Glueball in a chiral linear sigma model with vector mesons

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We present a two-flavor linear sigma model with global chiral symmetry and (axial-)vector mesons as well as an additional glueball degree of freedom. We study the structure of the well-established scalar resonances $f_0(1370)$ and $f_0(1500)$: by a fit to experimentally known decay widths we find that $f_0(1370)$ is predominantly a $\bar{q}q$ state and $f_0(1500)$ is predominantly a glueball state. The overall phenomenology of these two resonances can be well described. Other assignments for our mixed quarkonium-glueball states are also tested, but turn out to be in worse agreement with the phenomenology. As a by-product of our analysis, the gluon condensate is determined.

I. INTRODUCTION

Glueballs, the bound states of the gauge bosons of QCD, the gluons, are naturally expected in QCD due to the non-Abelian nature of the theory: the gluons interact strongly with themselves and thus they can bind and form colorless states, analogously to what occurs in the quark sector. The existence of glueballs has been studied in the framework of the effective bag model for QCD already four decades ago [1] and it has been further investigated in a variety of approaches [2]. Numerical calculations of the Yang-Mills sector of QCD also find a full glueball spectrum in which the scalar glueball is the lightest state [3].

Glueballs can mix with quarkonium ($\bar{q}q$) states with the same quantum numbers. This makes the experimental search for glueballs more complicated, because physical resonances emerge as mixed states. The scalar sector $J^{PC} = 0^{++}$ has been investigated in many works in the past. The resonance $f_0(1500)$ is relatively narrow when compared to other scalar-isoscalar states: for this reason it has been considered as a convincing candidate for a glueball state. Mixing scenarios in which two quark-antiquark isoscalar states $\bar{n}n$ and $\bar{s}s$ and one scalar glueball gg mix and generate the physical resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ have been discussed in Refs. [4,5].

The aim of this work is to extend the linear chiral model of Refs. [6,7] by including the dilaton/glueball field. The first attempt to incorporate a glueball into a linear sigma model was performed long ago in Ref. [8]. The novel features of the present study are the following: (i) The glueball is introduced as a dilaton field within a theoretical framework where not only scalar and pseudoscalar mesons, but also vector and axial-vector mesons, are present from the very beginning. This fact allows also for a calculation of decays into vector mesons. The model is explicitly evaluated for the case of $N_f = 2$, for which only one scalar-isoscalar quarkonium state exists: $\sigma \equiv \bar{n}n$ which mixes with the glueball. The two emerging mixed states are assigned to the resonances $f_0(1370)$ which is, in

accordance with Ref. [6], predominantly a $\bar{q}q$ state, and with $f_0(1500)$, which is predominantly a glueball state. (ii) We consequently test—to our knowledge for the first time—this mixing scenario above 1 GeV in the framework of a chiral model.

The model under consideration is built in accordance with the symmetries of the QCD Lagrangian. It possesses the known degrees of freedom of low-energy QCD [(pseudo)scalar and (axial-)vector mesons] as well as the same global chiral invariance. Another feature of the QCD Lagrangian is scale (or dilatation) invariance $x^\mu \rightarrow \lambda^{-1} x^\mu$ (where x^μ is a Minkowski-space coordinate and λ the scale parameter of the conformal group). It is realized at the classical level but broken at the quantum level due to the loop corrections in the Yang-Mills sector (scale anomaly). In this work the breaking of scale invariance is implemented at tree-level by means of a dilaton field (representing a glueball) with the usual logarithmic dilaton potential [8]. However, all the other interaction terms (with the exception of the chiral anomaly) are dilatation-invariant in the chiral limit.

