$Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states

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In the framework of the one-boson-exchange model, we have studied the interaction of the $B^*\bar{B}$ and $B^*\bar{B}^*$ systems. After considering the S-wave and D-wave mixing, we notice that both $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ can be interpreted as the $B^*\bar{B}$ and $B^*\bar{B}^*$ molecular states quite naturally. The long-range one-pion-exchange force alone is strong enough to form these loosely bound molecular states, which ensures the numerical results are quite model independent and robust.

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Very recently, the Belle Collaboration announced two charged bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$. These two states were observed in the invariant mass spectra of $h_b(nP)\pi^{\pm}$ ($n = 1, 2$), and $\Upsilon(mS)\pi^{\pm}$ ($m = 1, 2, 3$) of the corresponding $\Upsilon(5S) \to h_b(nP)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(mS)\pi^+\pi^-$ hidden-bottom decays [1]. With the above five hidden-bottom decay channels, Belle extracted the $Z_b(10610)$ and $Z_b(10650)$ parameters. The obtained averages over all five channels are $M_{Z_b(10610)} = 10608.4 \pm 2.0\ \mathrm{MeV}/c^2$, $\Gamma_{Z_b(10610)} = 15.6 \pm 2.5\ \mathrm{MeV}/c^2$, $M_{Z_b(10650)} = 10653.2 \pm 1.5\ \mathrm{MeV}/c^2$, $\Gamma_{Z_b(10650)} = 14.4 \pm 3.2\ \mathrm{MeV}/c^2$ [1]. In addition, the analysis of the angular distribution indicates both $Z_b(10610)$ and $Z_b(10650)$ favor $I^G(J^P) = 1^+(1^+)$.

If $Z_b(10610)$ and $Z_b(10650)$ arise from the resonance structures, they are good candidates of nonconventional bottomonium-like states. The masses of the $J^{PC} = 1^{++}$ and $J^{PC} = 1^{+-}$ $b\bar{b}q\bar{q}$ tetraquark states were found to be around 10.1–10.2 GeV in the framework of QCD sum rule formalism [2], which are significantly lower than these two charged Z_b states. Therefore, it is hard to accommodate them as tetraquarks. When comparing the experimental measurement with the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, one notices that $Z_b(10610)$ and $Z_b(10650)$ are close to the thresholds of $B\bar{B}^*$ and $B^*\bar{B}^*$, respectively. One plausible explanation is that both $Z_b(10610)$ and $Z_b(10650)$ are either $B^*\bar{B}^*$ or $B^*\bar{B}^*$ molecular states, respectively.

Before the observations of two charged $Z_b(10610)$ and $Z_b(10650)$ states, there were many theoretical works that focused on the molecular systems composed of $B^{(*)}$ and $\bar{B}^{(*)}$ meson pairs and indicated that there probably exist loosely bound S-wave $B^*\bar{B}^*$ or $B^*\bar{B}^*$ molecular states [3,4]. To some extent, such studies were stimulated by a series of near-threshold charomonium-like X, Y, Z states in the past 8 years.

Molecular states involving charmed quarks were first proposed by Voloshin and Okun more than 30 years ago [5]. Later, De Rujula, Georgi, and Glashow speculated $\psi(4040)$ as a $D^*\bar{D}^*$ molecular charmonium [6]. Törnqvist calculated the possible deuteron-like two-meson bound states such as $D\bar{D}^*$ and $D^*\bar{D}^*$ using the quark-pion interaction model [7,8].

As the first observed charged bottomonium-like states, $Z_b(10610)$ and $Z_b(10650)$ have attracted the attention of many theoretical groups. The authors discussed the special decay behavior of the $J = 1$ S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states based on the heavy quark symmetry in Ref. [9]. Chen, Liu, and Zhu [10] found that the intermediate $Z_b(10610)$ and $Z_b(10650)$ contribution to $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$ naturally explains Belle's previous observation of the anomalous $\Upsilon(2S)\pi^+\pi^-$ production near the peak of $\Upsilon(5S)$ at $\sqrt{s} = 10.87$ GeV [11], where the resulting $d\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-)/dm_{\pi^+\pi^-}$ and $d\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-)/d\cos\theta$ distributions agree with Belle's measurement after inclusion of these Z_b states [10]. The authors of Ref. [12] tried to reproduce the masses of $Z_b(10610)$ and $Z_b(10650)$ using a molecular bottomonium-like current in the QCD sum rule calculation. Yang *et al.* studied the mass spectra of the S-wave $[\bar{b}q][b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]$, $[\bar{b}q]^*[b\bar{q}]^*$ in the chiral quark model and indicated that $Z_b(10610)$ and $Z_b(10650)$ are good candidates of the S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ bound states [13]. Bugg proposed a nonexotic explanation of $Z_b(10610)$ and $Z_b(10650)$, which are interpreted as the orthogonal linear combinations of the $q\bar{q}$ and meson-meson states, namely, $b\bar{b} + B\bar{B}^*$ and $b\bar{b} + B^*\bar{B}^*$ [14], respectively. Nieves and Valderrama suggested the possible existence of two positive C-parity isoscalar states: a ${}^3S_1 - {}^3D_1$ state with a binding energy of 90–100 MeV and a ${}^3P_0$ state located about 20–30 MeV below the $B\bar{B}^*$ threshold [15].

