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Neutrino masses and the scalar sector of a $B-L$ extension of the standard model

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We consider an electroweak model based on the gauge symmetry $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ which has right-handed neutrinos with different exotic $B-L$ quantum numbers. Because of this particular feature we are able to write Yukawa terms, and right-handed neutrino mass terms, with scalar fields that can develop vacuum expectation values belonging to different energy scales. We make a detailed study of the scalar and the Yukawa neutrino sectors to show that this model is compatible with the observed solar and atmospheric neutrino mass scales and the tribimaximal mixing matrix. We also show that there are dark matter candidates if a Z_2 symmetry is included.

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I. INTRODUCTION

The neutrino masses and mixing, which are required for giving a consistent explanation for the solar and atmospheric neutrino anomalies, are the most firm evidence of physics beyond the electroweak standard model (ESM). New physics can be implemented in a variety of different scenarios. There are basically two main schemes that are often followed: (i) new matter content is added to the model respecting the original ESM gauge symmetry and (ii) to consider a model with a larger gauge symmetry. Certainly both schemes can be implemented together. In this vein, extensions of the ESM having an extra $U(1)$ gauge symmetry factor are interesting for a variety of reasons. They are the simplest way of extending the ESM gauge symmetry and can be thought of as an intermediate energy scale symmetry coming from the breaking, at a higher energy scale, of a larger gauge symmetry describing some yet unknown physics. For instance, $U(1)$ gauge factors are contained in grand unified theories, supersymmetric models, and left-right models. One major feature of these models is the existence of an extra neutral vector boson, usually denoted by Z', whose mass is related to the energy scale of the extra $U(1)$ symmetry spontaneously broken. It is expected to have Z' signals at the TeV scale and its discovery is one of the goals of the LHC and future lepton colliders. Depending on the implementation of this kind of model, it can have a natural candidate for dark matter (DM) and/or furnish a mechanism for leptogenesis. Through the years much work has been done considering the features of this extra $U(1)$ gauge factor and some particular formulations of the model were made. See, for example, Refs. [1,2]. In particular, when the charge of the extra $U(1)$ factor is identified with $B-L$ (baryon number minus lepton number), there is extensive literature concerning the most different versions of the model and a large variety of phenomenological aspects.

In this paper we consider a $B-L$ gauge model which has the particularity of being rendered anomaly free by introducing right-handed neutrinos with exotic $B-L$ charges. The number of right-handed neutrinos and their $B-L$ exotic charges is fixed by the anomaly cancellation equations. Since in this model not all of these right-handed neutrinos have the same exotic charge, we can construct Yukawa terms with different $SU(2)_L$ scalar doublets. Appropriate $SU(2)_L$ scalar singlets are also introduced to write the most general mass terms for the right-handed neutrinos. We make a detailed study of the scalar potential, concerning the mass spectra and the physical Goldstones, and take advantage of this rich scalar sector to construct a seesaw mechanism at low energies (TeV scale) to give realistic masses to the light active neutrinos.

The outline of this paper is as follows. In Sec. II we present the particular $B-L$ gauge model under consideration. In Sec. III we analyze the scalar potential of the model—the symmetries, the mass spectra, and the model compatibility with experimental constraints—and introduce a Z_2 symmetry to allow the model to have DM candidates. In Sec. IV we study the neutrino mass generation and show the compatibility of the model with the observed neutrino masses and the tribimaximal mixing. Finally, our conclusions are given in Sec. V.

II. THE MODEL

We consider the model of Ref. [3] that we briefly summarize here. The model is an extension of the ESM based on the gauge symmetry $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ where B and L are the usual baryonic and leptonic numbers, respectively, and Y' is a new charge. The values of Y' are chosen to obtain the ESM hypercharge Y through the relation $Y = [Y' + (B-L)]$, after the first spontaneous symmetry breaking. In order to make the model anomaly free we have to introduce right-handed neutrinos (n_R). Solving the anomaly equations we find that the number of n_R cannot be less than 3, if we restrict ourselves to integer

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TABLE I. Quantum number assignment for the fermionic fields.

	I_3	I	Q	Y'	$B-L$	Y
ν_{eL}	1/2	1/2	0	0	-1	-1
e_L	$-1/2$	1/2	-1	0	-1	-1
e_R	0	0	-1	-1	-1	-2
u_L	1/2	1/2	2/3	0	1/3	1/3
d_L	$-1/2$	1/2	$-1/3$	0	1/3	1/3
u_R	0	0	2/3	1	1/3	4/3
d_R	0	0	$-1/3$	-1	1/3	$-2/3$
n_{1R}	0	0	0	4	-4	0
n_{2R}	0	0	0	4	-4	0
n_{3R}	0	0	0	-5	5	0

quantum numbers only. For the minimal number (3) these equations have two solutions: the usual one, where all right-handed neutrinos are identical and have $L=1$, and the exotic one, where two of them have $L=4$ and the third one has $L=-5$. The model under consideration has the right-handed neutrinos having such exotic lepton numbers.

The fermionic content of the model is the same as the ESM plus the right-handed neutrinos introduced above. The respective charge assignment is shown in Table I. In the framework of a gauge theory with spontaneous symmetry breaking, we at least have to introduce a scalar doublet, H, in order to give mass to the lighter massive neutral vector boson (Z) and the charged fermions, as in the ESM. However, more scalar fields are needed to give mass to the extra neutral vector boson (Z'), which is expected to be heavier than Z, and to the neutrinos of the model. Respecting gauge invariance, a general choice is to introduce two $SU(2)$ scalar doublets, $\Phi_{1,2}$, and three $SU(2)$ neutral scalar singlets, $\phi_{1,2,3}$, with the charge assignments shown in Table II.

TABLE II. Quantum number assignment for the scalar fields.

	I_3	I	Q	Y'	$B-L$	Y
$H^{0,+}$	$\mp 1/2$	1/2	0, 1	1	0	1
$\Phi_1^{0,-}$	$\pm 1/2$	1/2	0, -1	-4	$+3$	-1
$\Phi_2^{0,-}$	$\pm 1/2$	1/2	0, -1	5	-6	-1
ϕ_1	0	0	0	-8	$+8$	0
ϕ_2	0	0	0	10	-10	0
ϕ_3	0	0	0	1	-1	0

With these fields, and omitting summation symbols, the most general Yukawa Lagrangian respecting the gauge invariance is given by

$$-\mathcal{L}_{\rm Y} = Y_i^{(l)}\bar{L}_{Li}e_{Ri}H + Y_{ij}^{(d)}\bar{Q}_{Li}d_{Rj}H + Y_{ij}^{(u)}\bar{Q}_{Li}u_{Rj}\tilde{H} + \mathcal{D}_{im}\bar{L}_{Li}n_{Rm}\Phi_1 + \mathcal{D}_{i3}\bar{L}_{Li}n_{R3}\Phi_2 + \mathcal{M}_{mn}\overline{(n_{Rm})^c}n_{Rn}\phi_1 + \mathcal{M}_{33}\overline{(n_{R3})^c}n_{R3}\phi_2 + \mathcal{M}_{m3}\overline{(n_{Rm})^c}n_{R3}\phi_3, +\text{H.c.}, \tag{1}$$

where $i, j = 1, 2, 3$ are lepton family numbers and represent e, μ, and τ, respectively, $m, n = 1, 2$, and $\tilde{H} = i\tau_2 H^*$. The corresponding scalar potential is

$$V_{B-L} = -\mu_H^2 H^\dagger H + \lambda_H(H^\dagger H)^2 - \mu_{11}^2\Phi_1^\dagger\Phi_1 + \lambda_{11}|\Phi_1^\dagger\Phi_1|^2 - \mu_{22}^2\Phi_2^\dagger\Phi_2 + \lambda_{22}|\Phi_2^\dagger\Phi_2|^2 - \mu_{s\alpha}^2|\phi_\alpha|^2 + \lambda_{s\alpha}|\phi_\alpha^*\phi_\alpha|^2 + \lambda_{12}|\Phi_1|^2|\Phi_2|^2 + \lambda'_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \Lambda_{H\gamma}|H|^2|\Phi_\gamma|^2 + \Lambda'_{H\gamma}(H^\dagger\Phi_\gamma)(\Phi_\gamma^\dagger H) + \Lambda_{Hs\alpha}|H|^2|\phi_\alpha|^2 + \Lambda'_{\gamma\alpha}|\Phi_\gamma|^2|\phi_\alpha|^2 + [\Phi_1^\dagger\Phi_2(\beta_{13}\phi_1\phi_3^* + \beta_{23}\phi_2^*\phi_3) + \beta_{123}\phi_1\phi_2(\phi_3^*)^2 + \text{H.c.}] + \Delta_{\alpha\beta}(\phi_\alpha^*\phi_\alpha)(\phi_\beta^*\phi_\beta), \tag{2}$$

