. In other words, with the conditions imposed by Eqs. (35)–(37) for this Z_N discrete symmetry, none of Lagrangian terms, except the violating PQ terms, such as $f\epsilon^{ijk}\eta_i\rho_j\chi_k$, ϕ^2, ϕ^3, ϕ^4, etc., are prohibited from appearing.

Furthermore, to stabilize the axion solution from quantum gravitational effects [36,37] we will make use of the Z_N discrete symmetry with anomaly cancellation by a discrete version of the Green-Schwarz mechanism [38–41]. Quantum gravity effective operators, allowed by the gauge symmetry, of the form $\phi^N/M_{\rm Pl}^{N-4}$ can induce a nonzero $\bar{\theta}$ given by

$$\bar{\theta} \simeq \frac{f_a^N}{\Lambda_{\rm QCD}^4 M_{\rm Pl}^{N-4}}. \tag{38}$$

From the neutron electric dipole moment (EDM) experimental data $\bar{\theta} \lesssim 10^{-11}$ [29], and using $f_a \sim 10^{10}$ GeV, we find that $N \geq 10$, in order to keep the PQ solution stable. It means that effective operators with $N < 10$ must be forbidden by the Z_N symmetry.

The neutron EDM will also receive contributions that do not come from the $\bar{\theta}\tilde{G}G$ term. Those which are similar to the SM contributions will pose no problems since they will have approximately the same values and will give $d_n^{\rm SM\text{-}CKM} \sim 10^{-32}$ e cm [42], i.e., 6 to 7 orders of magnitude smaller than the experimental limit [43]. The other contributions, which are specific to the present 3-3-1 model, like the 1-loop contribution due to the exchange of a charged scalar (χ^-), can be used to constrain the still free parameters of the model, in order to be consistent with the experimental neutron EDM data [43,44]. We will return to this point later.

Before introducing the Z_N symmetry to stabilize the PQ mechanism, we calculate the axion state. With the introduction of the scalar singlet ϕ, the scalar potential gains the following extra terms:

$$V_{\phi,\rm extra} = -\mu_\phi^2 \phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 + \lambda_{15}(\rho^\dagger\rho)(\phi^\dagger\phi) + \lambda_{16}(\eta^\dagger\eta)(\phi^\dagger\phi) + \lambda_{17}(\chi^\dagger\chi)(\phi^\dagger\phi). \tag{39}$$

Now, to calculate the eigenstate of the axion field, we write the fields as

$$\rho = \begin{pmatrix} \rho^+ \\ \frac{1}{\sqrt{2}}(V_{\rho^0} + {\rm Re}\rho^0 + i{\rm Im}\rho^0) \\ \rho^{++} \end{pmatrix},$$
$$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(V_{\eta^0} + {\rm Re}\eta^0 + i{\rm Im}\eta^0) \\ \eta^- \\ \frac{1}{\sqrt{2}}(V_{\eta_1^0} + {\rm Re}\eta_1^0 + i{\rm Im}\eta_1^0) \end{pmatrix}, \tag{40}$$
$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(V_{\chi^0} + {\rm Re}\chi^0 + i{\rm Im}\chi^0) \\ \chi^- \\ \frac{1}{\sqrt{2}}(V_{\chi_1^0} + {\rm Re}\chi_1^0 + i{\rm Im}\chi_1^0) \end{pmatrix},$$
$$\phi = \frac{1}{\sqrt{2}}(V_\phi + {\rm Re}\phi + i{\rm Im}\phi).$$

The axion field must be isolated from the eight NG bosons that are absorbed by the gauge bosons in the unitary gauge. This is fundamental to do a correct phenomenological study of the axion properties. By following standard procedures, the axion field, $a(x)$, is determined to be

$$a(x) = \frac{1}{f_a}\left[\frac{V_-^2}{V_{\rho^0}}{\rm Im}\rho^0 - V_{\chi_1^0}{\rm Im}\eta^0 + V_{\chi^0}{\rm Im}\eta_1^0 + V_{\eta_1^0}{\rm Im}\chi^0 - V_{\eta^0}{\rm Im}\chi_1^0 - \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)V_\phi{\rm Im}\phi\right], \tag{41}$$

where

$$V_-^2 \equiv V_{\chi^0}V_{\eta_1^0} - V_{\chi_1^0}V_{\eta^0}, \tag{42}$$

$$V_+^2 \equiv V_{\chi^0}^2 + V_{\chi_1^0}^2 + V_{\eta^0}^2 + V_{\eta_1^0}^2, \tag{43}$$

and f_a is the normalization constant given by

$$f_a \equiv \sqrt{\left(\frac{V_-^2}{V_{\rho^0}}\right)^2 + V_+^2 + \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)^2 V_\phi^2}. \tag{44}$$