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Unfortunately, the quantum number of the above states does not match those of these two charged Z_b states. Danilkin, Orlovsky, and Simonov studied the interaction between a light hadron and heavy quarkonium through the transition to a pair of intermediate heavy mesons. Based on the above coupled-channel effect, the authors discussed the resonance structures close to the $B^{(*)}\bar{B}^*$ threshold [16]. Using the chromomagnetic interaction, the authors of Ref. [17] discussed the possibility of $Z_b(10610)$ and $Z_b(10650)$ being tetraquark states. In contrast, the $b\bar{b}q\bar{q}$ tetraquark states were predicted to be around 10.2–10.3 GeV using the color-magnetic interaction with the flavor symmetry breaking corrections [18], consistent with the values extracted from the QCD sum rule approach [2].

As emphasized in Ref. [10], future dynamical study of the mass and decay pattern of the S-wave $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states are very desirable. In this work, we perform a more thorough study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems using the one-boson-exchange (OBE) model. Different from our former work in Refs. [3,4], we not only consider S-wave interaction but also include D-wave contribution between $B^{(*)}$ and $\bar{B}^{(*)}$. Such a study will be helpful to answer whether the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular bottomonia exist or not.

Belle indicated that both $Z_b(10610)$ and $Z_b(10650)$ belong to the isotriplet states. If $Z_b(10610)$ and $Z_b(10650)$ are the $B\bar{B}^*$ or $B^*\bar{B}^*$ molecular states, respectively, the flavor wave functions of $Z_b(10610)$ and $Z_b(10650)$ are [3,4] $|Z_b(10610)^{\pm}\rangle=\frac{1}{\sqrt{2}}(|B^{*\pm}\bar{B}^0\rangle+|B^{\pm}\bar{B}^{*0}\rangle)$, $|Z_b(10610)^0\rangle=\frac{1}{2}[(|B^{*+}B^-\rangle-|B^{*0}\bar{B}^0\rangle)+(|B^+B^{*-}\rangle-|B^0\bar{B}^{*0}\rangle)]$, $|Z_b(10650)^{\pm}\rangle=|B^{*\pm}\bar{B}^{*0}\rangle$, $|Z_b(10650)^0\rangle=\frac{1}{\sqrt{2}}(|B^{*+}B^{*-}\rangle-|B^{*0}\bar{B}^{*0}\rangle)$, respectively.

In order to obtain the effective potential of the $B\bar{B}^*$ and $B^*\bar{B}^*$ system, we employ the OBE model, which is an effective framework to describe the $B\bar{B}^*$ or $B^*\bar{B}^*$ interaction by exchanging the light pseudoscalar, scalar, and vector mesons. In terms of heavy quark limit and chiral symmetry, the interactions of light pesudoscalar, vector, and scalar mesons interacting with S-wave heavy flavor mesons were constructed in Refs. [19–25]