where $\gamma = 1, 2$; $\alpha, \beta = 1, 2, 3$; and $\alpha < \beta$ in the last term. In $\mathcal{L}_{\rm Y}$, the motivation for introducing such scalar fields is to write the most general neutrino mass terms. Because of the fact that not all right-handed neutrinos have the same Y' and $(B-L)$ charges, the neutrino mass matrix will have entries proportional to vacuum expectation values (VEVs) which can, in principle, belong to different energy scales. The scalar potential is a consequence of the fields we have previously introduced, and the terms in Eq. (2) are only dictated by gauge invariance. Now, we have to observe that when we write terms based on general grounds, although correct, we may have introduced more symmetries than we need. Hence, we must do a detailed study of the scalar potential, and know the scalar mass spectra in order to avoid inconsistencies with the present phenomenology. The scalar doublets of the model, H and $\Phi_{1,2}$, contribute to the Z boson mass, so their vacuum expectation values are bounded by the electroweak energy scale. Hence, the largest energy scale of the model comes from the $SU(2)$ scalar singlets $\phi_{1,2,3}$. In this way, the pattern of the spontaneous symmetry breaking is

$$SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L} \xrightarrow{\langle\phi_{1,2,3}\rangle} SU(2)_L \otimes U(1)_Y \xrightarrow{\langle H,\Phi_{1,2}\rangle} U(1)_{em}. \tag{3}$$

III. THE SCALAR POTENTIAL ANALYSIS

Now, we focus on the analysis of the V_{B-L} scalar potential given in Eq. (2) when all neutral scalar fields develop nonvanishing VEVs, with the usual shifting $\varphi^0 = \frac{1}{\sqrt{2}}(V_\varphi + \text{Re}\varphi + i\,\text{Im}\varphi)$. By using standard procedures we are able to find the constraint equations coming from the linear terms in the scalar potential after the symmetry breaking. See the appendix. In the same way, we can construct the mass-squared matrices for the charged, real, and imaginary scalar fields. We start looking at the mass-squared matrix for the charged fields. It is a complete 3×3 symmetric matrix in the basis $(H^+, \Phi_1^+, \Phi_2^+)$ that can be easily diagonalized and, after taking into account the constraint equations, gives the following mass spectrum: two charged Goldstone bosons

$$G_W^\pm = \frac{1}{\sqrt{1 + \frac{V_H^2}{V_{\Phi_2}^2} + \frac{V_{\Phi_1}^2}{V_{\Phi_2}^2}}}\left(-\frac{V_H}{V_{\Phi_2}} H^\pm + \frac{V_{\Phi_1}}{V_{\Phi_2}} \Phi_1^\pm + \Phi_2^\pm\right), \quad (4)$$

and two massive states whose expressions we are not showing by shortness. The fields $G_W^\pm$ will be absorbed to form the longitudinal components of the charged massive vector bosons $W^\pm$. The other two physical states remain in the spectrum and are a prediction of the model. Later in the paper we approach numerically all the mass spectra in some different situations.

In the neutral imaginary scalar sector we have a 6×6 mass-squared matrix that, after the diagonalization procedure, shows two massive scalar and four massless fields. Two of them will become the longitudinal components of the Z and the Z' neutral vector bosons. The other two massless states remain in the physical spectrum. We show the two physical Goldstone bosons only in the limit where $V_{\phi_{1,2,3}} \gg V_H, V_{\Phi_{1,2}}$ and they are given by

$$G_{F_1}^0 \approx \sqrt{\frac{V_{\Phi_1}^2 + V_{\Phi_2}^2}{V_H^2 + V_{\Phi_1}^2 + V_{\Phi_2}^2}}\left(\mathrm{Im}H^0 + \frac{V_H V_{\Phi_1}}{V_{\Phi_1}^2 + V_{\Phi_2}^2}\mathrm{Im}\Phi_1^0 + \frac{V_H V_{\Phi_2}}{V_{\Phi_1}^2 + V_{\Phi_2}^2}\mathrm{Im}\Phi_2^0\right), \quad (5)$$

$$G_{F_2}^0 \approx \frac{1}{\sqrt{110}}(7\,\mathrm{Im}\phi_1 + 5\,\mathrm{Im}\phi_2 + 6\,\mathrm{Im}\phi_3). \quad (6)$$

From the expressions above we see that $G_{F_{1,2}}^0$ are mainly doublet and singlet, respectively. This fact will be analyzed later on.

For the neutral real scalar sector we have a symmetric 6×6 mass-squared matrix with a nonvanishing determinant. Hence the spectrum will not contain massless states. We will not show analytical expressions here but we will give numerical values below.

The number of Goldstone bosons we have found by doing explicit calculations can be easily understood by studying the global symmetries of the scalar potential before and after the spontaneous symmetry breaking (SSB). Before the SSB, the global symmetries of the scalar potential are (a) $SU(2)$ acting on H and $\Phi_{1,2}$ doublets, (b) $U(1)$ acting on H with charge $+1$, (c) $U(1)$ acting on $\Phi_{1,2}$ with charge $+1$, and (d) two independent $U(1)_{\beta,\gamma}$ transformations acting on the fields Φ_1, Φ_2, ϕ_1, ϕ_2, ϕ_3 with charges $(\frac{1}{2}, -\frac{1}{2}, 1, -1, 0)$ and $(-\frac{1}{2}, \frac{1}{2}, 0, +2, 1)$, respectively. After the SSB the global symmetries of the scalar potential are reduced to a single $U(1)_\alpha$ acting on the charged components of the doublets, $H^\pm$ and $\Phi_{1,2}^\pm$, with charges $(\pm 1, \pm 1)$. Following the Goldstone theorem, the number of Goldstone bosons is equal to the number of broken symmetry generators. In this case the original symmetry has $(3 + 4 \times 1) = 7$ generators and the remaining symmetry has 1. Then we must have 6 Goldstone bosons, which is exactly the number we have found just above: two charged and four neutral imaginary fields.

Notice that the scalar potential given in Eq. (2) corrects the one given in Eq. (16) of Ref. [3] in which the terms proportional to $\Lambda'_{H\gamma}$ are missing. The lack of these terms alters the global symmetries under which the scalar potential is invariant and, consequently, the number of Goldstone bosons in the spectra. In that case the symmetries before the SSB are (a) $O(4)$ acting on the four components of H, (b) $SU(2)$ acting on $\Phi_{1,2}$, (c) $U(1)$ acting on $\Phi_{1,2}$, (d) the two $U(1)_{\beta,\gamma}$ defined above. After the SSB the remaining symmetries are (i) $O(3)$ acting on the components $(\mathrm{Im}H^0, \mathrm{Re}H^+, \mathrm{Im}H^+)$ and (ii) $U(1)$ acting on $\Phi_{1,2}^\pm$ with charge ± 1. Therefore, we are left with $(6+3+3\times 1)-(3+1)=12-4=8$ Goldstone bosons. The same result is obtained by doing explicit calculations. From the mass-squared matrices we find that the number of Goldstone bosons is the expected one and also that in the charged sector we are left with four massless states given by

$$G_W^\pm = H^\mp, \quad (7)$$

$$G_C^\pm = \frac{1}{\sqrt{V_{\Phi_1}^2 + V_{\Phi_2}^2}}(V_{\Phi_1}\Phi_1^\pm + V_{\Phi_2}\Phi_2^\pm). \quad (8)$$

Hence, this result is in conflict with the present phenomenology since there are two extra charged massless scalars in the spectrum.

