PHYSICAL REVIEW D **84,** 055017 (2011)

Higgs-flavor groups, naturalness, and dark matter

S. M. Barr

Bartol Research Institute, University of Delaware, Newark, Delaware 19716

(Received 19 May 2011; published 16 September 2011)

In the absence of low-energy supersymmetry, a multiplicity of weak-scale Higgs doublets would require additional fine-tunings unless they formed an irreducible multiplet of a non-Abelian symmetry. Remnants of such symmetry typically render some Higgs fields stable, giving several dark matter particles of various masses. The non-Abelian symmetry also typically gives simple, testable mass relations.

DOI: 10.1103/PhysRevD.84.055017 PACS numbers: 12.10.Dm

The idea that there might be more than one Higgs doublet has been much investigated for a wide variety of reasons. These reasons include incorporating *CP* violation in the scalar sector (as in the old Weinberg model [1]), addressing the flavor problem of the quarks and leptons [2,3]), and obtaining a dark matter candidate, as in "inert Higgs" models [4]. (For a recent thorough review of models with two Higgs doublets, see [5]).

If (as will be assumed here) there is no low-energy supersymmetry, then a multiplicity of light Higgs-doublets could exacerbate the gauge hierarchy problem. Instead of the mass of just one Higgs doublet being "fine-tuned" to be much lighter than the Planck scale, the mass of each Higgs doublet would have to be separately tuned. The small mass of the standard model Higgs doublet relative to the Planck scale might be accounted for anthropically [6–8], but that does not appear to be the case for "extra" Higgs doublets, which do not contribute to the breaking of the electroweak gauge symmetry. (Throughout this paper, weak-doublet scalar fields will be called "Higgs doublets" even if their vacuum expectation values are zero.)

On the other hand, just one fine-tuning would be sufficient to make all the Higgs doublets light if all of them were in an irreducible multiplet of a non-Abelian symmetry, G_Φ. Then the tuning of the mass parameter μ^2 of the standard model Higgs field would simultaneously ensure the lightness of all the extra Higgs doublets, as long as the splitting of the G_Φ multiplet were small. The breaking of G_Φ can be dynamical, and so this splitting can be of any magnitude without creating a naturalness problem.

In short, a multiplicity of elementary Higgs doublets with masses near the weak scale would seem to require (in the absence of low-energy supersymmetry) the existence of a non-Abelian symmetry relating their masses. Such a symmetry can have several interesting consequences, as will be seen in a simple example. It can make some of the extra Higgs fields stable, giving dark matter candidates, and give testable relations among the masses of new particles.

If there is such a non-Abelian symmetry of the Higgs fields, there are two possibilities. Either the quarks and leptons of the standard model also transform nontrivially under G_Φ, or they are singlets under G_Φ. The former possibility, studied, for example, in [2,3] is interesting as an approach to understanding the flavor structure of the quarks and leptons. It would generally lead to flavor-changing effects from the exchange of extra Higgs doublets. Here, however, we study the case where the only standard model field that transforms nontrivially under G_Φ is the Higgs field. Hence we call such models "Higgs-flavor models" and the group G_Φ a "Higgs-flavor group".

We will show that in typical Higgs-flavor models there exist the following: (a) One or more extra Higgs doublets that couple to quarks and leptons proportionally to the standard model Higgs doublet, ensuring "natural flavor conservation" [9]. These may be light enough to be produced at accelerators. (The idea of an extra Higgs doublet whose Yukawa couplings to quarks and leptons are proportional to those of the standard model Higgs has been proposed recently by Pich and Tuzón and further developed by Serôdio [10].) (b) Several Higgs doublets that do *not* couple to quarks and leptons and that are absolutely stable due to an unbroken discrete subgroup of the Higgs-flavor symmetry. These are realistic dark matter candidates, some of which can have masses near present accelerator limits. (c) Numerous testable symmetry relations among the masses of the extra Higgs fields. These are of two types: relations among the masses of different Higgs doublets, and relations among the $SU(2)_L$-breaking mass splittings within the Higgs doublets.