Having constructed the Lagrangian of the effective model, we calculate the masses of the pure $\bar{q}q$ and glueball states in the $J^{PC} = 0^{++}$ channel, study their mixing and calculate the decay widths of the mixed states. Although we work with $N_f = 2$, the use of flavor symmetry enables us to calculate the decay widths of the scalar resonances into kaons and into both the η and η' mesons which contain the s-quark in their flavor wave functions. After the study of the already mentioned scenario where $f_0(1370)$ and $f_0(1500)$ are predominantly quarkonium and glueball, respectively, we also test the alternative scenario in which the resonance $f_0(1710)$ is predominantly glueball and scenarios in which $f_0(600)$ is predominantly quarkonium. They, however, lead to inconsistencies when compared to the present data and are therefore regarded as less favorable.

This paper is organized as follows. In Sec. II, the Lagrangian of the linear sigma model with (axial-)vector

and glueball degrees of freedom is constructed. In Sec. III, we discuss the results for the masses of the quarkonium-glueball mixed states and their decay widths. In Sec. IV, we present our conclusions.

II. THE MODEL

The Yang-Mills (YM) sector of QCD (QCD without quarks) is classically invariant under dilatations. This symmetry is, however, broken at the quantum level. The divergence of the corresponding current is the trace of the energy-momentum tensor $T_{YM}^{\mu\nu}$ of the YM Lagrangian

$$(T_{YM})_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0, \tag{1}$$

where $G_{\mu\nu}^a$ is the field-strength tensor of the gluon fields, $g = g(\mu)$ is the renormalized coupling constant at the scale μ, and the β-function is given by $\beta(g) = \partial g/\partial \ln\mu$. At the one-loop level $\beta(g) = -bg^3$ with $b = 11N_c/(48\pi^2)$. This implies $g^2(\mu) = [2b\ln(\mu/\Lambda_{YM})]^{-1}$, where $\Lambda_{YM} \simeq 200$ MeV is the Yang-Mills scale. A finite energy scale thus emerges in a theory which is classically invariant under dilatation (dimensional transmutation). The expectation value of the trace anomaly does not vanish and represents the so-called gluon condensate:

$$\langle T_{YM,\mu}^\mu \rangle = -\frac{11N_c}{48} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{48} C^4, \tag{2}$$

where

$$C^4 \simeq (300\text{–}600 \text{ MeV})^4. \tag{3}$$

The numerical values have been obtained through QCD sum rules (lower range of the interval) [9] and lattice simulations (higher range of the interval) [10].

At the composite level, one can build an effective theory of the YM sector of QCD by introducing a scalar dilaton field G which describes the trace anomaly. The dilaton Lagrangian reads [8]

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right). \tag{4}$$

The minimum G_0 of the dilaton potential is realized for $G_0 = \Lambda$. Upon shifting $G \to G_0 + G$ a particle with mass m_G emerges, which is interpreted as the scalar glueball. The numerical value has been evaluated in Lattice QCD and reads $m_G \sim 1.5$ GeV [3]. The logarithmic term of the potential explicitly breaks the invariance under a dilatation transformation. The divergence of the corresponding current reads $\partial_\mu J_{dil}^\mu = T_{dil,\mu}^\mu = -\frac{1}{4}m_G^2\Lambda^2$. This can be compared with the analogous quantity in Eq. (2), which implies $\Lambda = \sqrt{11}C^2/(2m_G)$.

QCD with quarks is also classically invariant under dilatation transformations in the limit of zero quark masses (chiral limit). The scale of all hadronic phenomena is given by the previously introduced energy scale Λ_{YM}. This fact holds true also when the small but nonzero values of the quark masses are considered. In order to describe these properties in a hadronic model we now extend the linear sigma model with $U(N_f)_R \times U(N_f)_L$ of Refs. [6,11–13] by including the dilaton. To this end, the following criteria are applied [14]: (i) With the exception of the chiral anomaly, the parameter Λ from Eq. (4), which comes from the Yang-Mills sector of the theory in accordance with QCD, is the only dimensionful parameter of the Lagrangian in the chiral limit. (ii) The Lagrangian is required to be finite for every finite value of the gluon condensate G_0. This, in turn, also assures that no singular terms arise in the limit $G_0 \to 0$. In accordance with the requirements (i) and (ii) only terms with dimension exactly equal to 4 are allowed in the chiral limit.