Now, we return to our present analysis. Since our analysis of the scalar potential shows the existence of two physical Goldstone bosons, it is time to care about the safety of the model. Before that, some remarks about the VEVs of the model are due. The $V_{\phi_{1,2,3}}$ are the largest energy scale of the model. The main contribution to the Z boson square mass comes from the doublets so that $(V_H^2 + V_{\Phi_1}^2 + V_{\Phi_2}^2) = V_{\mathrm{ESM}}^2 = (246)^2\ \mathrm{GeV}^2$. The doublet H is the one that couples to quarks and to charged leptons via Yukawa interactions, and hence, V_H must be close to V_{ESM} to give the correct tree level mass to the quark top, as the ESM do, for an $\mathcal{O}(1)$ top Yukawa coupling. We then conclude that $V_{\Phi_1}^2 + V_{\Phi_2}^2 \ll V_H^2$.

The major challenge to models with physical Goldstone bosons, also called Majorons (J), comes from the energy loss in stars through the processes $\gamma + e^- \to e^- + J$. This process is used to put limits on the $\bar{e}eJ$ coupling, and it is found that it has to be $g_{eeJ} \leq 10^{-10}$ for the Sun, and $g_{eeJ} \leq 10^{-12}$ for the red-giant stars [4]. However, the dynamics of the red giants has not the same level of confidence as that of the Sun, and this fact considerably weakens the second constraint.

The physical Goldstone $G^0_{F_2}$ has components only in the $SU(2)$ singlets $\phi_{1,2,3}$, which couple only to right-handed neutrinos. Therefore, it is safe since there is no tree level contribution to the energy loss process. The case for $G^0_{F_1}$ is not that simple. $G^0_{F_1}$ has a component in the ESM-like doublet H, and it contributes to $\bar{e}eJ$ through $\mathrm{Im}H^0$. The components in $\Phi_{1,2}$, which couple only to neutrinos at the tree level, pose no problem. Since in this case symmetry eigenstates and mass eigenstates are connected by orthogonal matrices, from Eq. (6) we find

$$\mathrm{Im}\,H^0 \approx \sqrt{\frac{V^2_{\Phi_1} + V^2_{\Phi_2}}{V^2_H + V^2_{\Phi_1} + V^2_{\Phi_2}}} G^0_{F_1} + \ldots, \tag{9}$$

and, hence,

$$g_{eeJ} \approx \frac{Y_e}{\sqrt{2}} \sqrt{\frac{V^2_{\Phi_1} + V^2_{\Phi_2}}{V^2_H + V^2_{\Phi_1} + V^2_{\Phi_2}}} = \left(\frac{\sqrt{2}m_e}{V_H}\right)\left(\frac{V_\Phi}{\sqrt{2}V_H}\right)$$
$$\approx 2 \times 10^{-6} \frac{V_\Phi}{V_H} \leq 10^{-12} - 10^{-10}, \tag{10}$$

where Y_e is the electron Yukawa coupling to the doublet H, we have defined $V^2_{\Phi_1} + V^2_{\Phi_2} \equiv V^2_\Phi$, and we have used the shift $H^0 \rightarrow \frac{1}{\sqrt{2}}(V_H + \mathrm{Re}H^0 + i\mathrm{Im}H^0)$. From the equation above we conclude that the VEVs of the $SU(2)$ doublets $\Phi_{1,2}$ must be less than 12 MeV to satisfy the Sun constraint or less than 120 KeV to satisfy rigorously the red-giant constraint. Let us adopt for practical purposes an intermediate scale: $V_\Phi = 1$ MeV.

Once we have established the energy scale of the VEVs of the model, and verified its safety up to now, we can make an exemplary study of the full scalar mass spectra. We will do it numerically since the excessive length of the analytical expressions make them useless.

In order to compute the masses we consider a set of parameters: the dimensional ones $V_H = 246$, $V_{\Phi_{1,2}} = 0.001$, $V_{\phi_{1,2,3}} = 1000$, in GeV, and $\lambda_H = 0.2$, $\lambda_{11,22} = \Lambda'_{13,21,22,H1,H2} = \lambda_{s\alpha} = 1$, $\Lambda_{H1,H2} = \Lambda_{Hs\alpha} = \Lambda'_{11,12,23}$, $\Delta_{12,13,23} = 0.1$, $\beta_{123} = -0.8$ which are dimensionless, for $\alpha = 1, 2, 3$. Note that the values for the μ parameters are found by solving the constraint equations for the scalar potential: $\mu^2_H = (402)^2$, $\mu^2_{11} = -(630)^2$, $\mu^2_{22} = (230)^2$, $\mu^2_{s1} = \mu^2_{s2} = (838)^2$, $\mu^2_{s3} = (550)^2$, in GeV^2. We also use $g_{Y'} = g_{B-L} = 0.4885$, and $g = 0.6298$, where $g_{Y'}$, g_{B-L}, and g are the coupling constants of the $U(1)_{Y'}$, $U(1)_{B-L}$, and $SU(2)_L$ gauge factors, respectively, which are related to the electric charge through $1/e^2 = 1/g^2_{Y'} + 1/g^2_{B-L} + 1/g^2$ [3].

The charged scalar sector gives the masses $m_{C_j} = (1424.9, 173.9, 0)$, also in GeV, where the massless complex field is responsible for the longitudinal components of the charged vector bosons W^+ and W^-.

In the imaginary neutral scalar sector we have a 6×6 square mass matrix and after diagonalization we find, in GeV,$m_{I_i} = (1549.2, 1414.13, 0, 0, 0, 0)$. Notice the correct number of the massless field: two are absorbed to form the longitudinal component of the neutral vector bosons of the model, Z and Z', and the other two are the physical Goldstone bosons $G^0_{F_{1,2}}$, as discussed above.

In the real neutral scalar sector, in the same way, we find $m_{R_i} = (1743.9, 1643.2, 1414.2, 1029.8, 150.0, 0.0014)$, in GeV. We have found a very light scalar of about 1.4 MeV which poses a new challenge to the model: the Z invisible decay width. The presence of such a light scalar field, say, R, and the $G^0_{F_1} \equiv J$ zero mass state, allows the decay $Z \rightarrow RJ \rightarrow JJJ$, which will contribute to the Z invisible decay width as half of the decay $Z \rightarrow \bar{\nu}\nu$, for a single flavor family [5]. According to the experimental data there is no room for such an extra contribution [6].

The light scalar we found above is not the result of a particular choice of the input parameters, as it could be thought at the first moment. Let us provide a qualitative but convincing argument. As was observed in Ref. [5], the reason is as follows. We have mentioned above that, before the SSB, the scalar potential has a $U(1)$ global symmetry acting on each of the $\Phi_{1,2}$ doublets, say, Φ. This means that we can rotate freely in the $\mathrm{Re}\Phi^0$-$\mathrm{Im}\Phi^0$ plane, so that as long as this $U(1)$ symmetry holds, the fields $\mathrm{Re}\Phi^0$ and $\mathrm{Im}\Phi^0$ are mass degenerate. However, this symmetry is broken when the real neutral component acquires a nonvanishing VEV and, hence, the fields are no longer mass degenerate. The square mass difference must be, then, of the order of the square of the energy scale responsible for breaking the symmetry, i.e., $m^2_{\mathrm{Re}\Phi^0} - m^2_{\mathrm{Im}\Phi^0} = \mathcal{O}(V^2_\Phi)$. When $\mathrm{Im}\Phi^0$ is a Goldstone, $m^2_{\mathrm{Im}\Phi^0} = 0$, we are left with $m^2_{\mathrm{Re}\Phi^0} = \mathcal{O}(V^2_\Phi)$, which, in our case, it is a very light scalar since V_Φ must be of the order of 1 MeV, in order to be consistent with the star energy loss data. Then, we must find a way to reconcile the present model with the experimental constraints.

Some attempts can be made to remove such inconsistency. Since the origin of the problem is in the breaking of the $U(1)$ symmetry acting on the doublets $\Phi_{1,2}$, let us consider the situation where $V_{\Phi_2} = 0$, and all other VEVs are different from zero. In this case we find the same number of neutral Goldstone bosons (4): two would be Goldstone bosons and two physical ones $G^0_{F_{1,2}}$. $G^0_{F_2}$ is given by the same expression as in Eq. (6), and

$$G^0_{F_1} \approx \frac{1}{\sqrt{V^2_H + V^2_{\Phi_1}}}(V_{\Phi_1}\mathrm{Im}H^0 + V_H \mathrm{Im}\Phi^0_1). \tag{11}$$

We also find that for the same input parameters, but now providing an input value for $\mu^2_{22} = (230)^2$ GeV^2, the mass spectra are practically not affected and we still have a light real scalar whose mass is about 1 MeV $\sim\mathcal{O}(V_{\Phi_1})$. We get the same conclusion if we consider $V_{\Phi_1} = 0$ and $V_{\Phi_2} \neq 0$. We only have to do the replacement $\Phi_1 \leftrightarrow \Phi_2$ in the above results.