We will illustrate these ideas in a model based on a continuous Higgs-flavor group, $G_\Phi = SO(3)$. It should be noted, however, that the Higgs-flavor group could also be a non-Abelian *discrete* group. Indeed, there has been great attention in recent years to non-Abelian discrete groups, such as A_4 and T', in the context of attempts to understand the flavor structure of quarks and leptons. In such models, there are often Higgs doublets in nontrivial multiplets of the non-Abelian discrete symmetry. (The literature on this subject is large and rapidly growing, and there is as yet no comprehensive review. Some early references on A_4 can be found in [11], on T' in [12], and on other groups in [13].) If the standard model quarks and leptons were singlets under a non-Abelian discrete "Higgs-flavor" group, then one would typically have the features we have described above. In particular, some of the extra

Higgs doublets could be rendered stable by the non-Abelian discrete symmetry,

We now describe a simple model with $G_\Phi = SO(3)$. In this model, the low-energy theory consists of the standard model, except that the Higgs doublet is part of a 5-plet of $SO(3)_\Phi$, denoted $\Phi^{(ab)}$, where a, b are vector indices of $SO(3)_\Phi$ and run from 1 to 3. (The 5-plet is, of course, the rank-2 symmetric traceless tensor of $SO(3)_\Phi$.) There is also a "messenger field" $\eta^{(ab)}$, which communicates the effects of $SO(3)_\Phi$ breaking to the standard model fields. $\eta^{(ab)}$ is a real 5-plet of $SO(3)_\Phi$, but a singlet under the standard model gauge group. The messenger field is superheavy, but has a vacuum expectation value (VEV) that is of order 100 GeV to 1 TeV. This small VEV can arise very simply and in a technically natural way from the coupling of the messenger field to the fermions of a sector in which $SO(3)_\Phi$ is dynamically broken by a fermion condensate. (Schematically, if the fermions of that sector are called Ψ and transform both under $SO(3)_\Phi$ and under an asymptotically free group with a confinement scale Λ, one can have terms of the form $\frac{1}{2}M_\eta^2\eta^2 + f\langle\bar{\Psi}\Psi\rangle\eta$, which gives $\langle\eta\rangle \sim f\Lambda^3/M_\eta^2$. Even for M_η^2 superheavy, this can be naturally small.)

Given that the Higgs doublets transform as a 5-plet under the Higgs-flavor group, whereas the standard model quarks and leptons are singlets under it, the quark and lepton masses must come from higher-dimension operators, the smallest of which have the form

$$\mathcal{L}_{yukawa} = Y_{ij}\bar{\psi}_i\psi_j(\Phi^{(ab)}\eta^{(ab)})/M, \tag{1}$$

where i, j are fermion family indices, and repeated indices of all types are summed over. Such operators can arise from integrating out vectorlike quarks and leptons that carry $SO(3)_\Phi$ indices and have mass of order $\langle\eta\rangle$, as will be discussed later.

An important point about the term in Eq. (1) is that any Higgs fields in $\Phi_{(ab)}$ that couple to standard model quarks and leptons do so with the same Yukawa coupling Y_{ij}. This guarantees that the only effects that violate quark and lepton flavor are through the CKM angles, i.e. "natural flavor conservation" (NFC) [9]. The reason for NFC in this model is that the messenger sector is very simple. This is a point to which we shall return at the end of the paper.

The splittings of the 5-plet of Higgs doublets is given by the coupling of $\Phi^{(ab)}$ to the messenger field $\eta^{(ab)}$, the most general renormalizable form of which is given by

$$\begin{aligned} V_2(\Phi^{(ab)}) &= \frac{1}{2}M_\Phi^2\Phi^{(ab)\dagger}\cdot\Phi^{(ab)} + \sigma_1\Phi^{(ab)\dagger}\cdot\Phi^{(ab)}\eta^{(cd)}\eta^{(cd)} + \sigma_2\Phi^{(ab)\dagger}\cdot\Phi^{(bc)}\eta^{(cd)}\eta^{(da)} \\ &\quad + \sigma_3\Phi^{(ab)\dagger}\cdot\Phi^{(cd)}\eta^{(bc)}\eta^{(da)} + \sigma_4\Phi^{(ab)\dagger}\cdot\Phi^{(bc)}\eta^{(ab)}\eta^{(cd)} + m_5\Phi^{(ab)\dagger}\cdot\Phi^{(bc)}\eta^{(ca)} \\ &= \frac{1}{2}M^2\,\mathrm{Tr}[\Phi_\lambda^\dagger\Phi^\lambda] + \sigma_1\,\mathrm{Tr}[\Phi_\lambda^\dagger\Phi^\lambda]\mathrm{Tr}[\eta\eta] + \sigma_2\,\mathrm{Tr}[\Phi_\lambda^\dagger\Phi^\lambda\eta\eta] + \sigma_3\,\mathrm{Tr}[\Phi_\lambda^\dagger\eta\Phi^\lambda\eta] \\ &\quad + \sigma_4\,\mathrm{Tr}[\Phi_\lambda^\dagger\eta]\mathrm{Tr}[\Phi^\lambda\eta] + m_5\,\mathrm{Tr}[\Phi_\lambda^\dagger\Phi^\lambda\eta], \end{aligned} \tag{2}$$