The hadronic Lagrangian obeying these requirements reads

$$\mathcal{L} = \mathcal{L}_{dil} + \text{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - m_0^2\left(\frac{G}{G_0}\right)^2 \Phi^\dagger\Phi - \lambda_2(\Phi^\dagger\Phi)^2\right] - \lambda_1(\text{Tr}[\Phi^\dagger\Phi])^2 + c[\det(\Phi^\dagger) + \det(\Phi)]$$

$$+ \text{Tr}[H(\Phi^\dagger + \Phi)] - \frac{1}{4}\text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2 \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{h_1}{2}\text{Tr}[\Phi^\dagger\Phi]\text{Tr}[L_\mu L^\mu + R_\mu R^\mu]$$

$$+ h_2\text{Tr}[\Phi^\dagger L_\mu L^\mu\Phi + \Phi R_\mu R^\mu\Phi^\dagger] + 2h_3\text{Tr}[\Phi R_\mu\Phi^\dagger L^\mu] + \ldots, \tag{5}$$

where Φ denotes the $N_f \times N_f$ (pseudo)scalar multiplet and L^μ and R^μ the left- and right-handed vector multiplets, respectively. The dots represent further terms which do not affect the processes studied in this work.

In the particular case of $N_f = 2$ studied in this manuscript, one has $\Phi = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ (our eta meson η_N contains only nonstrange degrees of freedom), $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ and

$R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$; t^0, $\vec{t}$ are the generators of $U(2)$. Moreover, $D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$, and $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$.

The explicit breaking of the global chiral symmetry is described by the term $\text{Tr}[H(\Phi + \Phi^\dagger)] \equiv h\sigma (h = \text{const.} \sim m_q^2)$, which allows us to take into account the nonvanishing value m_q of the quark mass. This term contains the dimensionful parameter h with

$[h] = [\text{energy}^3]$ and also explicitly breaks the dilatation invariance, just as the quark masses do in the underlying QCD Lagrangian. Finally, the chiral anomaly is described by the term $c(\det\Phi + \det\Phi^\dagger)$ [15]. For $N_f = 2$, the parameter c carries the dimension $[\text{energy}^2]$ and represents a further breaking of dilatation invariance. This term arises from instantons which are also a property of the Yang-Mills sector of QCD.

The identification of the fields of the models with the resonances listed in Ref. [16] is straightforward in the pseudoscalar and (axial-)vector sectors: the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, respectively, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by "unmixing" the physical η and η' mesons which also contain $\bar{s}s$ contributions. The fields ω^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, while the fields f_1^μ and $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical ω and f_1 states also contain $\bar{s}s$ contributions but their admixture is small.) As shown in Ref. [6], the σ field should be interpreted as a predominantly $\bar{q}q$ state because its decay width decreases as $1/N_c$ in the limit of a large number of colors. The σ and G fields mix: the physical fields σ' and G' are obtained through an $SO(2)$ rotation, as we shall show in the following. Then the first and most natural assignment is $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$, see Sec. III A. Note that the $\vec{a}_0$ state is assigned to the physical $a_0(1450)$ resonance in accordance with results of Ref. [6]. Other assignments for $\{\sigma', G'\}$ will be also tested in Sec. III B and III C and turn out to be less favorable.

In order to study the nonvanishing vacuum expectation values (VEV's) of the two $J^{PC} = 0^{++}$ scalar-isoscalar fields of the model σ and G, we set all the other fields in Eq. (5) to zero and obtain:

$$\mathcal{L}_{\sigma G} = \mathcal{L}_{\text{dil}} + \frac{1}{2}(\partial^\mu \sigma)^2 - \frac{1}{2}\left[m_0^2\left(\frac{G}{G_0}\right)^2 - c\right]\sigma^2 \\ - \frac{1}{4}\left(\lambda_1 + \frac{\lambda_2}{2}\right)\sigma^4 + h\sigma. \tag{6}$$