Note that in the limit $V_\phi \gg V_{\chi^0}, V_{\chi_1^0}, V_{\eta^0}, V_{\eta_1^0}$

$$a(x) \simeq -{\rm Im}\phi + \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)^{-1} V_\phi^{-1}\left[\frac{V_-^2}{V_{\rho^0}}{\rm Im}\rho^0 - V_{\chi_1^0}{\rm Im}\eta^0 + V_{\chi^0}{\rm Im}\eta_1^0 + V_{\eta_1^0}{\rm Im}\chi^0 - V_{\eta^0}{\rm Im}\chi_1^0\right]; \tag{45}$$

i.e., the axion is primarily composed of the ${\rm Im}\phi$ field. As is well-known, to make the invisible axion compatible with astrophysical and cosmological considerations, the axion decay constant, f_a, must be in the range 10^9 GeV $\leq f_a \leq 10^{12}$ GeV.

Now, returning to the stabilization of the axion by the Z_N symmetry, let us put that in a short way. If the Z_N symmetry that survives at low energies was part of an "anomalous" $U(1)_A$ gauge symmetry, the Z_N charges of the fermions in the low energy theory must satisfy nontrivial conditions: The anomaly coefficients for the full theory are given by the coefficients for the low energy sector, in our case $A_{3C} \equiv [SU(3)_C]^2 U(1)_A$ and $A_{3L} \equiv [SU(3)_L]^2 U(1)_A$, plus an integer multiple of $N/2$ [45,46], i.e.,

$$\frac{A_{3C} + pN/2}{k_{3C}} = \frac{A_{3L} + rN/2}{k_{3L}} = \delta_{\rm GS}, \qquad (46)$$

with p and r being integers. The k_{3C} and k_{3L} are the levels of the Kac-Moody algebra for the $SU(3)_C$ and $SU(3)_L$, respectively. In the present case these are positive integers. Finally, the $\delta_{\rm GS}$ is a constant that is not specified by the low energy theory alone. Other anomalies such as $[U(1)_A]^3$, $[U(1)_A]^2 U(1)_X$ do not give useful low energy constraints because these depend on some arbitrary choices concerning $U(1)_A$ [47]. This is why these do not appear in Eq. (46). Now, to identify that anomalous $U(1)_A$ symmetry, it is helpful to write it as a linear combination of the $U(1)_{\rm PQ}$ and the $U(1)_B$ symmetries, i.e.,

$$U(1)_A = \alpha[U(1)_{\rm PQ} + \beta U(1)_B], \qquad (47)$$

where α is a normalization constant used to make the $U(1)_A$ charges integer numbers. With the charges given in Table I, it is straightforward to calculate the anomaly coefficients A_{3C} and A_{3L},

$$A_{3C} = -\frac{3}{2}\alpha, \qquad A_{3L} = \left[-\frac{9}{4} + \frac{3}{2}\beta\right]\alpha. \qquad (48)$$

Thus, the β parameter that satisfies the condition given in Eq. (46) is

$$\beta = \frac{1}{3}\left[-3\frac{k_{3L}}{k_{3C}} + \frac{9}{2} + \frac{N}{\alpha}\left(\frac{k_{3L}}{k_{3C}}p - r\right)\right]. \qquad (49)$$

Taking the simplest possibility for the parameters k_{3C} and k_{3L}, i.e., $k_{3C} = k_{3L}$, the parameter β becomes

$$\beta = \frac{1}{3}\left[\frac{3}{2} + \frac{N}{\alpha}(p - r)\right]. \qquad (50)$$

Recalling that to stabilize the axion from the quantum gravity corrections we need $N \geqslant 10$, we show two possible solutions with $N = 10$ and 11. The corresponding charge assignment of these two discrete subgroups of the $U(1)_A$ symmetry is given in Table III. Also, it is important to remember that those charges are defined mod N.

It can be explicitly verified that the charges in Table III satisfy Eq. (46), as it should be, since Z_{10} and Z_{11} are discrete subgroups of $U(1)_A$, which is anomaly-free by the Green-Schwarz mechanism.