where the coefficients are real. In the first expression for V_2, the dot represents the contraction of $SU(2)_L$ indices, which are not shown. In the second expression, the $SU(2)_L$ indices are denoted by λ, and the traces are over the $SO(3)_\Phi$ indices, which are not shown. In the terms of the form $(\Phi^\dagger\Phi)(\eta\eta)$, the product of η with itself must be in the symmetric product $(5\times5)_S = 1+5+9$. Thus, of the four terms with coefficients σ_i in Eq. (2), only three are independent. In particular, denoting the operator with coefficient σ_i by O_i, one has $O_1 - 4O_2 - 2O_3 + 2O_4 = 0$. Consequently, there is a 1-parameter redundancy among the coefficients σ_i, with $([\sigma_1+\alpha], [\sigma_2-4\alpha], [\sigma_3-2\alpha], [\sigma_4+2\alpha])$ giving the same physics as $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$.

Because $\langle\eta^{(ab)}\rangle$ is a real symmetric matrix, an $SO(3)_\Phi$ basis can be chosen where it is real and diagonal. Without loss of generality, then, the VEV of the messenger field may be written

$$\langle\eta^{(ab)}\rangle = \begin{pmatrix} a+\frac{1}{\sqrt{3}}b & 0 & 0 \\ 0 & -a+\frac{1}{\sqrt{3}}b & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}b \end{pmatrix}. \tag{3}$$

Let us write the components of the 5-plet of Higgs doublets as

$$\Phi^{(ab)} = \begin{pmatrix} A+\frac{1}{\sqrt{3}}B & \Phi^{(12)} & \Phi^{(13)} \\ \Phi^{(12)} & -A+\frac{1}{\sqrt{3}}B & \Phi^{(23)} \\ \Phi^{(13)} & \Phi^{(23)} & -\frac{2}{\sqrt{3}}B \end{pmatrix}. \tag{4}$$

Then it is directly seen from Eq. (1) that the doublet that couples to the known quarks and leptons is the linear combination $\Phi^{(ab)}\langle\eta^{(ab)}\rangle = 2\sqrt{a^2+b^2}\Phi_+$, where $\Phi_+ \equiv \frac{1}{\sqrt{a^2+b^2}}(aA+bB)$. The orthogonal combination will be denoted $\Phi_- \equiv \frac{1}{\sqrt{a^2+b^2}}(bA-aB)$. We shall call Φ_+ and Φ_- the "diagonal Higgs doublets," and $\Phi^{(ab)}$ with $a\neq b$ the "off-diagonal Higgs doublets". (Note that we define these with respect to the basis in which the VEV of the messenger field has the form given in Eq. (3)).

The mass spectrum of the doublets that results from Eq. (2) is easily computed by substituting into it the forms given in Eqs. (3) and (4). Let us first write it in the simple case where the cubic term in Eq. (2) vanishes exactly, i.e. $m_5 = 0$. (This could arise from a Z_2 symmetry under which the messenger fields are odd.) Then we will consider the slightly more interesting case where $m_5 \neq 0$. If $m_5 = 0$, then the mass eigenstates and eigenvalues come out to be the following:

$$\begin{aligned}
M^2(\Phi^{(12)}) &= M_0^2 + (\sigma_2 - \sigma_3)2a^2 + (\sigma_2 + \sigma_3)\frac{2}{3}b^2,\\
M^2(\Phi^{(13)}) &= M_0^2 + \sigma_2 a^2 + \left(\frac{5}{3}\sigma_2 - \frac{4}{3}\sigma_3\right)b^2\\
&\quad + (\sigma_2 - 2\sigma_3)\frac{2}{\sqrt{3}}ab,\\
M^2(\Phi^{(23)}) &= M_0^2 + \sigma_2 a^2 + \left(\frac{5}{3}\sigma_2 - \frac{4}{3}\sigma_3\right)b^2\\
&\quad - (\sigma_2 - 2\sigma_3)\frac{2}{\sqrt{3}}ab,\\
M^2(\Phi_-) &= M_0^2 + (\sigma_2 + \sigma_3)\frac{2}{3}(a^2 + b^2),\\
M^2(\Phi_+) &= M_0^2 + (\sigma_2 + \sigma_3 + 2\sigma_4)2(a^2 + b^2),
\end{aligned} \tag{5}$$