Upon shifting the fields by their VEV's, $\sigma \to \sigma + \phi$ and $G \to G + G_0$, we obtain the masses of the states $\sigma = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $G = gg$ [6],

$$M_\sigma^2 = m_0^2 - c + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2,$$
$$M_G^2 = m_0^2\frac{\phi^2}{G_0^2} + m_G^2\frac{G_0^2}{\Lambda^2}\left(1 + 3\ln\left|\frac{G_0}{\Lambda}\right|\right). \tag{7}$$

Note that the pure glueball mass M_G depends also on the quark condensate ϕ, but correctly reduces to m_G in the limit $m_0^2 = 0$ (decoupling of quarkonia and glueball). In the presence of quarkonia, $m_0^2 \neq 0$, the VEV G_0 is given by the equation

$$-\frac{m_0^2\phi^2\Lambda^2}{m_G^2} = G_0^4\ln\left|\frac{G_0}{\Lambda}\right|. \tag{8}$$

The shift of the fields by their VEV's introduces a bilinear mixing term $\sim \sigma G$ in the Lagrangian (6). The physical fields σ' and G' can be obtained through an $SO(2)$ rotation,

$$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \sigma \\ G \end{pmatrix}, \tag{9}$$

with

$$M_{\sigma'}^2 = M_\sigma^2\cos^2\theta + M_G^2\sin^2\theta + 2m_0^2\frac{\phi}{G_0}\sin(2\theta), \tag{10}$$

$$M_{G'}^2 = M_G^2\cos^2\theta + M_\sigma^2\sin^2\theta - 2m_0^2\frac{\phi}{G_0}\sin(2\theta), \tag{11}$$

where the mixing angle θ reads

$$\theta = \frac{1}{2}\arctan\left[-4\frac{\phi}{G_0}\frac{m_0^2}{M_G^2 - M_\sigma^2}\right]. \tag{12}$$

The quantity m_0^2 can be calculated from the masses of the pion, η_N, and the bare σ mass [see Ref. [6]]:

$$m_0^2 = \left(\frac{m_\pi}{Z}\right)^2 + \frac{1}{2}\left[\left(\frac{m_{\eta_N}}{Z}\right)^2 - M_\sigma^2\right]. \tag{13}$$

If $m_0^2 - c < 0$, spontaneous breaking of chiral symmetry is realized.

III. RESULTS AND DISCUSSION

The Lagrangian (5) contains the following 12 free parameters: m_0, λ_1, λ_2, m_1, g_1, c, h, h_1, h_2, h_3, m_G, $\Lambda = \sqrt{11}C^2/(2m_G)$. The processes that we shall consider depend only on the combination $h_1 + h_2 + h_3$, thus reducing the number of parameters to ten. We replace the set of ten parameters by the following equivalent set: m_π, m_{η_N}, m_ρ, m_{a_1}, ϕ, Z, M_σ, m_G, m_1, C. The masses $m_\pi (= 139.57$ MeV$)$ and $m_\rho (= 775.49$ MeV$)$ are fixed to their Particle Data Group (PDG) values [16].

As outlined in Refs. [6,12], the mass of the η_N meson can be calculated using the mixing of strange and nonstrange contributions in the physical fields η and $\eta'(958)$:

$$\eta = \eta_N\cos\varphi + \eta_S\sin\varphi,$$
$$\eta' = -\eta_N\sin\varphi + \eta_S\cos\varphi, \tag{14}$$

where η_S denotes a pure $\bar{s}s$ state and $\varphi \simeq -36°$ [17]. In this way, we obtain the value $m_{\eta_N} = 716$ MeV. (Given the well-known uncertainty of the value of the angle φ, one could also consider other values, e.g., $\varphi = -41.4°$, as published by the KLOE Collaboration [18], which corresponds to $m_{\eta_N} = 755$ MeV. Variations of the pseudoscalar mixing angle affect the results presented in this paper only slightly.)

