# Pair production of $J/\psi$ mesons in the $k_t$ -factorization approach

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In the framework of  $k_t$ -factorization approach, we consider the production of  $J/\psi$  pairs at the LHC conditions. We give predictions on the differential cross sections and discuss the source and the size of theoretical uncertainties. We also present a comparison with collinear parton model showing a dramatic difference in the  $J/\psi$  transverse momentum spectrum and  $J/\psi - J/\psi$  azimuthal correlations. Finally, we give predictions on the polarization observables in the helicity and Collins-Soper systems.

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### **I. INTRODUCTION**

The interest in the production of  $J/\psi$  pairs in hadronic collisions has a long story. The first experimental evidence for this process was obtained in 1982 by the NA3 collaboration [1] at CERN in low-energy  $\pi^- p$  interactions, and it was followed by measuring the  $J/\psi$  pair production cross section in pp interactions in 1985 [2] by the same collaboration.

First theoretical calculations of the production cross section based on the present-day accepted approach (namely, perturbative QCD and nonrelativistic bound state formalism) were done in 1983 [3] for the quark-quark annihilation subprocess and in [4,5] for the gluon-gluon fusion subprocess. Some nonperturbative mechanisms were discussed in 1989 in [6].

Later on [7], this process was found to possess very interesting polarization properties and even has been proposed as a tool probing the gluon polarization in a polarized proton. However, the appropriate experimental research has never been carried out because of insufficient production rates. Very recently, the production of  $J/\psi$  pairs has attracted a renewal attention [8–10] as a possible indicator of double parton scattering processes.

Revisiting this topic in the present note was largely stimulated by the recent LHCb report [11] on the first measurement of the double  $J/\psi$  production cross section (indicating that the experimental exploration of this field now turns into reality) and by increasingly recognizable success of the  $k_t$ -factorization approach providing new tools for theoretical calculations [12-14]. Among the advantages of the latter we note that, even with using the leading-order (LO) matrix elements for the hard subprocess, the  $k_t$ -factorization includes a large piece of higher-order contributions, taking them into account in the form of evolution of parton densities. In this respect the  $k_t$ -factorization can be regarded as a convenient alternative to explicit high-order matrix elements calculations in the collinear scheme [15]. The situation in double quarkonium production is specific in the sense that

calculating even the LO matrix elements is already complicated enough, so that extending to higher orders does not seem feasible in the nearest future. Thus, the  $k_t$ -factorization remains the only way open to potentially important higher-order effects. Below we present a comparison between the  $k_t$ - and collinear calculations and demonstrate the new features brought to the production of  $J/\psi$  pairs.

#### **II. THEORETICAL FRAMEWORK**

Our approach is based on perturbative QCD, nonrelativistic bound state formalism [16], and the  $k_t$ -factorization ansatz [12–14] in the parton model. At the parton level, we consider the gluon-gluon fusion subprocess

$$g + g \rightarrow J/\psi + J/\psi.$$
 (1)

The corresponding 31 Feynman diagrams are depicted in Fig. 1; this set is identical to the one employed in Ref. [7]. As usual, the production amplitudes contain spin and color projection operators that guarantee the proper quantum numbers of the final state mesons. Only the color singlet channels are taken into consideration.

The evaluation of Feynman diagrams is straightforward and follows the standard QCD rules, with one reservation: in accordance with the  $k_t$ -factorization prescription [12], the initial gluon spin density matrix is taken in the form

$$\overline{\boldsymbol{\epsilon}_g^{\mu}\boldsymbol{\epsilon}_g^{*\nu}} = k_T^{\mu}k_T^{\nu}/|\boldsymbol{k}_T|^2, \qquad (2)$$

where  $k_T$  is the component of the gluon momentum perpendicular to the beam axis. In the collinear limit, when  $k_T \rightarrow 0$ , this expression converges to the ordinary  $\overline{\epsilon_g^{\mu} \epsilon_g^{*\nu}} = -\frac{1}{2}g^{\mu\nu}$ , while in the case of off shell gluons it contains an admixture of longitudinal polarization.

Another innovation made in the present study in comparison with [7] is in including the  $J/\psi$  decay step that gives access to the polarization observables. Then, the spin density matrix of  $J/\psi$  meson is determined by the momenta  $l_1$  and  $l_2$  of the decay leptons and has the form

$$\epsilon^{\mu}_{\psi}\epsilon^{*\nu}_{\psi} = 3(l_1^{\mu}l_2^{\nu} + l_2^{\mu}l_1^{\nu} - g^{\mu\nu}m_{\psi}^2/2)/m_{\psi}^2.$$
(3)

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FIG. 1. Feynman diagrams representing the partonic subprocess  $gg \rightarrow \psi \psi$ .

After averaging over the decay angles this expression becomes equivalent to the usual  $\epsilon_{\psi}^{\mu} \epsilon_{\psi}^{*\nu} = -g^{\mu\nu} + p_{\psi}^{\mu} p_{\psi}^{\nu} / m_{\psi}^2$ . All algebraic manipulations have been performed using the routine FORM [17].