where $M_0^2 \equiv M_\Phi^2 + 4\sigma_1(a^2 + b^2)$. (We are at the moment neglecting $SU(2)_L$-breaking effects.) One sees that the combination that couples to quarks and leptons, Φ_+, is actually a mass eigenstate. If the quarks and leptons are to obtain mass, then Φ_+ must obtain a VEV and must therefore be identified with the standard model Higgs doublet (which we will call $\Phi_{\rm SM}$) and have a negative mass-squared with magnitude of order $(100\ \text{GeV})^2$. The other four Higgs doublets must have positive mass-squared large enough to evade present limits. Therefore, Φ_+ must have the smallest mass-squared in Eq. (5), which can happen if, for example, σ_4 is sufficiently negative. (It should be emphasized that all four σ_i can be chosen independently. The redundancy of the σ_i noted after Eq. (2) simply means, as explained there, that not all such choices give different physics. As is evident from the expressions in Eq. (5), choosing σ_4 large and negative does lower the mass-squared of Φ_+ relative to the other Higgs doublets).

The fact that $M^2(\Phi_+)$ is negative and of order the weak scale, rather than its "natural" value of order the Planck scale, has to be explained anthropically, presumably in the context of a multiverse scenario. It is simplest to imagine that the parameter that varies or "scans" among the domains of the multiverse [8] is the $SO(3)_\Phi$-invariant mass parameter M_Φ^2 of the 5-plet.

At first glance, it looks like the five masses shown in Eq. (5) depend on five free parameters: M_0, $\sigma_2 a^2$, σ_3/σ_2, σ_4/σ_2, and b/a. However, as we saw earlier, there is a 1-parameter redundancy among the σ_i. So, these masses really only depend on four combinations of parameters, which means that there should be exactly one relation among them. That relation can be seen by taking the differences among the mass-squareds of the four extra Higgs doublets Φ_-, $\Phi^{(12)}$, $\Phi^{(13)}$, and $\Phi^{(23)}$:

$$\begin{aligned}
\Delta_{12}^2 &\equiv M^2(\Phi^{(23)}) - M^2(\Phi_-) = \frac{4}{3}(\sigma_2 - 2\sigma_3)a^2,\\
\Delta_{13}^2 &\equiv M^2(\Phi^{(12)}) - M^2(\Phi_-) = (\sigma_2 - 2\sigma_3)\left(\frac{1}{\sqrt{3}}a + b\right)^2,\\
\Delta_{23}^2 &\equiv M^2(\Phi^{(13)}) - M^2(\Phi_-) = (\sigma_2 - 2\sigma_3)\left(\frac{1}{\sqrt{3}}a - b\right)^2.
\end{aligned} \tag{6}$$

Note that these three splittings can be written in terms of just the following two combinations of parameters: $(\sigma_2 - 2\sigma_3)a^2$ and b/a. From this one sees that

$$\begin{aligned}
|\Delta_{12}^2|^{1/2} &= |\Delta_{13}^2|^{1/2} + |\Delta_{23}^2|^{1/2}, \quad \text{if } |b| < \frac{1}{\sqrt{3}}|a|,\\
|\Delta_{13}^2|^{1/2} &= |\Delta_{12}^2|^{1/2} + |\Delta_{23}^2|^{1/2}, \quad \text{if } |b| > \frac{1}{\sqrt{3}}|a|, \quad ab > 0,\\
|\Delta_{23}^2|^{1/2} &= |\Delta_{12}^2|^{1/2} + |\Delta_{13}^2|^{1/2}, \quad \text{if } |b| > \frac{1}{\sqrt{3}}|a|, \quad ab < 0.
\end{aligned} \tag{7}$$

In other words, the largest of the three mass-squared splittings among the extra Higgs doublets is simply related to the other two. (Incidentally, the fact that the only relation among the five masses is given by Eq. (7) confirms that there is the freedom to make $M^2(\Phi_+)$ small.) Another fact implied by Eq. (6) is that Φ_- is either heavier than all the off-diagonal Higgs doublets or lighter than them all, depending on the sign of $\sigma_2 - 2\sigma_3$.

Equations (6) and (7) allow us to draw some conclusions about the stability of the four extra Higgs doublets. In what follows, when we say that a doublet is stable, we mean that its lightest component is stable, since the heavier components within a weak doublet can decay into lighter ones by charged weak interactions.