As the problem persists, let us now consider the case where $V_{\Phi_1} = 0$ and $V_{\Phi_2} = 0$. In this case, the number of Goldstone bosons is reduced to 3. There is only one physical Goldstone, the $G^0_{F_2}$ given in Eq. (6), which is safe, as discussed above. The mass spectra are now considerably affected. For the same input parameters as above, and with $\mu^2_{11} = -(800)^2$, $\mu^2_{22} = (230)^2$, in GeV^2, the spectra, with all the masses in GeV, are the following. For the charged scalars we have $m_{C_j} = (1469.4, 380.1, 0)$. For the imaginary scalars we find $m_{I_i} = (1549.2, 1459.1, 337.9, 0, 0, 0)$, and for the real scalars $m_{R_i} = (1743.9, 1643.2, 1459.1, 1029.8, 337.8, 150.0)$. As we can see, there is no a light real scalar anymore. The lighter real scalar is heavier than the Z vector boson, so that the problematic decay $Z \to RJ$ is kinematically forbidden. Then, we have succeed in making the model safe. However, this solution is not satisfactory since the choice we have made for the doublet VEVs ($V_{\Phi_{1,2}} = 0$) does not allow the light neutrinos to get mass. It is easy to see that in this case there is a remaining $U(1)$ quantum symmetry, say, $U(1)_\zeta$, protecting the neutrino mass generation at any level. A possible ζ-charge assignment is $\zeta(\nu_{eL}, e_L, e_R, \Phi_{1,2}) = -1$, $\zeta(u_L, d_L, u_R, d_R) = 1/3$, and $\zeta(n_{(1,2,3)R}, \phi_{1,2,3}) = 0$. In order to make the model compatible with the experimental data and, hence, with massive neutrinos, we have to look for a new kind of solution since the symmetry breaking pattern above is not realistic.

Before continuing the search for a satisfactory solution, let us observe that before the SSB the model has a Z_2 exact symmetry with the transformation rules $Z_2(n_{R3}) = -n_{R3}$, $Z_2(\Phi_2) = -\Phi_2$, $Z_2(\phi_3) = -\phi_3$, and all the other fields being even under Z_2. It is interesting to preserve this symmetry after the SSB if we are looking for DM candidates. This is true when $V_{\Phi_2} = V_{\phi_3} = 0$. In this case the Z_2 symmetry is not spontaneously broken, and a mechanism similar to that of Ref. [7] can be implemented. The number of Goldstone bosons is 4, and the physical ones are given by the following: $G^0_{F_1}$ is given by the same expression in Eq. (11), and

$$G^0_{F_2} = \frac{1}{\sqrt{16V^2_{\phi_1} + 25V^2_{\phi_2}}}(5V_{\phi_2}\mathrm{Im}\phi_1 + 4V_{\phi_1}\mathrm{Im}\phi_2). \quad (12)$$

However, as we already know, there is a very light real scalar that, together with $G^0_{F_1}$, has severe implications on the Z invisible decay width. We will come to this Z_2 picture later on.

A. The solution

With the aim of constructing a consistent model, let us introduce a new $SU(2)$ neutral scalar singlet ϕ_X with the quantum numbers $Y' = -(B-L) = 3$. The Yukawa Lagrangian remains as in Eq. (1), but to the scalar potential in Eq. (2), besides extending the range of the indices to $\alpha, \beta = 1, 2, 3, X$, we have to add the following non-Hermitian terms,

$$V^X_{B-L} = -i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X - i\kappa_{H2X}(\Phi_2^T\tau_2 H)(\phi^*_X)^2 + \beta_X(\phi^*_X\phi_1)(\phi_2\phi_3) + \beta_{3X}(\phi^*_X\phi^3_3) + \mathrm{H.c.}, \quad (13)$$

in order to account for all the gauge invariant terms after the introduction of ϕ_X. The terms above reduce the number of global symmetries of the scalar potential, so that changes in the scalar spectra are expected.

Before the SSB, the global symmetries of the total scalar potential are (a) $SU(2)$ acting on H and $\Phi_{1,2}$ doublets, (b) $U(1)_\alpha$ acting on H and $\Phi_{1,2}$, and (c) $U(1)_\beta$ acting on the fields H, Φ_1, Φ_2, ϕ_1, ϕ_2, ϕ_3, ϕ_X with charges $(\frac{3}{8}, 0, -\frac{9}{8}, 1, -\frac{5}{4}, -\frac{1}{8}, -\frac{3}{8})$, respectively. After the SSB the global symmetries of the scalar potential are reduced to a single $U(1)$ acting on the charged components of the doublets, $H^\pm$ and $\Phi^\pm_{1,2}$, with charges $(\pm 1, \pm 1)$. The total number of Goldstone bosons will be given by the number of broken generators, i.e., $5 - 1 = 4$, which is the number of massless fields needed to form the longitudinal components of the charged (W^+, W^-) and neutral vector bosons (Z, Z'). In this case, there are no physical Goldstone bosons at all. It means that the inclusion of the $SU(2)$ scalar singlet ϕ_X has removed all physical massless states from the spectrum, and we have succeeded in finding a solution for the safety of the model.

Now, numerical applications require expanding the input parameters set to account for the new ones related to ϕ_X. We then choose $V_{\phi_X} = 1000$, and $\kappa_{H1X} = 0.01$ in GeV, and the dimensionless $\lambda_{sX} = \Lambda'_{1X} = 1$, $\Lambda_{HsX} = \Lambda'_{2X} = \Delta_{1X} = \Delta_{2X} = \Delta_{3X} = \beta_{3X} = 0.1$, $\beta_X = -0.6$, and $\kappa_{H2X} = 0.001$. As before the μ parameters are found by solving the constraint equations given in the appendix. With the above parameter set, plus the one we have used previously, we find $m_{C_j} = (11\,137.3, 1661.7, 0)$ for the charged scalar sector, $m_{I_i} = (11\,135.9, 1652.6, 1467.0, 973.6, 0.002, 0, 0)$ for the neutral imaginary sector, and $m_{R_i} = (11\,135.9, 1927.6, 1816.6, 1652.7, 1508.8, 900.5, 146.2)$ for the real scalar sector, in GeV. Notice that we have now a very light pseudoscalar, which has components mainly in the $SU(2)$ singlet fields $\phi_{1,2,3,X}$. For instance, its component in $\mathrm{Im}H^0$ is 7.3×10^{-12}, which implies $g_{eeJ} \approx 10^{-18}$. Hence, it is compatible with the astrophysical constraint, and poses no problem to the Z invisible decay width, since all the real scalar fields are heavier than the Z boson.

In this case the introduction of the ϕ_X scalar provides the right elements to make the model safe. Moreover, concerning the neutrino mass generation, from the Yukawa terms in Eq. (1) we are able to construct the most general neutrino mass matrix, since now all VEVs are different from zero.