The value of m_{a_1} is fixed to 1050 MeV according to the study of Ref. [13]. (We note that taking the present PDG estimate of 1230 MeV does not change our conclusions.) The chiral condensate is fixed as $\phi = Z f_\pi$ and the renormalization constant Z is determined by the study of the process $a_1 \to \pi\gamma$: $Z = 1.67 \pm 0.2$ [6].

A. Assigning σ' and G' to $f_0(1370)$ and $f_0(1500)$

The σ' field denotes an isoscalar $J^{PC} = 0^{++}$ state and its assignment to a physical state is a long-debated problem of low-energy QCD [2,3,5,19]. The two major candidates are the resonances $f_0(600)$ and the $f_0(1370)$ [16]. The study of Ref. [6] has shown that $f_0(1370)$ is favored to be a state which is predominantly $\bar{q}q$. As stated above, the resonance $f_0(1500)$ is a convincing glueball candidate. For these reasons, we first test the scenario in which $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$, which turns out to be phenomenologically successful, see below.

We are left with the following four free parameters: C, M_σ, m_G, and m_1. They can be obtained by a fit to the five experimental quantities of Table I. The five experimental quantities are the masses of the resonances $f_0(1500)$ $(M_{G'} \equiv M_{f_0(1500)} = 1505$ MeV [16]) and $f_0(1370)$ (for which we use the mean value $M_{\sigma'}^{ex} = (1350 \pm 150)$ MeV taking into account the PDG mass range between 1200 MeV and 1500 MeV [16]) and the three well-known decay widths of the well-measured resonance $f_0(1500)$: $f_0(1500) \to \pi\pi$, $f_0(1500) \to \eta\eta$, and $f_0(1500) \to K\bar{K}$.

Using the Lagrangian (5), these observables can be expressed as functions of the parameters listed above. Note that, although our framework is based on $N_f = 2$, we can calculate the amplitudes for the decays into mesons containing strange quarks by making use of the flavor symmetry $SU(N_f = 3)$ [4]. It is then possible to calculate the following $f_0(1500)$ decay widths into pseudoscalar mesons containing s quarks: $f_0(1500) \to K\bar{K}$, $f_0(1500) \to \eta\eta$, and $f_0(1500) \to \eta\eta'$.

The χ^2 method yields $\chi^2/\text{d.o.f.} = 0.29$ (thus very small), $C = (699 \pm 40)$ MeV, $M_\sigma = (1275 \pm 30)$ MeV, $m_G = (1369 \pm 26)$ MeV and $m_1 = (809 \pm 18)$ MeV. We have also examined the uniqueness of our fit. To this end, we have considered χ^2 fixing three of four parameters entering the fit at their best values and varying the

remaining fourth parameter. In each of the four cases, we observe only one minimum of the χ^2 function; each minimum leads exactly to the parameter values stated in Table I. We also observe no changes of the results for the errors of the parameters. These findings give us confidence that the obtained minimum corresponds to the absolute minimum of the χ^2 function.

The consequences of this fit are the following:

(i) The quarkonium-glueball mixing angle reads $\theta = (29.7 \pm 3.6)°$. This, in turn, implies that the resonance $f_0(1500)$ consists to 76% of a glueball and to the remaining 24% of a quark-antiquark state. An inverted situation holds for $f_0(1370)$.

(ii) Our fit allows us to determine the gluon condensate: $C = (699 \pm 40)$ MeV. This result implies that the upper value in Eq. (2) is favored by our analysis. It is remarkable that insights into this basic quantity of QCD can be obtained from the PDG data on mesons.

(iii) Further results for the $f_0(1500)$ meson are reported in the first two entries of Table II. The decay into 4π is calculated as a product of an intermediate $\rho\rho$ decay. To this end the usual integration over the ρ spectral function is performed. Our result yields 30 MeV in the 4π decay channel and is about half of the experimental value $\Gamma_{f_0(1500)\to 4\pi} = (54.0 \pm 7.1)$ MeV. However, it should be noted that an intermediate state consisting of two $f_0(600)$ mesons (which is also expected to contribute in this decay channel) is not included in the present model. The decay into the $\eta\eta'$ channel is also evaluated; this channel is subtle because it is exactly on the threshold of the $f_0(1500)$ mass. Therefore, an integration over the spectral function of the decaying meson $f_0(1500)$ is necessary. The result is in a qualitative agreement with the experiment.