First, consider the lightest two of the three off-diagonal Higgs doublets. These are rendered absolutely stable by unbroken discrete subgroups of the Higgs-flavor symmetry. (And this conclusion applies even if $m_5 \neq 0$.) The relevant symmetries are the parity transformations P_a, where P_a reverses the sign of the a^{th} component of an $SO(3)_\Phi$ vector. With respect to P_a, any field with an odd (even) number of $SO(3)_\Phi$ indices equal to a is odd(even). For example, $\Phi^{(12)}$ and $\Phi^{(13)}$ are odd under P_1, while $\Phi^{(23)}$ is even. These parities are unbroken by the VEV in Eq. (3). Thus the lightest P_a-odd fields are stable. For example, if $\Phi^{(12)}$ is the heaviest of the off-diagonal Higgs-doublets, then $\Phi^{(13)}$ is the lightest P_1-odd multiplet and $\Phi^{(23)}$ is the lightest P_2-odd multiplet. Therefore the lightest components of $\Phi^{(13)}$ and $\Phi^{(23)}$ are absolutely stable. These will contribute to the dark energy of the universe as will be discussed briefly later.

The heaviest of the three off-diagonal Higgs doublets is not prevented by these discrete symmetries from decaying into lighter Higgs doublets. Indeed quartic terms exist which would appear to allow such decays (for example, $\Phi^{(ab)\dagger} \cdot \Phi^{(bc)}\Phi^{(cd)\dagger} \cdot \Phi^{(da)}$, which contains $\Phi^{(12)\dagger} \cdot \Phi^{(23)}\Phi^{(31)\dagger} \cdot \Phi_{\pm}$). Whether such decays can occur depends on kinematics. One must consider separately two cases. As noted above, Φ_- is either the lightest or the heaviest of the four extra Higgs doublets, depending on the sign of $(\sigma_2 - 2\sigma_3)$. Call these Cases I and II, respectively.

Case I. If Φ_- is the lightest of the extra Higgs doublets, the decay of the heaviest off-diagonal Higgs doublet into lighter Higgs doublets is kinematically forbidden, as we will now show. (We are still neglecting the $SU(2)_L$-breaking contributions to the masses of the extra Higgs doublets.) The point is that for the decay of the heaviest off-diagonal Higgs doublet to be kinematically allowed its mass must be greater than the sum of the masses of the particles it is decaying into, i.e. a certain inequality involving these masses must be satisfied. But that inequality can be shown to be inconsistent with Eq. (7) in the case we are considering. Denoting the (positive) mass-squared of Φ_- by m_0^2, Eq. (7) is equivalent to the statement that the mass-squareds of the three off-diagonal Higgs doublets can be written (in ascending order) as $m_0^2 + x^2$, $m_0^2 + y^2$, and $m_0^2 + (x + y)^2$ for some x and y such that $y > x > 0$. For the heaviest off-diagonal Higgs doublet to decay into the two lightest off-diagonal Higgs doublets plus other particles (which is the only decay allowed it by the parity symmetries P_a), one must therefore have $\sqrt{m_0^2 + (x+y)^2} > \sqrt{m_0^2 + x^2} + \sqrt{m_0^2 + y^2}$. However, since $\sqrt{m_0^2 + y^2} > y$, this means that $\sqrt{m_0^2 + (x + y)^2} > \sqrt{m_0^2 + x^2} + y$. Squaring both sides gives $m_0^2 + (x + y)^2 > m_0^2 + x^2 + y^2 + 2y\sqrt{m_0^2 + x^2} \Rightarrow 2xy > 2y\sqrt{m_0^2 + x^2} \Rightarrow x > \sqrt{m_0^2 + x^2}$, which is clearly a contradiction since $m_0^2 > 0$. In other words, the kinematic condition cannot be satisfied for this decay because it conflicts with Eq. (7). Moreover, the heaviest off-diagonal Higgs doublet obviously cannot decay directly into quarks and leptons, since it has no Yukawa coupling to them. It is therefore stable.

In Case I, the Higgs doublet Φ_- is also stable, because it has no Yukawa couplings to the quarks and leptons, and because there turn out to be no quartic couplings that allow its decay into three Φ_+. Some of these conclusions are modified if $m_5 \neq 0$ as will be seen.

Case II. In Case II, whether Φ_- and the heaviest off-diagonal Higgs doublet are able to decay into lighter Higgs doublets depends on the values of parameters. The lightest two off-diagonal Higgs doublets are, however, absolutely stable due to the symmetries P_a (that is, their lightest components are).