B. A Z_2 symmetry and dark matter

Now let us consider the Z_2 symmetry again, after the introduction of the scalar ϕ_X. We had the field symmetry transformation rules

$$Z_2(n_{R3}) = -n_{R3}, \quad Z_2(\Phi_2) = -\Phi_2, \quad Z_2(\phi_3) = -\phi_3, \quad (14)$$

and all the other being even. It is easy to see that all the Hermitian terms in the scalar potential involving ϕ_X are invariant under Z_2. However, the non-Hermitian terms

$$-i\kappa_{H2X}(\Phi_2^T\tau_2 H)(\phi_X^*)^2, \qquad \beta_X(\phi_X^*\phi_1)(\phi_2\phi_3), \quad \text{and} \quad \beta_{3X}(\phi_X^*\phi_3^3), \quad (15)$$

in V_{B-L}^X, are not invariant. We could change the ϕ_X transformation rule to odd, in order to have some of them invariant. In this case, however, if we want to keep the Lagrangian invariant under Z_2 after the SSB, we must have $V_{\phi_X} = 0$, and this is not an option since we need $V_{\phi_X} \neq 0$ to have a consistent model, as discussed above. Motivated by the possibility of having DM candidates we impose the Z_2 symmetry to the entire Lagrangian. Then, the terms in Eq. (15) will be removed from the scalar potential and the only non-Hermitian terms allowed are

$$V_{B-L}^{NH} = \Phi_1^\dagger\Phi_2(\beta_{13}\phi_1\phi_3^* + \beta_{23}\phi_2^*\phi_3) + \beta_{123}\phi_1\phi_2(\phi_3^*)^2 - i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X + \text{H.c.} \quad (16)$$

After the SSB, the Z_2 symmetry is not broken if we have $V_{\Phi_2} = V_{\phi_3} = 0$, and we have mass eigenstates that are also eigenstates of this symmetry. However, we know from our previous analysis, before introducing ϕ_X, that this vacuum configuration challenges the safety of the model due to a physical Goldstone and a light real scalar. Now, after introducing ϕ_X we have four massless states in the neutral imaginary sector. However, in this case, both of the physical massless states are mainly singlets: $G_{F_2}^0$ is given by the same expression in Eq. (12), and

$$G_{F_1}^0 \approx \frac{1}{\sqrt{7093}}(12\,\text{Im}\phi_1 - 15\,\text{Im}\phi_2 + 82\,\text{Im}\phi_X). \quad (17)$$

In fact, for the parameter set we used above, the $G_{F_1}^0$ component in $\text{Im}H^0$ is $\approx 2\times10^{-12}$ which implies $g_{eeJ} \approx 4\times10^{-18}$; thus it is safe with respect to the star energy loss constraint. This main feature is due to the introduction of the trilinear term $-i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X$. Qualitative arguments to explain this behavior can be given. The number of $U(1)$ symmetries is the same in both situations, with and without ϕ_X, 4. Without ϕ_X, we have two independent $U(1)$ symmetries involving only the doublets, say, $U(1)_\sigma$ acting on H, and $U(1)_\alpha$ acting on $\Phi_{1,2}$. With ϕ_X, the trilinear term relates the $U(1)$ charge of H to that of Φ_1, reducing the number of $U(1)$ symmetries involving only doublets to just one, say, $U(1)_\alpha$ acting on H and $\Phi_{1,2}$, and at the same time, it introduces a new $U(1)$ symmetry acting on H and ϕ_X, say, $U(1)_\gamma$. The number of broken generators is the same in both situations, since we have the same number of massless states; however, the origin of these physical massless states is different. The introduction of ϕ_X works in a very similar way to the singlet introduced in Refs. [8,9] to form the terms $H_u^T\tau_2 H_d\phi$ and $H_u^T\tau_2 H_d\phi^2$, respectively, in order to make the axion invisible.

The numerical spectra for the different scalar sectors, in GeV, are $m_{C_j} = (1489.9, 1330.3, 0)$ for the charged scalar sector; $m_{I_i} = (1479.7, 1433.9, 1318.9, 0, 0, 0, 0)$ for the neutral imaginary sector, and $m_{R_i} = (1483.5, 1479.7, 1378.4, 1378.4, 1318.9, 675.3, 152.9)$ for the real scalar sector. The point here is that all real scalar fields are now heavier than the Z boson, avoiding in this way the Z invisible decay width constraint. Therefore, the model is safe from these most severe constraints.

As the Z_2 symmetry still holds after the SSB, due to this particular vacuum configuration, the model can present some DM candidates. In general, a candidate must be the lightest particle odd under Z_2, in order to be stable. In our case, it can be the lightest odd mass eigenstate of the n_{R3} or the lightest odd imaginary mass eigenstate, or its odd real counterpart. This subject will be considered elsewhere. Here we only estimate the relic abundance and the direct detection cross section for the case of the fermionic cold DM candidate n_{R3} (referred as χ from now on).

The most relevant annihilation process of χ occurs via the t-channel exchange of $\Phi_2^\pm$ (Φ_2^0) to charged (neutral) leptons' final states. The thermally averaged χ annihilation cross section, $\langle\sigma v\rangle$, is given by [10]

$$\langle\sigma v\rangle \approx a + b\langle v^2\rangle \approx \frac{1}{16\pi}\sum_{ij} G_{\text{eff},ij}^2 c_c M_\chi^2\langle v^2\rangle, \quad (18)$$

where $i, j = e, \mu, \tau$ and c_c are the color factors, equal to 1 for leptons. Also we have neglected the lepton masses. The $G_{\text{eff},ij} = D_{i3}D_{j3}^*/(m_{C_1,R_2}^2 + M_\chi^2)$ are the effective couplings, where $m_{C_1} \approx 1489$ GeV ($m_{R_2} \approx 1489$ GeV), the mass to be considered when charged (neutral) leptons are produced. The relic abundance of χ is approximately given by [11]

$$\Omega h^2 \approx \frac{1.04\times10^{-9}x_f}{M_{\text{Pl}}\sqrt{g_*}(a + 3b/x_f)}, \quad (19)$$

where, in this model, $g_* = 107.75$ is the number of relativistic degrees of freedom available at the freeze-out temperature, T_f, and $x_f = M_\chi/T_f$ is given by

$$x_f = \ln\left[c\sqrt{\frac{45}{8}}\frac{g_\chi M_{\text{Pl}}M_\chi(a + 6b/x_f)}{2\pi^3\sqrt{x_f g_*}}\right], \quad (20)$$

with $c = 5/4$ and $g_\chi = 2$. Using the following set of parameters, $D_{e3} = 0.06$, $D_{\mu3} = 0.9$, $D_{e3} = 1$, and for $M_\chi = 750$ GeV, we find $x_f = 24.81$ and $\Omega h^2 = 0.11$, which is in agreement with the experimental bounds [12]. The same interaction allowing the χ annihilation in charged leptons, which are proportional D_{i3}, also induces lepton flavor violation (LFV) such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$ (see below).

The next task is to compute the direct detection cross section. In our case, the elastic scattering of χ with nuclei occurs via the t-channel $\chi + N \to \chi + N$ process due to the exchange of the scalar mass eigenstate R_7, which is the Higgs scalar boson in the model with mass $m_{R_7} \approx$ 152.9 GeV. The spin-independent cross section is given by [13]

$$\sigma_{\chi N} = \frac{4}{\pi}\frac{M_\chi^2 m_N^2}{(M_\chi + m_N)^2}[Zf_p + (A - Z)f_n]^2, \tag{21}$$

where the effective couplings to protons and neutrons, $f_{p,n}$, are

$$f_{p,n} = \sum_{q=u,d,s}\frac{G_{\text{eff},q}}{\sqrt{2}}f_{Tq}^{(p,n)}\frac{m_{p,n}}{m_q} + \frac{2}{27}f_{TG}^{(p,n)}\sum_{q=c,b,t}\frac{G_{\text{eff},q}}{\sqrt{2}}\frac{m_{p,n}}{m_q}. \tag{22}$$

In this case $G_{\text{eff},q} = G_0 \times m_q \equiv [C_{\phi_2 R_7}C_{HR_7}M_\chi/(V_{\phi_2}V_H m_{R_7}^2)] \times m_q$, where $C_{\phi_2 R_7} \approx 0.01$ and $C_{HR_7} \approx 0.99$ are the coefficients relating the symmetry eigenstates (ϕ_2, H) to the relevant mass eigenstate R_7, respectively. By using $f_{Tq}^{(p,n)}$ and $f_{TG}^{(p,n)}$ given in Ref. [14] we find

$$\sigma_{\chi,p} \approx 3 \times 10^{-7}\ \text{pb} \times \left[\frac{G_{\text{eff},q} \times (1\ \text{GeV}/m_q)}{10^{-7}\ \text{GeV}^{-2}}\right]^2, \tag{23}$$

which gives $\sigma_{\chi,p} \approx 4.74 \times 10^{-11}$ pb, for $M_\chi = 750$ GeV, which is in agreement with the most recent present bounds [15–17]. The parameter set we have used in all the cases above is compatible with the following requirements: (i) the constraint equations are satisfied, (ii) all obtained masses are $m^2 > 0$, and (iii) results for the already known fields are consistent with those of the SM at the tree level.