(iv) The results for the $f_0(1370)$ meson are reported in the last four rows of Table II. They are in agreement with the experimental data regarding the full width: $\Gamma_{f_0(1370)} = (200-500)$ MeV [16]. Unfortunately, the experimental results in the different channels are not yet conclusive. Our theoretical results point towards a dominant direct $\pi\pi$ and a

TABLE I. Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$. Note that the $f_0(1370)$ mass ranges between 1200 MeV and 1500 MeV [16] and therefore, as an estimate, we are using the value $m_{\sigma'} = (1350 \pm 150)$ MeV in the fit.

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1200–1500
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

TABLE II. Further results regarding the $\sigma' \equiv f_0(1370)$ and $G' \equiv f_0(1500)$ decays.

Quantity	Our Value [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	54 ± 7.1
$G' \to \eta\eta'$	0.6	2.1 ± 1
$\sigma' \to \pi\pi$	284 ± 43	$\cdots$
$\sigma' \to \eta\eta$	72 ± 6	$\cdots$
$\sigma' \to K\bar{K}$	4.6 ± 2.1	$\cdots$
$\sigma' \to \rho\rho \to 4\pi$	0.09	$\cdots$

non-negligible $\eta\eta$ contribution; these results correspond well to the experimental analysis of Ref. [20] where $\Gamma_{f_0(1370)\to\pi\pi} = 325$ MeV and $\Gamma_{f_0(1370)\to\eta\eta}/\Gamma_{f_0(1370)\to\pi\pi} = 0.19 \pm 0.07$ are obtained. We find that the four-pion decay of $f_0(1370) \to \rho\rho \to 4\pi$ is strongly suppressed (as was also found in Ref. [6]). This is unlike Ref. [20], where a small but non-negligible value of about 50 MeV is found. However, it should be noted that due to interference effects, our result for this decay channel varies strongly when the parameters are even slightly modified.

(v) The mass of the ρ meson can be expressed as $m_\rho^2 = m_1^2 + \phi^2(h_1 + h_2 + h_3)/2$. In order that the contribution of the chiral condensate is not negative, the condition $m_1 \leq m_\rho$ should hold. In the framework of our fit this condition is fulfilled at the two-sigma level. This result points towards a dominant m_1 contribution to the ρ mass. This property, in turn, means that the ρ mass is predominantly generated from the gluon condensate and not from the chiral condensate. It is therefore expected that the ρ mass in the medium scales as the gluon condensate rather than as the chiral condensate. In view of the fact that m_1 is slightly larger than m_ρ, we have also repeated the fit by fixing $m_1 = m_\rho$: the minimum has a $\chi^2/\text{d.o.f.} \simeq 1$ and the results are very similar to the previous case. The corresponding discussion about the phenomenology is unchanged.

(vi) As already stressed in Refs. [6,21], the inclusion of (axial-)vector mesons plays a central role to obtain the present results. The artificial decoupling of (axial-)vector states would generate a, by far, too-wide $f_0(1370)$ state. For this reason the glueball-quarkonium mixing scenario above 1 GeV has been previously studied only in phenomenological models with flavor symmetry [2,4] but not in the context of chirally invariant models.

Given that the resonance $f_0(1370)$ has a large mass uncertainty, we have also examined the behavior of the fit at different points of the PDG mass interval. Considering the minimal value $m_{f_0(1370)}^{\min} = (1220 \pm 20)$ MeV, we obtain $\chi^2 = 0.2/\text{d.o.f.}$ The resulting value of the mixing angle $\theta = (30.3 \pm 3.4)°$ is practically the same as the value $\theta = (29.7 \pm 3.6)°$ obtained in the case where $m_{f_0(1370)} = (1350 \pm 150)$ MeV was considered. Other results are also qualitatively similar to the case of $m_{f_0(1370)} = (1350 \pm 150)$ MeV.