The parameter setting used in numerical calculations is as follows. The charmed quark mass is set to one half of the  $J/\psi$  mass,  $m_c = m_{\psi}/2$ ; the  $J/\psi$  radial wave function is supposed to be known from leptonic decay width [18] and set to  $|\mathcal{R}_{\psi}(0)|^2 = 0.8 \text{ GeV}^3$ ; the renormalization and factorization scales are chosen equal to each other and  $\mu_F^2 =$  $\mu_R^2 = \hat{s}/4$ , with  $\hat{s}$  being the invariant energy of the partonic subprocess  $(\hat{s}/4 \text{ is numerically close to the "standard" choice <math>m_{\psi}^2 + p_{\psi,t}^2$ , but is preferable here since it is Lorentz-invariant and symmetric with respect to both  $J/\psi$ 's). The ordinary (collinear) gluon density is taken from Ref. [19], and the unintegrated gluon density  $\mathcal{F}(x, k_t^2, \mu^2)$  is given by the A0 set from Ref. [20]. This will be the default setting throughout the paper; to show the theoretical uncertainty band we will also use the A+ and A- gluon parametrizations from [20].

## **III. NUMERICAL RESULTS**

Our numerical results are exhibited in Figs. 2–9. First we show the  $J/\psi$  transverse momentum, rapidity, and  $J/\psi + J/\psi$ -invariant mass distributions in Fig. 2. The most significant numerical uncertainties come from the renormalization scale in the strong coupling  $\alpha_s(\mu_R^2)$ . Variations



FIG. 2. Effect of the renormalization scale in the strong coupling on the  $J/\psi$  transverse momentum (upper panel),  $J/\psi$  rapidity (middle panel), and  $J/\psi + J/\psi$ -invariant mass (lower panel) distributions. Notation of the curves: solid,  $\mu_R^2 = \hat{s}/4$ ; dashed,  $\mu_R^2 = \hat{s}/2$ ; dash-dotted,  $\mu_R^2 = \hat{s}/8$ . A0 gluon parametrization [20] with  $\mu_F^2 = \hat{s}/4$  is assumed everywhere.



FIG. 3. Effect of the different gluon parametrizations on the  $J/\psi$  transverse momentum (upper panel),  $J/\psi$  rapidity (middle panel), and  $J/\psi + J/\psi$ -invariant mass (lower panel) distributions. Notation of the curves: solid, A0 density; dashed, A- density; dash-dotted, A+ density; dotted, collinear parton model with LO GRV [19] gluon set.  $\mu_R^2 = \mu_F^2 = \hat{s}/4$  is assumed everywhere.

in the range  $\hat{s}/8 < \mu_R^2 < \hat{s}/2$  around the default value  $\mu_R^2 = \hat{s}/4$  make a factor of 1.6 increasing or decreasing effect on the total production rate.

A comparison between the different parametrizations of gluon densities is presented in Fig. 3 along with a comparison with collinear parton model. It is worth noting that the asymptotic shapes of the  $p_t$  spectra are qualitatively different in the collinear and  $k_t$ -factorization approaches. Here is another repetition of the scenario seen already in the inclusive  $J/\psi$  production. Taken solely, the hard subprocess matrix element possesses the behavior  $|\mathcal{M}(gg \rightarrow \psi \psi)|^2 \propto 1/p_T^8$  (as is dictated by the internal propagators in the relevant Feynman diagrams), whereas the transverse momentum generated in the partonic evolution cascade behaves like  $1/p_T^4$  (due to the *t*-channel gluon propagators). Thus, including the initial state radiation



FIG. 4. Transverse momentum distribution of  $J/\psi$  pairs. Notation of the curves: solid, A0 density; dashed, A- density; dash-dotted, A+ density.  $\mu_R^2 = \mu_F^2 = \hat{s}/4$  is assumed everywhere.



FIG. 5. Azimuthal angle difference between the two  $J/\psi$  mesons. Notation of the curves: solid, A0 density; dashed, A- density; dash-dotted, A+ density.  $\mu_R^2 = \mu_F^2 = \hat{s}/4$  is assumed everywhere.



FIG. 6.  $J/\psi$  spin alignment in pair production as seen in the helicity frame. Left column,  $k_t$ -factorization approach; right column, colinear parton model. Notation of the curves: dashed, at least one  $J/\psi$  meson has longitudinal polarization; dotted, both  $J/\psi$  mesons have longitudinal polarization. Default parameter setting is assumed everywhere.



FIG. 7. Same as Fig. 6, but for Collins-Soper frame.

(or employing the  $k_t$ -dependent gluon distributions  $\mathcal{F}(x, k_t^2, \mu^2)$ ) completely changes the whole kinematics.

The shape of the initial gluon  $k_t$  spectrum can be directly measured by measuring the transverse momenta of  $J/\psi$ pairs, see Fig. 4. (In the absence of the initial  $k_t$ , the latter would be just a  $\delta$ -function at  $p_T(\psi \psi) = 0$ .) The effect of this  $k_t$  on the event kinematics is vividly demonstrated in Fig. 5. No evidence of the original back-to-back  $J/\psi$ configuration can be found in the azimuthal correlations.