At this point, an important remark is in order. In its most general form, this model possesses other CP-violating sources apart from the strong CP-violating $\bar{\theta}$ term, which can give additional contributions to the electric dipole moment of the neutron. The reason is that not all phases can be absorbed into the quark and lepton field definitions. Therefore, it is necessary to estimate if these additional contributions do not require tuning the model parameters at the same order of the $\bar{\theta}$ parameter. Then, let us compute a representative case: the up-quark electric dipole moment, d_u^e. One dominant diagram contributing to d_u^e is derived from the one given in Fig. 1(b), when an external photon line is attached. To compute the resultant diagram, we need to know the mixing of the scalar fields, $\mathcal{C}_{ij}$, coming from the diagonalization of the scalar mass matrix. However, we will consider $\mathcal{C}_{ij} \sim \mathcal{O}(1)$, which is the worst case. Standard calculations lead to

$$d_u^e|_{m \ll m_{u'}, m_\chi} \approx \frac{eG_3^5|G_1|\sin\phi}{48\pi^2}\frac{m_{u'}}{m_\chi^2}\mathcal{K}(r), \qquad (51)$$

where

$$\mathcal{K}(r) = \frac{1}{2r} - \frac{1}{r^2} + \frac{1}{r^3}\ln(1 + r), \qquad (52)$$

with $r = \frac{m_{u'}^2}{m_\chi^2} - 1$; $m_{u'}$ and m_χ are the exotic quark and scalar masses, respectively; G_3^5 and G_1 are the Yukawa couplings given in Eq. (5); and $\sin\phi$ is the sine of the CP-violating phase ϕ related to the complex parameter G_1. Also, we have taken the limit $m \ll m_\chi$ and $m \ll m_{u'}$, with m the up-quark mass. Furthermore, to give numerical results, it is interesting to consider $m_{u'} \approx m_\chi$ in Eq. (51), since these two exotic particles obtain mass from the same vev, $V_{\chi_1^0}$. In this case, we have

$$G_3^5|G_1|\sin\phi \times \left(\frac{1\ \text{TeV}}{m_\chi}\right) \lesssim 2.1 \times 10^{-6}, \qquad (53)$$

since $\mathcal{K}(0) = 1/3$. To obtain the bound in Eq. (53), we have used $d_n^e \sim \frac{4}{3}d_d^e - \frac{1}{3}d_u^e \approx \mathcal{O}(d_u^e) < 0.29 \times 10^{-25}$ e · cm [48]. Now, for instance, let us assume that the CP-violating phase is such that $\sin\phi \sim 10^{-2}$ and $m_\chi \sim 1$ TeV. In this case the parameters $G_3^5 \sim 10^{-2}$ and $|G_1| \sim 10^{-2}$ satisfy the upper bound given in Eq. (53). In the general case, i.e., $m_\chi \neq m_{u'}$, it can be shown that when $m_\chi > m_{u'}$ the limit on the couplings is softer than the one given in Eq. (53).

Hence, since the order of the model parameters differs from $\bar{\theta} \lesssim 10^{-11}$ by several order of magnitude, a solution to the strong CP problem, as the one presented above, is required.

TABLE III. The charge assignments for Z_{10} and Z_{11} that stabilize the axion, for $\alpha = 6$.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	(χ, η)	ϕ
Z_{10}	+5	+7	+1	+1	+7	+1	+6	+6	+2
Z_{11}	+6	+7	+1	+1	+8	+2	+6	+6	+4

IV. CONCLUSIONS

In this paper we have shown a detailed and comprehensive study concerning the implementation of the PQ symmetry into a 3-3-1 model in order to solve the strong *CP* problem. We have considered a version of the 3-3-1 model in which the scalar sector is minimal. In its original form this version has only two scalar triplets (χ, ρ) and it is found that the model presents an automatic PQ-like symmetry. However, for this scalar content, there is a $U(1)$ subgroup of $U(1)_X \otimes U(1)_{\rm PQ}$ that remains unbroken and hence no axion field, $a(x)$, arises. Therefore, the strong *CP* problem cannot be solved by the dynamical properties of the axion field. However, as we have shown in the text, the problem can be solved due to the appearance of three massless quark states. We show explicitly that those massless quark states remain massless to all orders in perturbation theory. This solution is disfavored since results from lattice and current algebra do not point in that direction. When the model is slightly extended by the addition of a third scalar triplet η, with the same quantum numbers as χ, we do not have massless quarks anymore but we cannot implement a PQ symmetry in a natural way. The trilinear term in the scalar potential forbids this symmetry. We can resort to a Z_2 symmetry to remove the trilinear term. In this case, we can define a PQ symmetry and an axion field appears in the physical scalar spectrum. Unfortunately this axion is visible since it is related to the V_{ρ^0} energy scale, which is of the order of the electroweak scale. Therefore, the model must be extended. We have succeeded in implementing a stable PQ mechanism by introducing a ϕ scalar singlet and a Z_N discrete gauge symmetry. The introduction of the ϕ scalar makes the axion invisible provided $V_\phi \gtrsim 10^9$ GeV, i.e., $a(x) \simeq {\rm Im}\phi$. On the another hand, the Z_N protects the axion against quantum gravity effects because both it is anomaly-free, as it was shown by using a discrete version of the Green-Schwarz mechanism, and it forbids all effective operators of the form $\sim\phi^N/M_{\rm Pl}^{N-4}$, with $N < 10$, which could destabilize the PQ mechanism.

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