For the upper boundary of the $f_0(1370)$ mass, the error interval of ± 20 MeV turns out to be too restrictive as it leads to unacceptably large χ^2 values. Consequently, increasing the error interval decreases the χ^2 values— we observe that $m_{f_0(1370)}^{\max} = (1480 \pm 120)$ MeV leads to an acceptable χ^2 value of $1.14/\text{d.o.f.}$ We then obtain

$\theta = (30.0 \pm 3.5)°$, practically unchanged in comparison with the value $\theta = (29.7 \pm 3.6)°$ in the case where $m_{f_0(1370)} = (1350 \pm 150)$. Also other quantities remain basically the same as in the case of $m_{f_0(1370)} = (1350 \pm 150)$ MeV.

We have also considered the fit at several points between the lower and upper boundaries of the $m_{f_0(1370)}$ mass range. We have chosen points of 50 MeV difference starting at $m_{f_0(1370)} = 1250$ MeV (i.e., we have considered $m_{f_0(1370)}\{1250, 1300, 1350, 1400, 1450\}$ MeV) with errors chosen such that the $\chi^2/\text{d.o.f.}$ becomes minimal (error values are between ± 30 MeV for $m_{f_0(1370)} = 1250$ MeV and ± 100 MeV for $m_{f_0(1370)} = 1450$ MeV). We observe that the previous results presented in this section do not change significantly; most notably, the mixing angle θ attains values between 30.2° and 30.7°, with an average error value of $\pm 3.4°$.

We therefore conclude that considering different values of $m_{f_0(1370)}$ within the (1200–1500) MeV interval does not change the results significantly. In particular, the quarkonium-glueball mixing angle θ changes only slightly (by approximately 1°) and thus we confirm our conclusion that $f_0(1370)$ is predominantly a quarkonium and $f_0(1500)$ is predominantly a glueball.

B. Assigning σ' and G' to $f_0(1370)$ and $f_0(1710)$

Although the resonance $f_0(1710)$ has also been regarded as a glueball candidate in a variety of works [22], its enhanced decay into kaons and its rather small decay width make it compatible with a dominant $\bar{s}s$ contribution in its wave function. Nonetheless, we have also tested the assumption that the pure quarkonium and glueball states mix to produce the resonances $f_0(1370)$ and $f_0(1710)$.

The resonance $f_0(1710)$ is experimentally well-known. Decays into $\pi\pi$, $\bar{K}K$, and $\eta\eta$ have been seen, while no decays into $\eta\eta'$ and into 4π have been detected. Using the total decay width $\Gamma_{f_0(1710)} = (135 \pm 8)$ MeV and the branching ratios reported in Ref. [16], it is possible to deduce the decay widths into $\pi\pi$, $\bar{K}K$, and $\eta\eta$, see Table III.

A fit analogous to the one in Table I yields too-large errors for the decay width $\sigma' \equiv f_0(1370) \to \pi\pi$. For this reason we repeat our fit by adding the following constraint: $\Gamma_{\sigma'\to\pi\pi} = (250 \pm 150)$ MeV. The large error assures that

TABLE III. Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1710)\}$.

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1386 ± 134	1350 ± 150
$M_{G'}$	1720 ± 6	1720 ± 6
$G' \to \pi\pi$	29.7 ± 6.5	29.3 ± 6.5
$G' \to \eta\eta$	6.9 ± 5.8	34.3 ± 17.6
$G' \to K\bar{K}$	16 ± 14	71.4 ± 29.1
$\sigma' \to \pi\pi$	379 ± 147	250 ± 150

TABLE IV. Further results from the fit with $\{\sigma', G'\} = \{f_0(1370), f_0(1710)\}$.