The model presented in this paper differs from most models of dark matter in that there are several stable particles that contribute to the dark matter density of the universe. (For some other recent papers that consider the possibility of multiple dark matter particle types, see [14].) Most of the dark matter density would come from the heaviest stable extra Higgs particle, because of both its smaller annihilation cross-section and its larger mass. However, in the present model, the lightest stable extra Higgs particle can be much lighter than heaviest one. For example, in Case I, Φ_- can be much lighter than the heaviest stable Higgs doublet. In Case II, the lightest off-diagonal Higgs doublet can be much lighter than all three of the other extra Higgs doublets. Thus, even if the particle which is the dominant component of the dark matter has a mass of order a TeV, there can be other stable Higgs particles several times lighter than that. The calculation of the dark matter density is obviously quite involved as it depends on the numerous quartic couplings of Eqs. (2) and (12), and will be considered in detail elsewhere.

Now let us consider the model when $m_5 \neq 0$. This term in Eq. (2) has no affect on the masses of the off-diagonal Higgs doublets, but modifies the 2×2 mass matrix of the diagonal Higgs doublets, so that the eigenstates are mixtures of what we called $\Phi_{\pm}$:

$$\begin{aligned} \Phi_{SM} &= \Phi'_+ = \cos\theta_H \Phi_+ - \sin\theta_H \Phi_-, \\ \Phi'_- &= \sin\theta_H \Phi_+ + \cos\theta_H \Phi_-, \end{aligned} \tag{8}$$

where, for small m_5,

$$\tan\theta_H \cong \frac{m_5 a(-a^2 + 3b^2)/(a^2 + b^2)}{M^2(\Phi'_-) - M^2(\Phi'_+)}. \tag{9}$$

Thus the diagonal extra Higgs doublet Φ'_- now couples to the quarks and leptons with a strength that is simply $\tan\theta_H$ times that of the standard model Higgs doublet and is no longer stable. From the decays of Φ'_- into quarks, the value $\tan\theta$ is in principle directly measurable.

There is also a shift in the mass of Φ'_- from the value predicted in Eq. (7). For small m_5, this shift is given by

$$\delta M^2(\Phi_-) = -\frac{2}{\sqrt{3}} m_5 b \frac{3a^2 - b^2}{a^2 + b^2}. \tag{10}$$

Thus one has the prediction

$$\delta M^2(\Phi_-) = -r\frac{3 - r^2}{3r^2 - 1}\tan\theta_H[M^2(\Phi_-) - M^2(\Phi_+)], \tag{11}$$

where $r \equiv b/a$ can be extracted from Eqs. (6) and (7). In particular $|(r + \frac{1}{\sqrt{3}})/(r - \frac{1}{\sqrt{3}})| = \sqrt{\Delta_{13}^2/\Delta_{23}^2}$. This shifting of the mass of Φ_- has the effect that in Case I for certain values of parameters the heaviest off-diagonal Higgs boson can decay into other Higgs doublets.

In the case $m_5 \neq 0$, the two "diagonal" Higgs-doublets both couple to the quarks and leptons, but avoid FCNC effects because their Yukawa coupling matrices are proportional to each other (i.e. "aligned"), as in the 2-Higgs-doublet models of Ref. [10]. In the case $m_5 = 0$, one of the two "diagonal Higgs doublets decouples from the quarks and leptons, as in the "inert Higgs" models of Ref. [4]. The phenomenology of the extra Higgs doublet in those kinds of models will therefore be similar to the phenomenology of the doublet we call Φ_- here. We leave a detailed discussion of that phenomenology to a later paper.

Up to this point, we have neglected the $SU(2)_L$-breaking effects in computing the masses of the extra Higgs doublets. This breaking gives splitting with each doublet between the charged, neutral scalar, and neutral pseudoscalar components. The splitting occurs through the coupling of the standard model Higgs doublet to the extra Higgs doublets in the quartic terms in the Higgs potential. Since the Higgs-flavor group $SO(3)_\Phi$ significantly constrains the form of the those quartic terms, testable predictions arise for the pattern of $SU(2)_L$-breaking splittings. The most general form for the quartic part of the Higgs potential is