Now we turn to the polarization observables. The fractions of longitudinally polarized  $J/\psi$  mesons as functions



FIG. 8. Decay muon angular distributions as seen in the helicity (left column) and Collins-Soper (right column) frames. Notation of the curves: solid,  $k_t$ -factorization approach (with default parameter setting); dashed, collinear parton model.



FIG. 9.  $J/\psi$  meson angular distributions as seen in the helicity (left column) and Collins-Soper (right column) frames.

of the  $J/\psi$  transverse momentum and  $J/\psi$ -pair invariant mass are shown in Figs. 6 and 7 for the helicity and Collins-Soper frames, respectively. Here we also present a comparison with collinear parton model. These properties result in the decay muon angular distributions shown in Fig. 8. Here we do not see significant difference between the collinear and  $k_t$ -factorization predictions.

Finally, it may be of interest to look at the angular distributions of  $J/\psi$  mesons measured in the rest frame of the  $J/\psi$  pair, see Fig. 9. These distributions bear information on the polarization of the system formed by two initial gluons and decaying into two  $J/\psi$  's. In this case we can only make predictions for the  $k_t$ -factorization approach, because in the one-dimensional collinear kinematics one cannot unambiguously define the coordinate system (i.e., helicity, Collins-Soper, etc.).

### **IV. CONCLUSIONS**

We have considered the production of  $J/\psi$  pairs at the LHC conditions in the framework of the  $k_t$ -factorization approach. Our predictions on the integral production cross section range from ~10 nb to ~27 nb, depending on the choice of the unintegrated gluon density  $\mathcal{F}(x, k_T^2, \mu_F^2)$  and the renormalization scale in the strong coupling  $\alpha_s(\mu_R^2)$ . Although these uncertainties are rather large, they mainly affect the overall normalization, but not the shape of the kinematical distributions; and so, they do not mask important physical effects inherently connected with the  $k_t$ -factorization approach.

We have performed a comparison with colinear calculations and found an impressive difference in the behavior of the  $J/\psi$  transverse momentum spectrum and in the  $J/\psi - J/\psi$  azimuthal correlations. If supported

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experimentally, our results would demonstrate the importance of the initial state radiation effects that are contained in the  $k_t$ -factorization approach as part of the initial gluon evolution. At the same time, we have found no significant difference in the polarization observables.

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- J. Badier *et al.* (NA3 Collaboration), Phys. Lett. B **114**, 457 (1982).
- [2] J. Badier *et al.* (NA3 Collaboration), Phys. Lett. B 158, 85 (1985).
- [3] V. G. Kartvelishvili and S. N. Esakiya, Sov. J. Nucl. Phys. 38, 722 (1983).
- [4] B. Humpert and P. Mèry, Z. Phys. C 20, 83 (1983); Phys. Lett. B 124, 265 (1983).
- [5] R.E. Ecclestone and D.M. Scott, Z. Phys. C 19, 29 (1983).
- [6] V. V. Kiselev, A. K. Likhoded, S. R. Slabospitsky, and A. V. Tkabladze, Yad. Fiz. 49, 1681 (1989) [Sov. J. Nucl. Phys. 49, 1041 (1989)].
- [7] S.P. Baranov and H. Jung, Z. Phys. C 66, 647 (1995).
- [8] C. H. Kom, A. Kulesza, and W. J. Stirling, arXiv:1105.4186 [Phys. Rev. Lett. (to be published)].
- [9] S. P. Baranov, A. M. Snigirev, and N. P. Zotov, arXiv:1105.6276.
- [10] A. A. Novoselov, arXiv:1106.2184.
- [11] LHCb Collaboration, LHCb-CONF-2011-009.

- [12] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. 100, 1 (1983); E. M. Levin and M. G. Ryskin, Phys. Rep. 189, 268 (1990).
- [13] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B 242, 97 (1990); Nucl. Phys. B366, 135 (1991).
- [14] J.C. Collins and R.K. Ellis, Nucl. Phys. B360, 3 (1991).
- [15] Giving some phenomenological  $k_t$  to the initial gluons in collinear calculations [21] can mimic the  $k_t$ -factorization to some extent, though not completely.
- [16] C.-H. Chang, Nucl. Phys. B172, 425 (1980); R. Baier and R. Rückl, Phys. Lett. B 102, 364 (1981); E. L. Berger and D. Jones, Phys. Rev. D 23, 1521 (1981).
- [17] J. A. M. Vermaseren, *Symbolic Manipulations with FORM* (Computer Algebra Nederland, Kruislaan, SJ Amsterdaam, 1991), ISBN 90-74116-01-9.
- [18] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B 667, 1 (2008).
- [19] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. C 5, 461 (1998).
- [20] H. Jung, http://www.desy.de/~jung/cascade/updf.html.
- [21] A.K. Likhoded et al., arXiv:1101.5881.