Decay Width	Our Value [MeV]	Experimental value [MeV]
$G' \to 4\pi$	115	$\cdots$
$G' \to \eta\eta'$	16	$\cdots$
$\sigma' \to \eta\eta$	153 ± 79	$\cdots$
$\sigma' \to K\bar{K}$	$2.1^{+13.6}_{-2.1}$	$\cdots$

this value is in agreement with experimental data on this decay width. The results of the fit are reported in Table III.

The χ^2 fit yields $\chi^2/\text{d.o.f.} = 1.72$, $C = (764 \pm 256)$ MeV, $M_\sigma = (1516 \pm 80)$ MeV, $m_G = (1531 \pm 233)$ MeV, and $m_1 = (827 \pm 36)$ MeV. The mixing angle between the pure quarkonium σ and the pure glueball G calculated from Eq. (12) is $\theta = (37.2 \pm 21.4)°$. The χ^2 is worse than in the previous case, but the overall agreement is acceptable. The mixing angle is large and could also overshoot the value of $45°$, which would imply a somewhat unexpected and unnatural reversed ordering, in which $f_0(1370)$ is predominantly glueball and $f_0(1710)$ predominantly quarkonium.

In Table IV we report the decay widths $G' \to 4\pi$, $G' \to \eta\eta'$, $\sigma' \to \eta\eta$, and $\sigma' \to K\bar{K}$, which can be calculated as a consequence of the fit of Table III.

A clear problem of this scenario emerges: the decay width $G' \equiv f_0(1710) \to 4\pi$ is large, while experimentally it has not been seen. Therefore, we conclude that this scenario is not favored. Moreover, in this scenario the remaining resonance $f_0(1500)$ should then be interpreted as a predominantly $\bar{s}s$ state, contrary to what its experimentally dominant $\pi\pi$ decay pattern suggests. Consequently, $f_0(1710)$ is unlikely to be predominantly a glueball state; this is also in accordance with the results from the ZEUS Collaboration [23].

C. Scenarios with $\sigma' \equiv f_0(600)$

The scenarios $\{\sigma', G'\} = \{f_0(600), f_0(1500)\}$ and $\{\sigma', G'\} = \{f_0(600), f_0(1710)\}$ have also been tested. In both cases, the mixing angle turns out to be small

($\lesssim 15°$), thus the state $f_0(600)$ is predominantly quarkonium. Then, in these cases, the analysis of Ref. [6] applies: a simultaneous description of the $\pi\pi$ scattering lengths and the $\sigma \to \pi\pi$ decay width cannot be achieved. For these reasons, the mixing scenarios with the resonance $f_0(600)$ as a quarkonium state are not favored.

IV. CONCLUSIONS AND OUTLOOK

We have presented a globally chirally invariant linear sigma model with (axial-)vector mesons and a dilaton/glueball degree of freedom. We have studied the phenomenology of the scalar states for the case $N_f = 2$: in the favored scenario, the resonance $f_0(1500)$ is predominantly a glueball with a subdominant $\bar{q}q$ component and, conversely, $f_0(1370)$ is predominantly a quark-antiquark $(\bar{u}u + \bar{d}d)/\sqrt{2}$ state with a subdominant glueball contribution. It is interesting to observe that the success of the phenomenological description of these scalar resonances is due to the inclusion of the (axial-)vector mesons in the model. The gluon condensate is also an outcome of our study and turns out to be in agreement with lattice QCD results. Different scenarios in which $f_0(1710)$ is predominantly glueball and/or $f_0(600)$ is predominantly quarkonium do not seem to be in agreement with the present experimental data.

Natural extensions of the model are the case $N_f = 3$ [24] and the inclusion of a nonet of tetraquark states as additional low-lying scalar states. In this general scenario, a mixing of five scalar-isoscalar states takes place, which allows us to describe all relevant scalar-isoscalar resonances listed in the PDG below 1.8 GeV [16]. Applications of the model at nonzero temperature and density are also important because the presence of the dilaton field allows us to study the restoration of both the dilatation and the chiral symmetry of QCD.

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