$$\begin{aligned} V_4(\Phi^{(ab)}) &= \lambda_1[\Phi^{(ab)\dagger}\cdot\Phi^{(ab)}]^2 + \lambda_2[\Phi^{(ab)\dagger}\cdot\Phi^{(cd)}][\Phi^{(cd)\dagger}\cdot\Phi^{(ab)}] + \lambda_3[\Phi^{(ab)\dagger}\cdot\Phi^{(cd)}][\Phi^{(ab)\dagger}\cdot\Phi^{(cd)}] \\ &\quad + \lambda_4[\Phi^{(ab)\dagger}\cdot\Phi^{(cd)}][\Phi^{(ac)\dagger}\cdot\Phi^{(bd)}] + \lambda_5[\Phi^{(ab)\dagger}\cdot\Phi^{(bc)}][\Phi^{(cd)\dagger}\cdot\Phi^{(da)}] + \lambda_6[\Phi^{(ab)\dagger}\cdot\Phi^{(bc)}][\Phi^{(da)\dagger}\cdot\Phi^{(cd)}] \\ &= \lambda_1[\mathrm{Tr}(\Phi^\dagger_\lambda\Phi^\lambda)]^2 + \lambda_2\,\mathrm{Tr}[\Phi^\dagger_\lambda\Phi^\mu]\mathrm{Tr}[\Phi^\dagger_\mu\Phi^\lambda] + \lambda_3\,\mathrm{Tr}[\Phi^\dagger_\lambda\Phi^\dagger_\mu]Tr[\Phi^\lambda\Phi^\mu] + \lambda_4\,\mathrm{Tr}[\Phi^\dagger_\lambda\Phi^\mu\Phi^\lambda\Phi^\dagger_\mu] \\ &\quad + \lambda_5\,\mathrm{Tr}[\Phi^\dagger_\lambda\Phi^\lambda\Phi^\dagger_\mu\Phi^\mu] + \lambda_6\,\mathrm{Tr}[\Phi^\dagger_\lambda\Phi^\lambda\Phi^\mu\Phi^\dagger_\mu] \end{aligned} \tag{12}$$

where the notation is the same as in Eq. (2). By the Hermiticity of V_4 the λ_i are real. Since the product $(\Phi^\dagger\Phi)$ must be in $5\times 5 = 1+3+5+7+9$, the six terms in Eq. (12) depend on only five invariant combinations.

It is easy to show that V_4 given in Eq. (12) contains no terms of the form $\Phi^\dagger_-\Phi_+\Phi^\dagger_+\Phi_+$. Consequently, when Φ_+ acquires a VEV it does not mix Φ_- and Φ_+. Therefore, the conclusion reached earlier that for $m_5 = 0$ the doublet Φ_- does not couple to quarks and leptons still holds.

When $\Phi_{\rm SM}$ acquires a vacuum expectation value, the terms in Eq. (12) (except for the λ_1 term) give $SU(2)_L$-breaking contributions to the masses of the Higgs fields. Each Higgs doublet has a charged, neutral scalar, and neutral pseudoscalar component, and thus two splittings. So the four extra Higgs doublets have altogether eight $SU(2)_L$-breaking splittings, which are determined by the five parameters λ_i, $i = 2,\ldots,6$ (of which only four are independent of each other). There are therefore four testable mass relations.

We have said that there are testable relations among masses in this model, namely, both the relations among $SU(2)_L$-breaking splittings that we have just discussed and the relations given in Eq. (7). Of course, in principle, the masses of all the extra Higgs fields are observable, and these relations testable. There is the question, however, of how one could go about measuring these masses in practice, especially in the case of the Higgs doublets that do not couple to quarks and leptons, i.e. the three "off-diagonal" Higgs doublets (and in the case of $m_5 = 0$ also Φ_-). In an e^+e^- machine one could directly pair produce the charged components of any of the extra doublets. In a hadron collider, one could pair produce either the charged or the neutral components via WW fusion. If the heaviest component of a Higgs doublet is produced in one of these ways, the lighter components would appear in its β decay products. Since the extra Higgs fields in these models do not lead to flavor-changing effects, they do not have to be extremely heavy, as is typically the case in models with extra Higgs doublets; and so they could well have masses in the range of 200 GeV to a few TeV. It is possible, therefore, that they could be produced at the LHC. We will discuss the production and detection of such particles more in another place.