Notice that, although we are considering a multi-Higgs model, the values we have found for the lightest real scalar, the Higgs boson, are in agreement with the last combined CDF and D0 results for the ESM Higgs boson, which have excluded, at the 95% C.L., a region at high mass in $158 < m_H < 175$ GeV [18].

IV. NEUTRINO MASSES

The model without the Z_2 symmetry already has a satisfactory solution to the neutrino masses, since we are able to construct a general neutrino mass matrix. However, we are going to consider the case with this symmetry because the model becomes more attractive due to the presence of stable candidates to DM.

The Yukawa Lagrangian in Eq. (1) gives the following neutrino mass terms:

$$-\mathcal{L}_{m_\nu} = \mathcal{D}_{im}\overline{\nu_{Li}}n_{Rm}V_{\Phi_1} + \mathcal{M}_{mn}\overline{(n_m^c)_L}n_{Rn}V_{\phi_1} + \mathcal{M}_{33}\overline{(n_3^c)_L}n_{R3}V_{\phi_2} + \text{H.c.}, \tag{24}$$

where $i, j = 1, 2, 3$ (or e, μ, τ, respectively, when convenient) and $m, n = 1, 2$. In matrix form Eq. (24) reads

$$-\mathcal{L}_{m_\nu} = \begin{bmatrix}\overline{\nu_L} & \overline{(n^c)_L}\end{bmatrix}\begin{bmatrix}0 & M_D\\ M_D^T & M_M\end{bmatrix}\begin{bmatrix}(\nu^c)_R\\ n_R\end{bmatrix}, \tag{25}$$

with

$$\nu_L = [\nu_e\nu_\mu\nu_\tau]_L^T, \qquad n_R = [n_1 n_2 n_3]_R^T. \tag{26}$$

The Majorana mass matrix (M_M) and the Dirac mass matrix (M_D) are given by

$$M_M = V_{\phi_1}\begin{pmatrix}\mathcal{M}_{11} & \mathcal{M}_{12} & 0\\ \mathcal{M}_{12} & \mathcal{M}_{22} & 0\\ 0 & 0 & \frac{V_{\phi_2}}{V_{\phi_1}}\mathcal{M}_{33}\end{pmatrix},\quad M_D = V_{\Phi_1}\begin{pmatrix}\mathcal{D}_{11} & \mathcal{D}_{12} & 0\\ \mathcal{D}_{21} & \mathcal{D}_{22} & 0\\ \mathcal{D}_{31} & \mathcal{D}_{32} & 0\end{pmatrix}, \tag{27}$$

since $V_{\Phi_2} = V_{\phi_3} = 0$, where $M_M = M_M^T$. For $V_{\Phi_1} \ll V_{\phi_1}$, the mass matrix in Eq. (25) can be diagonalized by an approximate scheme. The masses of the heavy neutrinos are related to the energy scale of the VEVs of the singlets ϕ_1 and ϕ_2, and are given by the eigenvalues of M_M:

$$M_{1,2} = \frac{(\mathcal{M}_{11} + \mathcal{M}_{22}) \mp \sqrt{4\mathcal{M}_{12}^2 + (\mathcal{M}_{11} - \mathcal{M}_{22})^2}}{2}V_{\phi_1},\quad M_3 = \mathcal{M}_{33}V_{\phi_2}.$$

The masses of the light neutrinos are given by the eigenvalues of the matrix

$$M_\nu \approx M_D M_M^{-1} M_D^T, \tag{28}$$

which are

$$m_{1,2} = \frac{1}{2D_M}[\Delta \mp \sqrt{\Delta^2 + r}]\frac{V_{\Phi_1}^2}{V_{\phi_1}}, \qquad m_3 = 0, \tag{29}$$

where the following definitions have been used:

$$\begin{aligned}\vec{C}_1 &= (\mathcal{D}_{11}, \mathcal{D}_{21}, \mathcal{D}_{31}), \quad \vec{C}_2 = (\mathcal{D}_{12}, \mathcal{D}_{22}, \mathcal{D}_{32}),\\ r &= 4D_M[(\mathcal{D}_{12}\vec{C}_1 - \mathcal{D}_{11}\vec{C}_2)^2 + D_D^2],\\ \Delta &= \mathcal{M}_{11}(\vec{C}_2)^2 + \mathcal{M}_{22}(\vec{C}_1)^2 - 2\mathcal{M}_{12}(\vec{C}_1.\vec{C}_2),\\ D_M &= \mathcal{M}_{12}^2 - \mathcal{M}_{11}\mathcal{M}_{22}, \quad D_D = \mathcal{D}_{21}\mathcal{D}_{32} - \mathcal{D}_{22}\mathcal{D}_{31}.\end{aligned} \tag{30}$$

The parameters in M_M and M_D have to be chosen in order to have light neutrino masses consistent with the solar and atmospheric experimental data. However, since there is no a standard procedure to do that, we will present a particular solution to show that this model can generate realistic active neutrino masses.

A particular solution

From the experimental neutrino data it is found that the neutrino mixing matrix is compatible with the so-called tribimaximal (TB) one [19], which is given by

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{31}$$

and where it is assumed that the neutrino mixing angles are in a good approximation given by $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, and $\sin^2\theta_{13} = 0$. Working in a basis where the charged lepton mass matrix is already diagonal, the $U_{\rm TB}$ matrix diagonalizes the light neutrino mass matrix in Eq. (28): $U_{\rm TB}^T M_\nu U_{\rm TB} = \hat{M}_\nu = {\rm diag}(m_1, m_2, m_3)$. It can be shown that the most general neutrino mass matrix that can be exactly diagonalized by $U_{\rm TB}$ has the form

$$M_{\rm TB} = \begin{pmatrix} x & y & y \\ y & x+\nu & y-\nu \\ y & y-\nu & x+\nu \end{pmatrix}, \tag{32}$$

using the same notation as in Ref. [19].

The $M_{\rm TB}$ mass eigenstates are

$$m_1 = x - y, \quad m_2 = x + 2y, \quad m_3 = 2\nu + x - y. \tag{33}$$

The square mass differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$, needed to explain the solar and atmospheric neutrino anomalies, can be obtained by imposing conditions on x, y, and ν. The simplest way to apply this analysis to our particular case is as follows. We consider

$$\mathcal{M}_{11} = \mathcal{M}_{22} = \frac{V_{\phi 2}}{V_{\phi_1}}\mathcal{M}_{33}, \qquad \mathcal{M}_{12} = 0, \tag{34}$$

so that the Majorana and Dirac mass matrices are now given by

$$M_M = \mathcal{M}_{11} V_{\phi_1} \mathbf{1}_{3\times 3}, \quad M_D = V_{\Phi_1} \begin{pmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & 0 \\ \mathcal{D}_{21} & \mathcal{D}_{22} & 0 \\ \mathcal{D}_{31} & \mathcal{D}_{32} & 0 \end{pmatrix}. \tag{35}$$

Then, the light neutrino mass matrix becomes

$$M_\nu = M_D M_M^{-1} M_D^T = \frac{V^2_{\Phi_1}}{V_{\phi_1}} \frac{1}{\mathcal{M}_{11}} M_D M_D^T = \frac{V^2_{\Phi_1}}{\mathcal{M}_{11} V_{\phi_1}} \begin{pmatrix} \mathcal{D}_{11}^2 + \mathcal{D}_{12}^2 & \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} & \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32} \\ \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} & \mathcal{D}_{21}^2 + \mathcal{D}_{22}^2 & \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32} \\ \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32} & \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32} & \mathcal{D}_{31}^2 + \mathcal{D}_{32}^2 \end{pmatrix}. \tag{36}$$

The matrix above has a null determinant and, therefore, a zero mass eigenstate. Hence, in order to make both matrices compatible, we must have a vanishing eigenvalue in Eq. (33). We choose $m_3 = 0$, i.e., $2\nu + x - y = 0$, and, hence, $x + \nu = y - \nu$.