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{P}}=-i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\lambda\dagger}\partial^\nu\mathbb{P}_{ba}+i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\tilde{\mathcal{P}}_a^{*\mu\dagger}\tilde{\mathcal{P}}_b^{*\lambda}\partial^\nu\mathbb{P}_{ab}, \tag{1}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{P}}=-\frac{2g}{f_\pi}(\mathcal{P}_b\mathcal{P}_{a\lambda}^{*\dagger}+\mathcal{P}_{b\lambda}^*\mathcal{P}_a^\dagger)\partial^\lambda\mathbb{P}_{ba}+\frac{2g}{f_\pi}(\tilde{\mathcal{P}}_{a\lambda}^{*\dagger}\tilde{\mathcal{P}}_b+\tilde{\mathcal{P}}_a^\dagger\tilde{\mathcal{P}}_{b\lambda}^*)\partial^\lambda\mathbb{P}_{ab}, \tag{2}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}}=-\sqrt{2}\beta g_V\mathcal{P}_b\mathcal{P}_a^\dagger v\cdot\mathbb{V}_{ba}+\sqrt{2}\beta g_V\tilde{\mathcal{P}}_a^\dagger\tilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab}, \tag{3}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}}=-2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\mathcal{P}_b\mathcal{P}_a^{*\mu\dagger}+\mathcal{P}_b^{*\mu}\mathcal{P}_a^\dagger)(\partial^\alpha\mathbb{V}^\beta)_{ba}-2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\tilde{\mathcal{P}}_a^{*\mu\dagger}\tilde{\mathcal{P}}_b+\tilde{\mathcal{P}}_a^\dagger\tilde{\mathcal{P}}_b^{*\mu})(\partial^\alpha\mathbb{V}^\beta)_{ab}, \tag{4}$$

$$\begin{aligned}\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}}=&\sqrt{2}\beta g_V\mathcal{P}_b^*\cdot\mathcal{P}_a^{*\dagger}v\cdot\mathbb{V}_{ba}\\&-i2\sqrt{2}\lambda g_V\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\nu\dagger}(\partial_\mu\mathbb{V}_\nu-\partial_\nu\mathbb{V}_\mu)_{ba}\\&-\sqrt{2}\beta g_V\tilde{\mathcal{P}}_a^{*\dagger}\cdot\tilde{\mathcal{P}}_b^*v\cdot\mathbb{V}_{ab}\\&-i2\sqrt{2}\lambda g_V\tilde{\mathcal{P}}_a^{*\mu\dagger}\tilde{\mathcal{P}}_b^{*\nu}(\partial_\mu\mathbb{V}_\nu-\partial_\nu\mathbb{V}_\mu)_{ab},\end{aligned} \tag{5}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma}=-2g_s\mathcal{P}_b\mathcal{P}_b^\dagger\sigma-2g_s\tilde{\mathcal{P}}_b\tilde{\mathcal{P}}_b^\dagger\sigma, \tag{6}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\sigma}=2g_s\mathcal{P}_b^*\cdot\mathcal{P}_b^{*\dagger}\sigma+2g_s\tilde{\mathcal{P}}_b^*\cdot\tilde{\mathcal{P}}_b^{*\dagger}\sigma, \tag{7}$$

where $g=0.59$ is extracted from the experimental width of D^{*+} [26]. The parameter β relevant to the vector meson can be fixed as $\beta=0.9$ by the vector meson dominance mechanism while $\lambda=0.56\ \mathrm{GeV}^{-1}$ was obtained by comparing the form factor calculated by the light cone sum rule with the one obtained by lattice QCD. As the coupling constant related to the scalar meson σ, $g_s=g_\pi/(2\sqrt{6})$ with $g_\pi=3.73$ was given in Refs. [4,24].

Generally, the scattering amplitude $i\mathcal{M}(J,J_Z)$ is related to the interaction potential in the momentum space in terms of the Breit approximation. The potential in the coordinate space $\mathcal{V}(\mathbf{r})$ is obtained after performing the Fourier transformation, where we need to introduce the monopole form factor $\mathcal{F}(q^2,m_E^2)=(\Lambda^2-m_E^2)/(\Lambda^2-q^2)$ to reflect the structure effect of the vertex of the heavy mesons interacting with the light mesons. m_E denotes the exchange meson mass. In reality, the phenomenological cutoff Λ is around one to several GeV, which also plays the role of regulating the effective potential.

The general expressions of the total effective potentials of the $Z_b(10610)$ systems are

$$\mathcal{V}^{Z_b(10610)}=V_\sigma^{\mathrm{Direct}}-\tfrac{1}{2}V_\rho^{\mathrm{Direct}}+\tfrac{1}{2}V_\omega^{\mathrm{Direct}}+\tfrac{1}{4}\Big(-2V_\pi^{\mathrm{Cross}}+\tfrac{2}{3}V_\eta^{\mathrm{Cross}}-2V_\rho^{\mathrm{Corss}}+2V_\omega^{\mathrm{Cross}}\Big), \tag{8}$$