There are some technical points about symmetry breaking to be considered. We analyzed two cases $m_5 = 0$ and $m_5 \neq 0$. The case $m_5 = 0$ can be realized if there is a Z_2 under which the messenger field $\eta^{(ab)}$ is odd and all the standard model fields are even. The sector that dynamically breaks $SO(3)_\Phi$ could then (for example) have the form $\frac{1}{2}M^2_\eta\eta^{(ab)}\eta^{(ab)} + \sum_{I=1}^5 f_I(\bar\Psi^{(ab)}\Psi_I)\eta^{(ab)}$, where under $SU(N)_{DSB}\times SO(3)_\Phi\times Z_2$, one has $\bar\Psi^{(ab)} = (\bar N, 5, +)$, $\Psi_I = (N, 1, -)$, and $\eta^{(ab)} = (1, 5, -)$. The confining group $SU(N)_{DSB}$ causes a $\bar\Psi\Psi$ condensate to form that induces a linear term, and thus a VEV, for the messenger field. In the case $m_5 \neq 0$ one must explain why m_5 is of order the weak scale rather than the Planck scale. Here too one can invoke a Z_2. In this case, one could have two messenger fields, $\eta^{(ab)}$ and η' that are, respectively, a 5-plet and a singlet under $SO(3)_\Phi$ and that are both odd under Z_2. The dynamical symmetry breaking sector could (for example) have the form $\frac{1}{2}M^2_\eta\eta^{(ab)}\eta^{(ab)} + \frac{1}{2}M^2_{\eta'}\eta'\eta' + \sum_{I=1}^5 f_I(\bar\Psi^{(ab)}\Psi_I)\eta^{(ab)} + f'(\bar\Psi'\Psi')\eta'$. If the confining scale of $SU(N)_{DSB}$ is Λ, and both $M^2_\eta \sim M^2_{\eta'}$ are of superheavy scale, then both the 5-plet and singlet messenger fields will naturally have VEVs of the same order of magnitude, namely Λ^3/M^2_η.

The effective Yukawa operators given in Eq. (1) can arise through integrating out fermions that have mass of order $\langle\eta\rangle$. For example, if there is a set of fermions $\psi^{(ab)} = (1, 5, +)$ that has the quantum numbers of a family under the standard model gauge group, and $\bar\psi^{(ab)} = (1, 5, -)$ that has the quantum numbers of an antifamily, then there can be renormalizable Yukawa couplings of the form $\psi_i\psi^{(ab)}\Phi^{(ab)}$, $\bar\psi^{(ab)}\psi^{(ab)}\eta'$, and $\bar\psi^{(ab)}\psi_j\eta^{(ab)}$. When the $\psi^{(ab)} + \bar\psi^{(ab)}$ is integrated out, it leads at tree level to effective terms of the form $\psi_i\psi_j(\Phi^{(ab)}\eta^{(ab)}/\langle\eta'\rangle$, as given in Eq. (1). As they arise at tree level, there is no reason why some of the coefficients of such effective terms could not be of order one (as would be needed for the t quark mass).

One further point: the Higgs-flavor group can be local. The gauge bosons of G_Φ would obtain mass of order the condensate $\Lambda \sim (\langle \eta \rangle M_\eta^2)^{1/3} \gg \langle \eta \rangle$. Therefore, they would have negligible effect at low energies.

We conclude by noting that the "Higgs flavor" model presented here is typical but hardly unique. There are different possibilities for the non-Abelian Higgs-flavor group G_Φ (including both continuous and discrete groups), and various possibilities for the G_Φ representations for the Higgs doublets and messenger fields. One would expect, however, that typical features of such models would include the existence of one or more absolutely stable extra Higgs doublets that contribute to dark matter and some of which can be quite light, and the existence of other Higgs doublets that couple to the known quarks and leptons proportionally to the standard model Higgs.

One expects these features to be typical because they tend to follow from having a very simple messenger sector, and a simple messenger sector is required to ensure "natural flavor conservation" of the quarks and leptons. For example, in the model described in this paper, if there were two messenger fields, $\eta^{(ab)}$ and $\eta'^{(ab)}$, instead of just one, then instead of the single Yukawa term of Eq. (1), there would be two: $\mathcal{L}_{yukawa} = Y_{ij}\bar{\psi}_i \psi_j(\Phi^{(ab)}\eta^{(ab)})/M + Y'_{ij}\bar{\psi}_i\psi_j(\Phi^{(ab)}\eta'^{(ab)})/M$. Since Y_{ij} and Y'_{ij} would have no reason to be simultaneously diagonalizable, potentially large quark and lepton flavor-changing mediated by scalar exchange would result. Ensuring natural flavor conservation in Higgs-flavor models thus generally requires a minimal set of messenger fields. But this in turn makes the G_Φ-breaking in the low-energy Higgs sector very simple and tends to leave unbroken in that sector a discrete remnant of the Higgs-flavor symmetry that can render some of the Higgs fields absolutely stable, as we have seen.

The general lesson, then, is that the non-Abelian Higgs-flavor symmetry required to make extra elementary Higgs doublets naturally light in the absence of low-energy supersymmetry, together with the simplicity of the messenger sector needed to avoid excessive quark and lepton flavor changing, tends to result in stable Higgs particles that can play the role of dark matter. It also can give rise to unstable extra Higgs fields that couple to quarks and leptons proportionally to the standard model Higgs field. And, finally, it tends to yield simple and testable mass relations among the extra Higgs fields.

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