Comparing Eq. (32) with Eq. (36) we have the following equations:

$$\frac{x}{K} = \mathcal{D}_{11}^2 + \mathcal{D}_{12}^2, \tag{37}$$

$$\frac{y}{K} = \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} = \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32}, \tag{38}$$

$$\frac{x+\nu}{K} = \mathcal{D}_{21}^2 + \mathcal{D}_{22}^2 = \mathcal{D}_{31}^2 + \mathcal{D}_{32}^2, \tag{39}$$

$$\frac{y-\nu}{K} = \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32}, \tag{40}$$

where we have defined the dimensional constant $K = \frac{V^2_{\Phi_1}}{\mathcal{M}_{11} V_{\phi_1}}$. A solution for the above equations is

$$\mathcal{D}_{21} = \mathcal{D}_{31}, \quad \text{and} \quad \mathcal{D}_{22} = \mathcal{D}_{32}. \tag{41}$$

From the above equations we see that the condition to have $m_3 = 0$, $x + \nu = y - \nu$, is automatically satisfied. We have the following equations to fit the atmospheric and solar neutrino data,

$$m_1 = x - y, \qquad m_2 = x + 2y, \qquad m_3 = 0, \tag{42}$$

and, therefore,

$$\Delta m^2_{\rm sol} = m_2^2 - m_1^2 = 3y(2x + y) > 0, \tag{43}$$

$$|\Delta m^2_{\rm atm}| = |m_3^2 - m_1^2| = (x - y)^2. \tag{44}$$

Assuming that $x - y > 0$ we have to solve the equations

$$3y(2x + y) = 7.67 \times 10^{-5}\ ({\rm eV})^2, \quad \text{and} \quad x - y = (2.4 \times 10^{-3})^{1/2}\ {\rm eV}, \tag{45}$$

which are satisfied by $x = 0.0492487$ and $y = 0.000258887$, in eV. The corresponding mass eigenvalues are then given by $m_1 = 0.0489898$, $m_2 = 0.0497665$, and $m_3 = 0$, in eV, showing an inverse hierarchy pattern. We can now solve Eqs. (37) and (38) for the $\mathcal{D}_{ij}$ parameters. In order to do that we have to know the value of the dimensional constant K. For $V_{\Phi_1} = 1$ MeV, $V_{\phi_1} = 1$ TeV, and assuming $\mathcal{M}_{11} = 1$, we have $K = 1$ eV. Choosing the input values $\mathcal{D}_{22} = 0.25$ and $\mathcal{D}_{21} = 0.15$, we find $\mathcal{D}_{11} = 0.190751$, and $\mathcal{D}_{12} = -0.113415$. Experiments on $0\nu\beta\beta$ can put bounds on $|m_{ee}|$, and the strongest one is $|m_{ee}| < 0.26(0.34)$ eV at 68% (90%) C.L. [20]. This quantity is related to the mass eigenvalues through $|m_{ee}| = |c_{13}^2(m_1 c_{12}^2 e^{i\delta_1} + m_2 s_{12}^2 e^{i\delta_2}) + m_3 e^{2i\phi_{CP}} s_{13}^2|$. In our case, with no CP violation nor phases in the leptonic mixing matrix, we find $|m_{ee}| \approx 0.05$ eV. Future experiments, however, expect to improve sensitivity up to ≈ 0.01 eV [21].

The procedure we have followed for finding a particular solution for the light neutrino masses can also be realized by using, instead of the matrices given in Eqs. (31) and (32), the ones given in Ref. [22], provided we make, in the notation of this reference, $c = -d/2$, and the identifications $\nu = d - (a + b)$, $y = d$, $x = a + 2b - d$. It results in $-a = x - y + 2\nu = m_3$, and we take $a = 0$.

The results showed above demonstrate that the model is fully compatible with the experimental neutrino data, and that light neutrino masses can be generated neither appealing for very large energy scales nor imposing fine-tuning. Now, we have to verify if the set of parameters we have used above is in agreement with the LFV constraints coming from a process like $l_i \to l_j + \gamma$, where $i = 2, 3 = \mu, \tau$ and $j = 1, 2 = e, \mu$, respectively. This model has one loop contributions to such a process since charged leptons couple to charged scalars and right-handed heavy neutrinos. The branching ratio is estimated as [23]

$$B(l_i \to l_j + \gamma) = \frac{96\pi^3\alpha}{G_F^2 m_{l_i}^4}(|f_{M1}|^2 + |f_{E1}|^2), \tag{46}$$

where $\alpha \simeq 1/137$ and $G_F \simeq 1.16 \times 10^{-5}$ GeV^{-2} is the Fermi constant and

$$f_{M1} = f_{E1} = \sum_{k=1}^{3} \frac{\mathcal{D}_{ik}\mathcal{D}_{jk}}{4(4\pi)^2} \frac{m_{l_i}^2}{m_\Phi^2} F_2\left(\frac{m_{N_k}^2}{m_\Phi^2}\right), \tag{47}$$

with $F_2(x)$ being

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}. \tag{48}$$

Using the parameters needed to fit the neutrino masses and the ones to estimate $\Omega h^2 \simeq 0.11$ ($\mathcal{D}_{e3} \simeq 0.06$, $\mathcal{D}_{\mu 3} \simeq 0.9$, $\mathcal{D}_{\tau 3} \simeq 1$, $m_{n_{R3}} = 750$ GeV, $m_{C_1} \simeq m_{\Phi_1^\pm} = 1.33$ TeV, $m_{C_2} = m_{\Phi_2^\pm} = 1.48$ TeV), we can give an estimate for the branching ratio $B(\mu \to e + \gamma) \simeq 7.9 \times 10^{-12}$ and $B(\tau \to \mu + \gamma) \simeq 2.5 \times 10^{-9}$. These values are in agreement with the present upper bounds $B(\mu \to e + \gamma) < 1.2 \times 10^{-11}$ and $B(\tau \to \mu + \gamma) \simeq 6.8 \times 10^{-8}$ [24,25].

The ratio between the VEVs we have used for finding the neutrino mass eigenvalues is $V_{\Phi_1}/V_{\phi_1} = 10^{-6}$. This is of the same order as the ratio $m_e/m_{\text{top}} = Y_e/Y_{\text{top}} \approx 10^{-6}$, and it is comparable with $m_u/m_{\text{top}} = Y_u/Y_{\text{top}} \approx 10^{-5}$. We have chosen those values for V_{Φ_1} and V_{ϕ_1} in order to have light neutrino masses without resorting to very tiny neutrino Yukawa coupling constants, or fine-tuning, and, at the same time, to have the Z' vector boson not extremely heavy. This is a kind of seesaw mechanism where the heaviest scale, V_{ϕ_1}, is constrained by the Z' vector boson, which should be not too heavy in order to not decouple from the spectrum. The light scale, V_{Φ_1}, is then used to fix the absolute neutrino mass scale through the ratio $V_{\Phi_1}^2/V_{\phi_1}$. In this picture we are substituting a hierarchy in the VEVs, which would have a possible explanation based on the dynamics of the fields, for one in the Yukawa coupling constants, for which we cannot find any natural explanation. This is basically the philosophy behind the work in Refs. [26,27].

As we have discussed above, the absolute neutrino mass scale depends on the ratio $V_{\Phi_1}^2/V_\phi$, where V_{Φ_1} is a tiny value. Although this value can be affected by radiative corrections, it can be argued that, when the Z_2 symmetry is considered, setting V_{Φ_1} to a tiny value, at the tree level, is natural because if it were in fact taken to be zero this would increase the symmetry of the entire Lagrangian ('t Hooft's principle of naturalness). This can be seen considering the constraint equations with $V_{\Phi_1} \to 0$. It implies that $\kappa_{H1X} = 0$, since V_H and V_{ϕ_X} differ from zero. Then the term $-i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X$ does not appear in the scalar potential, Eq. (16), and the entire Z_2 invariant Lagrangian is now invariant under an additional global quantum symmetry, say, $U(1)_\zeta$. A possible ζ-charge assignment is $\zeta(\nu_{eL}, e_L, e_R, \Phi_{1,2}) = -1$, $\zeta(u_L, d_L, u_R, d_R) = 1/3$, and $\zeta(n_{(1,2,3)R}, \phi_{1,2,3}) = 0$. Thus, it is expected that the VEV hierarchy will remain stable when radiative corrections are taken into account.