where the subpotentials from the π, η, σ, ρ and ω meson exchanges are written as

$$V_\pi^{\mathrm{Cross}}=-\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_2,m_2,r)+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_2,m_2,r)\bigg], \tag{9}$$

$$V_\eta^{\rm Cross} = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_3, m_3, r) + \frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger)T(\Lambda_3, m_3, r)\bigg], \tag{10}$$

$$V_\sigma^{\rm Direct} = -g_s^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\sigma, r), \tag{11}$$

$$V_\rho^{\rm Direct} = -\tfrac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\rho, r), \tag{12}$$

$$V_\rho^{\rm Cross} = 2\lambda^2 g_V^2[\tfrac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_0, m_0, r) - \tfrac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger)T(\Lambda_0, m_0, r)], \tag{13}$$

$$V_\omega^{\rm Direct} = -\tfrac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\omega, r), \tag{14}$$

$$V_\omega^{\rm Cross} = 2\lambda^2 g_V^2[\tfrac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_1, m_1, r) - \tfrac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger)T(\Lambda_1, m_1, r)]. \tag{15}$$

In the above expressions, we define

$$\Lambda_{0,1,2,3}^2 = \Lambda^2 - (m_{B^*} - m_B)^2,$$
$$m_{0,1,2,3}^2 = m_{\rho,\omega,\pi,\eta}^2 - (m_{B^*} - m_B)^2,$$

and $S(\hat{\boldsymbol{r}}, \mathbf{a}, \mathbf{b}) = 3(\hat{\boldsymbol{r}}\cdot\mathbf{a})(\hat{\boldsymbol{r}}\cdot\mathbf{b}) - \mathbf{a}\cdot\mathbf{b}$. Additionally, functions $Y(\Lambda, m, r)$, $Z(\Lambda, m, r)$, and $T(\Lambda, m, r)$ are defined as

$$Y(\Lambda, m_E, r) = \frac{1}{4\pi r}(e^{-m_E r} - e^{-\Lambda r}) - \frac{\Lambda^2 - m_E^2}{8\pi\Lambda}e^{-\Lambda r}, \tag{16}$$

$$Z(\Lambda, m_E, r) = \nabla^2 Y(\Lambda, m_E, r) = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}Y(\Lambda, m_E, r), \tag{17}$$

$$T(\Lambda, m_E, r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y(\Lambda, m_E, r). \tag{18}$$

We consider both S-wave and D-wave interactions between the B and $\bar{B}^*$ mesons. Finally the total effective potential can be obtained by making the replacement in the subpotentials

$$\left.\begin{matrix}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)\\(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)\end{matrix}\right\}\mapsto\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad {\rm S}(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_2, \boldsymbol{\epsilon}_3^\dagger)\mapsto\begin{pmatrix}0 & -\sqrt{2}\\ -\sqrt{2} & 1\end{pmatrix},$$

which results in the total effective potential of the $B\bar{B}^*$ system, i.e., a 2×2 matrix.

For the $Z_b(10650)$ system, the general expression of the total effective potential is

$$\mathcal{V}^{Z_b(10650)} = W_\sigma - \tfrac{1}{2}W_\rho + \tfrac{1}{2}W_\omega - \tfrac{1}{2}W_\pi + \tfrac{1}{6}W_\eta, \tag{19}$$

where the π, η, σ, ρ, and ω meson exchanges can contribute to the effective potentials. The corresponding subpotentials are expressed as

$$W_\pi = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\pi, r) + \frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\pi, r)\bigg], \tag{20}$$

$$W_\eta = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\eta, r) + \frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\eta, r)\bigg], \tag{21}$$

$$W_\sigma = -g_s^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\sigma, r), \tag{22}$$

$$W_\rho = -\tfrac{1}{4}\{2\beta^2 g_V^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\rho, r) - 8\lambda^2 g_V^2[\tfrac{2}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\rho, r) - \tfrac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\rho, r)]\}, \tag{23}$$

$$W_\omega = -\tfrac{1}{4}\{2\beta^2 g_V^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\omega, r) - 8\lambda^2 g_V^2[\tfrac{2}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\omega, r) - \tfrac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\omega, r)]\}. \tag{24}$$

In this work, we consider both S-wave and D-wave interactions between the B^* and $\bar{B}^*$ mesons. Thus, the total effective potential of the $Z_b(10650)$ with $J = 1$ is a 3×3 matrix, which can be obtained by replacing the corresponding terms in the subpotentials, i.e.,