V. CONCLUSIONS

In this paper we have studied in detail the scalar and the neutrino Yukawa sectors of an extension of the electroweak standard model which has an extra $U(1)$ gauge factor, as described in Sec. II. We have analyzed the scalar spectra of the potential given in Eq. (2) and found that it is inconsistent with the experimental data coming from the star energy loss and the Z invisible decay width. We would like to stress that this is a general result for this scalar potential.

We find that the more suitable solution to this problem is the addition of a new $SU(2)$ scalar singlet, called ϕ_X in the text. The new terms introduced by ϕ_X are able to remove all the physical Goldstone bosons and, at the same time, to have all the real mass eigenstates heavier than the Z boson. This solution is particularly interesting since, in this case, all VEVs can be different from zero, which allows for the construction of a general neutrino mass matrix.

In order to have a still more attractive model we consider the possibility of having DM candidates by including a Z_2 symmetry. Before the SSB the only fields having odd transformation under Z_2 are n_{R3}, Φ_2, and ϕ_3. Z_2 will still be a symmetry if the scalar fields Φ_2 and ϕ_3 do not develop VEVs. Hence, after the SSB we will have states which are mass and Z_2 eigenstates simultaneously. It opens the possibility of having DM fields since the lighter Z_2 odd eigenstate will be stable. Moreover, we show in a preliminary study that the fermionic field n_{R3} is a viable cold DM candidate.

We consider in detail the neutrino mass generation in the framework of the model with the Z_2 symmetry. In this case we found an inverted hierarchy compatible with the solar and atmospheric neutrino data and the tribimaximal mixing matrix. Two appealing features are (i) the absolute scale of the neutrino masses is obtained by a seesaw mechanism at $\mathcal{O}$(TeV) energy scale, which is the scale of the first symmetry breaking, and (ii) the observed mass-squared differences are obtained without resorting to fine-tuning the neutrino Yukawa couplings.

The model has also some phenomenological implications. One of them is the existence of an extra neutral vector boson, Z', which can be in principle detected at the LHC or International Linear Collider. In fact, there are studies showing that the Z' of this particular model can be distinguished from that of other models by comparing, for instance, the forward-backward asymmetry for the process $p + p \to \mu^+ + \mu^- + X$ as a function of the dilepton invariant mass, or the muon transverse momentum distribution at the LHC [28], and the same asymmetry for the process $e^+ + e^- \to f + \bar{f}$ $(f = q, l)$ at the International Linear Collider [29]. At first glance, another interesting feature is that the model seems to indicate that the LFV and DM are closely related. It implies that when the parameters are appropriate to satisfy the DM requirements, the LFV is relatively close to the present experimental bounds. In this way, the model can be confronted by the next generation of LFV experiments.

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APPENDIX: THE CONSTRAINT EQUATIONS

Here we show the constraint equations for the scalar potential given in Eq. (2) plus the terms after the ϕ_X introduction and without the Z_2 symmetry. These equations are obtained by considering, after the spontaneous symmetry breaking, the linear terms $(t_\varphi \varphi)$ in the scalar potential, and the solutions to the equations $t_\varphi = 0$ are the critical points of the scalar potential.

$$
\begin{aligned}
t_H &= V_H(2\lambda_H V_H^2 + \Lambda_{H1}V_{\Phi_1}^2 + \Lambda_{H2}V_{\Phi_2}^2 + \Lambda_{Hs1}V_{\phi_1}^2 + \Lambda_{Hs2}V_{\phi_2}^2 + \Lambda_{Hs3}V_{\phi_3}^2 + \Lambda_{HsX}V_{\phi_X}^2 - 2\mu_H^2) - \sqrt{2}\kappa_{H1X}V_{\Phi_1}V_{\phi_X} \\
&\quad - \kappa_{H2X}V_{\Phi_2}V_{\phi_X}^2, \\
t_{\Phi_1} &= V_{\Phi_1}(\Lambda_{H1}V_H^2 + 2\lambda_{11}V_{\Phi_1}^2 + (\lambda_{12}+\lambda'_{12})V_{\Phi_2}^2 + \Lambda'_{11}V_{\phi_1}^2 + \Lambda'_{12}V_{\phi_2}^2 + \Lambda'_{13}V_{\phi_3}^2 + \Lambda'_{1X}V_{\phi_X}^2 - 2\mu_{11}^2) - \sqrt{2}\kappa_{H1X}V_H V_{\phi_X} \\
&\quad + V_{\Phi_2}V_{\phi_3}(\beta_{13}V_{\phi_1} + \beta_{23}V_{\phi_2}), \\
t_{\Phi_2} &= V_{\Phi_2}(\Lambda_{H2}V_H^2 + (\lambda_{12}+\lambda'_{12})V_{\Phi_1}^2 + 2\lambda_{22}V_{\Phi_2}^2 + \Lambda'_{21}V_{\phi_1}^2 + \Lambda'_{22}V_{\phi_2}^2 + \Lambda'_{23}V_{\phi_3}^2 + \Lambda'_{2X}V_{\phi_X}^2 - 2\mu_{22}^2) - \kappa_{H2X}V_H V_{\phi_X}^2 \\
&\quad + V_{\Phi_1}V_{\phi_3}(\beta_{13}V_{\phi_1} + \beta_{23}V_{\phi_2}), \\
t_{\phi_1} &= V_{\phi_1}(\Lambda_{Hs1}V_H^2 + \Lambda'_{11}V_{\Phi_1}^2 + \Lambda'_{21}V_{\Phi_2}^2 + 2\lambda_{s1}V_{\phi_1}^2 + \Delta_{12}V_{\phi_2}^2 + \Delta_{13}V_{\phi_3}^2 + \Delta_{1X}V_{\phi_X}^2 - 2\mu_{s1}^2) + \beta_{13}V_{\Phi_1}V_{\Phi_2}V_{\phi_3} \\
&\quad + V_{\phi_2}V_{\phi_3}(\beta_{123}V_{\phi_3} + \beta_X V_{\phi_X}), \\
t_{\phi_2} &= V_{\phi_2}(\Lambda_{Hs2}V_H^2 + \Lambda'_{12}V_{\Phi_1}^2 + \Lambda'_{22}V_{\Phi_2}^2 + \Delta_{12}V_{\phi_1}^2 + 2\lambda_{s2}V_{\phi_2}^2 + \Delta_{23}V_{\phi_3}^2 + \Delta_{2X}V_{\phi_X}^2 - 2\mu_{s2}^2) + \beta_{23}V_{\Phi_1}V_{\Phi_2}V_{\phi_3} \\
&\quad + V_{\phi_1}V_{\phi_3}(\beta_{123}V_{\phi_3} + \beta_X V_{\phi_X}), \\
t_{\phi_3} &= V_{\phi_3}(\Lambda_{Hs3}V_H^2 + \Lambda'_{13}V_{\Phi_1}^2 + \Lambda'_{23}V_{\Phi_2}^2 + \Delta_{13}V_{\phi_1}^2 + \Delta_{23}V_{\phi_2}^2 + 2\lambda_{s3}V_{\phi_3}^2 + \Delta_{3X}V_{\phi_X}^2 + 3\beta_{3X}V_{\phi_3}V_{\phi_X} - 2\mu_{s3}^2) \\
&\quad + V_{\Phi_1}V_{\Phi_2}(\beta_{13}V_{\phi_1} + \beta_{23}V_{\phi_2}) + V_{\phi_1}V_{\phi_2}(2\beta_{123}V_{\phi_3} + \beta_X V_{\phi_X}), \\
t_{\phi_X} &= V_{\phi_X}(\Lambda_{HsX}V_H^2 + \Lambda'_{1X}V_{\Phi_1}^2 + \Lambda'_{2X}V_{\Phi_2}^2 + \Delta_{1X}V_{\phi_1}^2 + \Delta_{2X}V_{\phi_2}^2 + \Delta_{3X}V_{\phi_3}^2 + 2\lambda_{sX}V_{\phi_X}^2 - 2\kappa_{H2X}V_H V_{\Phi_2} - 2\mu_{sX}^2) \\
&\quad - \sqrt{2}\kappa_{H1X}V_H V_{\Phi_1} + \beta_X V_{\phi_1}V_{\phi_2}V_{\phi_3} + \beta_{3X}V_{\phi_3}^3.
\end{aligned}
$$

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