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{25}$$

$$(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}, \tag{26}$$

$$S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)\mapsto\begin{pmatrix}0 & -\sqrt{2} & 0\\ -\sqrt{2} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{27}$$

With the obtained effective potentials, we can find the bound state solution by solving the coupled-channel Schrödinger equation. The kinetic terms for the $Z_b(10610)$ and $Z_b(10650)$ systems are

$$K_{Z_b(10610)} = {\rm diag}\bigg(-\frac{\triangle}{2\tilde{m}_1}, -\frac{\triangle_2}{2\tilde{m}_1}\bigg), \tag{28}$$

$$K_{Z_b(10650)} = {\rm diag}\bigg(-\frac{\triangle}{2\tilde{m}_2}, -\frac{\triangle_2}{2\tilde{m}_2}, -\frac{\triangle_2}{2\tilde{m}_2}\bigg), \tag{29}$$

respectively. Here, $\triangle = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $\triangle_2 = \triangle - \frac{6}{r^2}$. $\tilde{m}_1 = m_B m_{B^*}/(m_B + m_{B^*})$ and $\tilde{m}_2 = m_{B^*}/2$ are the reduced masses of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems, where m_B and m_{B^*}

TABLE I. The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for the $Z_b(10610)$ and $Z_b(10650)$ systems. Here, we discuss two situations, i.e., including all OME contribution and only considering OPE potential.

	OME			OPE		
State	Λ	E (MeV)	$r_{\rm RMS}$ (fm)	Λ	E (MeV)	$r_{\rm RMS}$ (fm)
$Z_b(10610)$	2.1	-0.22	3.05	2.2	-8.69	0.62
	2.3	-1.64	1.31	2.4	-20.29	0.47
	2.5	-4.74	0.84	2.6	-38.54	0.36
$Z_b(10650)$	2.2	-0.81	1.38	2	-2.17	1.15
	2.4	-3.31	0.95	2.2	-8.01	0.68
	2.6	-7.80	0.68	2.4	-19.00	0.48
	2.8	-14.94	0.52	2.6	-36.36	0.38

denote the masses of the pseudoscalar and vector bottom mesons [27], respectively. In this work, the FESSDE program [28,29] is adopted to produce the numerical values for the binding energy and the relevant root-mean-square r with the variation of the cutoff in the region of $0.8 \leq \Lambda \leq 5$ GeV. Moreover, we also use MATSCE [30], a MATLAB package for solving coupled-channel Schrödinger equations, to perform an independent cross-check.

Throughout this work, we will first present the numerical results of the obtained bound state solutions when all types of the one-meson-exchange (OME) potentials are included. The one-pion-exchange (OPE) force contributes to the long-range interaction between the heavy meson pair, which is clear and well known. In contrast, the scalar and vector meson exchanges are used to mimic the intermediate and short-range interaction between the heavy mesons, which are not determined very precisely. In order to find out whether the existence of the possible bound molecular states is sensitive to the details of the short-range interaction, we will also study the case when only the OPE contribution is considered. In the following illustration, we use OME and OPE to distinguish such two cases. If the OPE force alone is strong enough to form a loosely bound state, such a case is particularly interesting phenomenologically.

For the $Z_b(10610)$ state, we find the bound state solution with Λ around 2.2 GeV. Our result shows that $Z_b(10610)$ could be as a molecular state with a very shallow binding energy. Although to some extent its binding energy is still sensitive to Λ, the result obtained by the OBE model shows that it is possible to interpret $Z_b(10610)$ as a $B\bar{B}^*$ molecular state with isospin $I = 1$. For the $Z_b(10610)$, we need to decrease the Λ value to obtain the same binding energy as that from OME. The one pion meson exchange potential indeed plays a crucial role in the formation of the BB^* bound states. A loosely bound state also exists for $Z_b(10650)$ with Λ slightly above 2 GeV. Only considering the OPE potential, the obtained binding energy becomes deeper with the same Λ value (see Table I for the details of numerical result).

In summary, stimulated by the newly observed bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$, we have carried out a systematical study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ system using the OBE model in our work. We have considered both the S-wave and D-wave interaction between the $B^{(*)}$ and $\bar{B}^*$ mesons, which results in the mixing of the S-wave and D-wave contribution. Our numerical results indicate that the $Z_b(10610)$ and $Z_b(10650)$ signals can be interpreted as the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states with $I^G(J^P) = 1^+(1^+)$, respectively.

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